Flavor Distribution of UHE Cosmic Neutrino Oscillations at Neutrino Telescopes

Zhi-zhong Xing

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, China
(E-mail: xingzz@ihep.ac.cn)

Abstract

If the ultrahigh-energy (UHE) cosmic neutrinos produced from a distant astrophysical source can be measured at a km^3-size neutrino telescope such as the IceCube or KM3NeT, they will open a new window to understand the nature of flavor mixing and to probe possible new physics. Considering the conventional UHE cosmic neutrino source with the flavor ratio $\phi_e : \phi_\mu : \phi_\tau = 1 : 2 : 0$, I point out two sets of conditions for the flavor democracy $\phi_e^{\rm T} : \phi_\mu^{\rm T} : \phi_\tau^{\rm T} = 1 : 1 : 1$ to show up at neutrino telescopes: either $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ (CP invariance) or $\delta = \pm\pi/2$ and $\theta_{23} = \pi/4$ (CP violation) in the standard parametrization of the 3×3 neutrino mixing matrix V. Allowing for slight μ-τ symmetry breaking effects characterized by $\Delta \in [-0.1, +0.1]$, I find $\phi_e^{\rm T} : \phi_\mu^{\rm T} : \phi_\tau^{\rm T} = (1-2\Delta) : (1+\Delta) : (1+\Delta)$ as a good approximation. Another possibility to constrain Δ is to detect the $\overline{\nu}_e$ flux of $E_{\overline{\nu}_e} \approx 6.3$ PeV via the Glashow resonance channel $\overline{\nu}_e e \to W^- \to$ anything. I also give some brief comments on (1) possible non-unitarity of V in the seesaw framework and its effects on the flavor distribution at neutrino telescopes, and (2) a generic description and determination of the cosmic neutrino flavor composition at distant astrophysical sources.

Key words: UHE Neutrinos, Flavor Mixing, Neutrino Oscillations, Neutrino Telescopes
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1. Introduction

Current neutrino experiments have provided us with very convincing evidence for the existence of neutrino oscillations and opened a window to new physics beyond the standard model. The neutrino mixing is usually described by a unitary matrix V,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \qquad (1)$$

In the "standard" parametrization of V [1], one defines $V_{e2} = \sin\theta_{12}\cos\theta_{13}$, $V_{e3} = \sin\theta_{13} e^{-i\delta}$ and $V_{\mu 3} = \sin\theta_{23}\cos\theta_{13}$. Here I have omitted the Majorana CP-violating phases from V, because they are irrelevant to the properties of neutrino oscillations to be discussed. A global analysis of current experimental data (see, e.g., Ref. [2]) points to $\theta_{13} = 0$ and $\theta_{23} = \pi/4$, which have motivated a number of authors to consider the μ-τ permutation symmetry for model building.

My talk focuses on the oscillations of ultrahigh-energy (UHE) cosmic neutrinos at neutrino telescopes. Its main purpose is three-fold: (a) to point out two sets of conditions under which the flavor distribution of UHE neutrinos at neutrino telescopes is exactly democratic; (b) to analyze how the effect of μ-τ symmetry breaking can show up at a neutrino telescope; and (c) to comment on the possibility of probing the non-unitarity of V or the flavor ambiguity of an astrophysical neutrino source by using neutrino telescopes. I assume that IceCube [3] and

KM3NeT [4] are both able to detect the fluxes of UHE cosmic ν_e ($\overline{\nu}_e$), ν_μ ($\overline{\nu}_\mu$) and ν_τ ($\overline{\nu}_\tau$) neutrinos generated from very distant astrophysical sources.

2. Flavor democracy

For most of the currently-envisaged sources of UHE cosmic neutrinos [5], a general and canonical expectation is that the initial neutrino fluxes are produced via the decay of charged pions created from pp or $p\gamma$ collisions and their flavor content can be expressed as

$$\{\phi_e \ , \ \phi_\mu \ , \ \phi_\tau\} \ = \ \left\{\frac{1}{3} \ , \ \frac{2}{3} \ , \ 0\right\}\phi_0 \ , \qquad (2)$$

where ϕ_α (for $\alpha = e, \mu, \tau$) denotes the sum of ν_α and $\overline{\nu}_\alpha$ fluxes, and $\phi_0 = \phi_e + \phi_\mu + \phi_\tau$ is the total flux of neutrinos and antineutrinos of all flavors. Let me define $\phi^{(\rm T)}_\alpha \equiv \phi^{(\rm T)}_{\nu_\alpha} + \phi^{(\rm T)}_{\overline{\nu}_\alpha}$ (for $\alpha = e, \mu, \tau$), where $\phi^{(\rm T)}_{\nu_\alpha}$ and $\phi^{(\rm T)}_{\overline{\nu}_\alpha}$ denote the ν_α and $\overline{\nu}_\alpha$ fluxes to be measured at a neutrino telescope, respectively. As for the UHE neutrino fluxes produced from the pion-muon decay chain with $\phi_{\nu_\tau} = \phi_{\overline{\nu}_\tau} = 0$, the relationship between ϕ_{ν_α} (or $\phi_{\overline{\nu}_\alpha}$) and $\phi^{\rm T}_{\nu_\alpha}$ (or $\phi^{\rm T}_{\overline{\nu}_\alpha}$) is given by $\phi^{\rm T}_{\nu_\alpha} = \phi_{\nu_e} P_{e\alpha} + \phi_{\nu_\mu} P_{\mu\alpha}$ or $\phi^{\rm T}_{\overline{\nu}_\alpha} = \phi_{\overline{\nu}_e} \bar{P}_{e\alpha} + \phi_{\overline{\nu}_\mu} \bar{P}_{\mu\alpha}$, in which $P_{\beta\alpha}$ and $\bar{P}_{\beta\alpha}$ (for $\alpha = e, \mu, \tau$ and $\beta = e$ or μ) stand respectively for the oscillation probabilities $P(\nu_\beta \to \nu_\alpha)$ and $P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$. Because the Galactic distances far exceed the observed neutrino oscillation lengths, $P_{\beta\alpha}$ and $\bar{P}_{\beta\alpha}$ are actually averaged over many oscillations. Then I obtain $\bar{P}_{\beta\alpha} = P_{\beta\alpha}$ and

$$P_{\beta\alpha} \ = \ \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\beta i}|^2 \ , \qquad (3)$$

where $V_{\alpha i}$ and $V_{\beta i}$ (for $\alpha, \beta = e, \mu, \tau$ and $i = 1, 2, 3$) denote the matrix elements of V defined in Eq. (1). The relationship between ϕ_α and $\phi^{\rm T}_\alpha$ turns out to be

$$\phi^{\rm T}_\alpha \ = \ \phi_e P_{e\alpha} + \phi_\mu P_{\mu\alpha} \ . \qquad (4)$$

To be explicit, I have

$$\begin{aligned} \phi^{\rm T}_e &= \frac{\phi_0}{3}\left(P_{ee} + 2P_{\mu e}\right) \ , \\ \phi^{\rm T}_\mu &= \frac{\phi_0}{3}\left(P_{e\mu} + 2P_{\mu\mu}\right) \ , \\ \phi^{\rm T}_\tau &= \frac{\phi_0}{3}\left(P_{e\tau} + 2P_{\mu\tau}\right) \ . \end{aligned} \qquad (5)$$

It is then possible to evaluate the relative sizes of $\phi^{\rm T}_e$, $\phi^{\rm T}_\mu$ and $\phi^{\rm T}_\tau$ by using Eqs. (1), (3) and (5).

I want to figure out the necessary conditions under which the cosmic neutrino flavor democracy

$$\{\phi^{\rm T}_e, \ \phi^{\rm T}_\mu, \ \phi^{\rm T}_\tau\} \ = \ \left\{\frac{1}{3} \ , \ \frac{1}{3} \ , \ \frac{1}{3}\right\}\phi_0 \qquad (6)$$

exactly holds at a neutrino telescope. With the helps of Eqs. (1), (3) and (5), Zhou and I find that Eq. (6) requires $P_{e\mu} = P_{e\tau}$ and $P_{\mu\mu} = P_{\mu\tau} = P_{\tau\tau}$ [6], which can be derived from

$$\sum_{i=1}^{3} |V_{\alpha i}|^2 \left(|V_{\mu i}|^2 - |V_{\tau i}|^2\right) = 0 \ . \qquad (7)$$

The solution to Eq. (7) is

$$|V_{\mu i}|^2 = |V_{\tau i}|^2 \ , \quad (\text{for } i = 1, 2, 3) \ . \qquad (8)$$

Adopting the standard parametrization of V, we are then left with two sets of conditions for the cosmic neutrino flavor democracy at neutrino telescopes [6]:

$$\theta_{13} = 0 \ , \quad \theta_{23} = \frac{\pi}{4} \ ; \qquad (9)$$

or

$$\delta = \pm\frac{\pi}{2} \ , \quad \theta_{23} = \frac{\pi}{4} \ . \qquad (10)$$

The condition in Eq. (9) is already known [7]. It reflects the μ-τ permutation symmetry of the neutrino mass matrix. Because of vanishing θ_{13}, there is no CP violation in neutrino oscillations. A typical example for this type of flavor mixing is the tri-bimaximal neutrino mixing pattern [8], in which $\theta_{12} = \arctan(1/\sqrt{2}) \approx 35.3^\circ$ together with Eq. (9). The condition shown in Eq. (10) is more interesting in the sense that it may lead to appreciable effects of CP violation in neutrino oscillations, provided the value of θ_{13} is not strongly suppressed. One obtains $\mathcal{J} = \pm c_{12} s_{12} c^2_{13} s_{13}/2$ in this case and finds $|\mathcal{J}| \sim \mathcal{O}(10^{-2})$ if $\theta_{12} \sim 34^\circ$ and $\theta_{13} \geq 3^\circ$. An example for this type of flavor mixing is the tetra-maximal neutrino mixing pattern [9], in which $\theta_{12} = \arctan(2 - \sqrt{2}) \approx 30.4^\circ$ and $\theta_{13} = \arcsin[(\sqrt{2} - 1)/(2\sqrt{2})] \approx 8.4^\circ$ together with Eq. (10).

Slight deviations from the above conditions are possible in realistic neutrino mass models, no matter whether the seesaw mechanism is taken into account or not. They can in general give rise to the breaking of cosmic neutrino flavor democracy at neutrino telescopes, as we shall see later on.

3. μ-τ symmetry breaking

Starting from the hypothesis given in Eq. (2) and allowing for the slight breaking of μ-τ symmetry, I am going to show that

$$\phi_e^{\rm T} : \phi_\mu^{\rm T} : \phi_\tau^{\rm T} = (1-2\Delta) : (1+\Delta) : (1+\Delta) \qquad (11)$$

holds to a good degree of accuracy, where Δ characterizes the effect of μ-τ symmetry breaking (i.e., the combined effect of $\theta_{13} \neq 0$ and $\theta_{23} \neq \pi/4$) [10]. One can obtain $-0.1 \leq \Delta \leq +0.1$ from the present neutrino oscillation data.

In order to clearly show the effect of μ-τ symmetry breaking on the neutrino fluxes to be detected at neutrino telescopes, I define

$$\varepsilon \equiv \theta_{23} - \frac{\pi}{4} , \qquad (|\varepsilon| \ll 1) . \qquad (12)$$

Namely, ε measures the slight departure of θ_{23} from $\pi/4$. Using small θ_{13} and ε, I express $|V_{\alpha i}|^2$ (for $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$) as follows:

$$\begin{aligned}\left[|V_{\alpha i}|^2\right] = & \frac{1}{2}A + \varepsilon B + \frac{1}{2}\left(\theta_{13} \sin 2\theta_{12} \cos\delta\right) C \\ & + \mathcal{O}(\varepsilon^2) + \mathcal{O}(\theta_{13}^2) . \end{aligned} \qquad (13)$$

where

$$A = \begin{bmatrix} 2\cos^2\theta_{12} & 2\sin^2\theta_{12} & 0 \\ \sin^2\theta_{12} & \cos^2\theta_{12} & 1 \\ \sin^2\theta_{12} & \cos^2\theta_{12} & 1 \end{bmatrix} ,$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ -\sin^2\theta_{12} & -\cos^2\theta_{12} & 1 \\ \sin^2\theta_{12} & \cos^2\theta_{12} & -1 \end{bmatrix} ,$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix} . \qquad (14)$$

Eqs. (3) and (13) allow me to calculate $P_{\beta\alpha}$:

$$\begin{aligned} P_{ee} + 2P_{\mu e} &= 1 + \frac{\theta_{13}}{2}\sin 4\theta_{12}\cos\delta \\ &\quad -\varepsilon \sin^2 2\theta_{12} , \\ P_{e\mu} + 2P_{\mu\mu} &= 1 - \frac{\theta_{13}}{4}\sin 4\theta_{12}\cos\delta \\ &\quad +\frac{\varepsilon}{2}\sin^2 2\theta_{12} , \\ P_{e\tau} + 2P_{\mu\tau} &= 1 - \frac{\theta_{13}}{4}\sin 4\theta_{12}\cos\delta \\ &\quad +\frac{\varepsilon}{2}\sin^2 2\theta_{12} , \end{aligned} \qquad (15)$$

where the terms of $\mathcal{O}(\varepsilon^2)$ and $\mathcal{O}(\theta_{13}^2)$ are omitted. Substituting Eq. (15) into Eq. (5), I get [10]

$$\begin{aligned} \phi_e^{\rm T} &= \frac{\phi_0}{3}(1-2\Delta) , \\ \phi_\mu^{\rm T} &= \frac{\phi_0}{3}(1+\Delta) , \\ \phi_\tau^{\rm T} &= \frac{\phi_0}{3}(1+\Delta) , \end{aligned} \qquad (16)$$

where

$$\Delta = \frac{1}{4}\left(2\varepsilon\sin^2 2\theta_{12} - \theta_{13}\sin 4\theta_{12}\cos\delta\right) . \qquad (17)$$

Eq. (11) is therefore proved by Eqs. (16) and (17). One can see that $\phi_e^{\rm T} + \phi_\mu^{\rm T} + \phi_\tau^{\rm T} = \phi_0$ holds. More accurate results for Eqs. (13)—(17) can be found in Refs. [6,11]. Some discussions are in order.

(1) The small parameter Δ characterizes the overall effect of μ-τ symmetry breaking. Allowing δ to vary between 0 and π, I obtain the lower and upper bounds of Δ for given values of θ_{12} ($< \pi/4$), θ_{13} and ε: $-\Delta_{\rm bound} \leq \Delta \leq +\Delta_{\rm bound}$, where

$$\Delta_{\rm bound} = \frac{1}{4}\left(2|\varepsilon|\sin^2 2\theta_{12} + \theta_{13}\sin 4\theta_{12}\right) . \qquad (18)$$

It is obvious that $\Delta = -\Delta_{\rm bound}$ when $\varepsilon < 0$ and $\delta = 0$, and $\Delta = +\Delta_{\rm bound}$ when $\varepsilon > 0$ and $\delta = \pi$. A global analysis of current neutrino oscillation data indicates $30^\circ < \theta_{12} < 38^\circ$, $\theta_{13} < 10^\circ$ (≈ 0.17) and $|\varepsilon| < 9^\circ$ (≈ 0.16) [2] at the 99% confidence level, but the CP-violating phase δ is entirely unrestricted. Using these constraints, I analyze the allowed range of Δ and its dependence on δ. The maximal value of $\Delta_{\rm bound}$ (i.e., $\Delta_{\rm bound} \approx 0.098$) appears when $|\varepsilon|$ and θ_{13} approach their respective upper limits and $\theta_{12} \approx 33^\circ$ holds [10]. $\Delta_{\rm bound}$ is not very sensitive to the variation of θ_{12} in its allowed region.

If $\theta_{13} = 0$ holds, $\Delta_{\rm bound} = 0.5|\varepsilon|\sin^2 2\theta_{12} < 0.074$ when θ_{12} approaches its upper limit. If $\varepsilon = 0$ (i.e., $\theta_{23} = \pi/4$) holds, I obtain $\Delta_{\rm bound} = 0.25\theta_{13}\sin 4\theta_{12} < 0.038$ when θ_{12} approaches its lower limit. Thus $\Delta_{\rm bound}$ is more sensitive to the deviation of θ_{23} from $\pi/4$.

(2) Of course, $\Delta = 0$ exactly holds when $\theta_{13} = \varepsilon = 0$ is taken. Because the sign of ε and the range of δ are both unknown, we are now unable to rule out the nontrivial possibility $\Delta \approx 0$ in the presence of $\theta_{13} \neq 0$ and $\varepsilon \neq 0$. In other words, Δ may be vanishing or extremely small if its two leading terms cancel each other. It is easy to arrive at $\Delta \approx 0$ from Eq. (17), if the condition

$$\frac{\varepsilon}{\theta_{13}} = \cot 2\theta_{12}\cos\delta \qquad (19)$$

is satisfied. Because of $|\cos\delta| \leq 1$, Eq. (19) imposes a strong constraint on the magnitude of ε/θ_{13}. The dependence of ε/θ_{13} on δ is illustrated in Ref. [10], where θ_{12} varies in its allowed range. I find that $|\varepsilon|/\theta_{13} < 0.6$ is necessary to hold, such that a large cancellation between two leading terms of Δ is possible to take place.

The implication of the above result on UHE cosmic neutrino telescopes is two-fold. On the one hand, an observable signal of $\Delta \neq 0$ at a neutrino telescope implies the existence of significant μ-τ symmetry breaking. If a signal of $\Delta \neq 0$ does not show up at a neutrino telescope, on the other hand, one cannot conclude that the μ-τ symmetry is an exact or almost exact symmetry. It is therefore meaningful to consider the complementarity between neutrino telescopes and terrestrial neutrino oscillation experiments [12], in order to finally pin down the parameters of neutrino mixing and leptonic CP violation.

(3) To illustrate, I define the flux ratios

$$R_e \equiv \frac{\phi_e^{\rm T}}{\phi_\mu^{\rm T} + \phi_\tau^{\rm T}} \; , \quad R_\mu \equiv \frac{\phi_\mu^{\rm T}}{\phi_\tau^{\rm T} + \phi_e^{\rm T}} \; , \quad R_\tau \equiv \frac{\phi_\tau^{\rm T}}{\phi_e^{\rm T} + \phi_\mu^{\rm T}} \; , \qquad (20)$$

which may serve as the *working* observables at neutrino telescopes [13]. At least, R_μ can be extracted from the ratio of muon tracks to showers at IceCube and KM3NeT, even if those electron and tau events cannot be disentangled. Taking account of Eq. (16), I approximately obtain

$$R_e \approx \frac{1}{2} - \frac{3}{2}\Delta \; , \quad R_\mu \approx \frac{1}{2} + \frac{3}{4}\Delta \; , \quad R_\tau \approx \frac{1}{2} + \frac{3}{4}\Delta \; . \qquad (21)$$

It turns out that R_e is most sensitive to the effect of μ-τ symmetry breaking.

Due to $\phi_\mu^{\rm T} = \phi_\tau^{\rm T}$ shown in Eq. (16), $R_\mu = R_\tau$ holds no matter whether Δ vanishes or not. This observation implies that the "μ-τ" symmetry between R_μ and R_τ is actually insensitive to the breaking of μ-τ symmetry in the neutrino mass matrix. If both R_e and R_μ are measured, one can then extract Δ from their difference:

$$R_\mu - R_e = \frac{9}{4}\Delta \; . \qquad (22)$$

Taking $\Delta = \Delta_{\rm bound} \approx 0.1$, we get $R_\mu - R_e \leq 0.22$.

4. On the Glashow resonance

I proceed to discuss the possibility to probe the breaking of μ-τ symmetry by detecting the $\overline{\nu}_e$ flux from distant astrophysical sources through the so-called Glashow resonance (GR) channel $\overline{\nu}_e e \rightarrow W^- \rightarrow$ anything [14]. The latter can take place over a very narrow energy interval around the $\overline{\nu}_e$ energy $E_{\overline{\nu}_e}^{\rm GR} \approx M_W^2/2m_e \approx 6.3$ PeV. A neutrino telescope may measure both the GR-mediated $\overline{\nu}_e$ events ($N_{\overline{\nu}_e}^{\rm GR}$) and the $\nu_\mu + \overline{\nu}_\mu$ events of charged-current (CC) interactions ($N_{\nu_\mu + \overline{\nu}_\mu}^{\rm CC}$) in the vicinity of $E_{\overline{\nu}_e}^{\rm GR}$. Their ratio, defined as $R_{\rm RG} \equiv N_{\overline{\nu}_e}^{\rm GR}/N_{\nu_\mu + \overline{\nu}_\mu}^{\rm CC}$, can be related to the ratio of $\overline{\nu}_e$'s to ν_μ's and $\overline{\nu}_\mu$'s entering the detector,

$$R_0 \equiv \frac{\phi_{\overline{\nu}_e}^{\rm T}}{\phi_{\nu_\mu}^{\rm T} + \phi_{\overline{\nu}_\mu}^{\rm T}} \; . \qquad (23)$$

Note that $\phi_{\overline{\nu}_e}^{\rm T}$, $\phi_{\nu_\mu}^{\rm T}$ and $\phi_{\overline{\nu}_\mu}^{\rm T}$ stand respectively for the fluxes of $\overline{\nu}_e$'s, ν_μ's and $\overline{\nu}_\mu$'s before the RG and CC interactions occur at the detector. In a recent paper [15], $R_{\rm GR} = aR_0$ with $a \approx 30.5$ has been obtained by considering the muon events with contained vertices [16] in a water- or ice-based detector. An accurate calculation of a is crucial for a specific neutrino telescope to detect the GR reaction rate, but it is beyond the scope of this talk. Here I only concentrate on the possible effect of μ-τ symmetry breaking on R_0.

Provided the initial neutrino fluxes are produced via the decay of π^+'s and π^-'s created from high-energy pp collisions, their flavor composition can be expressed in a more detailed way as

$$\left\{\phi_{\nu_e}, \; \phi_{\nu_\mu}, \; \phi_{\nu_\tau}\right\} = \left\{\frac{1}{6} \; , \; \frac{1}{3} \; , \; 0\right\}\phi_0 \; , \quad \left\{\phi_{\overline{\nu}_e}, \; \phi_{\overline{\nu}_\mu}, \; \phi_{\overline{\nu}_\tau}\right\} = \left\{\frac{1}{6} \; , \; \frac{1}{3} \; , \; 0\right\}\phi_0 \; . \qquad (24)$$

In comparison, the flavor content of UHE neutrino fluxes produced from $p\gamma$ collisions reads

$$\left\{\phi_{\nu_e}, \; \phi_{\nu_\mu}, \; \phi_{\nu_\tau}\right\} = \left\{\frac{1}{3} \; , \; \frac{1}{3} \; , \; 0\right\}\phi_0 \; , \quad \left\{\phi_{\overline{\nu}_e}, \; \phi_{\overline{\nu}_\mu}, \; \phi_{\overline{\nu}_\tau}\right\} = \left\{0 \; , \; \frac{1}{3} \; , \; 0\right\}\phi_0 \; . \qquad (25)$$

For either Eq. (24) or Eq. (25), the sum of ϕ_{ν_α} and $\phi_{\overline{\nu}_\alpha}$ is consistent with ϕ_α in Eq. (2).

Due to neutrino oscillations, the $\overline{\nu}_e$ flux at the detector of a neutrino telescope is given by $\phi_{\overline{\nu}_e}^{\rm T} =$

$\phi_{\overline{\nu}_e} \bar{P}_{ee} + \phi_{\overline{\nu}_\mu} \bar{P}_{\mu e}$. With the help of Eqs. (3), (4), (13), (24) and (25), I explicitly obtain

$$\phi^{\rm T}_{\overline{\nu}_e}(pp) \;=\; \frac{\phi_0}{6}\left(1 - 2\Delta\right) \; ,$$
$$\phi^{\rm T}_{\overline{\nu}_e}(p\gamma) \;=\; \frac{\phi_0}{12}\left(\sin^2 2\theta_{12} - 4\Delta\right) \; . \qquad (26)$$

The sum of $\phi^{\rm T}_{\nu_\mu}$ and $\phi^{\rm T}_{\overline{\nu}_\mu}$, which is defined as $\phi^{\rm T}_\mu$, has been given in Eq. (16). It is then straightforward to calculate R_0 by using Eq. (26) for two different astrophysical sources:

$$R_0(pp) \;\approx\; \frac{1}{2} \;-\; \frac{3}{2}\Delta \; ,$$
$$R_0(p\gamma) \;\approx\; \frac{\sin^2 2\theta_{12}}{4} \;-\; \frac{4+\sin^2 2\theta_{12}}{4}\Delta \; . \qquad (27)$$

This result indicates that the dependence of $R_0(pp)$ on θ_{12} is hidden in Δ and suppressed by the smallness of θ_{13} and ε. In addition, the deviation of $R_0(pp)$ from 1/2 can be as large as $1.5\Delta_{\rm bound} \approx 0.15$. It is obvious that the ratio $R_0(p\gamma)$ is very sensitive to the value of $\sin^2 2\theta_{12}$. A measurement of $R_0(p\gamma)$ at IceCube and KM3NeT may therefore probe the mixing angle θ_{12} [15]. Indeed, the dominant production mechanism for ultrahigh-energy neutrinos at Active Galactic Nuclei (AGNs) and Gamma Ray Bursts (GRBs) is expected to be the $p\gamma$ process in a tenuous or radiation-dominated environment [17]. If this expectation is true, the observation of $R_0(p\gamma)$ may also provide us with useful information on the breaking of μ-τ symmetry.

5. Unitarity violation of *V*

If the tiny masses of three known neutrinos (ν_1, ν_2, ν_3) are attributed to the popular seesaw mechanism (either type-I or type-II), in which there exist a few heavy (right-handed) Majorana neutrinos N_i, then the 3×3 neutrino mixing matrix V must be non-unitary. The effect of unitarity violation of V depends on the mass scale of N_i, and it can be of $\mathcal{O}(10^{-2})$ if N_i are at the TeV scale [18] — an energy frontier to be explored by the LHC. Indeed, a global analysis of current neutrino oscillation data and precision electroweak data yields some stringent constraints on the non-unitarity of V, but its effect is allowed to be of $\mathcal{O}(10^{-2})$ [19] and may have some novel implications on neutrino oscillations [20]—[22]. I shall show that the flavor democracy of cosmic neutrinos at neutrino telescopes can be broken at the several percent level, just due to the non-unitarity of V.

In the presence of small unitarity violation, I write V as $V = \mathcal{A}V_0$, where V_0 is a unitary matrix containing 3 rotation angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and 3 phase angles [1], and $\mathcal{A}$ is a quasi-identity matrix which can in general be parametrized in terms of 9 rotation angles θ_{ij} and 9 phase angles δ_{ij} (for $i = 1, 2, 3$ and $j = 4, 5, 6$) [20]. For simplicity, here we adopt the expression of $\mathcal{A}$ shown in Eq. (11) of Ref. [20] and take V_0 to be the popular tri-bimaximal mixing pattern [8] without any CP-violating phases. Then Zhou and I obtain nine elements of the non-unitary neutrino mixing matrix $V = \mathcal{A}V_0$ as follows [6]:

$$V_{e1} = \frac{2}{\sqrt{6}}\left(1 - W_1\right) \; ,$$
$$V_{e2} = \frac{1}{\sqrt{3}}\left(1 - W_1\right) \; ,$$
$$V_{e3} = 0 \; ,$$
$$V_{\mu 1} = -\frac{1}{\sqrt{6}}\left(1 - W_2 + 2X\right) \; ,$$
$$V_{\mu 2} = \frac{1}{\sqrt{3}}\left(1 - W_2 - X\right) \; ,$$
$$V_{\mu 3} = \frac{1}{\sqrt{2}}\left(1 - W_2\right) \; ,$$
$$V_{\tau 1} = \frac{1}{\sqrt{6}}\left(1 - W_3 - 2Y + Z\right) \; ,$$
$$V_{\tau 2} = -\frac{1}{\sqrt{3}}\left(1 - W_3 + Y + Z\right) \; ,$$
$$V_{\tau 3} = \frac{1}{\sqrt{2}}\left(1 - W_3 - Z\right) \; , \qquad (28)$$

where $W_i = (s^2_{i4} + s^2_{i5} + s^2_{i6})/2$ (for $i = 1, 2, 3$), and

$$X = \hat{s}_{14}\hat{s}^*_{24} + \hat{s}_{15}\hat{s}^*_{25} + \hat{s}_{16}\hat{s}^*_{26} \; ,$$
$$Y = \hat{s}_{14}\hat{s}^*_{34} + \hat{s}_{15}\hat{s}^*_{35} + \hat{s}_{16}\hat{s}^*_{36} \; ,$$
$$Z = \hat{s}_{24}\hat{s}^*_{34} + \hat{s}_{25}\hat{s}^*_{35} + \hat{s}_{26}\hat{s}^*_{36} \; . \qquad (29)$$

Here $s_{ij} \equiv \sin\theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}} s_{ij}$ have been defined, and higher-order terms of s_{ij} have been neglected. The mixing angles in θ_{ij} can at most be of $\mathcal{O}(0.1)$, but the CP-violating phases δ_{ij} are entirely unrestricted. If both θ_{ij} and δ_{ij} are switched off, the tri-bimaximal neutrino mixing pattern will be reproduced from Eq. (28). With the help of Eqs. (3), (5) and (28), we arrive at the flavor distribution at neutrino telescopes [6]:

$$\phi^{\rm T}_e = \frac{\phi_0}{3}\left[1 - \frac{4}{9}\left(7W_1 + 2W_2 - {\rm Re}X\right)\right] ,$$

$$\phi_\mu^{\rm T} = \frac{\phi_0}{3}\left[1 - \frac{2}{9}\left(2W_1 + 16W_2 + {\rm Re}X\right)\right] ,$$
$$\phi_\tau^{\rm T} \equiv \frac{\phi_0}{3}\left[1 - \frac{2}{9}\left(2W_1 + 7W_2 + 9W_3 + {\rm Re}X\right)\right] . \quad (30)$$

The flavor democracy of $\phi_\alpha^{\rm T}$ (for $\alpha = e, \mu, \tau$) is clearly broken. Because of the non-unitarity of V, the total flux of cosmic neutrinos at the telescope is not equal to that at the source:

$$\sum_\alpha \phi_\alpha^{\rm T} = \phi_0 \left[1 - \frac{2}{3}\left(2W_1 + 3W_2 + W_3\right)\right] . \quad (31)$$

This sum is apparently smaller than ϕ_0, and it approximately amounts to $0.96\phi_0$ if $W_i \sim 0.01$ (for $i = 1, 2, 3$).

Note that ${\rm Re}X$ receives the most stringent constraint from current experimental data, $|X| < 7.0 \times 10^{-5}$ [19]. Hence the dominant effects of unitarity violation on $\phi_\alpha^{\rm T}$ come from W_i. The breaking of cosmic neutrino flavor democracy can be at the several percent level. Although the strength of non-unitarity is very small and certainly difficult to be observed in realistic experiments, it *does* illustrate how sensitive a neutrino telescope will be to this kind of new physics. More discussions can be found in Ref. [6].

6. On the ratio $\phi_e : \phi_\mu : \phi_\tau$

What I have so far considered is the canonical or conventional astrophysical source, from which the UHE neutrino flux results from the pion decays and thus has the flavor composition $\phi_e : \phi_\mu : \phi_\tau = 1 : 2 : 0$. In reality, however, this simple flavor content could somehow by contaminated for certain reasons (e.g., a small amount of ν_e, ν_μ and ν_τ and their antiparticles might come from the decays of heavier hadrons produced by pp and $p\gamma$ collisions) [23]. Following a phenomenological approach, Zhou and I proposed a generic parametrization of the initial flavor composition of an UHE neutrino flux [13]:

$$\begin{pmatrix} \phi_e \\ \phi_\mu \\ \phi_\tau \end{pmatrix} = \begin{pmatrix} \sin^2\xi \cos^2\zeta \\ \cos^2\xi \cos^2\zeta \\ \sin^2\zeta \end{pmatrix} \phi_0 , \quad (32)$$

where $\xi \in [0, \pi/2]$ and $\zeta \in [0, \pi/2]$. Then the conventional picture, as shown in Eq. (2), corresponds to $\zeta = 0$ and $\tan\xi = 1/\sqrt{2}$ (or $\xi \approx 35.3^\circ$) in our parametrization. It turns out that any small departure of ζ from zero will measure the existence of cosmic ν_τ and $\overline{\nu}_\tau$ neutrinos, which could come from the decays of D_s and $B\overline{B}$ mesons produced at the source [7]. On the other hand, any small deviation of $\tan^2\xi$ from $1/2$ will imply that the pure pion-decay mechanism for the UHE neutrino production has to be modified.

After defining three neutrino flux ratios R_α (see Eq. (20) for $\alpha = e, \mu, \tau$) as our working observables at a neutrino telescope, we have shown that the source parameters ξ and ζ can in principle be determined by the measurement of two independent R_α and with the help of accurate neutrino oscillation data [13]:

$$\begin{aligned} \sin^2\xi &= K\left[r_e\left(P_{\tau\mu} - P_{\mu\mu}\right) - r_\mu\left(P_{\tau e} - P_{\mu e}\right) \right. \\ &\quad \left. + \left(P_{\mu\mu}P_{\tau e} - P_{\mu e}P_{\tau\mu}\right)\right] , \\ \tan^2\zeta &= K\left[r_e\left(P_{\mu\mu} - P_{e\mu}\right) - r_\mu\left(P_{\mu e} - P_{ee}\right) \right. \\ &\quad \left. + \left(P_{e\mu}P_{\mu e} - P_{ee}P_{\mu\mu}\right)\right] , \end{aligned} \quad (33)$$

where

$$\begin{aligned} K^{-1} &= + \left(r_e - P_{\tau e}\right)\left(P_{e\mu} - P_{\mu\mu}\right) \\ &\quad - \left(r_\mu - P_{\tau\mu}\right)\left(P_{ee} - P_{\mu e}\right) , \end{aligned} \quad (34)$$

and

$$\begin{aligned} r_e &\equiv \frac{R_e}{1 + R_e} , \\ r_\mu &\equiv \frac{R_\mu}{1 + R_\mu} . \end{aligned} \quad (35)$$

Indeed, it is easy to check that $r_e = \phi_e^{\rm D}/\phi_0$ and $r_\mu = \phi_\mu^{\rm D}/\phi_0$ hold. One may in principle choose either (R_e, R_μ) or (r_e, r_μ) as a set of working observables to inversely determine ξ and ζ.

We have also examined the dependence of R_α upon the smallest neutrino mixing angle θ_{13} and upon the Dirac CP-violating phase δ. Our numerical examples in Ref. [13] indicate that it is promising to determine or (at least) constrain the initial flavor content of UHE neutrino fluxes with the second-generation neutrino telescopes.

7. Concluding remarks

I have discussed why and how a km^3-size neutrino telescope can serve as a striking probe of the flavor distribution of UHE cosmic neutrinos. Based on the conventional mechanism for UHE cosmic neutrino production at a distant astrophysical source and the standard picture of neutrino oscillations, I have shown that the flavor composition of cosmic neutrino fluxes at a terrestrial detector may deviate from the naive expectation $\phi_e^{\rm T} : \phi_\mu^{\rm T} : \phi_\tau^{\rm T} = 1 : 1 : 1$. This expectation can only be true when either $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ (CP invariance) or $\delta = \pm\pi/2$ and $\theta_{23} = \pi/4$ (CP violation) are satisfied in the standard parametrization of V. Instead, $\phi_e^{\rm T} : \phi_\mu^{\rm T} : \phi_\tau^{\rm T} = (1 - 2\Delta) : (1 + \Delta) : (1 + \Delta)$ holds, where Δ characterizes the effect of μ-τ symmetry breaking. The latter is actually a reflection of $\theta_{13} \neq 0$ and $\theta_{23} \neq \pi/4$ in V. I have examined the sensitivity of Δ to the deviation of θ_{13} from zero and to the departure of θ_{23} from $\pi/4$, and obtained $-0.1 \leq \Delta \leq +0.1$ from current data. I find that it is also possible to probe the breaking of μ-τ symmetry by detecting the $\overline{\nu}_e$ flux of $E_{\overline{\nu}_e} \approx 6.3$ PeV via the Glashow resonance channel $\overline{\nu}_e e \to W^- \to$ anything. Furthermore, I give some brief comments on possible effects of the non-unitarity of V on the flavor distribution at neutrino telescopes, and on a generic description of the cosmic neutrino flavor composition at distant astrophysical sources.

This work reveals the combined effect of $\theta_{13} \neq 0$, $\theta_{23} \neq \pi/4$ and $\delta \neq \pi/2$ which can show up at the detector. Even if $\Delta \neq 0$ is established from the measurement of UHE neutrino fluxes, the understanding of this μ-τ symmetry breaking signal requires more precise information about θ_{13}, θ_{23} and δ. Hence it makes sense to look at the complementary roles played by neutrino telescopes and terrestrial neutrino oscillation experiments (e.g., the reactor experiments to pin down θ_{13} and the neutrino factories or superbeam facilities to measure δ) in the era of precision measurements.

The feasibility of the above ideas depends on the assumption that we have correctly understood the production mechanism of cosmic neutrinos from a distant astrophysical source (i.e., via pp and $p\gamma$ collisions) with little uncertainties. It is also dependent upon the assumption that the error bars associated with the measurement of relevant neutrino fluxes or their ratios are much smaller than Δ. The latter is certainly a challenge to the sensitivity or precision of IceCube or KM3NeT, unless the effect of μ-τ symmetry breaking is unexpectedly large. Nevertheless, any constraint on Δ to be obtained from neutrino telescopes will be greatly useful in diagnosing the astrophysical sources and in understanding the properties of neutrinos themselves. Much more effort is therefore needed in this direction.

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