Les étoiles à neutrons, du modèle aux observations

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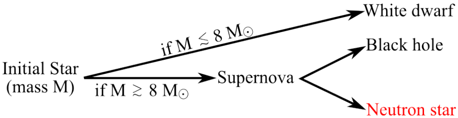
- 1920 Theoretical prediction of the neutron by Rutherford
- 1931 Some discussions about a "neutron core" by Landau or Langer and Rosen
- 1932 Discovery of the neutron by Chadwick
- 1934 Theoretical prediction of neutron stars by Baade and Zwicky
- 1939 First realistic model of neutron stars, Tolman, Oppenheimer and Volkov
- 1967 First observation of a pulsar by Hewish and Bell

Figure: Chadwick, Baade and Zwicky

Neutron stars are formed after a gravitational supernova type II.



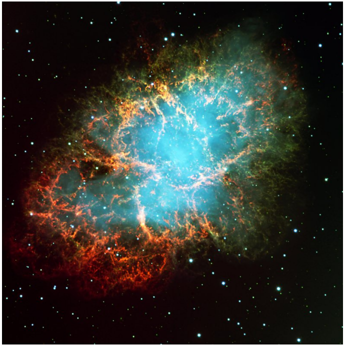


Figure: Crab nebula with a Pulsar in the center.

Neutron star characteristics are:

- A radius: $R \simeq 10 - 15$ km
- A mass: $M \simeq 1 - 3\,\mathrm{M}_{\odot}$
- Compacity: $\Xi = \dfrac{GM}{Rc^2} \simeq 0.2$
- Average density: $\rho \simeq 2.10^{14}\,\mathrm{g.cm}^{-3}$
- Temperature: $T \simeq 10^6 - 10^{10}$ K
- Period of rotation: $P \simeq 0.001 - 10\,\mathrm{s}$
- Magnetic field: $B \simeq 10^7 - 10^{15}$ G

$\Longrightarrow$ Test for fundamental physics

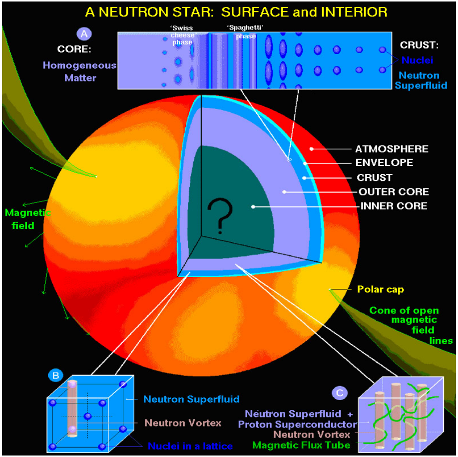


Figure: Neutron star structure, D. Page

Pulsar Observation

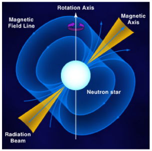
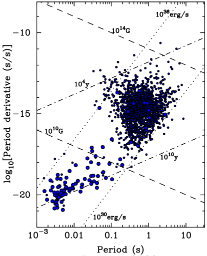
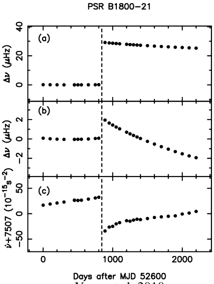


Diagram $P/\dot{P}$



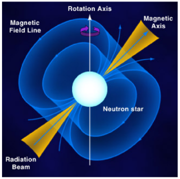
Lorimer 2008

Glitches

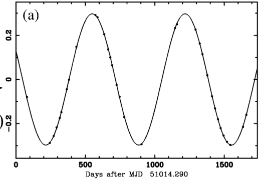


Yuan et al. 2010

Binary with a pulsar Observation

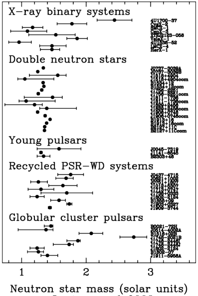


Period shift (micro second)



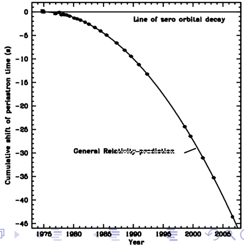
Lorimer et al. 2008

Mass



Lorimer et al. 2008

Test for GR



Weisberg et al. 2010

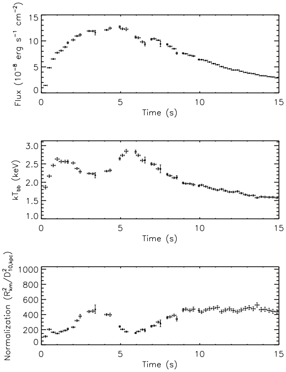


Figure: Thermonuclear bursts observation at the surface of a neutron star, Guver et al. 2008

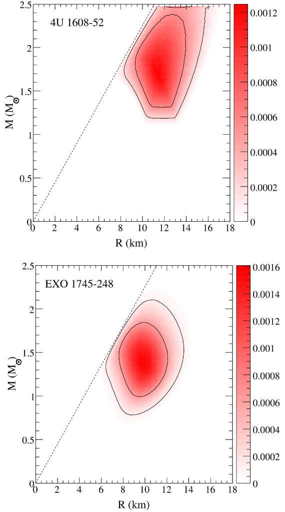


Figure: Probability for Mass-Radius,

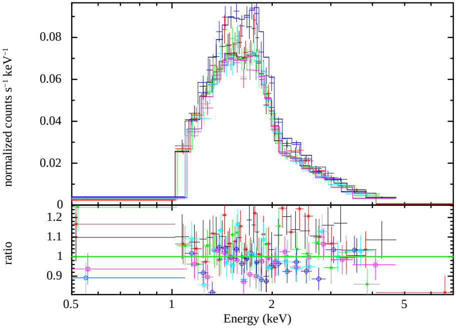


Figure: Chandra observations of Cassiopeia A Neutron Star between 2000 and 2009, Heinke et al. 2010

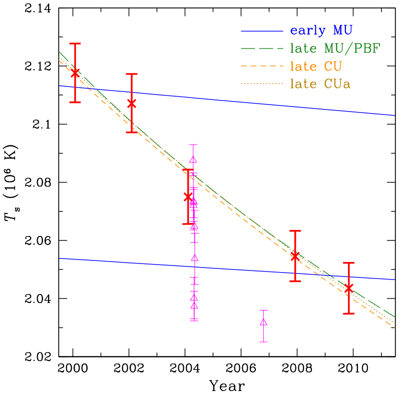


Figure: Temperature evolution, Heinke et al. 2010

OBSERVATIONS $\leftrightarrows$ MODELS

Nuclear matter is largely **unknown**

- No QCD for nuclear matter
- Phenomenological interaction

$$B(A,Z) = a_V A + a_s A^{\frac{2}{3}} + (a_I + a_{IS} A^{-\frac{1}{3}}) \frac{(N-Z)^2}{A} + a_c \frac{Z^2}{A^{\frac{1}{3}}} - \delta_p + E_D \quad (1)$$

- Many body problem?
- Large number of neutrons?
- Composition of nuclear matter at very high density?
- Bulk and shear viscosities for description of macroscopic dynamics?

$\rightarrow$ Hundreds Equation of States for nuclear matter

Mass-Radius diagrams obtained with TOV equations are signature of Equation of States.

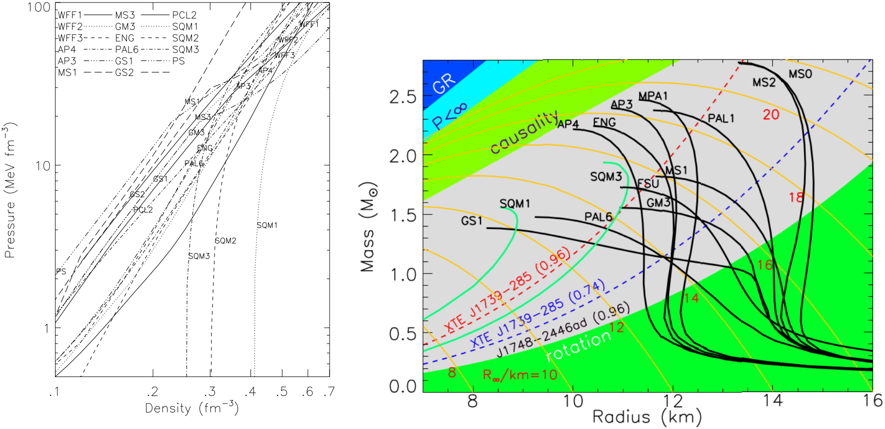


Figure: EOS and corresponding Mass-Radius diagrams, Lattimer 2001

Extracting equation of state informations from gravitational waves:

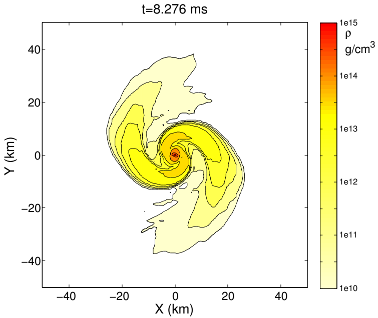


Figure: Isodensity contours for two merged NS after 8.276 ms, Rezzola et al. 2010

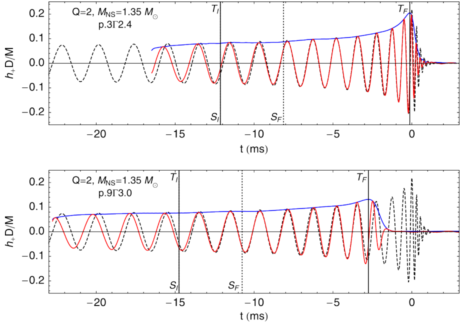


Figure: BH-NS merger waveform for to EOS, Lackey et al. 2012

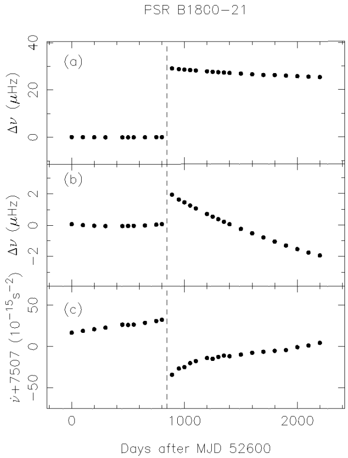


Figure: Yuan et al. 2010

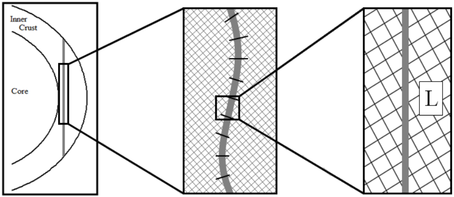


Figure: Vortices in neutron stars, Grill and Pizzochero 2012

→ Glitches as a proof of superfluidity in neutron stars

Thermal emission + estimated age $\Longrightarrow$ constraint on thermal evolution models

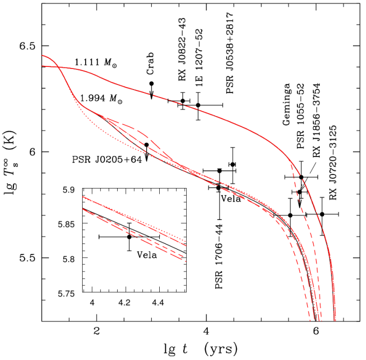


Figure: Cooling curve and observational data (Gusakov et al. 2004)

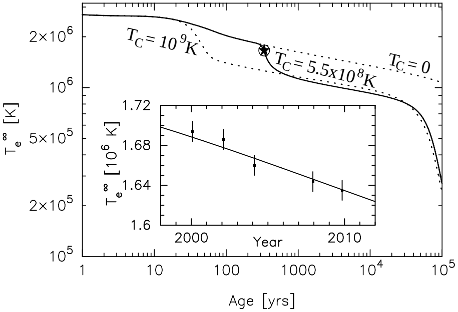


Figure: Cooling in Cassiopeia A (Page et al. 2011)

→ Accurate thermal evolution model to interpret these constraints.

Specific heat + thermal conductivity + ν emissivity $\Longrightarrow$ Thermal identity of the matter

- Specific heat is a sum over the different contributions from the different excitations (nuclei, phonons, electrons,...)
- The crust is important for thermal evolution models (Gnedin et al. 2001, Brown and Cumming 2009)
- Nucleonic contribution in the inner crust is strongly suppressed

$\rightarrow$ Investigation of a new contribution to the specific heat from the collective excitations.

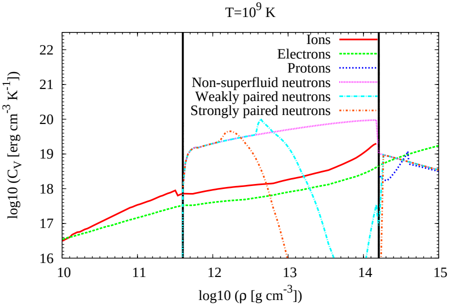


Figure: Specific heat contribution as a function of the density at $T = 10^9$K (Fortin et al. 2010)

MOTIVATIONS

- At low temperature collective modes are present
- Pairing energy of the order of 10^{10} K
  $\Longrightarrow$ No excitations coming from pair breaking of nucleons at $T < 10^{10}$ K
- Due to pairing $\Longrightarrow$ matter is superfluid
- Superfluidity $\Longrightarrow$ Collective excitation at low energy

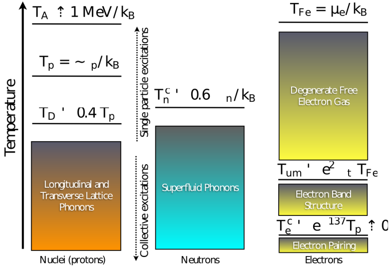


Figure: Collective excitations regime VS single particle excitation regime function of the temperature (Page and Reddy 2012)

→ Hydrodynamic approximation to model collective behaviour of nucleons

Two basic equations to derive non-relativistic hydrodynamics of uncharged superfluids (Prix 2004):

- Conservation of particle number: $\partial_t \; n_{\mathrm{a}} + \nabla . \; \mathbf{j}_{\mathrm{a}} = 0$ with $\mathrm{a} = \mathrm{n,p}$
- Euler equation: $\partial_t \mathbf{P}^{\mathrm{a}} = \nabla \pi^{\mathrm{a}}$ with $\mathrm{a} = \mathrm{n,p}$

$$-\pi^{\mathrm{a}} = \mu^{\mathrm{a}} - \frac{1}{2} m^{\mathrm{a}} \mathbf{v}_{\mathrm{a}}^2 + \mathbf{v}_{\mathrm{a}} \cdot \mathbf{p}^{\mathrm{a}} \tag{2}$$

Characteristics of superfluids come from quantum properties:

- No viscosity
- Locally irrotational
- No entropy transport
- Entrainment between the two fluids (n,p): non dissipative interaction which misalign velocities and momenta $\Longrightarrow$ coupling between fluids

$\rightarrow$ Microscopic input from nuclear interaction

Few hypothesis:

- Zero temperature
- β equilibrium $\Longrightarrow$ proportion of n,p
- Hydrostatic equilibrium
- Non relativistic hydrodynamics

Two linearised equations:

- Conservation of particle number: $\partial_t\, \delta n_\mathrm{a} + n_\mathrm{a} \nabla .\, \delta \mathbf{v}_\mathrm{a} = 0$ with a = n,p
- Euler equation: $\partial_t \delta \mathbf{P}^\mathrm{a} = -\nabla \delta \mu^\mathrm{a}$ with a = n,p

$\Longrightarrow$ Two eigenvectors $(U^{\pm})$ with associated sound velocity $(u_{\pm})$

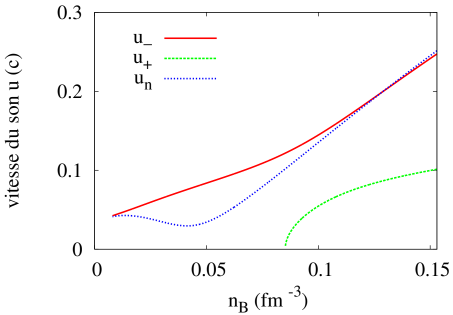


Figure: Sound velocities as a function of the density

→ Hydrodynamic modes in the inner crust

The inner crust structures:

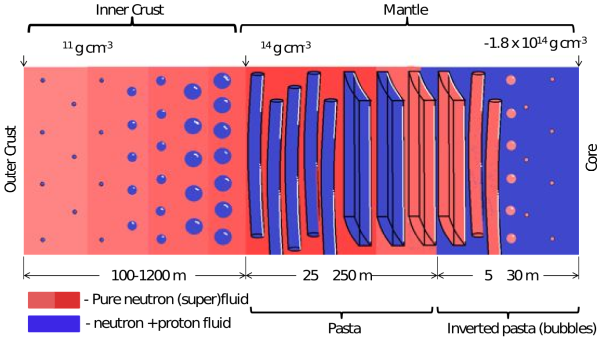


Figure: Neutron star inner crust, Newton et al 2011

- Inner crust = transition from homogeneous matter to a lattice of atomic nuclei
- Inner crust = lattice of nuclei immersed in a neutron fluid
- Pasta phase = very deformed nuclei

The inner crust structures:

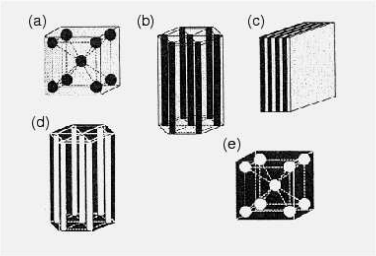


Figure: Pasta structures, G. Watanabe

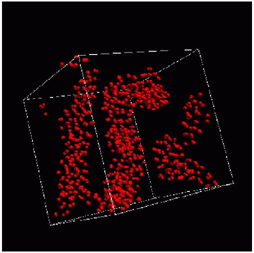


Figure: Proton distribution in cylinder at $n_b = 0.033\ \mathrm{fm}^{-3}$ from a numerical simulation (Watanabe et al. 2003)

- Inner crust = transition from homogeneous matter to the lattice of atomic nuclei
- Inner crust = lattice of nuclei immersed in a neutron fluid
- Pasta phase = very deformed nuclei

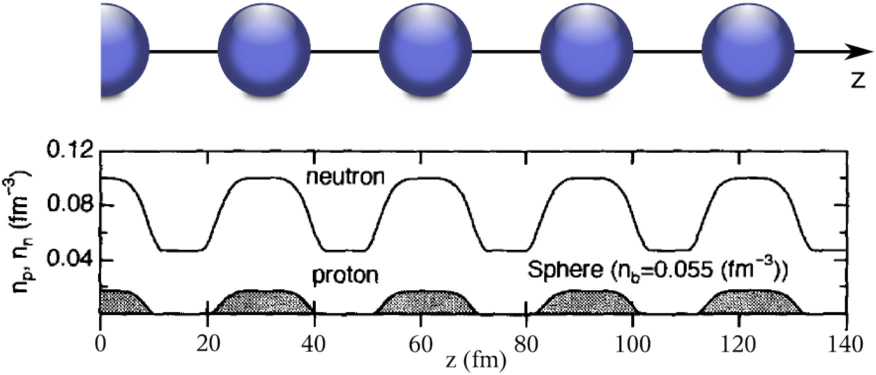


Figure: The neutron (upper) and proton (lower, shaded) distributions along the straight lines joining the centers of the nearest spherical nuclei at $n_b = 0.055$ fm^{-3} (Oyamatsu 1993)

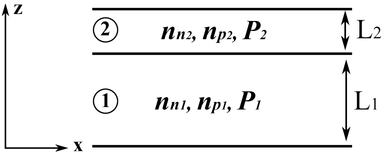
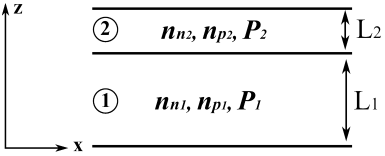


Figure: Representation of "1D structure"

Description of the model:

- Model with 1D geometry
- Structures correspond to a periodic alternance of two slabs ("gaseous" and "liquid") with different proton and neutron densities
- Superfluid hydrodynamics approximation in each slab



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$\Longrightarrow$ We need 4 boundary conditions based on behaviour of matter at interfaces in order to describe transmission/reflection coefficients

Several compatible sets of four boundary conditions are possible.
The most probable is :

- Two continuity of perpendicular fluid velocities (neutrons and protons)
- A common surface for protons and neutrons
- Continuity of the pressure

To complete the set of equations:

- Invariance along $\mathbf{r}_{||} \Longrightarrow$ Snell-Descartes laws for angles $\left(k^{\pm}_{||1} = k^{\pm}_{||2} = k^{\pm}_{||3} = q_{||} = q\cos(\theta)\right)$
- We use the Floquet-Bloch theorem to take into account the periodicity $U(\mathbf{r}+\mathbf{L}) = U(\mathbf{r})e^{i\mathbf{q}.\mathbf{L}}$ where $\mathbf{L}$ is the periodicity

$\rightarrow$ The problem is formulated with a 6×6 matrix. Solutions are the zeros of the matrix determinant

Lasagna at $n_{\mathrm{B}} = 0.08\ \mathrm{fm}^{-3} \sim \frac{\rho_0}{2}$. Excitations spectrum at zero angle of propagation:

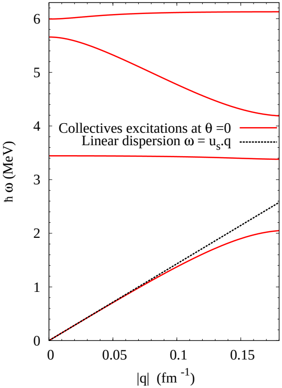

Figure: Excitation spectrum at $n_{\mathrm{B}} = 0.08\ \mathrm{fm}^{-3}$ and $\theta = 0$

A basic model of lasagne leads to:

$$\frac{L}{n_{\mathrm{B}} u_s^2} = \frac{L_1}{n_{\mathrm{B}1} u_{s1}^2} + \frac{L_2}{n_{\mathrm{B}2} u_{s2}^2} \qquad (3)$$

with

$$u_{si}^2 = \frac{1}{m} \left. \frac{\partial P_i}{\partial n_{\mathrm{B}i}} \right|_{Y_{pi}} \qquad (4)$$

This model gives $u_s = 0.073\, c$.
$\rightarrow$ The acoustic branch has a small enough slope to contribute significantly to the specific heat

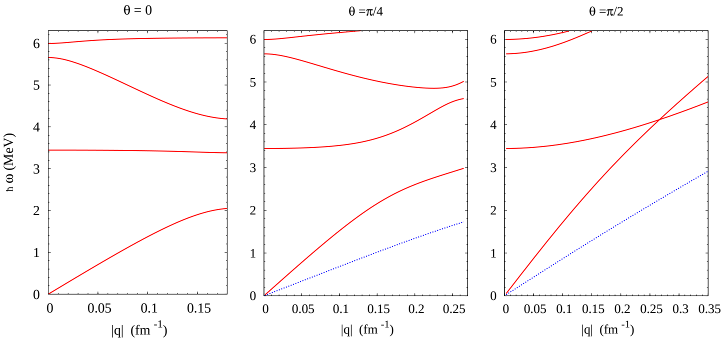

Figure: Excitation spectrum at $n_{\rm B} = 0.08$ fm^{-3} for $\theta = 0$ (left), $\theta = \pi/4$ (middle), $\theta = \pi/2$ (right)

→ The second mode is close to the dispersion relation $\omega \simeq u'_s q_{||}$ with a slope varying from $u'_s = 0.041\,c$ to $u'_s = 0.051\,c$. Both acoustic mode can be associated to a Goldstone mode

Collective modes have a Bose distribution with zero chemical potential. We can integrate over all momenta in order to obtain energy density:

$$\mathcal{E}(T) = \int_{-\pi/L}^{\pi/L} \frac{dq_z}{2\pi} \int \frac{d^2 q_{||}}{(2\pi)^2} \; \hbar\omega(\mathbf{q}) \frac{1}{e^{\hbar\omega(\mathbf{q})/k_B T} - 1} \tag{5}$$

Temperature derivation leads to the specific heat:

$$C_v(T) = \left. \frac{\partial \mathcal{E}}{\partial T} \right|_V \tag{6}$$

For a linear dependence $\omega = u_s q$ of the energy:

$$C_v^s = \frac{2\pi^2 k_B^4 T^3}{15\hbar^3 u_s^3} \equiv bT^3 \tag{7}$$

For a linear dependence $\omega = u_s' q_{||}$:

$$C_v^{s'} = \frac{3\zeta(3) k_B^3 T^2}{\pi \hbar^2 {u_s'}^2 L} \equiv aT^2 \tag{8}$$

with $\zeta(3) = 1.202$

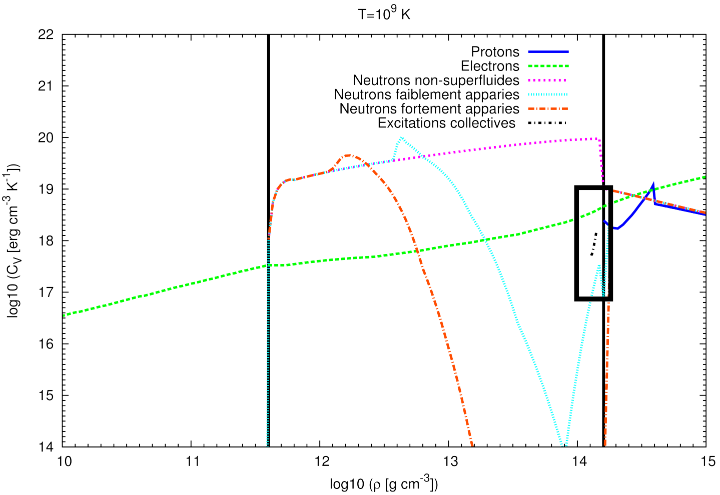

Figure: Specific heat at $T = 10^9$ K as a function of the density from Fortin et al. 2010

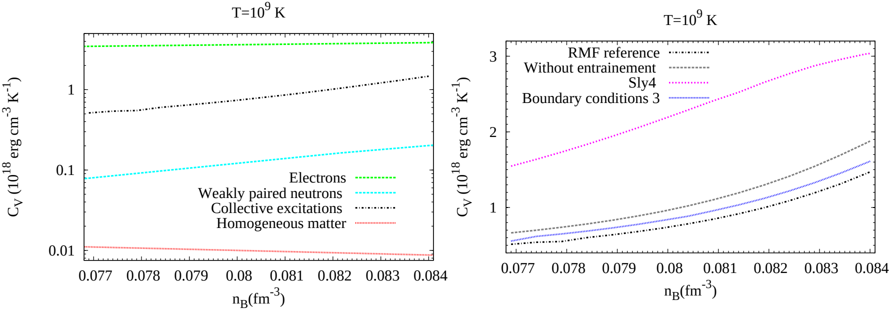

Figure: Specific heat at $T = 10^9$ K as a function of the density compare with other contributions from Fortin et al. 2010

$\rightarrow$ Collective excitations contribute significantly to the specific heat

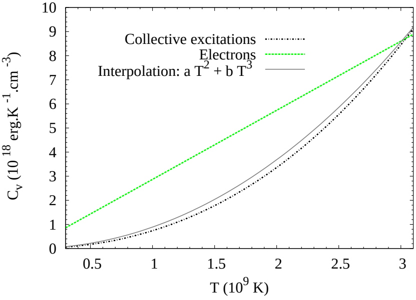

Figure: Specific heat contribution as a function of the temperature at $n_{\rm B} = 0.08$ fm^{-3}

The electronic contribution is:

$$C_v^{el.} = \frac{k_B^2 \mu_e^2 T}{3(\hbar c)^3} \qquad (9)$$

Collective excitations contribution is interpolated by

$$C_v = aT^2 + bT^3 \qquad (10)$$

with $u_s = 0.079\, c$ and $u_s' = 0.046\, c$.
$\rightarrow$ Collective excitations contribution can dominate electrons contribution. Contribution from acoustic mode and good agreement between exact solutions and interpolation.

Conclusion

- Neutron stars are fascinating objects with lot of physics phenomenon
- It is essential to develop enough complete models to prepare future observations
- Keep in mind that theoretical considerations are not the truth since potential observations can constrain and also undermine such theories

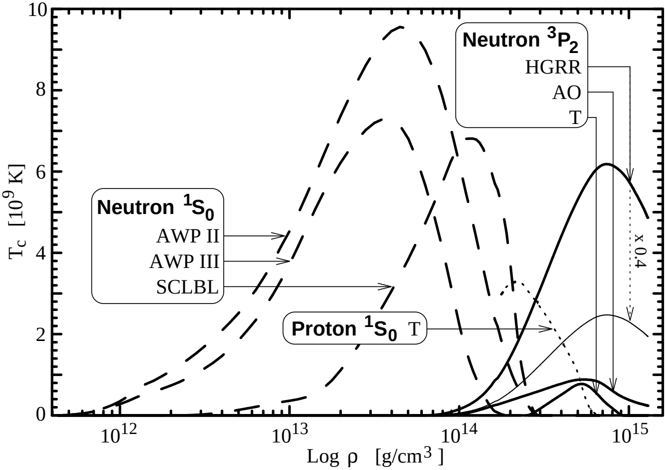

Figure: Critical temperature for several models of pairing (Page 1997)