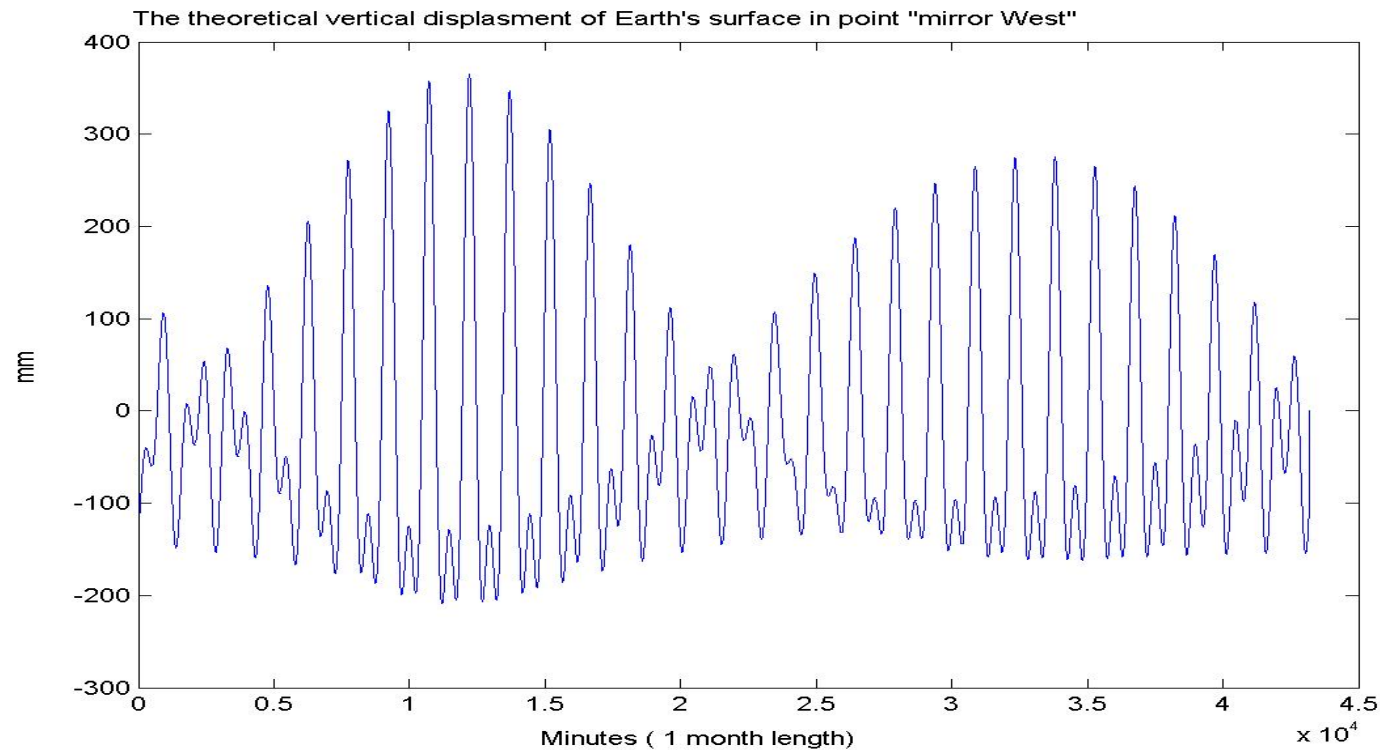


Long-base Hydrostatic Incline Meter.

gravitational antenna

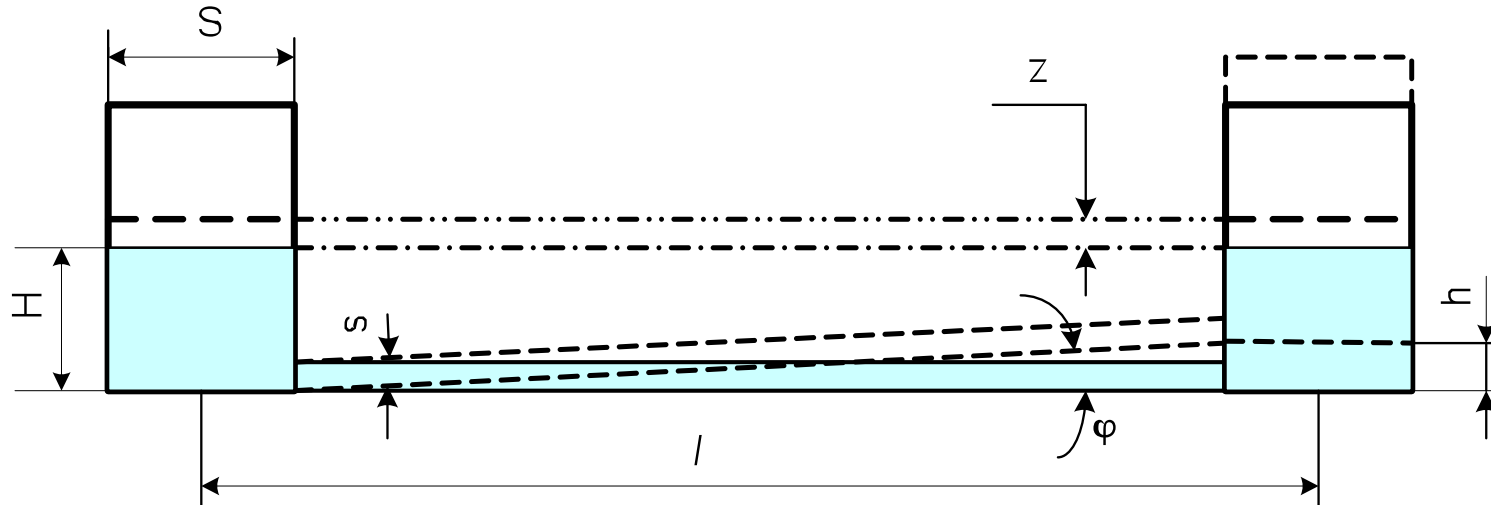
- There are many reasons resulting in relative vertical displacements of end-capping and central mirrors of gravitational antenna. Firstly is passing of tidal waves. Amplitude of relative vertical displacements of mirrors arrives at 0.5 mm.



- Vertical displacements of mirrors during 1 month

- Secondly are deformation processes: influence of variations of temperature and its gradients, change of atmospheric pressure at passing of atmospheric fronts etc. Amplitudes of relative vertical displacements can arrive at a few millimeters.
- These relative vertical motions of mirrors result in vertical motions of rays of measuring laser on the surfaces of mirrors, that causes the change of length of ray and other effects.
- The automatic system of vertical correction of positions of mirrors, based on the use of data about motions from mechanical seismic filters, does not arrive at the necessary error of setting and with holding of vertical positions of end-capping and central mirrors.
- Therefore it is necessary an additional exact device for independent monitoring of the vertical position of the mirrors.
- Such device can be Long-base Hydrostatic Incline Meter.

- The principle of operation of the Hydrostatic Incline Meter is simple - this is the principle of communicating vessels (Fig.1)



- S – the cross-sectional area of a measuring vessel, s – the cross-sectional area of a connecting tube, l – length of base of the device, h – vertical moving of a measuring vessel, H – initial level of a liquid in a vessel, z – size of displacement of level of a liquid in a vessel, ϕ – angle of tilt of base of the device.

In a quasistatic mode at base $l = 3$ km, a base inclination as whole on 1 arc sec ($5 \cdot 10^{-6}$ rad) will correspond to relative change of level of a liquid in vessels $\sim 1,5$ cm, and amplitude of tidal inclinations (0,02 arc. sec) corresponds to relative change of the level of 0,3 mm! This is a great value.

Radio engineering methods of measurement of small mechanical movements allow to measure variations of level of a liquid in end vessels with the resolution of 10^{-7} cm (10^{-3} μ). Therefore maintain continuous observation of variations of the vertical relative movements of the base unit, separated by 3 km with a resolution of 1 micron formally is not a problem. All is defined by influence of variations of atmospheric pressure, temperature and its gradient. We will consider influence of each factor separately.

- 1. Influence of variations of atmospheric pressure.
- For dielectric oils (condenser, transformer, polymethylsilicone) compressibility factor $\chi \approx 6 \cdot 10^{-5} \text{ 1/bar}$. At cross-sectional area of a connecting tube s , cross-section of vessel S and length of base l change of level h in each measuring vessel when the air pressure changes on the value of ΔP will be

- $$h = (s/2S) l \chi \Delta P$$

- If $S/s = 250$, $l = 3 \cdot 10^3 \text{ m}$, $\Delta P = 1 \text{ Torr}$, then $h \approx 5 \cdot 10^{-4} \text{ mm}$.
- It should be noted that this value will change once the level in both vessels, so that the compressibility of the liquid will not change the relative levels of the liquid, i.e. to an apparent relative vertical displacement of the measuring vessel.

- 2. Thermal expansion of the dielectric oil and connecting tube
- If β_o - coefficient of volume expansion of oil, β_T - coefficient of linear expansion of a material of a connecting tube, change of level in each vessel

$$h \approx \left(\frac{s}{2S}\right) \cdot l \cdot (\beta_i - 3\beta_o) \Delta T$$

-
- If $\beta_o \sim 6 \cdot 10^{-4} \text{ 1/K}$, $\beta_T \sim 6 \cdot 10^{-4} \text{ 1/K}$ (for a rubber tube),
- $\Delta T = 1^\circ \text{C}$, then $h \approx 0,3 \text{ mm}$.
- Temperature change also as well as pressure change does not lead to seeming vertical motions of vessels, but demands maintenance of a considerable dynamic range of the sensors of the displacement of the liquid. So, at temperature change on 20°C , change of level of a liquid in measuring vessels reaches about 6 mm at the chosen parameters.

- 3. Influence of temperature gradient along the base of the device.
-
- If the temperature gradient T_l is constant, the temperature along the base of the instrument obviously changes according to the formula
- $$T(x) = (1 + T_l \cdot x)$$
- Difference between levels is equal to $\Delta h \approx T_l \cdot H \beta_v \cdot l$
-
- where H – the initial height of the liquid level in the vessels.
- If $T_l = 10^{-3} \text{ } ^\circ\text{C/M}$, ($\Delta \dot{O} = \dot{O}_1 - \dot{O}_2 = 3^0 \tilde{N}$), $H \approx 10\text{cm}$, then $\Delta h = 0,2 \text{ mm}$.
-
- A constant temperature gradient along the base of the device does not lead to errors in the measurements.
- If change of a gradient of temperature leads to change of a difference of temperatures of end vessels $\Delta(\Delta T)$, then $\Delta(\Delta h) = \Delta(\Delta H \beta_v)$. So when $\Delta(\Delta T) = 1^\circ\text{C}$,
- $\Delta(\Delta h) \approx 0,06 \text{ mm} = 60 \text{ } \mu$.
- Precision measurements of temperature near end vessels allow to reduce this influence to a fraction of a micron.
-

- Dynamic parameters of a Hydrostatic Incline Meter

- Hydrostatic Incline Meter is a complex oscillatory system. The equation of movement of a liquid:

- $$\frac{4}{3} \rho s l \left(\frac{S}{s} \right)^2 \ddot{z} + 8\pi\mu \cdot l \left(\frac{S}{s} \right)^2 \dot{z} + 2\rho g S \cdot z = -\rho \cdot l S \ddot{X}(t) - \rho S g l \varphi(t) - \rho S H l \ddot{\varphi}(t) \quad (1)$$

- In the right part of equation - the external influences leading to variations of level in measuring vessels.

- The first term of the equation $a = \rho \cdot l S \ddot{X}(t)$ defines the impact of horizontal movements of the whole device;

- $b = \rho S g l \varphi(t)$ - the influence of inclinations of base;

- $c = \rho S H l \ddot{\varphi}(t)$ - the influence of the angular accelerations when slopes the base unit

- The effective mass, the coefficient of friction and rigidity:

- $$m_{eff} = \frac{4}{3} \rho s l \left(\frac{S}{s} \right)^2 \quad h_{eff} = 8\pi\mu \cdot l \left(\frac{S}{s} \right)^2 \quad K_{eff} = 2\rho g S \quad (2)$$

- Hence natural resonant frequency is defined by the equation:

$$\omega_0 = \sqrt{\frac{K_{eff}}{m_{eff}}} = \sqrt{\frac{3gs}{2lS}} \approx 1,4 \cdot 10^{-2} \frac{rad}{s}; \Rightarrow \text{natural period } T_0 \approx 400 s$$

- The damping decrement is equal to

$$\delta = \frac{3\pi\mu}{\rho \cdot s} = \frac{3\pi\nu}{s}$$

- where ν is the kinematic viscosity of the fluid. Unevident estimation - in δ not include neither the density of the liquid, or tube length, but only the kinematic viscosity and a cross-section of the tube.
- If $\nu = 10^{-2}$ St, $s = 3 \text{ cm}^2$, then $\delta = 3 \cdot 10^{-2} \text{ 1/s}$.
- Let's consider influence of horizontal movements of whole device. The response to the harmonic influence ($X = x_0 e^{-ipt}$) on frequency p is defined as:

$$z = \frac{\rho l S x_0}{m_{eff} \sqrt{\left(1 - \frac{w_0^2}{p^2}\right)^2 + 4 \frac{\delta^2}{p^2}}};$$

- For a estimation x_0 we will use article [*Fix J.E. Ambient earth motion in the period range from 0,1 to 2560 sec. – Bull. Soc. Am., 62, 1753, 1972.*], which shows spectral density
- of seismic noise in the periods from 10-2 s up to 2000 s. In according with [*Gusev G. Manukin A. Limiting sensitivity of the gravinertial measurements. Proceeding AS USSR, Physics of the Earth, №9, 1985 c. 90-95.*], the dependence of the power spectral density
- from frequency can be approximated by the formula:

$$\overline{\delta_p^2} \sim \frac{B}{p^4};$$

- The value $\sqrt{z(\delta)^2}$ in the frequency band Δp depends on the value of p and, for example,

- for the case of $p \sim \omega_0 \sim \delta$, given that $\Delta p = 2\pi/T$ (T – time measurement) looks like:

$$\sqrt{z(\delta)^2} \approx \frac{l}{2g} \sqrt{\frac{\pi B}{T}}$$

- The variable B characterizes the seismicity of the selected area of measurements. If $B = 10^{-10}$
- $\text{cm}^2 \text{ s}^{-3}$, $l = 3 \text{ km}$, $T = 1000 \text{ s}$, then $\approx 8 \cdot 10^{-5} \text{ cm} = 0,8 \mu$.

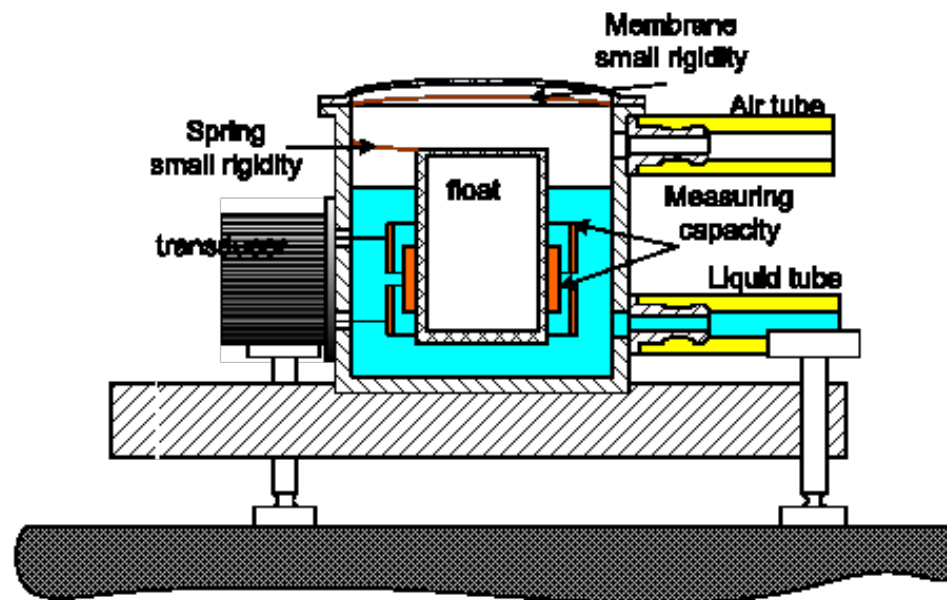
- The estimation of size of intrinsic thermal noise shows that their

$$\sqrt{z(\delta)^2} = \sqrt{\frac{k\Theta}{2\rho g S}} \approx 10^{-9} \text{ cm for earlier}$$

- chosen parameters of device.

The variant of the system for the measurement of changes of level of a liquid in the vessel of Hydrostatic Incline Meter

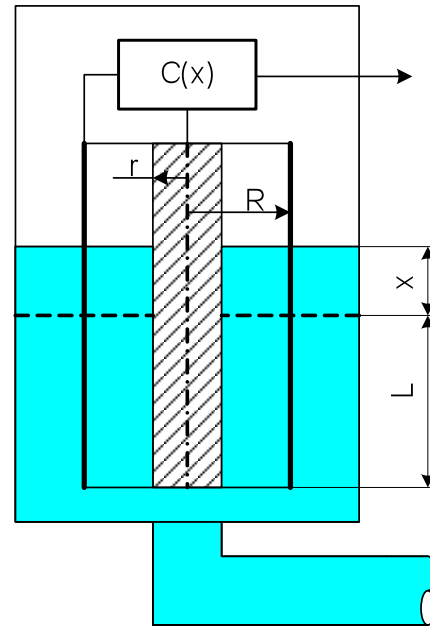
- There are many ways of building systems for measurement of variations of the level of liquid in the vessel. In figure one of them, which allows to perform measurements with a resolution of $5 \cdot 10^{-3} \mu$.



- If you change the level of a liquid in a vessel, the float in a measuring vessel changes the position that leads to change of size of two differential condensers of the capacitive sensor. When you move the float the area of overlap of the fixed plates and plates on the float changes, which leads to increase of one and reduce of other capacity.

- We used a frequency capacitance sensor with self compensate the drift as a transducer [*Gusev G., Manukin A. Two-axis inclinometer pendulum. The Physics of the Earth, №7, 1982, p. 86-88*]. Sensitivity of the device allows to measure variations of level of a liquid to $5 \cdot 10^{-3} \mu$, however a dynamic range is about 5 mm.
-
- Feature of the Hydrostatic Incline Meter (with base in some kilometers) is necessity of essential increase in a dynamic range of the sensor of level of a liquid in the measuring vessels.
- The new version, which differs simplicity of design, it is proposed to measure the change in capacitance, whose value is directly related to variations in liquid level.

- As the measuring capacity is proposed to use a cylindrical capacitor with the following parameters: r - radius of the inner electrode, R - radius of the outer electrode (Fig. 2).



- If the length of the capacitor is equal to $2L$ (first half of the capacitor is filled with liquid), then when the liquid level vary in the value of x the value of capacitor is:

$$C(x) = \frac{1}{2 \ln(R/r)} [L(\varepsilon + 1) + \tilde{o}(\varepsilon - 1)]$$

Relative change of capacity: $\frac{\Delta \tilde{N}}{\tilde{N}} = \frac{\varepsilon - 1}{\varepsilon + 1} \cdot \frac{x}{L}$

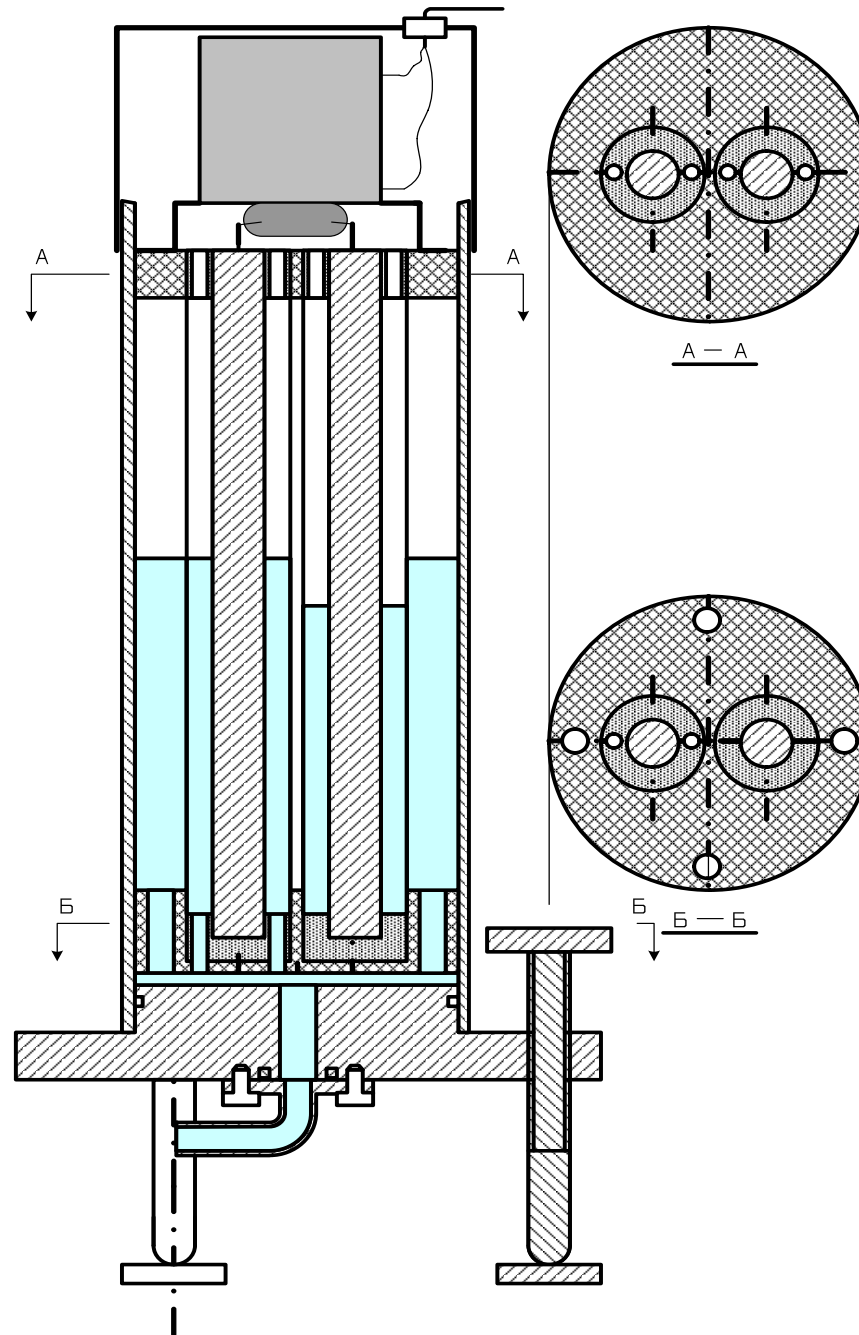
- where ε - the coefficient of dielectric permeability of a liquid in a measuring vessel.

- The table shows an example of calculation of such capacity, the geometrical parameters which are given in cm, and the value of the capacitance C - also in cm (1 cm = 0.9 pF)
- The table shows that when the liquid level changes on 1 micron (0.0001 cm), the relative change of capacity $\Delta C/C$ is about 10^{-5} . This is a large and easily measurable value. When using a capacitor frequency transducer with a frequency of 10 MHz, the frequency change at $x = 1 \mu$ is about 100 Hz. The resolution of this transducer was 0.01 μ for a measurement time of 1 s. Even if the base unit will be only 10 meters, this resolution will match the resolution of the angle of inclination of 10^{-9} rad или 0,2 msec arc. If the base is increased to 3 km, the resolution of the inclination will be $3 \cdot 10^{-12}$ or 0,6 ns arc!

R(cm)	r(cm)	L(cm)	x(μ)	C(cm)	dC/C
1	0,5	3	1	6,5	$1,1 \cdot 10^{-5}$
1	0,5	5	1	10,82	$6,7 \cdot 10^{-6}$
1,5	1	3	1	11,1	$1,1 \cdot 10^{-5}$
1,5	1	5	1	18,5	$6,7 \cdot 10^{-6}$
3	2	3	1	12,33	$1,1 \cdot 10^{-5}$
2	1	10	1	21,6	$3,3 \cdot 10^{-6}$
1	0,5	10	1	21,6	$3,3 \cdot 10^{-6}$

- To reduce the influence of temperature on the dielectric permeability, in the device 2 capacities will be used: one measuring, and the second - basic. In basic capacity the dielectric liquid is not contact with a liquid in a vessel.
- Fig. 3 shows the sketch of a design of a measuring vessel with etalon, basic capacity, which is used to compensate influence of changes of dielectric permeability of a liquid.
- As the liquid can be used Polymethylsilicone liquid, transformer or capacitor oil.

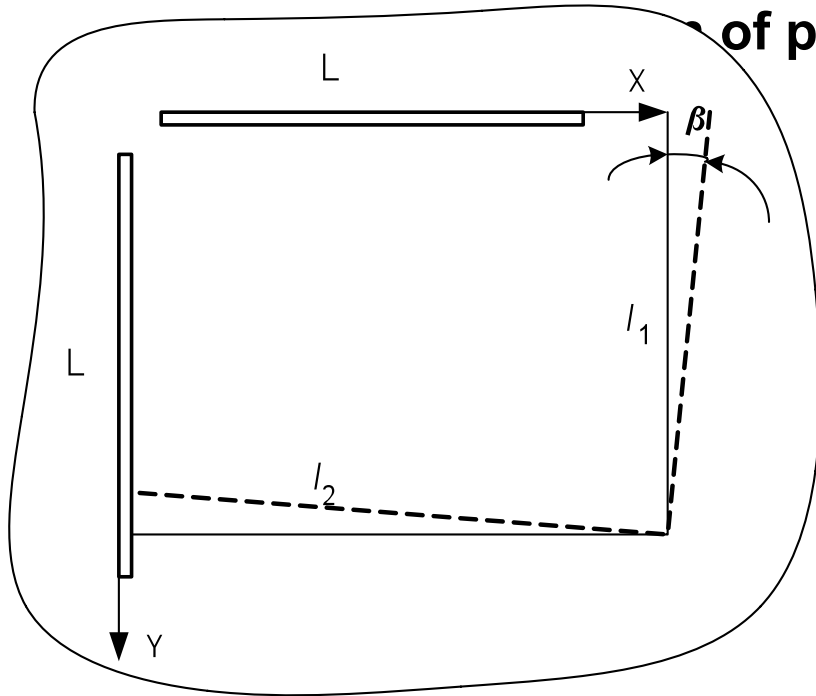
- Fig. 3. Sketch design of the measuring vessel of the Long-base Hydrostatic Incline Meter



possibilities of geophysical measurements

- **1. Measurement of irregular rotation of the Earth within days.**
- If to assume that relative change of angular speed of the Earth $\Delta\omega/\omega = 10^{-6}$
- and passes during $\Delta T = 10^4$ c, that tangential acceleration
- $a_T = R_E \Delta\omega / \Delta T = 4 \cdot 10^{-6} \text{ cm/c}^2$.
- Thus level of liquid will change on size $z = a_T / 2g \approx 6 \cdot 10^{-4} \text{ cm} = 6 \mu$.

2. Use of two hydrostatic levels for measurement of rotational motion of plates of the lithosphere



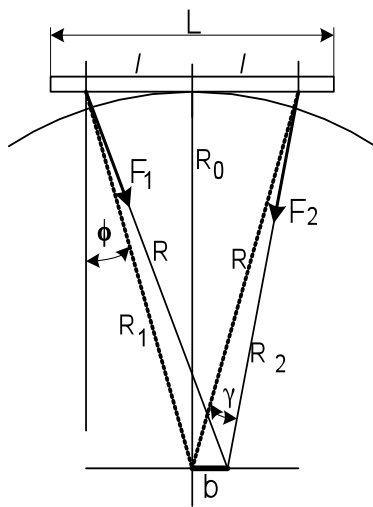
$$x = \beta_0 e^{i\Omega t} l_1; \ddot{x} = \beta_0 \Omega^2 e^{i\Omega t} l_1; z_x = \frac{L \ddot{x}}{2g} = \frac{L \beta_0 \Omega^2 l_1}{2g}$$

$$y = \beta_0 e^{i\Omega t} l_2; \ddot{y} = \beta_0 \Omega^2 e^{i\Omega t} l_2; z_y = \frac{L \ddot{y}}{2g} = \frac{L \beta_0 \Omega^2 l_2}{2g}$$

If $L = 3 \text{ km}$, $\Omega = 2\pi \cdot 10^{-5} \text{ rad/s}$,

$l_1 = l_2 = 10^3 \text{ km}$, $z = 10^{-6} \text{ m} = 1 \text{ } \mu$,
 $\beta_0 = 1,5 \cdot 10^{-7} \text{ rad} = 3 \cdot 10^{-2} \text{ angle sec}$

The response to the movement of Earth's inner- core



$$R_1^2 = R^2 + b^2 + 2bR \cos\left(\frac{\pi}{2} - \varphi\right) = R_0^2 \left(1 + \left(\frac{b}{R_0}\right)^2 + \frac{2b}{R_0} \varphi\right)$$

$$R_2^2 = R^2 + b^2 - 2bR \cos\left(\frac{\pi}{2} - \varphi\right) = R_0^2 \left(1 + \left(\frac{b}{R_0}\right)^2 - \frac{2b}{R_0} \varphi\right)$$

$$dF_1 = \frac{GM_E \rho s dl}{R_1^2}; dF_{1x} = dF_1 \sin(\varphi + \gamma)$$

$$dF_2 = \frac{GM_E \rho s dl}{R_2^2}; dF_{2x} = dF_2 \sin(\varphi - \gamma)$$

$$R = \frac{R_0}{\tilde{n} \cos \gamma}$$

- b – displacement center of gravity of the Earth, if displacement of inner-core – D
- G – gravity constant, M_E – mass of the Earth, R_0 – radius of the Earth

$$dF_1 - dF_2 = \rho g s \frac{2b}{R_0} dl \quad F = \rho g s \frac{b}{R_0} L$$

$$b = D \frac{V_{ic} (\rho_{ic} - \rho_l)}{M_E} \quad V_{ic} - \text{volume of inner core.}$$

- From equation $F = 2z \rho g s$, $z \approx bL/2R_0$. If $\Delta \rho = 0,5 \text{ g/cm}^3$, $D = 1 \text{ m}$,

$$z \approx 2,5 \cdot 10^{-5} \text{ cm} = 0,25 \mu$$

CONCLUSION

- The use of Long-base Hydrostatic Incline Meter allows:
- 1. To solve the problem of measurement of vertical displacements of the grounds mirrors of the gravitational detector with an accuracy of 1 micron.
- 2. To get an unique instrument for the geophysical measuring. With his help it is possible:
 - observe the irregularity of the Earth's rotation with a resolution. $\Delta\omega/\omega \approx 10^{-6}$;
 - measure the horizontal oscillations of the blocks of the lithosphere with the resolution ~ 0.01 arc. sec. for a day;
 - The study of tidal effects in solid Earth
 - realistic approach to the observation of the translational oscillations of the inner core of the Earth.

**Thank you for your
attention!**