

Measurement of Geophysical Effects with large scale gravitational interferometers

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Part II :

**Technical aspects geodynamical measurements
with Virgo**

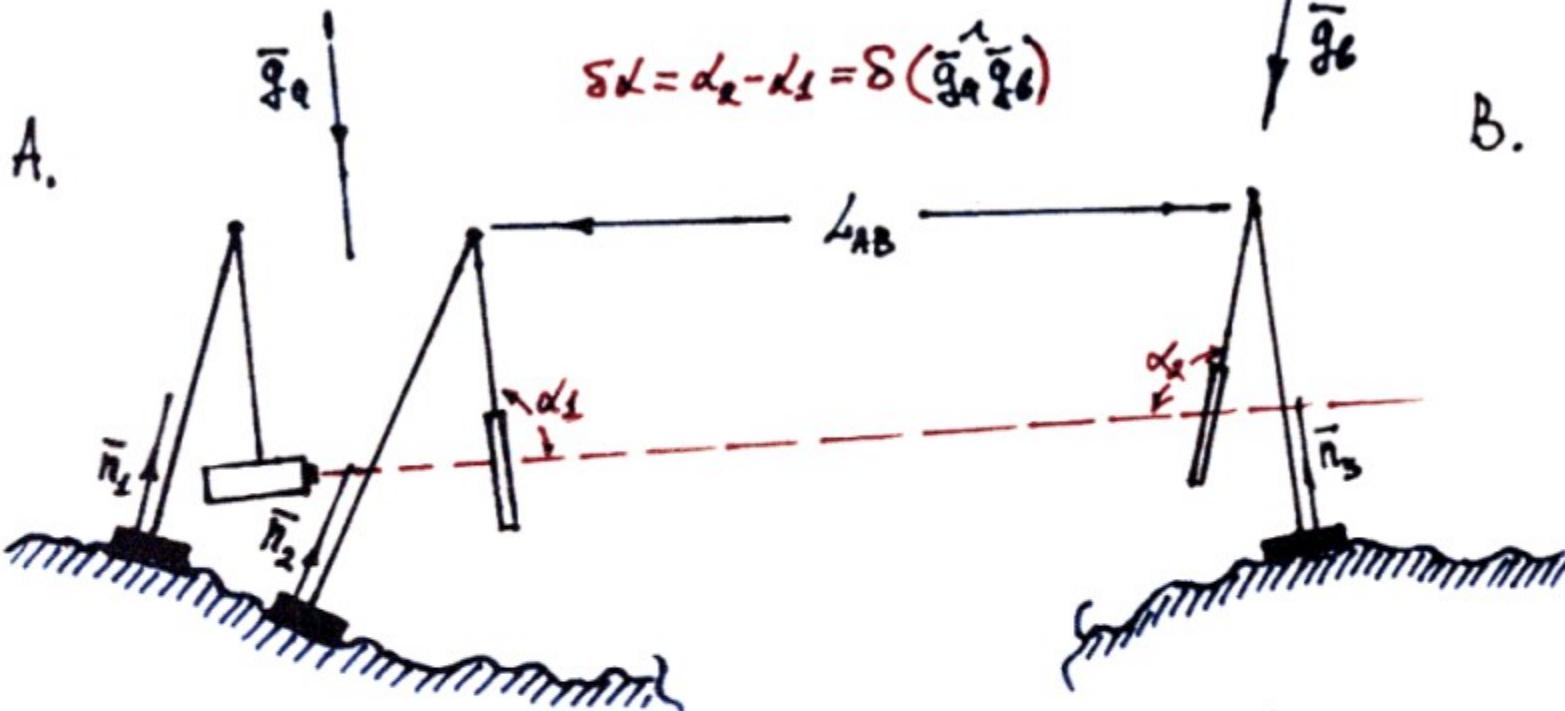
GGD Workshop 17-18 April 2012, IPGP, Paris (France)

Virgo accuracy limits

- strain $\delta L/L < 10^{-16}$
 - tilts $\Delta\alpha < 10^{-13}$ rad

inner core oscillation effect

- Polar mode ($a = 1M$) $5 \cdot 10^{-14}$ rad (theory)
- Translation mode $5 \cdot 10^{-15}$ rad (measurement)



Sensing of pure
gravity field
variations

Idea of the angular gravity gradiometer



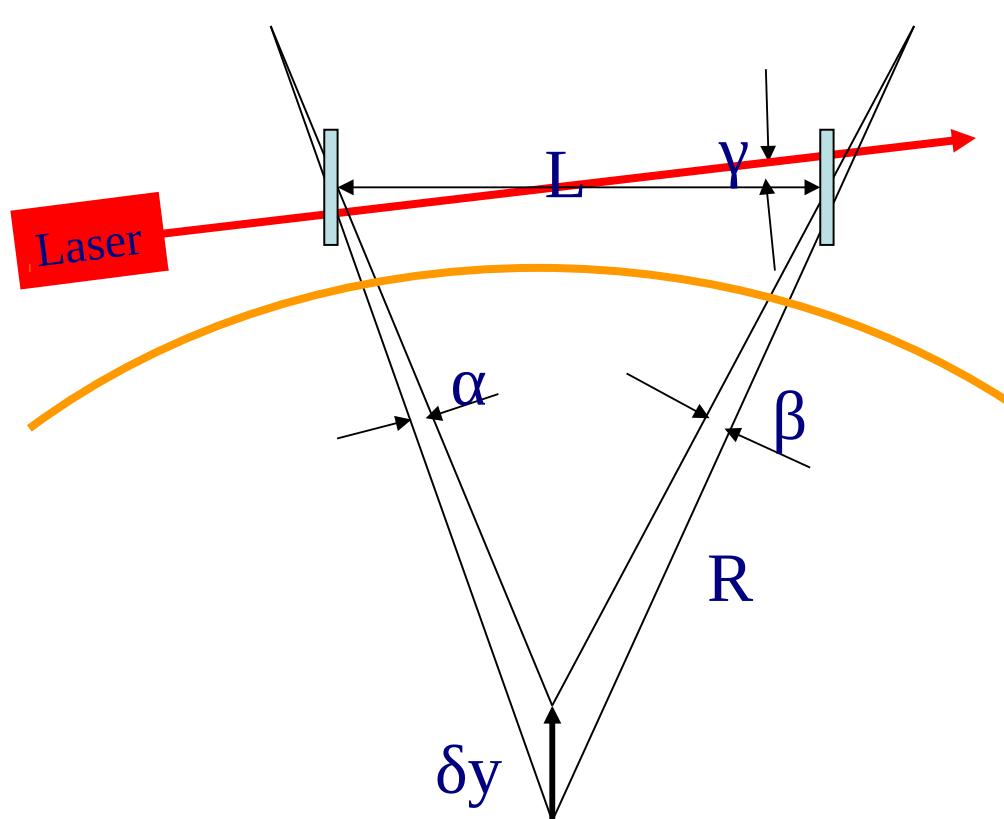
conventional
tiltmeter
is
a type of
defromograph

$$\alpha \rightarrow (\hat{n} \hat{g}) \approx \alpha(t)$$

$$\bar{g} \approx \text{Const}; \rightarrow \bar{n} = \bar{n}(t)$$

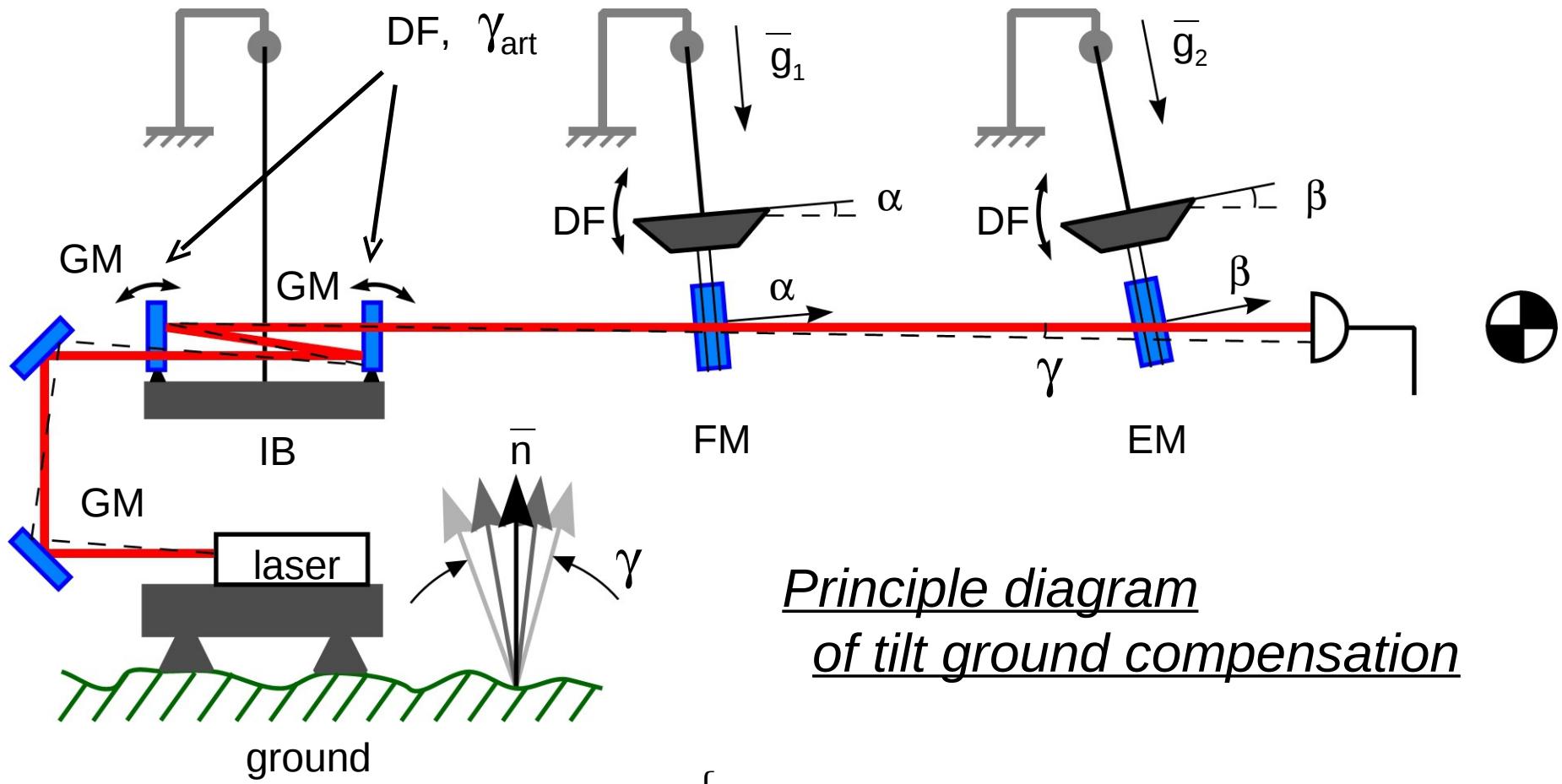
Can we measure Earth Core Movements with Virgo?

Virgo Mirrors are suspended with a single wire suspension, hence angle $\alpha + \beta$ is independent from ground tilt and from Laser beam angle γ .



$$\left\{ \begin{array}{l} \Delta y = \frac{2(\alpha + \beta)R^2}{L} \\ \frac{\partial \Delta y}{\partial \gamma} = 0 \end{array} \right.$$

If $\alpha, \beta > 10^{-11}\text{rad}$, $\Delta y > 1\text{m}$



Principle diagram
of tilt ground compensation

GM – guide mirrors

FM – front mirror, EM – end mirror

DF – driver force

IB – injection bench

γ – tilt deformation

$$\left\{ \begin{array}{l} \delta\alpha \approx \delta\gamma + \delta\gamma_{art} + \theta_1(\bar{g}_1); \quad \delta\beta \approx \delta\gamma + \delta\gamma_{art} + \theta_2(\bar{g}_2) \\ \delta\alpha + \delta\beta = 2(\delta\gamma + \delta\gamma_{art}) + \theta\left(\hat{\bar{g}_1 \bar{g}_2}\right) \approx 2(\delta\gamma + \delta\gamma_{art}) \\ \delta\alpha - \delta\beta = \theta\left(\hat{\bar{g}_1 \bar{g}_2}\right) \end{array} \right. \xrightarrow{\text{gravitational signal !}}$$

Conception of angular measurements

(Virgo as angular gravity meter)

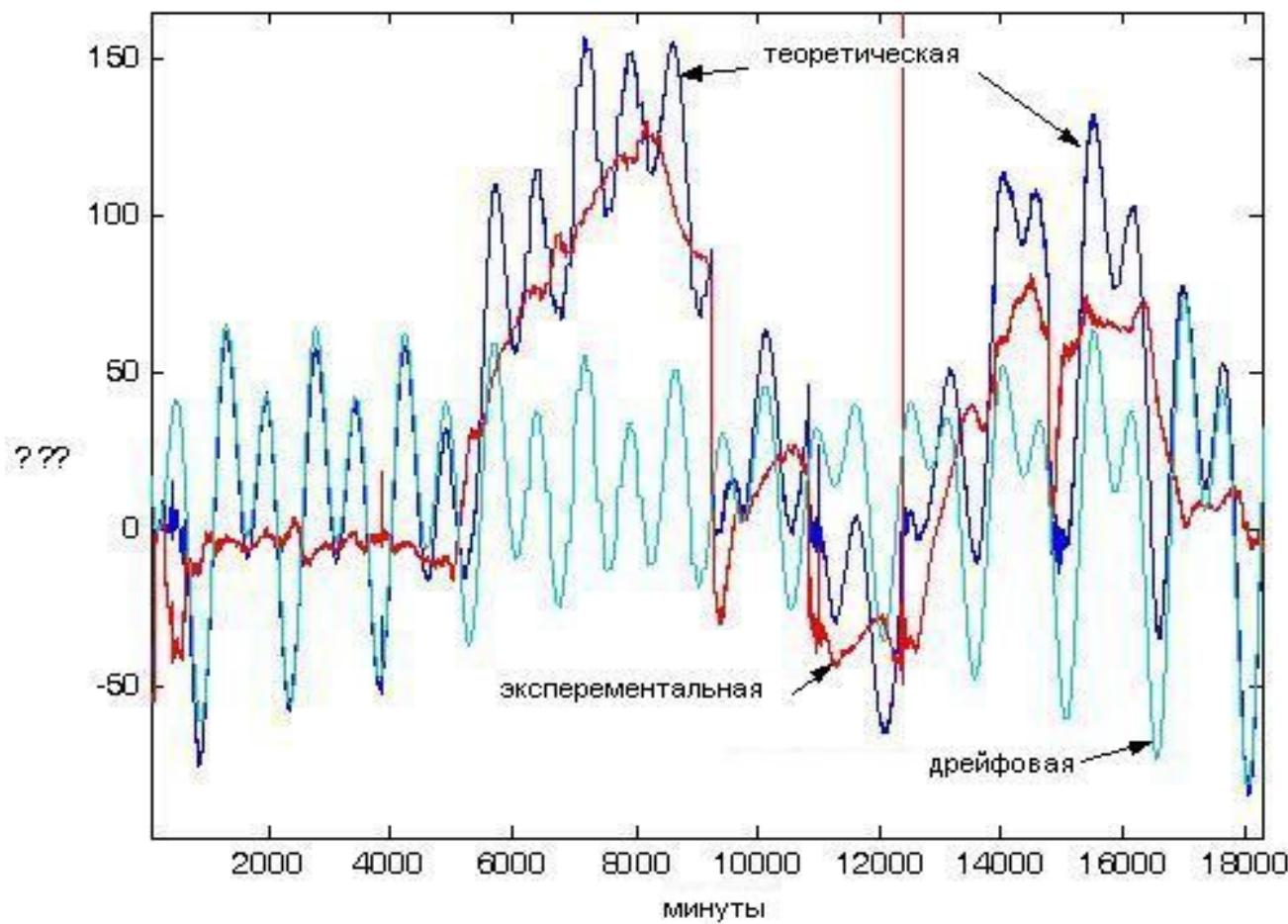
- Laser is not suspended:
so land angular deformations (tilts of the norm vector) are transferred to beam deflections ; also there are artificial beam inclinations to lock its position in the mirror's center ; - all together it composes an “*angular noise of the beam*” contaminating the gravity plumb line variations.
- A. Measurement of the “sum of internal angles”
composed by beam with *front* and *end* mirrors filters the “*angular noise of the beam*”, so again the plumb lines are affected only by gravity force variations
- B. Measurement of a single angle “beam ^plumb line”
presents the principle of conventional tilt meter: variations of the gravity force vector can be measured only at the beam deformational angular noise background; one has to find an additional ability to reduce this noise.

Virgo strain data

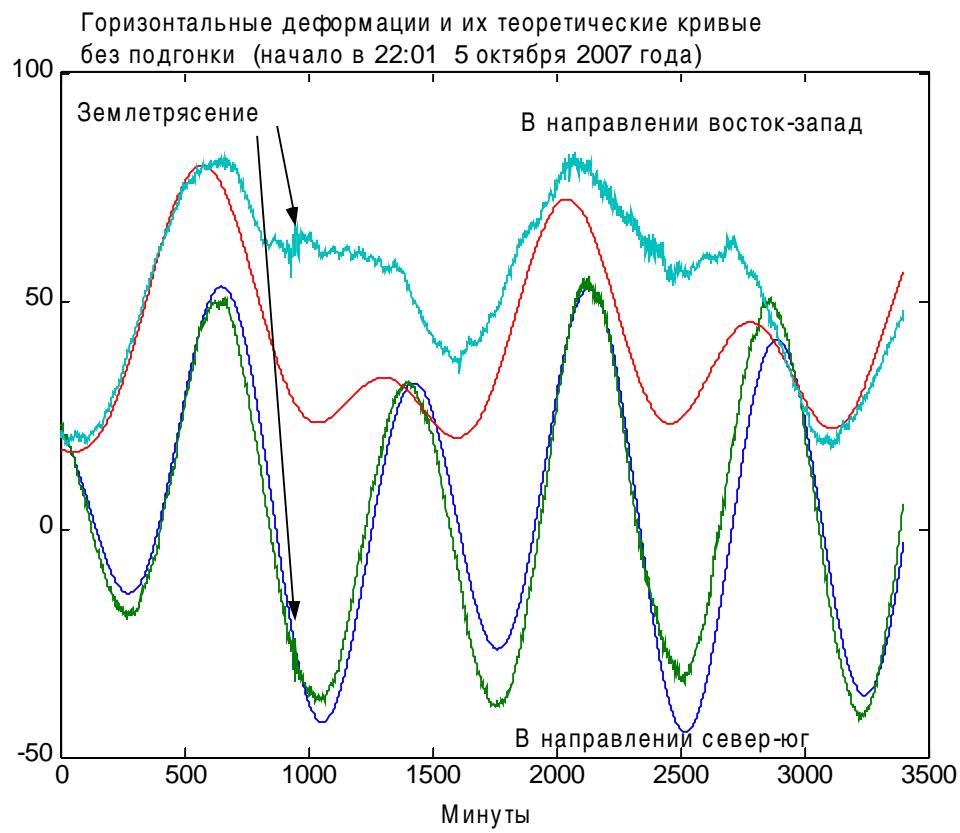
Raw strain data from “inverted pendulum” LVDT sensors were processed by LSM-fitting with theoretical tidal curve.

It results in estimations of

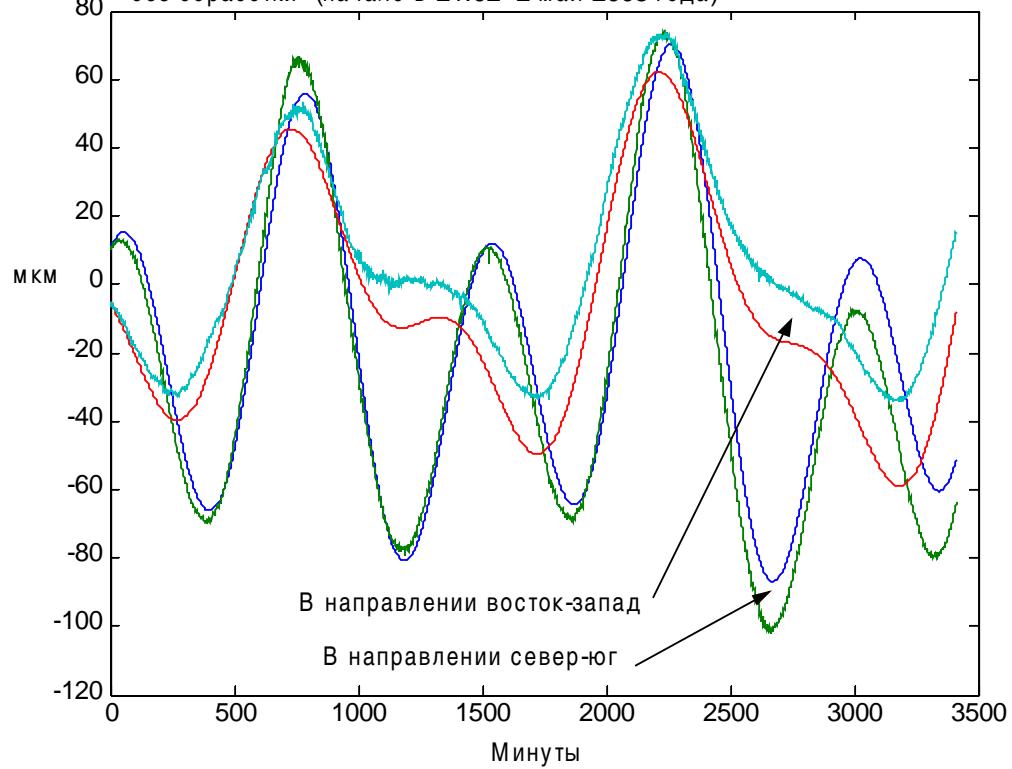
- f - virgo transform (scale) factor,
- τ - delay time (phase shift),
- Δl – deform. shift (“lock to lock”)

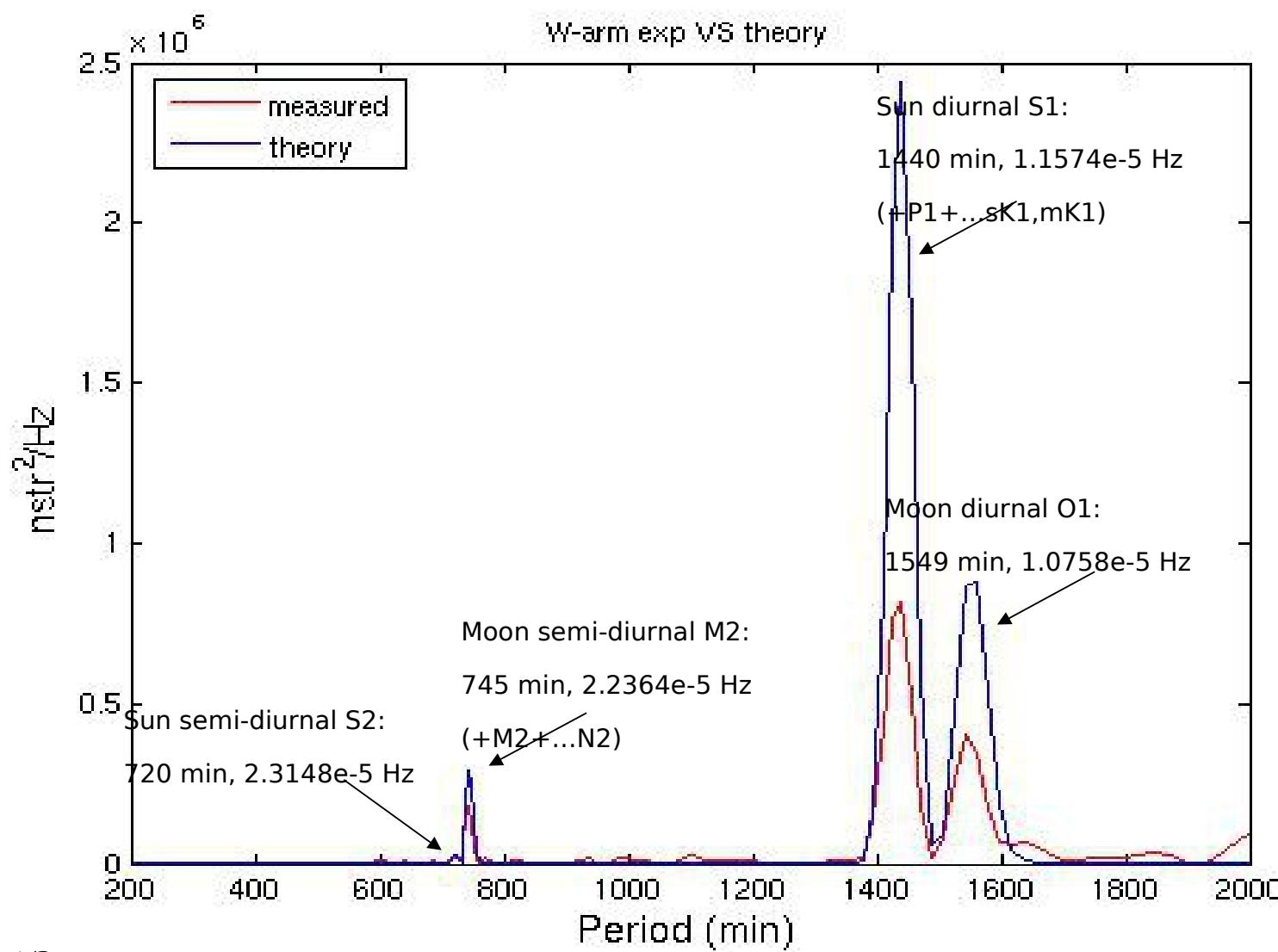


Theoretical curve , experimental curve (raw data) and
drift curve for North arm during 13 days VSR-2



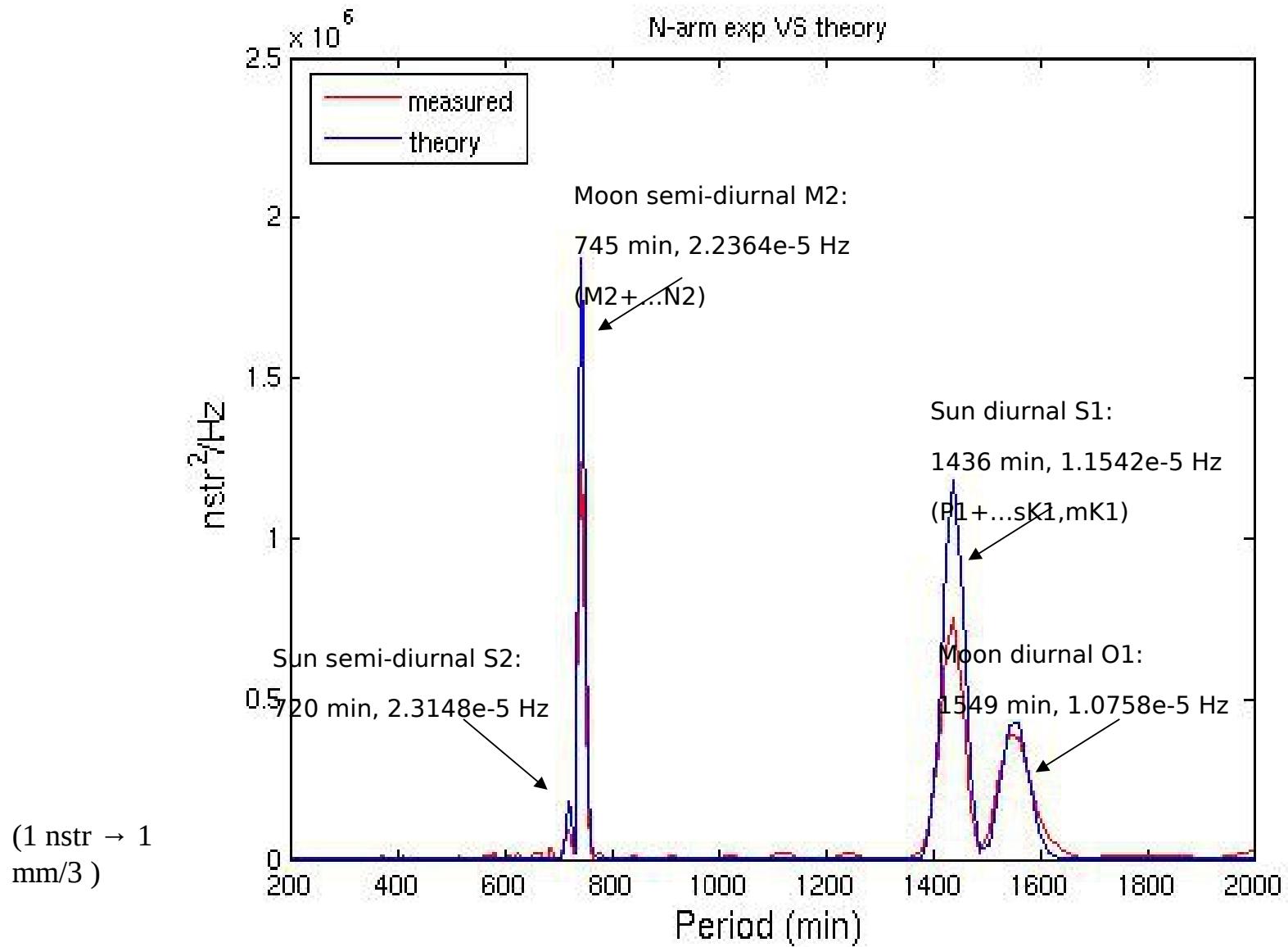
Горизонтальные деформации и их теоретические кривые
без обработки (начало в 21:02 2 мая 2008 года)





1instr → 1/3 mm

relative amplitude $d = (a_{\text{obs}}/a_{\text{the}})$; phase shift $\Delta\Phi \approx 2\Omega\tau$
experimental estimates $d = 1.15 \pm 0.03$, $\Delta\Phi = 2.4 \text{--} 4.0$ deg.

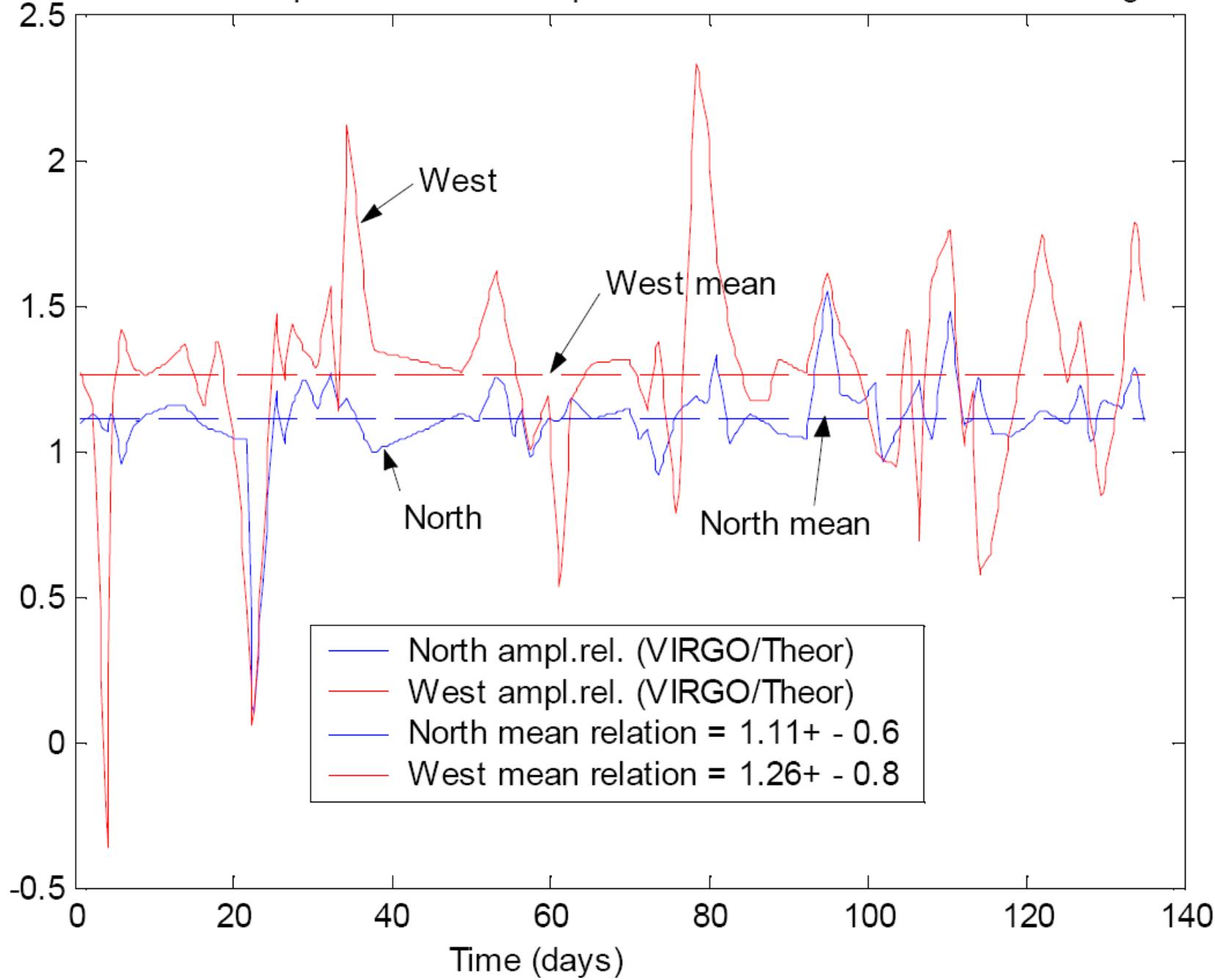


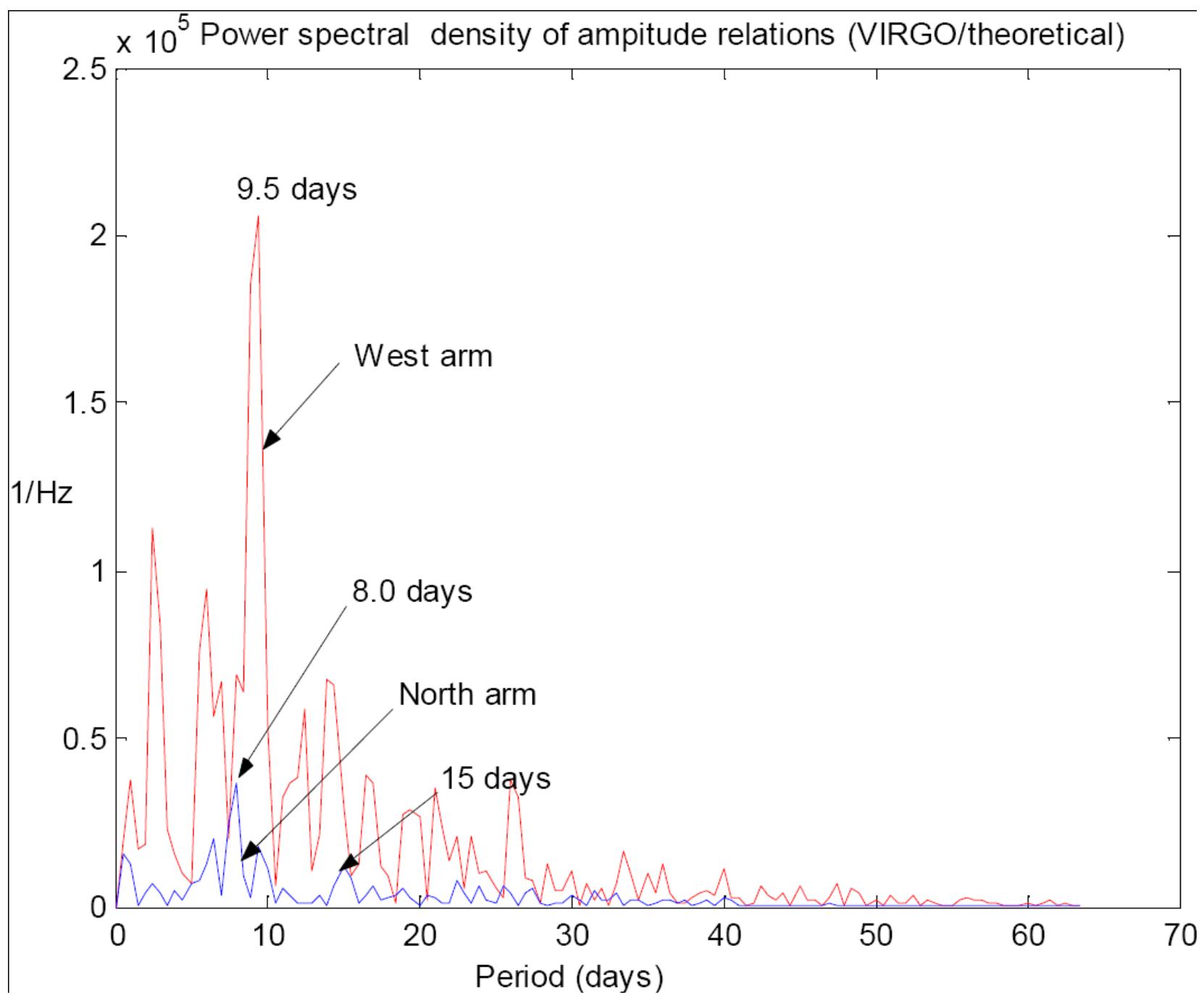
VSR-2

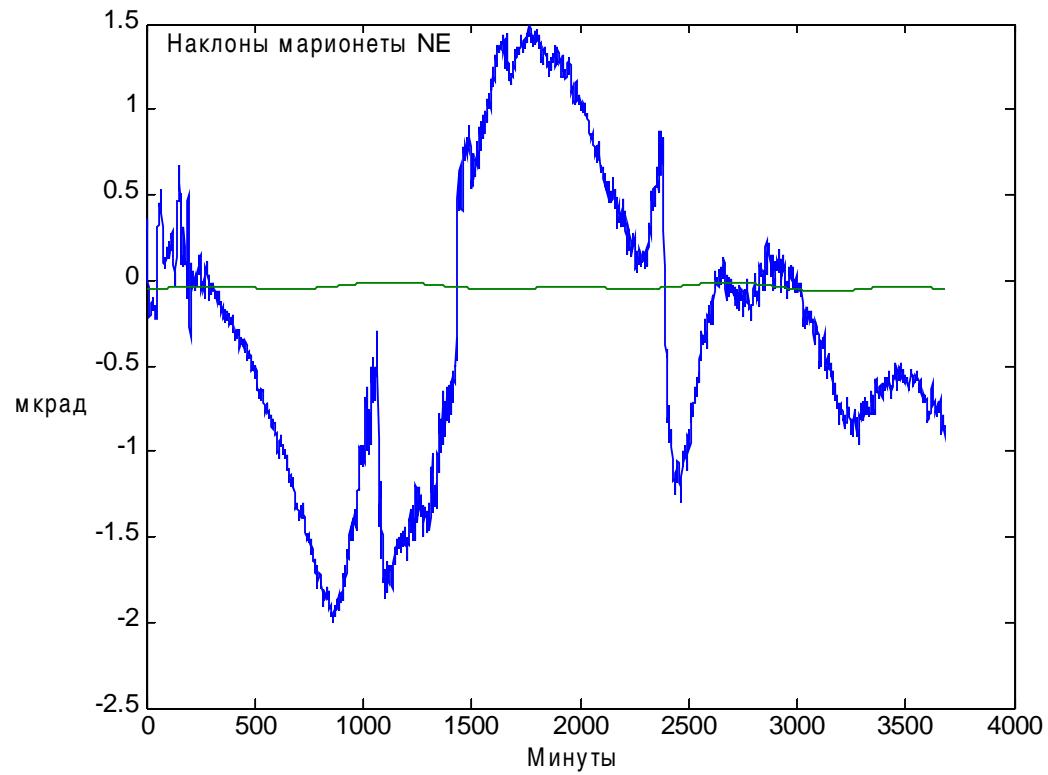
Волны			Теория	Теория: NE,162сут		Вирго:NE,162сут		Теория: WE162сут		Вирго:WE162сут	
Группы		Имя	Период,	Период,	Амплит.	Период,	Амплит.	Период,	Амплит.	Период,	Амплит.
От	До		Час	Час	МКМ	час	Мкм	час	МКМ	час	МКМ
1	2	3	4	5	6	7	8	9	10	11	12
Полусуточные											
1009	1021	K2	11,96	11,995	16	11,993	13	11,995	6	11,998	5
988	1008	S2	12,00								
948	987	L2	12,19								
891	947	M2	12,42	12,420	33	12,421	30	12,420	13	12,422	11
840	890	N2	12,65								
Суточные											
559	592	K1	23,93	23,912	20	23,907	15	23,912	30	23,902	14
555	558	S1	24,00								
538	554	P1	24,06					24,115	7	24,095	9
429	488	O1	25,82	25,817	15	25,831	14	25,817	22	25,794	17
286	428	Q1	26,87								

Main tidal harmonics estimates: periods and amplitudes

Variation of amplitude relations experimental and theoretical curve during 135 days







Tilt of North arm end mirror in compare
with theoretical curve

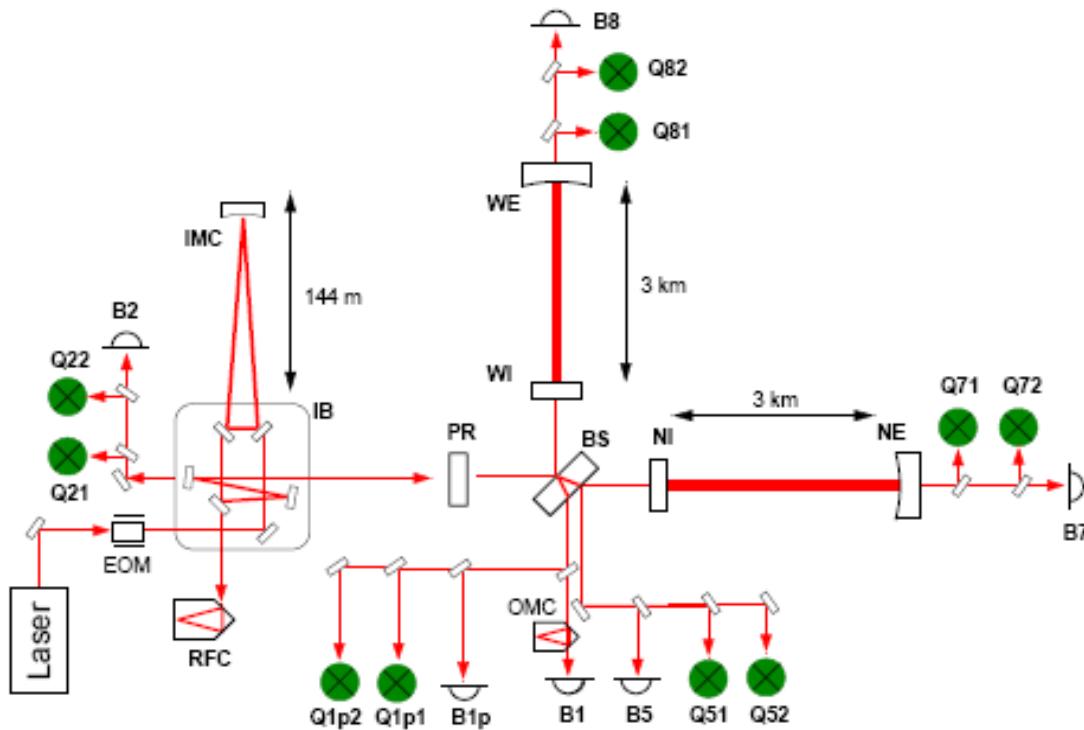
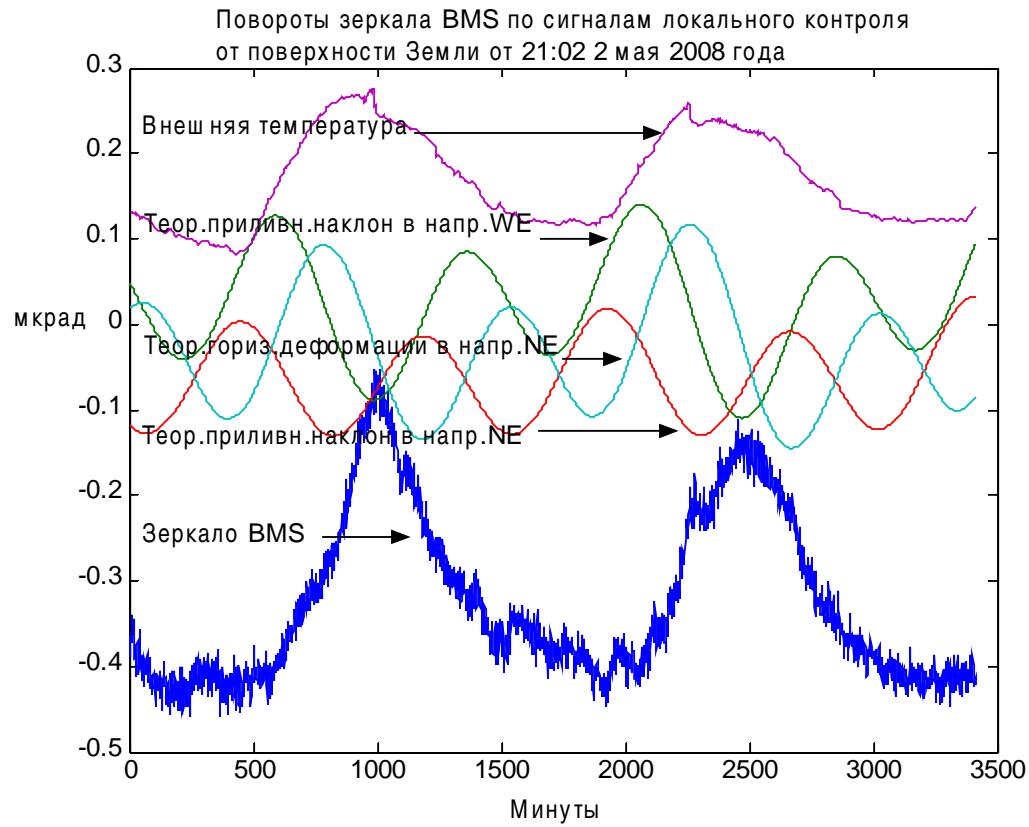


Figure 1: Simplified scheme of the optical design of Virgo. The laser beam is passing through a 144 m long *input mode-cleaner* cavity (IMC), then it is split at the BS mirror (BS) in two orthogonal Fabry-Perot cavities (the North and the West cavity). The longitudinal position of the mirrors is controlled in order to have a destructive interference at the asymmetric port (B1p). In this condition most of the light is reflected to the bright port, thus to enhance the power circulating in the interferometer a power recycling mirror is inserted (PR). The gravitational wave signal is detected at the asymmetric port (B1) after having been filtered by an output mode cleaner (OMC). The photo-diodes used for longitudinal (with names starting with B) and alignment (starting with Q) control system are shown.



Angular rotation of BMS mirror (ITF input)
along the vertical (normal to beam) axis

Angular effect of the inner core oscillations (amplitude 1 m)

- Plumb line deflection:

$$10^{-10} \text{ rad.}, \tau = 3.5 \text{ h} \sim 10^4 \text{ sec.} \rightarrow 10^{-8} \text{ Hz}^{-1/2}$$

A. “single angle measurement”:

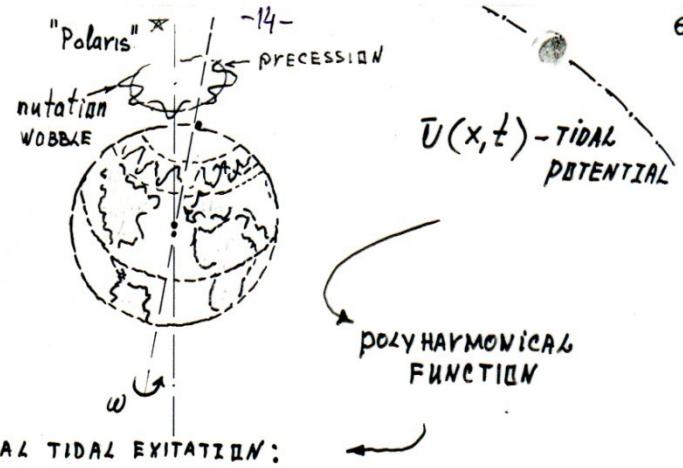
Virgo mirror’s angle noise: 10^{-7} - 10^{-8} rad. $\text{Hz}^{-1/2}$

- Mutual angle of two plumb line deflection:
 10^{-13} rad

B. “sum of internal angles measurement”:

Virgo angle noise depends on the level of *beam angular noise depression*;
the required level 10^{-11} rad. $\text{Hz}^{-1/2}$

Method of extracting the right “sum angle variable” has to be addressed (!)



PERIODICAL TIDAL EXCITATION:

$$T \sim \sum_i k_i A_i \sin(\omega_i t + \phi_i + \alpha_i)$$

amplitude ← factors → phase

SUBJECT OF INTEREST

GRAVIMETRY



TILTMETRY

$k_i \rightarrow \delta$ -factor

α_i

$k_i \rightarrow \gamma$ -factor

$$\frac{\Delta g}{g} \sim \delta = 1 + h - \frac{3}{2} k ; \quad \frac{\Delta \alpha}{\alpha} \sim \gamma = 1 + k - l$$

h, k, l - Love's numbers

modern state of art: $\Delta \delta_{\min} \sim 0.1\%$; $\Delta \gamma \sim 1\%$ accuracy of measurements

• TIDES $\Delta R \simeq [35 \div 40] \text{ cm}$; $\Delta g \simeq [100 \div 200] \mu\text{kg}$; $\Delta \alpha \simeq [0.01 \div 0.03] \text{ act. sec.}$
 $\sim 45^\circ \text{ N. lat}$

• Sensitivity: $[2 \div 5] \cdot 10^{-12} \text{ g}$, $\tau_m \sim 10^5 \text{ sec.}$; $[1 \div 5] \cdot 10^{-4} \text{ act. sec.}$, $\zeta \sim 1\%$.
 \downarrow $0.1 \mu\text{kg}/\text{Hz}^{1/2}$ \downarrow $1 \cdot 10^{-4} \text{ act. sec.} \simeq 5 \cdot 10^{-10} \text{ rad}/\text{Hz}^{1/2}$

FUNDAMENTAL GRAVIMETRY

Objects for research → "Earth spectroscopy"

7

1. eigen modes of the Earth

- crust modes $\tau_0 \leq 54 \text{ min}$
- core modes $\tau_0 \gtrsim 100 \text{ min}$ ↗ gravimeters could be uneffective

2 induced oscillations of the Earth

• tidal crust deformations

- liquid core resonance $\tau_0 \sim 24 \text{ h}$

it had been registered with accuracy: $\Delta\chi \sim 3\%$, $\delta \sim 0.2\%$

by gravimeters: [W. Zurn, P. Rydelek. et.al., p141]

tiltmeters: [I. Venedictov, P. Melchior et.al., p149]

Proc. 10th Symp. of Earth Tidals. 1986.

To study in detail an increasing of sensitivity

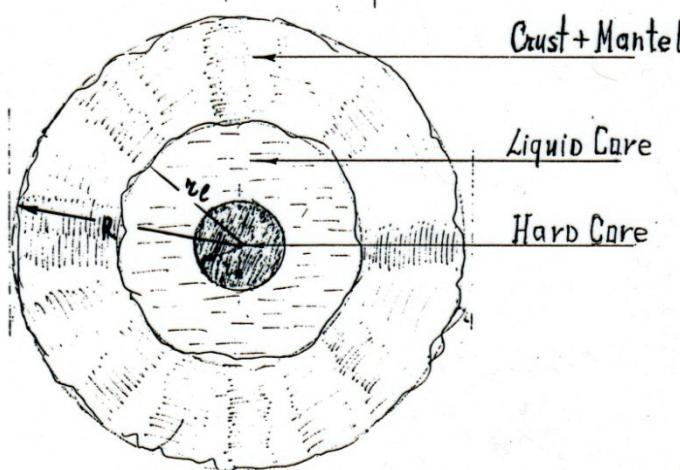
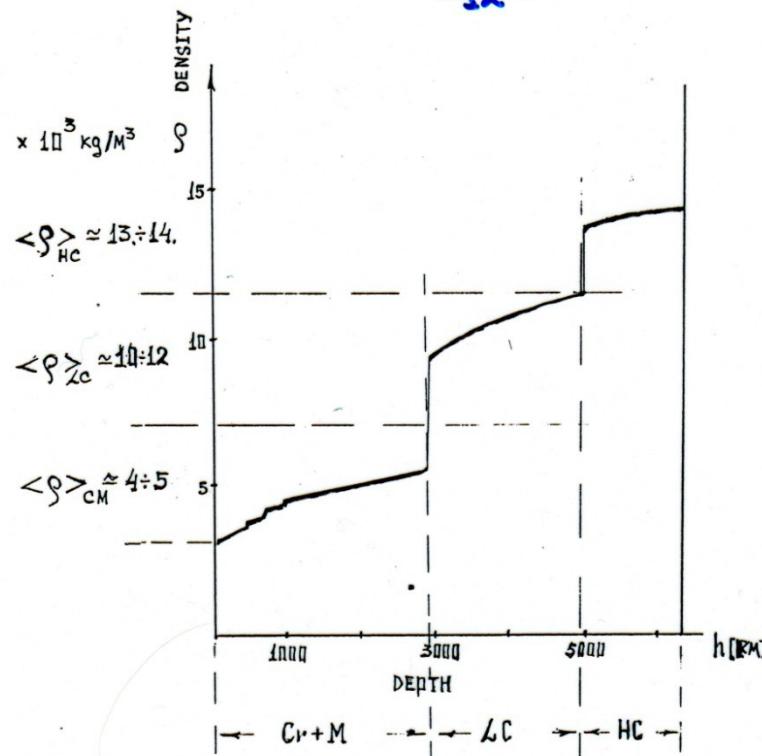
→ up to $\Delta\chi \simeq 0.01\%$ ($\chi_i \simeq 1\%$)
is desirable

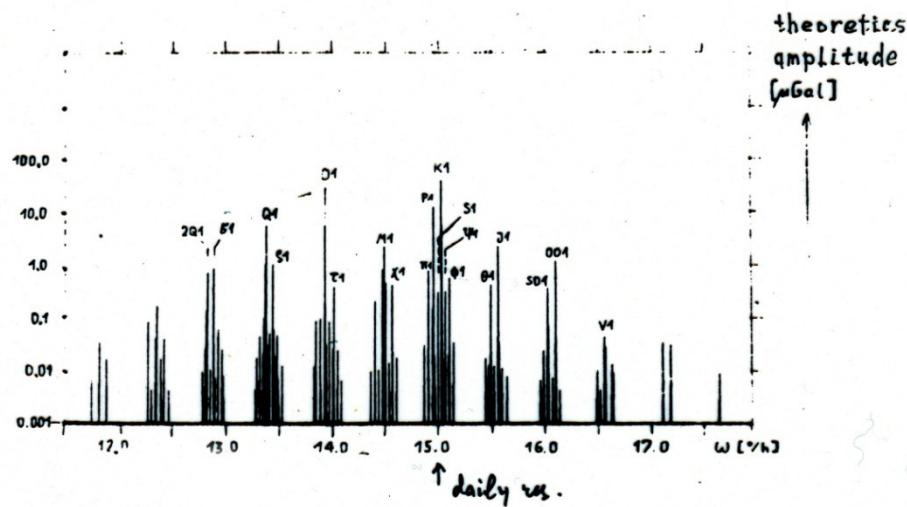
• special resonances

a) $\tau_0 = 3 \text{ h } 18 \text{ m}$ perturb. barycenter of Earth-Moon Sy.
- never h.b. detected (Y. Avsyuk 1976).

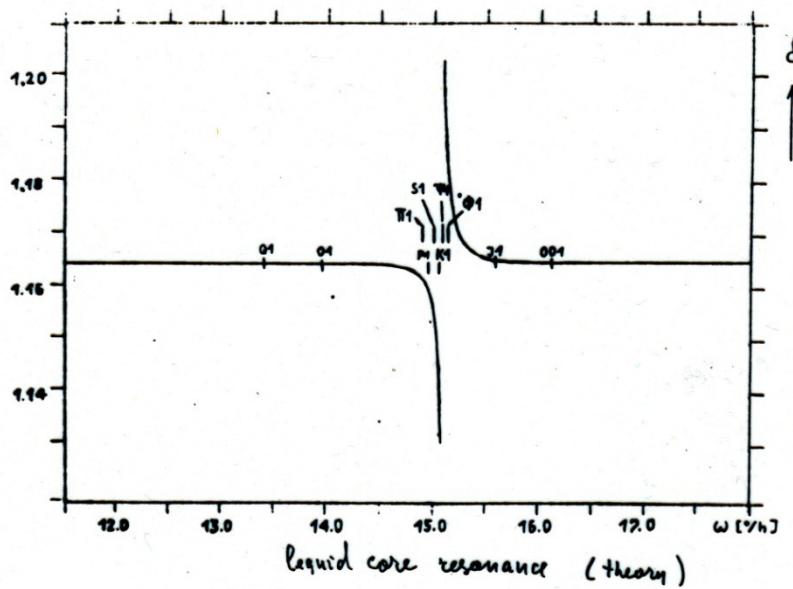
b) $\tau_0 = 14 \text{ h}$ hard core detection? (Melchior et.al. 1985).

TRIPLE-STRATUM EARTH MODEL

-12-



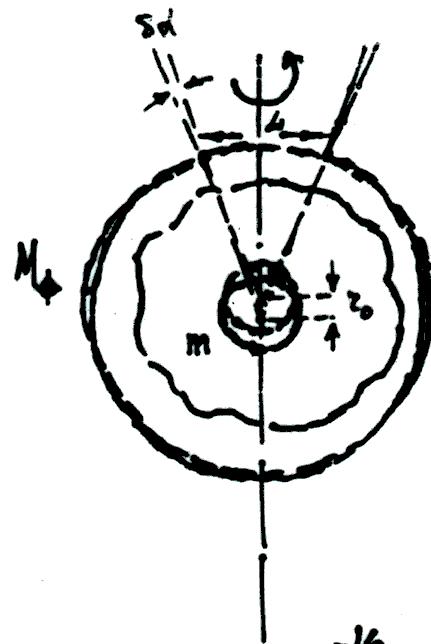
spectrum of the tidal (potential) force



liquid core resonance (theory)

Polar inner core oscillations

polar inner core
oscillation mode
estimate



$$S_d \approx \frac{m}{M_4} \cdot \frac{r_0}{R_4} \cdot \frac{\omega}{\omega_0}$$

\downarrow
 $7.5 \cdot 10^{-7}$
 \downarrow
 $2.5 \cdot 10^{-7}$
 \downarrow
 $5 \cdot 10^{-4}$

$$; S_d \approx 5 \cdot 10^{-14} \text{ rad.}$$

$$\omega_0 \approx 3 \cdot (G \Delta g)^{-1/2} \sim 10^4 \text{ sec}^{-1} \sim 3 \text{ h.} ;$$

$$\Delta g = g_i - g_0 \approx 2 \text{ g/cm}^3 ;$$

$$; \frac{S_d}{g} \sim 10^{-11} \sim 100 \text{ nGal}$$

translation mode triplet

$\tau_i \sim 4 \text{ h}$ (Smalley, Durney...)

Smylie D.E.

inner core translation modes forecast

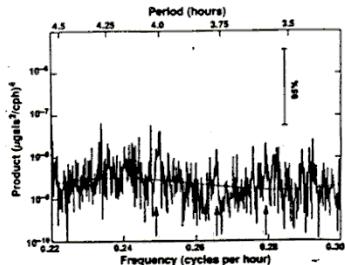


Fig. 1. Product spectrum of the four superconducting gravimeter records described in the text. A sinusoid is shown fitted across the whole spectrum to provide a reference noise level. The locations of the resonances identified by their rotational splitting as the triplet of inner core translational oscillations are shown by the arrows. Statistically, the spectrum represents the equivalent of 24.3 years of independent hourly samples. Vertical bar shows 95 percent confidence interval (CI).

Exploiting the expectation that all three oscillations should appear in the observations, I developed and applied stringent tests that claimed resonances must satisfy before they can be associated with the inner core translational triplet. These tests are based on the rotational splitting that the modes are known to have.

Rotational splitting arises from the effect of Coriolis acceleration, which scales according to the ratio of the period to half the length of the sidereal day. Thus, if the three periods are calculated for a given Earth model, their values will be offset for other Earth models, and for correctly identified observed periods, in nearly the same proportions as those calculated for the given Earth model.

Recent theoretical and numerical advances (9) allow the translational eigenperiods and displacement fields, as well as those for other long-period oscillations of the fluid outer core, to be accurately calculated. The forms of the displacement fields are found to vary little with Earth model (Fig. 2), but the translational eigenperiods are highly model-dependent.

For a widely accepted Earth model, CORE11 of Widmer *et al.* (18) (Fig. 2), the period of the retrograde equatorial mode (T_R) is 3.7195 hours, that of the axial mode (T_A) is 3.5056 hours, and that of the prograde equatorial mode (T_P) is 3.3432 hours. The period of the axial mode is offset by 0.02575 hours from T_M , the mean of T_R and T_P ($T_M = 3.53135$ hours), whereas T_R and T_P are offset from the mean by 0.18815 hours.

Below I fit resonances to the candidate spectral features indicated by arrows in Fig. 1 and obtain the observed values $T_R = 4.015 \pm 0.001$ hours, $T_C = 3.7677 \pm 0.0006$ hours, and $T_P = 3.5820 \pm 0.0008$ hours. The offset of T_C from the mean of T_R and T_P , $T_M = 3.7985$ hours, is 0.0308 hours. If the offsets follow the expected proportionality to those for Earth model CORE11, we would then expect to find the observed retrograde resonance at 4.0235 hours and the prograde resonance at 3.5735 hours. The relative errors in the forecasts of the locations of the observed equatorial mode resonances are 0.21 and 0.24 percent, respectively. Although more generally based, this test is numerically equivalent to the known quadratic dependence of the eigenfrequencies on azimuthal number given by second-order perturbation theory (3).

The small variation of the displacement field forms shown in Fig. 2, over Earth models with widely different eigenperiods, permits the construction of an even more stringent test of whether the observed resonances can be associated with the translational triplet. Under

~18~ Smylie D.E. Science 255, 1678, (1992)

general conditions (9, 10), the period T of a core oscillation obeys the equation

$$\left(\frac{T}{T_0}\right)^2 + 2 \frac{g}{T_S} \left(\frac{T}{T_0}\right) T_0 - 1 = 0 \quad (1)$$

where T_0 is the period neglecting rotation, T_S is the length of a sidereal day, and g is a dimensionless form factor representing a weighted average of the product of the meridional and azimuthal components of the displacement fields, compared to a weighted average of the squared magnitude of the full displacement vector. The form factor obeys $|g| \leq 1$ and for a particular mode is nearly invariant with the Earth model.

On the basis of the form factors of CORE11, three curves (branches of hyperbolae) can be constructed along which all possible translational periods must lie (Fig. 3). For example, the locations of the periods for an older model 1066A (19) are accurately predicted (Table 1). Forecast locations for the observed periods with this identification test are 4.0166, 3.7687, and 3.5813 hours. The

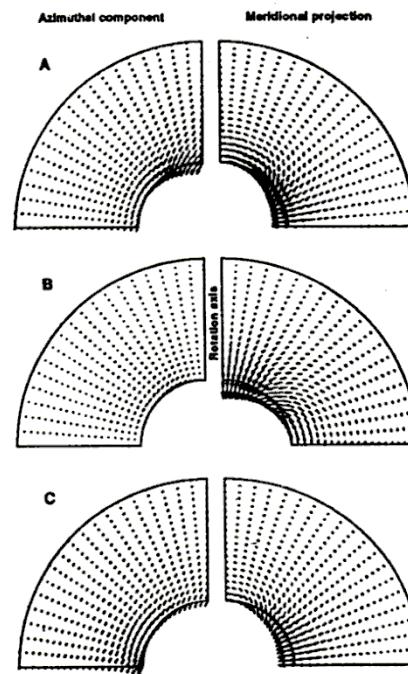


Fig. 2. Displacement vector fields in the fluid outer core of the three translational modes of the solid inner core. The azimuthal components are shown in perspective. (A) Retrograde equatorial mode. (B) Axial mode. (C) Prograde equatorial mode. In each case, the azimuthal component leads the meridional component by 90°.



Global superconducting gravimeter observations and the search for the translational modes of the inner core

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inner core
 translation modes
 detection
 with super
 conducting
 gravimeter net
 (!?)

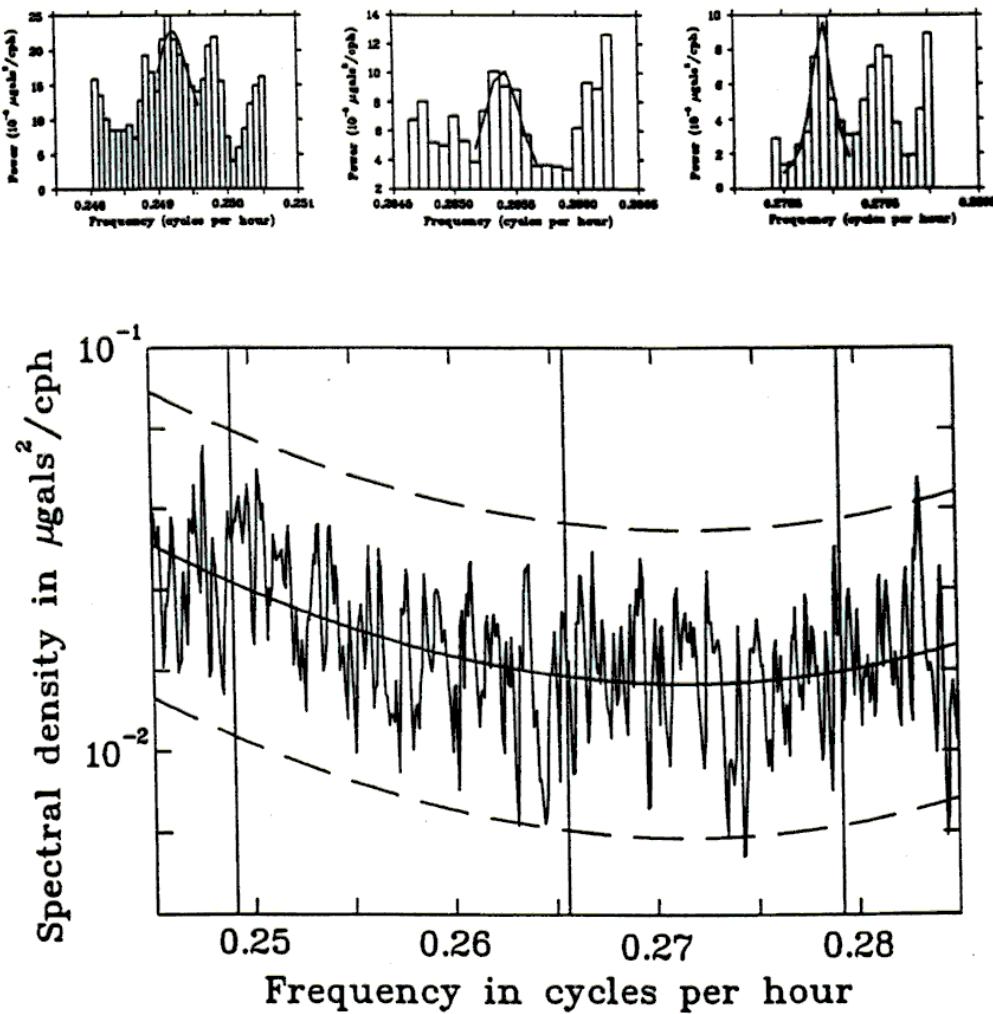
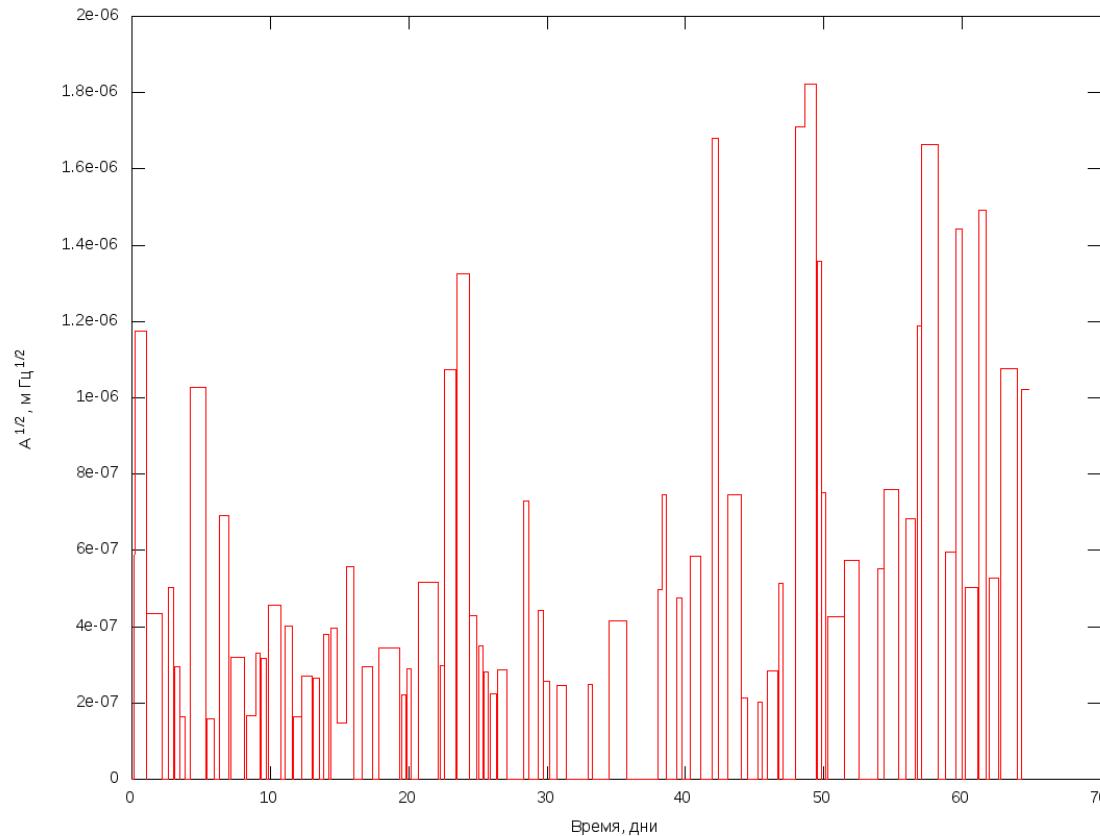


Fig. 15. Fitted resonances in the spectra near the translational resonances for the multistation experiment are shown in the top row. Recovered central periods are 3.5855 ± 0.00004 (retrograde), 3.7680 ± 0.00005 (axial) and 4.0125 ± 0.00011 (prograde) h. These differ from those found in records confined to Europe by 0.092, 0.064 and -0.062% , respectively. The lower plot displays the sum of all three spectra from the global experiment.

$$\Delta g_s \sim 0.2 \cdot \text{Agal}/\sqrt{\text{cph}} \rightarrow \frac{\Delta g_s}{g} \approx 3 \cdot 10^{-12} \text{ s}^{-1}/\sqrt{\text{Hz}}$$

time evolution of the sample of deformation
 noise standard $A^{1/2} = \sigma I$ in mHz $^{-1/2}$

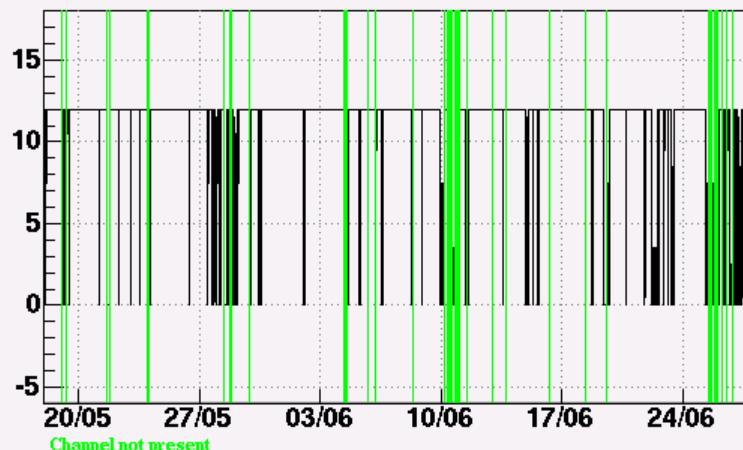


5 months series VSR-2 measured time evolution of variance of deformation noise; estimate for upper limit core oscillation amplitude $(\Delta g)_{\min} \geq g_0 (\Delta a)_{\min} / L \approx 2, 8 \mu\text{Gal}$. (in two orders worse the GSGr result)

Literature.

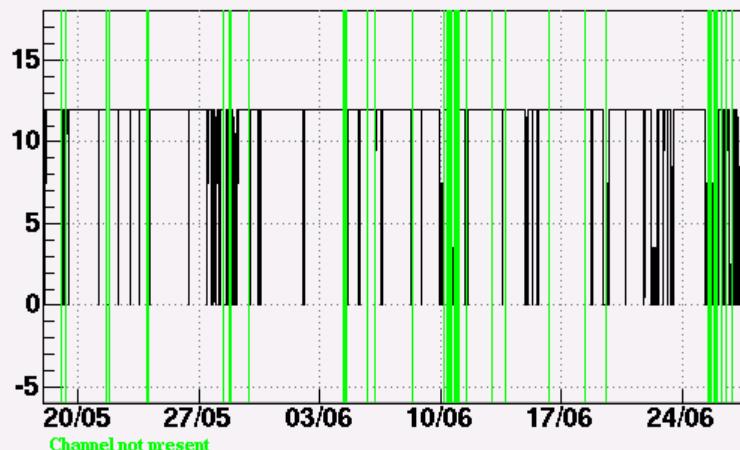
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- Grishchuk L.P., Kulagin V.V., Rudenko V.N., Serdobolskii A.V *Gravitational studies with laser beam detectors - of gravitational waves*", Class.Quantum Grav. v.22, (№2995), p 245-269, 2005.
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Alp_Main_LOCK_STEP_STATUS_TIME



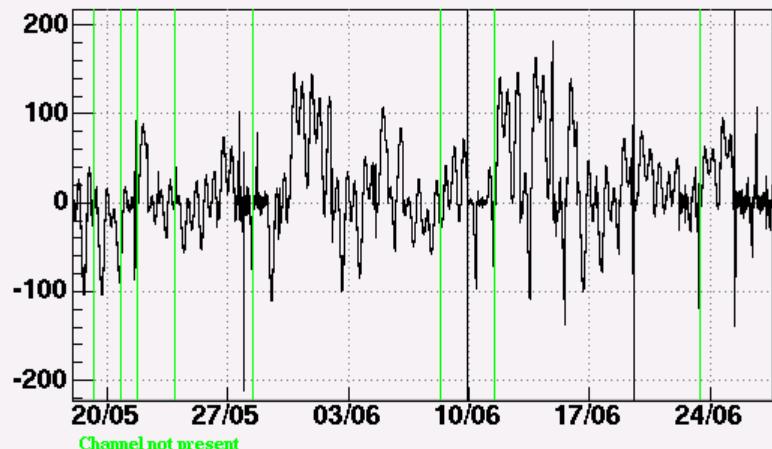
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V1:Alp_Main_LOCK_STEP_STATUS_TIME



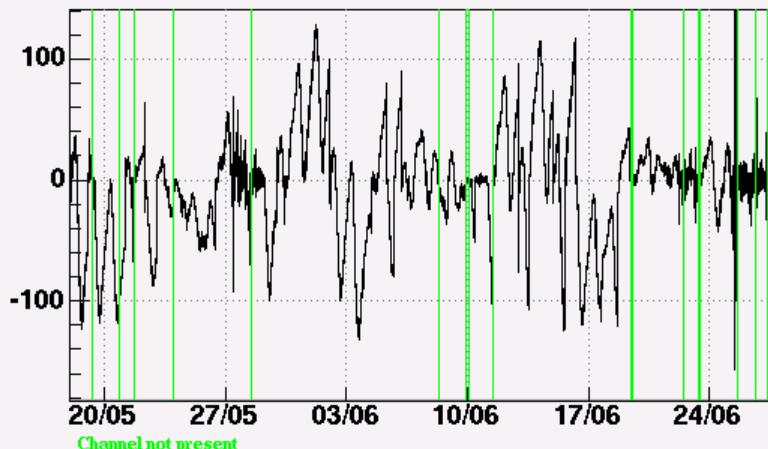
863558086 : May 18 2007 21:14:32 UTC

V1:Sa_NE_F0_zLvdt_500Hz_mean_TIME



863558086 : May 18 2007 21:14:32 UTC

V1:Sa_WE_F0_zLvdt_500Hz_mean_TIME



863558086 : May 18 2007 21:14:32 UTC

Items to be studied first

- Alignment technique details:
(nonlinear alignm., linear alignm, drivers, marionette etc.)
- Injection bench tilt compensation
- Vertical mirror's shift control circuits
- Tidal calibration:
 $\Delta\alpha_T \sim 3 \cdot 10^{-8} \text{ rad, } [10^{-5} \text{ rad} \cdot \text{Hz}^{-1/2}]$
the point is an accuracy of the tidal reconstruction
 $\delta\alpha = \varepsilon \Delta\alpha, \quad \varepsilon = 0.1; 0.01; \dots 0.003$ (core effect)

Current Problems

Tidal calibration of the Virgo instrument

- - new software for “mixed (strain + tilt) tidal perturbations”...?
- - reconstruction of the phase information after feed back “lock off” ?
- - filtering a “gravity signal” from deformations and beam jitters..?
- - filtering of vertical shifts..?

Sorry for attention!