

# Measurement of Geophysical Effects with large scale gravitational interferometers

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Part I :

Physical mechanisms for geodynamical  
signals in Virgo.

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# Introduction

# Spectroscopy of vibrations of the earth by means of gravitational-wave interferometers

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*Pis'ma Zh. Eksp. Teor. Fiz.* **59**, No. 9, 630–633 (10 May 1994)

The possibility of using large-baseline, free-mass gravitational-wave antennas as highly sensitive tiltmeters to study global geodynamic characteristics is discussed.

Funding has now started for the construction of two laser free-mass gravitational-wave antennas with baselines of several kilometers: the LIGO project<sup>1</sup> in the U.S. and the Franco-Italian project VIRGO.<sup>2</sup> These installations were designed to detect metric perturbations from  $h=10^{-21}$  to  $h=10^{-23}$  as the measurement apparatus is refined. The reception bandwidth is to be  $\simeq 10^3$  Hz; the range of frequencies which can be received is to be<sup>1-3</sup> from  $\simeq 10$  Hz to 2 kHz. Realizing these extreme characteristics in practice will require some nontrivial technical facilities,<sup>1,2</sup> so these projects are extremely expensive undertakings. They are justified by the fundamental nature of only one ultimate goal: to master a new gravitational-wave channel for astrophysical information.

In this letter we wish to call attention to the possibility in principle of another, independent application of these installations: for solving problems of interest in geophysics, in particular, for studying the global characteristics of the earth's core-mantle system. Below we point out some specific geodynamic effects, explain the idea of measuring them by means of gravitational-wave interferometers, and discuss some instrumental and background limitations on the sensitivity.

at quasi static frequencies VIRGO interferometer can be considered as  
a conventional *two coordinate strain meter*, *four points tilt meter*  
and

as a new type of *gravity field gradiometer*



# Gravitational free mass antenna as an angular gravity gradiometer

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## Abstract

In the low frequency region  $10^{-4}$ – $10^{-5}$  Hz a long base gravitational-wave interferometer is considered as a differential tiltmeter insensitive to perturbations of the local normal to the land. The effect of variations of the reciprocal deflection of two pendulum-mirrors is estimated for three conceivable scenarios of the earth core movements. The possibility of measurement of this effect is discussed according to the noise background estimate typical for modern gravitational antenna projects.

PACS: 04.80.N; 95.55.Y; 91.N; 93.85; 42.62.E

Keywords: Gravitational wave antenna; Long base laser interferometer; Gravity gradiometer; Differential tiltmeter; Global geodynamics

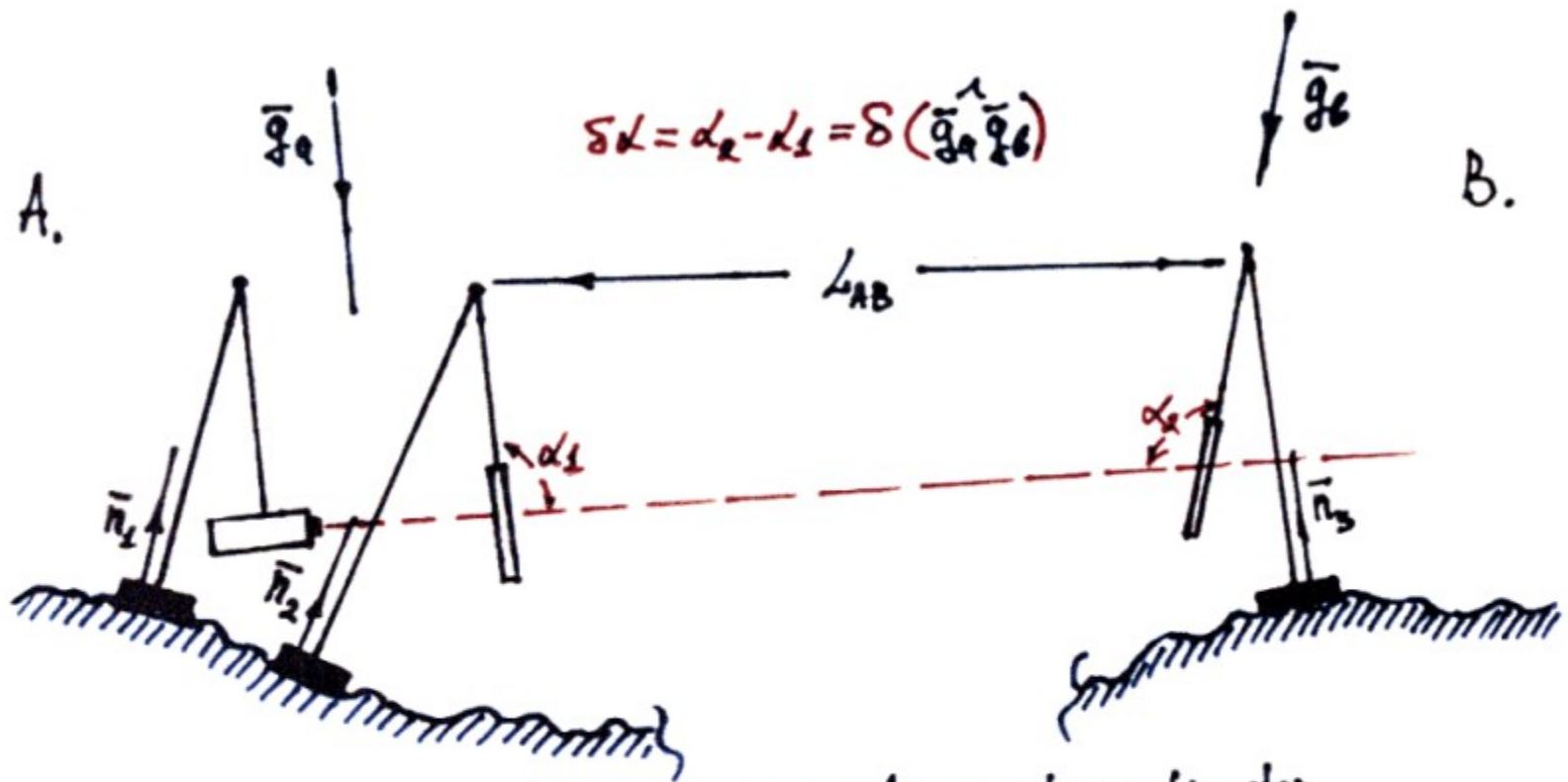
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## 1. Introduction

In Refs. [1,2] the idea was proposed to use a gravitational wave interferometer on free masses for observations and measurements of global geodynamical processes induced by internal energy sources and the external tidal potential as well. In this case the gravitational wave interferometer was considered as a differential tiltmeter in the very low frequency range

(see also Ref. [2], p. 86). The corresponding voltage compensation, the so-called “error signal”, could be a carrier of desired information on slow geodynamical variations of the gravity force vector. The idea looks very attractive in many aspects and first of all for the problems of global geodynamics and astrometry, because movements on the earth’s core influence the precession and nutation of the earth’s axis [5,6].

It was noted in Refs. [1,2] that the potential (in-



Idea of the angular gravity gradiometer

Sensing of pure  
gravity field  
variations



conventional  
tiltmeter  
is  
a type of  
deformograph

$$\alpha \rightarrow (\hat{\vec{n}} \hat{\vec{g}}) \approx \alpha(t)$$

$$\vec{g} \approx \text{Const}; \rightarrow \vec{n} = \vec{n}(t)$$

# Geophysical studies with laser-beam detectors of gravitational waves

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## Abstract

The existing high technology laser-beam detectors of gravitational waves may find very useful applications in an unexpected area—geophysics. To make possible the detection of weak gravitational waves in the region of high frequencies of astrophysical interest,  $\sim 30\text{--}10^3$  Hz, control systems of laser interferometers must permanently monitor, record and compensate much larger external interventions that take place in the region of low frequencies of geophysical interest,  $\sim 10^{-5}\text{--}3 \times 10^{-3}$  Hz. Such phenomena as tidal perturbations of land and gravity, normal mode oscillations of Earth, oscillations of the inner core of Earth, etc will inevitably affect the performance of the interferometers and, therefore, information about them will be stored in the data of control systems. We specifically identify the low-frequency information contained in distances between the interferometer mirrors (deformation of Earth) and angles between the mirrors' suspensions (deviations of local gravity vectors and plumb lines). We show that the access to angular information may require some modest amendments to the optical scheme of the interferometers, and we suggest the ways of doing that. The detailed evaluation of environmental and instrumental noises indicates that they will not prevent, even if only marginally, the detection of interesting geophysical phenomena. Gravitational-wave instruments seem to be capable of reaching, as a by-product of their continuous operation, very ambitious geophysical goals, such as observation of the Earth's inner core oscillations.

PACS numbers: 04.80.Nn, 07.10.Fq, 01.50.Pa



## GEOPHYSICAL OBSERVATIONS WITH VIRGO

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Estimates of  
Virgo intrinsic  
noises

Longbase gravitational free mass interferometer is considered for a registration of the low frequency Earth gravity gradients. Such measurements are possible because a variation of the Earth gravity force vector cause the corresponding misalignment of the interferometer mirrors. Principal instrumental noises which can limit the sensitivity of installation as the geophysical device are considered and the instrumental sensitivity is estimated.

The idea of measuring the low frequency Earth gravity gradients (below 0.01 Hz) with the help of longbase gravitational interferometers such as VIRGO installation was discussed in [1] and different geophysical noises with simple computer simulation were considered in [2]. In this paper we shall address the instrumental sensitivity of the VIRGO installation.

The optical noises in gravitational interferometer are suppressed by frequency and power stabilization systems. The estimations of paper [1] give the photon noise restriction for angular measurements at low frequencies at a level  $5 \cdot 10^{-14}$  rad/Hz<sup>1/2</sup> for moderate power 5 W, finesse 100, angular divergence of the beam  $10^{-5}$  rad/Hz<sup>1/2</sup>. If the pump will be increased up to the design power 300 W then the expected value will be about  $10^{-15}$  rad/Hz<sup>1/2</sup>.



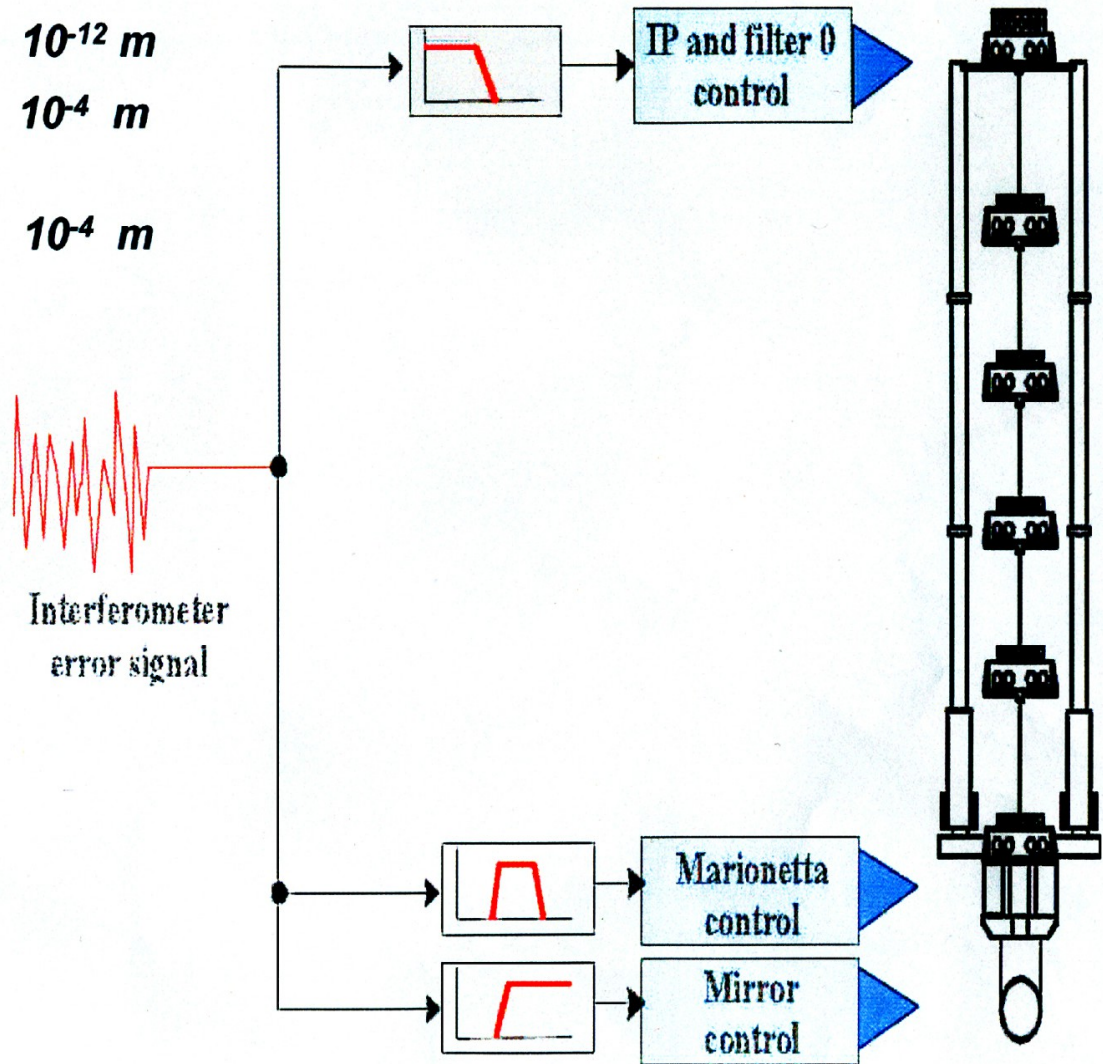
Virgo control circuit signals

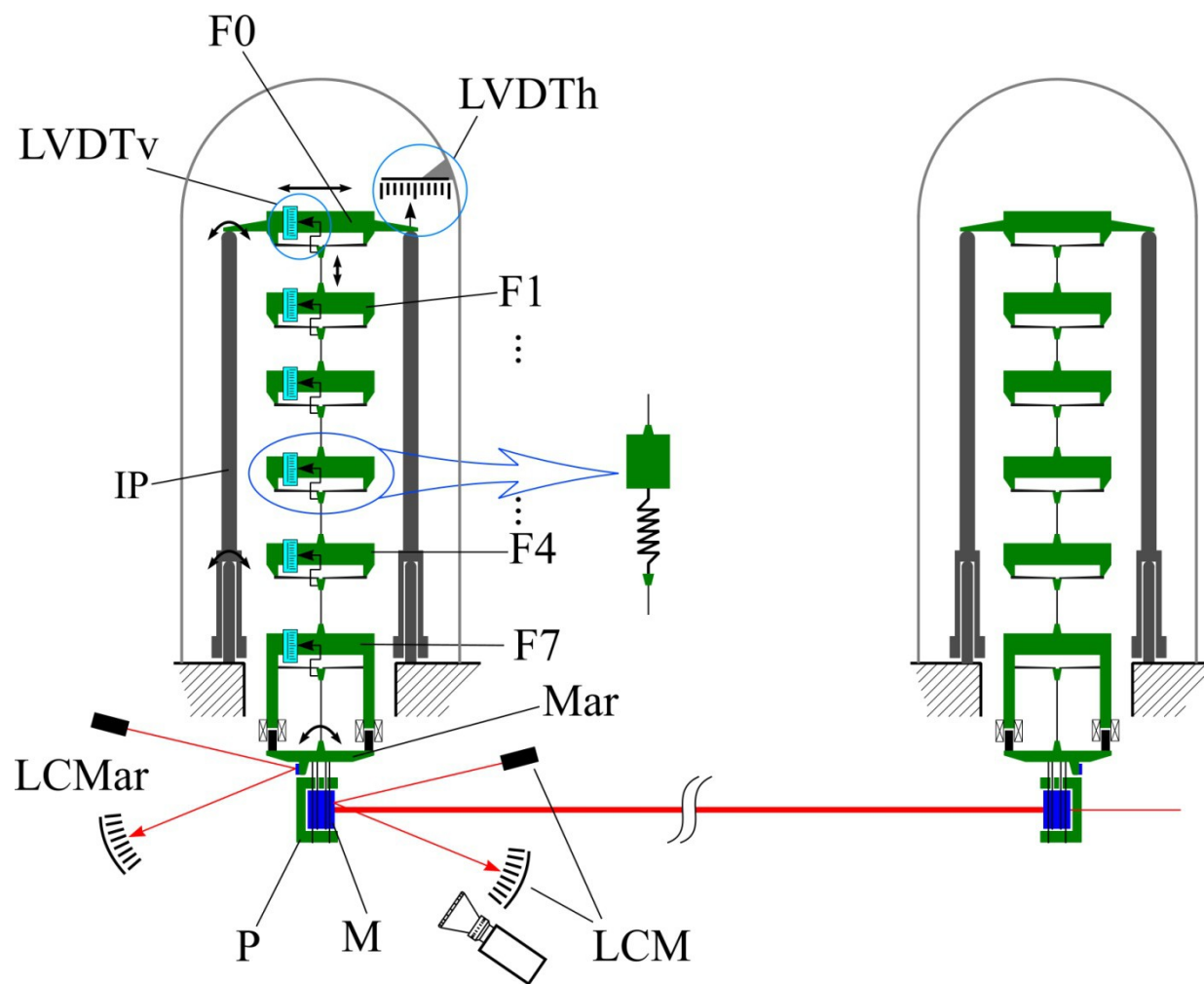
# Virgo suspension control strategy

- Locking requirement:  $10^{-12} \text{ m}$
- Tidal strain over 3 km:  $10^{-4} \text{ m}$
- Resonant motions of the mirrors  $10^{-4} \text{ m}$

*Need control of the mirror position over a very big range.*

*Need hierarchical control.*







# The vacuum chambers of Virgo Central Area

Power Recycling

West input

Nord input

Beam Splitter

Signal Recycling





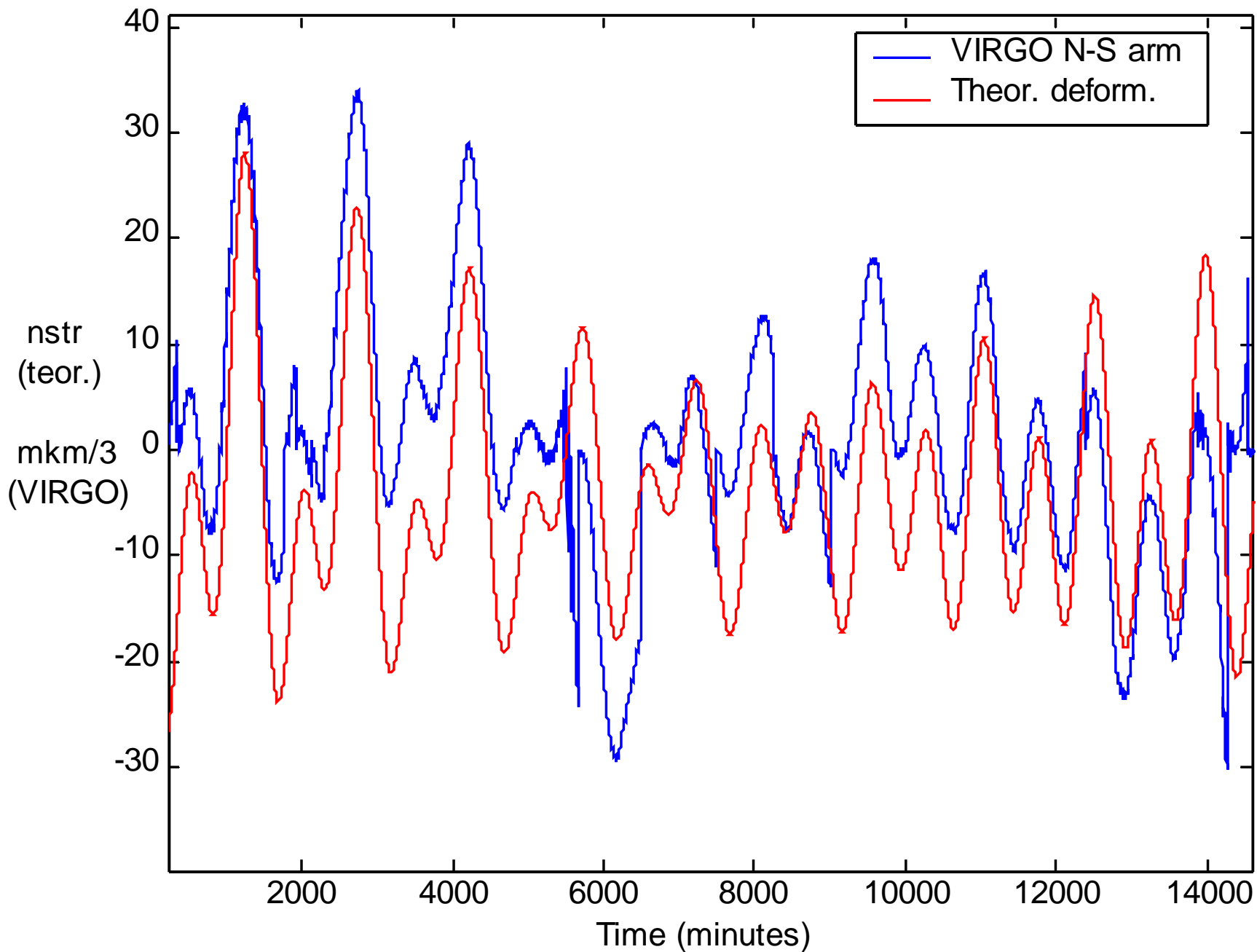
## Geo-applications of Virgo: basic principles and preliminary measurements

A. Giazotto, A.V. Gusev, S. Braccini, E. Majorana, M. Mantovani, P. Ruggi, V.N. Rudenko\*,  
A.A. Samoilenko\*, I.N. Tsybankov, V.D. Yushkin\*

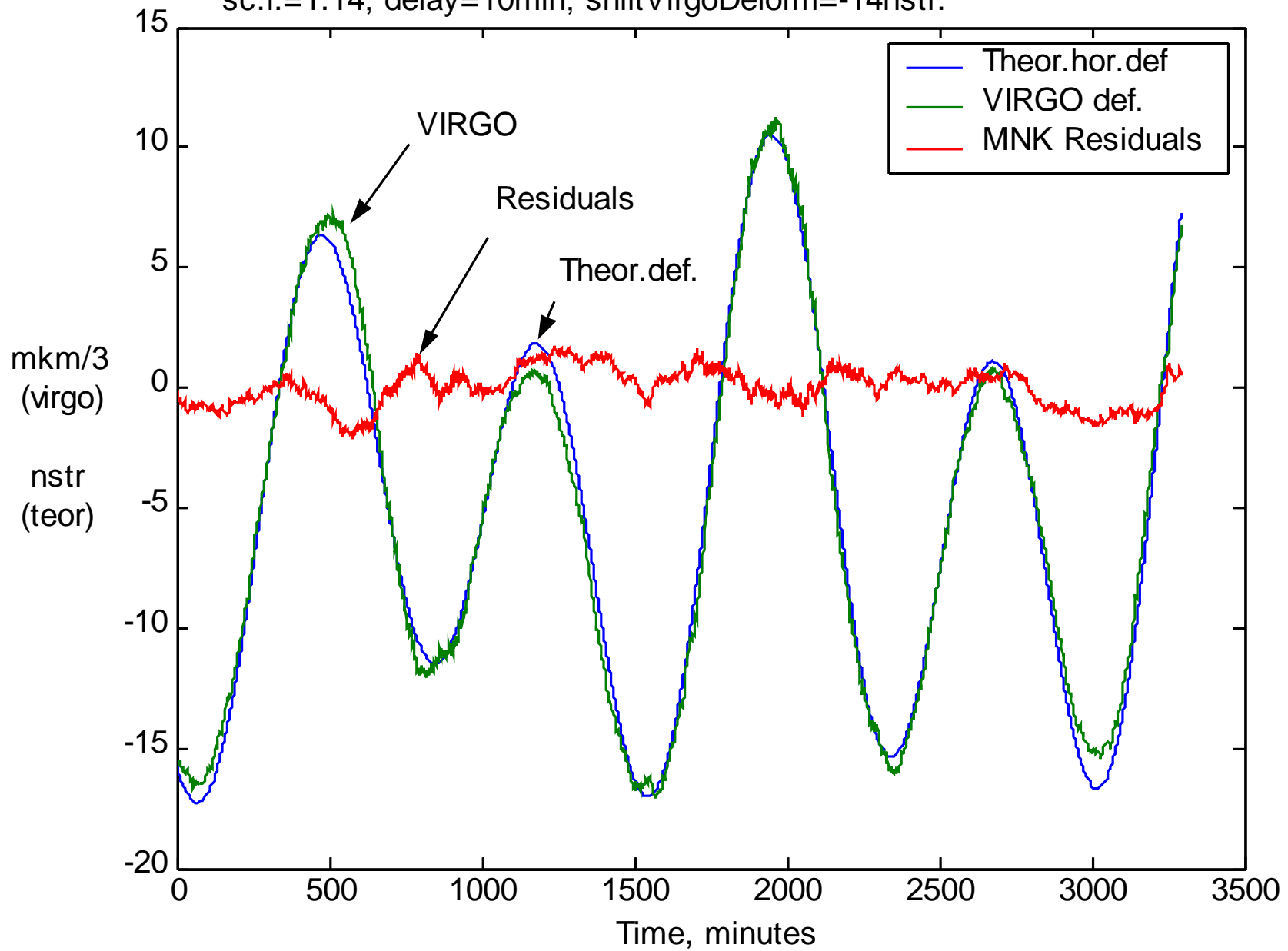
**VIR-003B-09**

Date : 7th February 2009

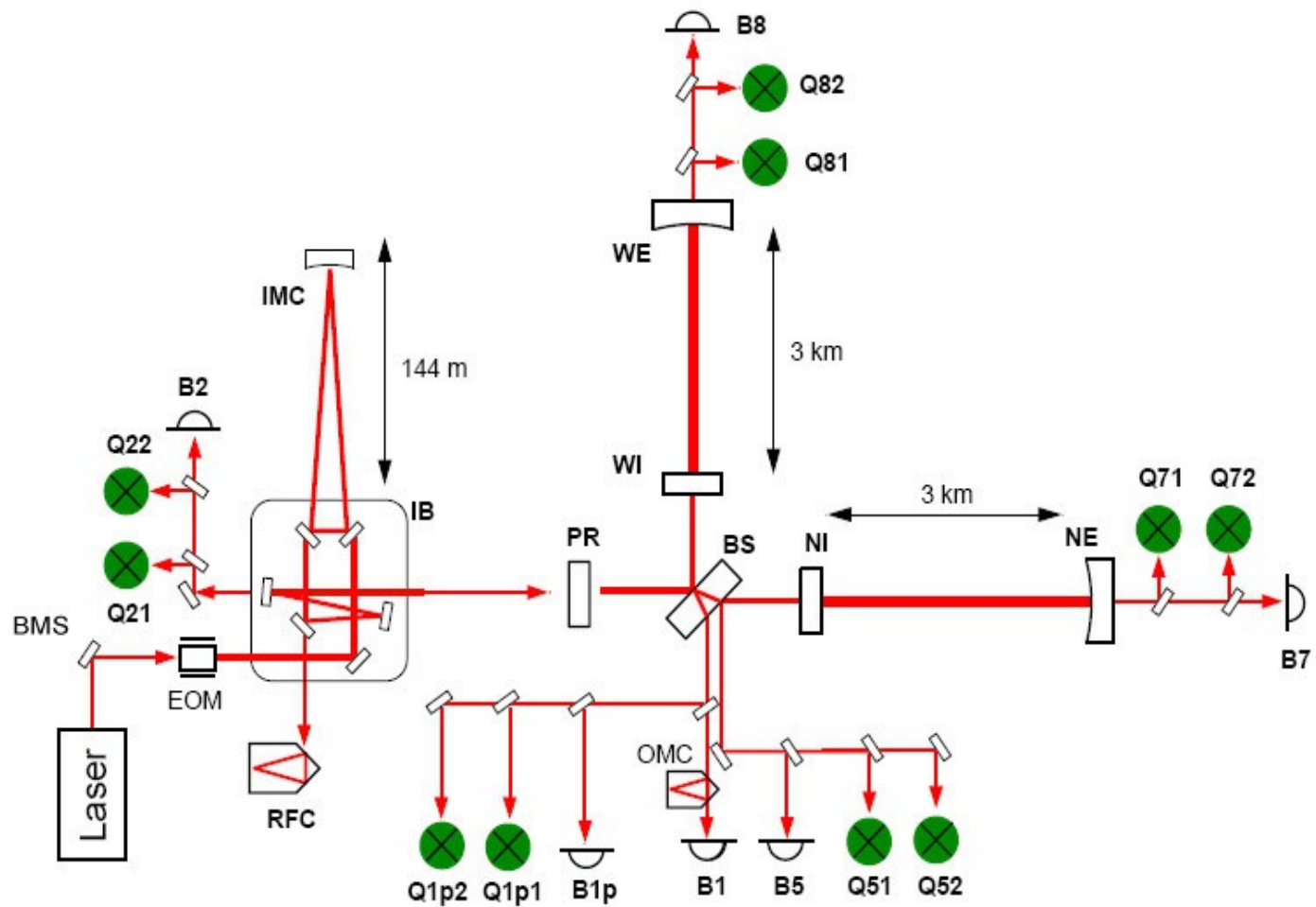
Horizontal deformations of N-S arm during 10 days



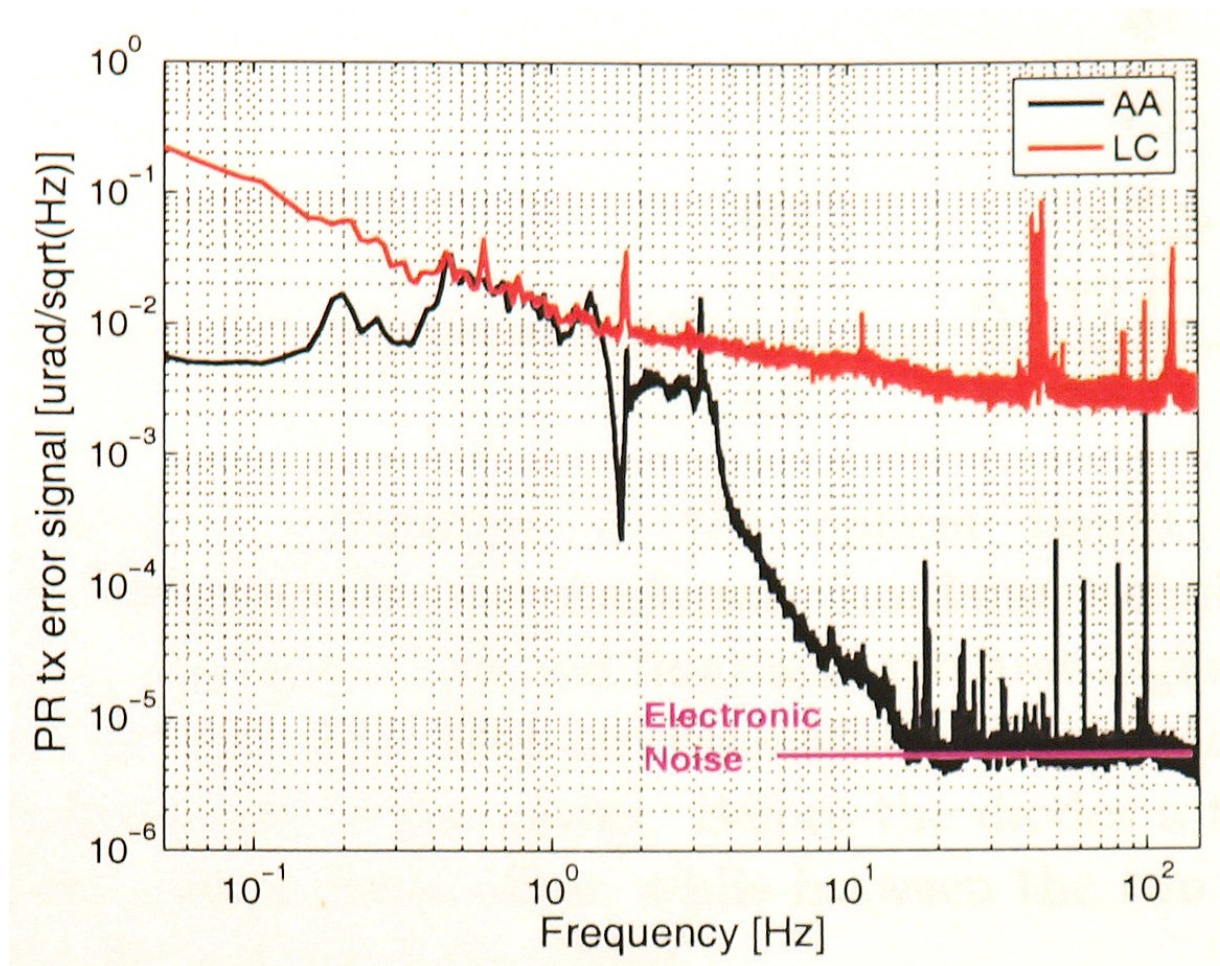
Horizontal deformations North arm (18101-21613), drift=0.007nstr/min,  
sc.f.=1.14; delay=10min; shiftVirgoDeform=-14nstr.







Principle scheme of the VIRGO angular control system



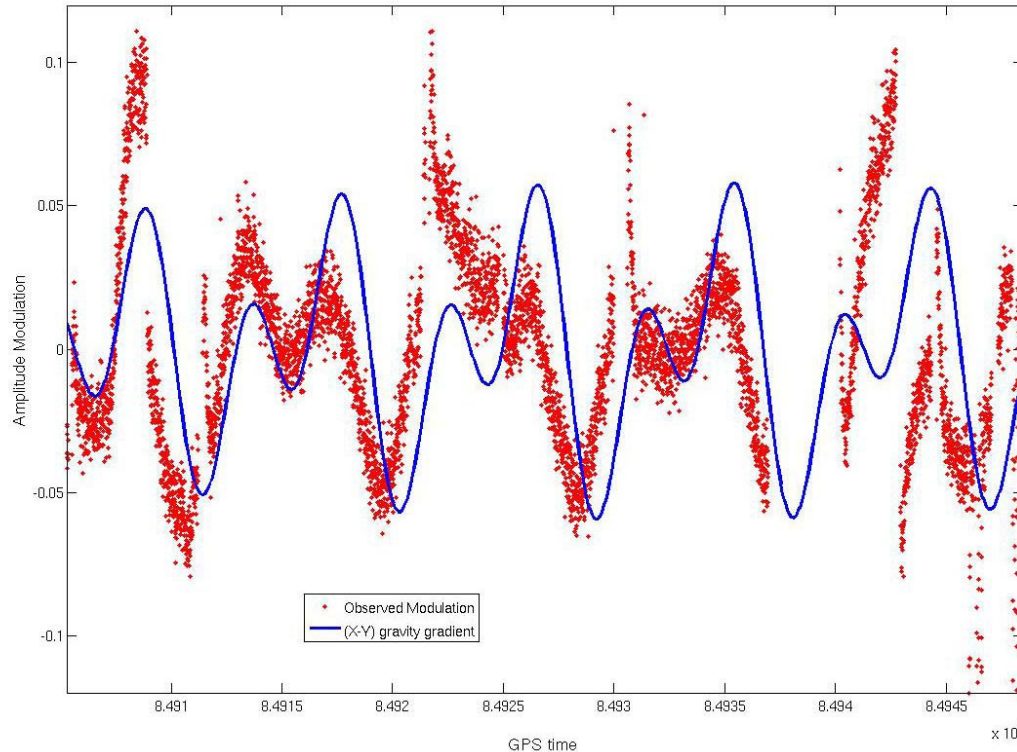
recycling mirror angular noise (around the beam direction) in  $\mu\text{rad}/\text{Hz}^{-1/2}$ ;  
LC (red) – local control regime ; GC (black) – global control regime..

Geo signals at the main output

# (LIGO) data vs. prediction for 5 days, December 7, 2006

(Neither the phase or amplitude have been normalized)

It is assumed:  $h_t = g_{||}L/c^2$



LIGO tidal signal at FSR-frequency  $\sim 37$  kHz

Meliessinos A., MG-12 Paris, July 12–18, 2009; <http://xxx.lanl.gov/PS,cache/arxiv/pdf/1001/1001.558> v.2.pdf.  
Forrest C.V., *Tidal Effects on Laser Gravitational Wave Detectors*, Thesis Univ. of Rorester, LIGO Document P09 0000 v1 (2009).

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# **Gravitational Modulation of the Optical Length of Long Baseline Laser Interferometers**

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The effect of quasi static variations of the Earth's gravitational field on the output signal of long baseline gravitational-wave interferometers has been considered. The relativistic representation of the gravitational field in the form of a varying refractive index is used. An analytical calculation with numerical estimates of the tidal effect has been performed to explain the recent results observed on the LIGO interferometers

## Pump with a finite width of line

$$\nu_0 \sim 3 \cdot 10^{14} \text{ Hz} \quad \text{pump frequency}$$

$$\Delta \nu \sim 100 \cdot \text{kHz} \quad \text{pedestal width}$$

$$\nu_{fsr} = c / 2Ln \approx 37 \cdot \text{kHz} \quad \text{FSR interval}$$

$$\text{for } L = 4 \cdot \text{km}$$

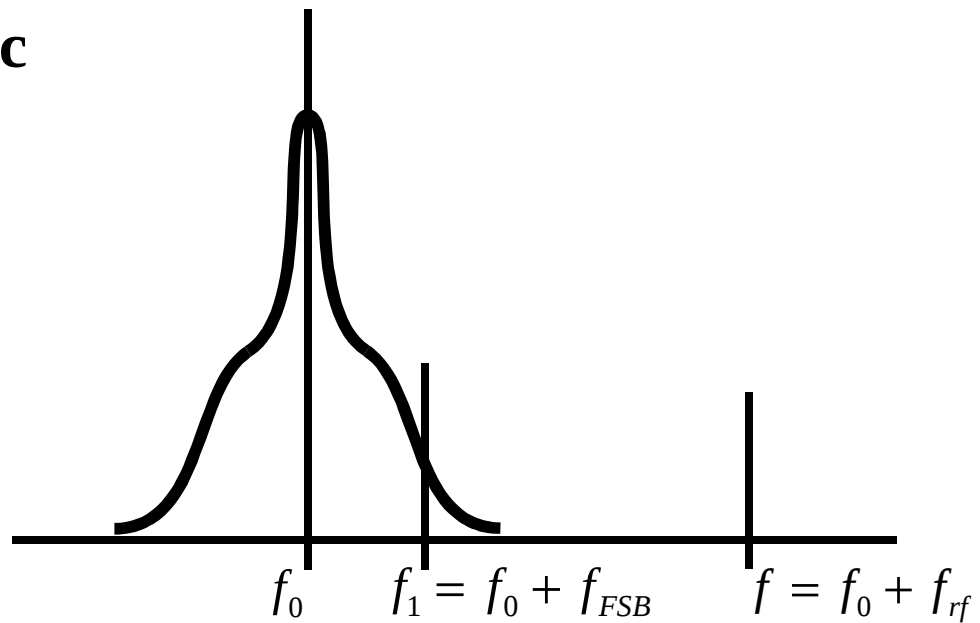
$$\nu_1 = \nu_0 + \nu_{fsr} \quad \text{neighbors mode}$$

## Mechanism FSR-harmonic at the main output

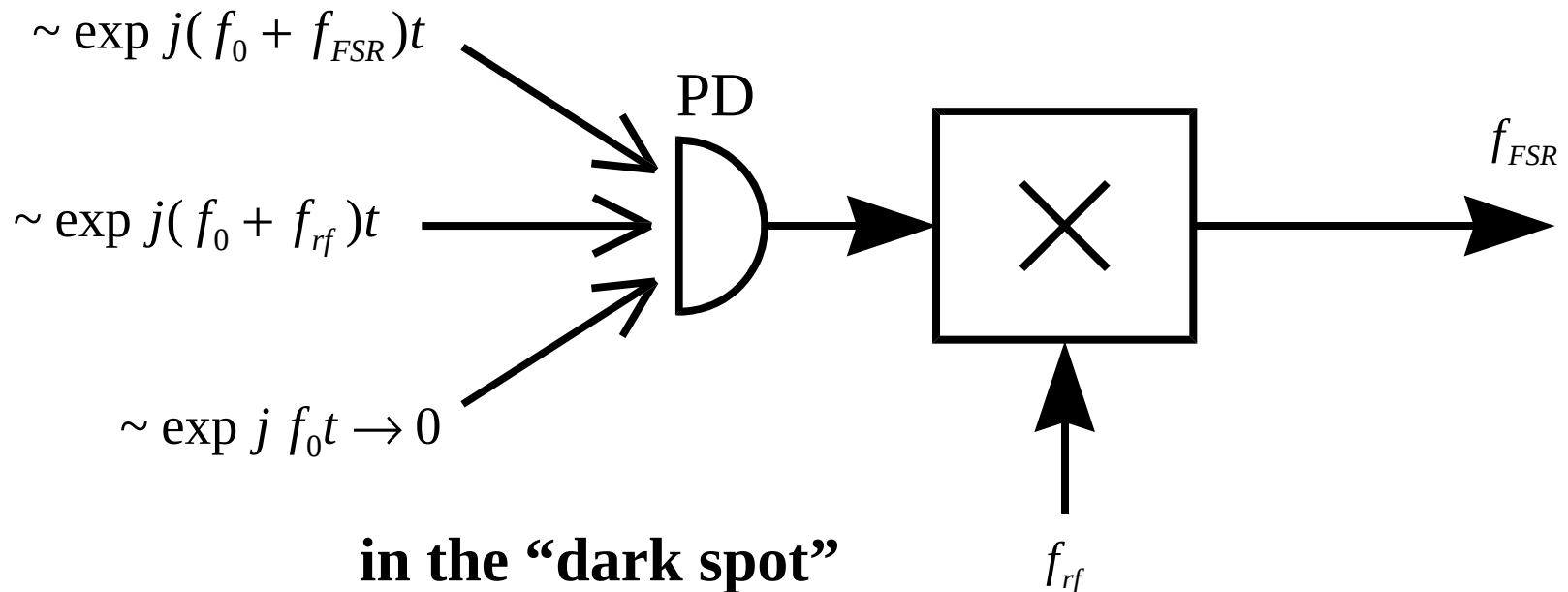
$$L_{opt} = Ln; \quad L = \text{const.}$$

$$n_{eff} \cong n_0(1 \pm h_t)$$

$$h_t = \frac{g_{\perp} L}{c^2}.$$



## relativistic gravity effect !





$$\Delta\varphi \cong \frac{1}{1-r} \cdot \left( 2\pi \frac{\Delta L}{L} \cdot + \cdot \frac{4\pi L}{\lambda} \cdot \frac{2\delta n}{n} \right) \quad |V(t)|_{fsr} \sim \Delta\varphi|$$

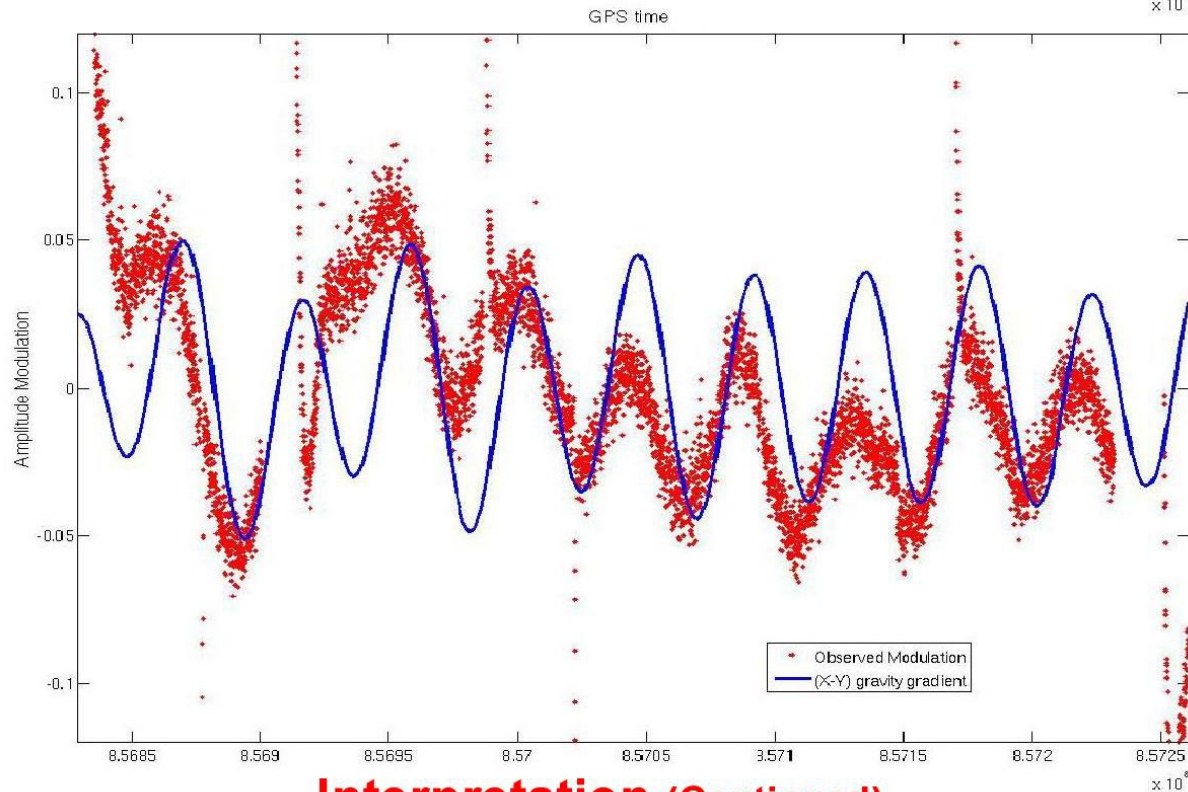
**phase difference at the PD**

$$\frac{\delta n}{n} \cong \frac{\Delta U_t}{2c^2} \sim 10^{-7} g_0 \frac{L}{c^2} \quad \text{n – tidal variation}$$

$$\frac{\Delta P}{P} \sim \frac{2\Delta A}{A} \sim 10^{-7} g_0 \frac{2L^3}{\lambda \cdot \Delta L \cdot c^2} \quad \text{modulation index} \\ \sim 0.02 \%$$

**observable value  $\sim 1 \%$**

observable effect was too big  
in respect of RG estimate



## Interpretation (Continued)

A.Melissinos GR  
«red shift model»

Because of tidal forces the direction of the gravitational acceleration, “g”, at the Earth’s surface deviates from the vertical.

The horizontal component of g is along the arms and of order  $g_{//} \sim 10^{-7}g$ . The horizontal component varies in time at the tidal frequencies, and induces a phase shift on the carrier.

Order of magnitude estimate for  $h_t$ ; by analogy to Pound-Rebka effect,  $\Delta f_{(ITM-ETM)}/f = g_{//} L/c^2$

$$h_t = (\Delta f/f) \cdot B = (g_{//} L/c^2) \cdot B \sim 4 \times 10^{-18}$$

$$\frac{1}{2} \Delta P/P = 1.6 \times 10^{15} h_t = 6.4 \times 10^{-3}$$

If it is not GR effect, ... then what ?

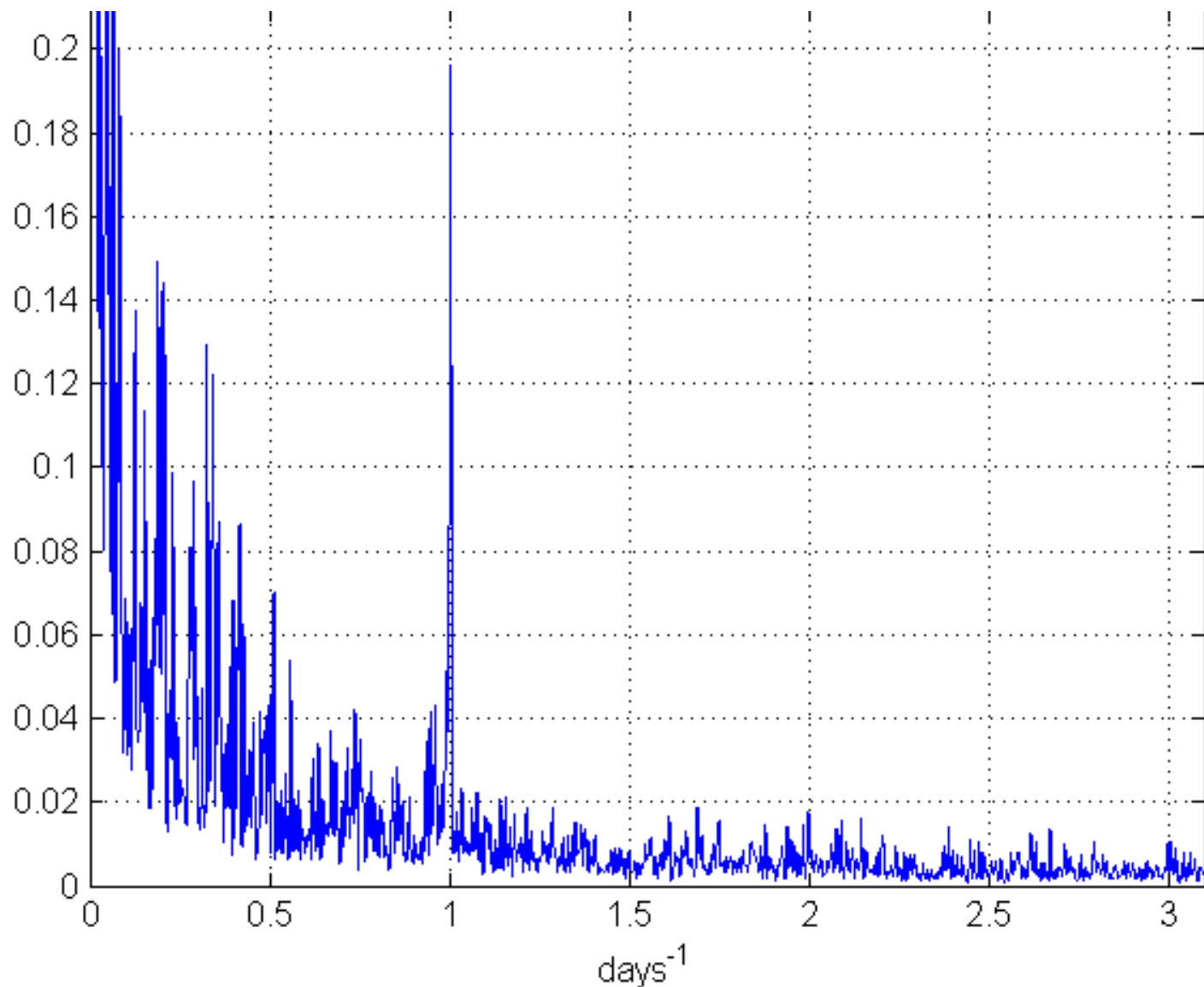
the only hypothesis  $\rightarrow$  it is a residual arm shift...

$L \neq \text{const.}$  behind the control accuracy  $\sim \Delta L = 10^{-10}$  cm

tidal  $\Delta L_T \sim 10^{-7} L$

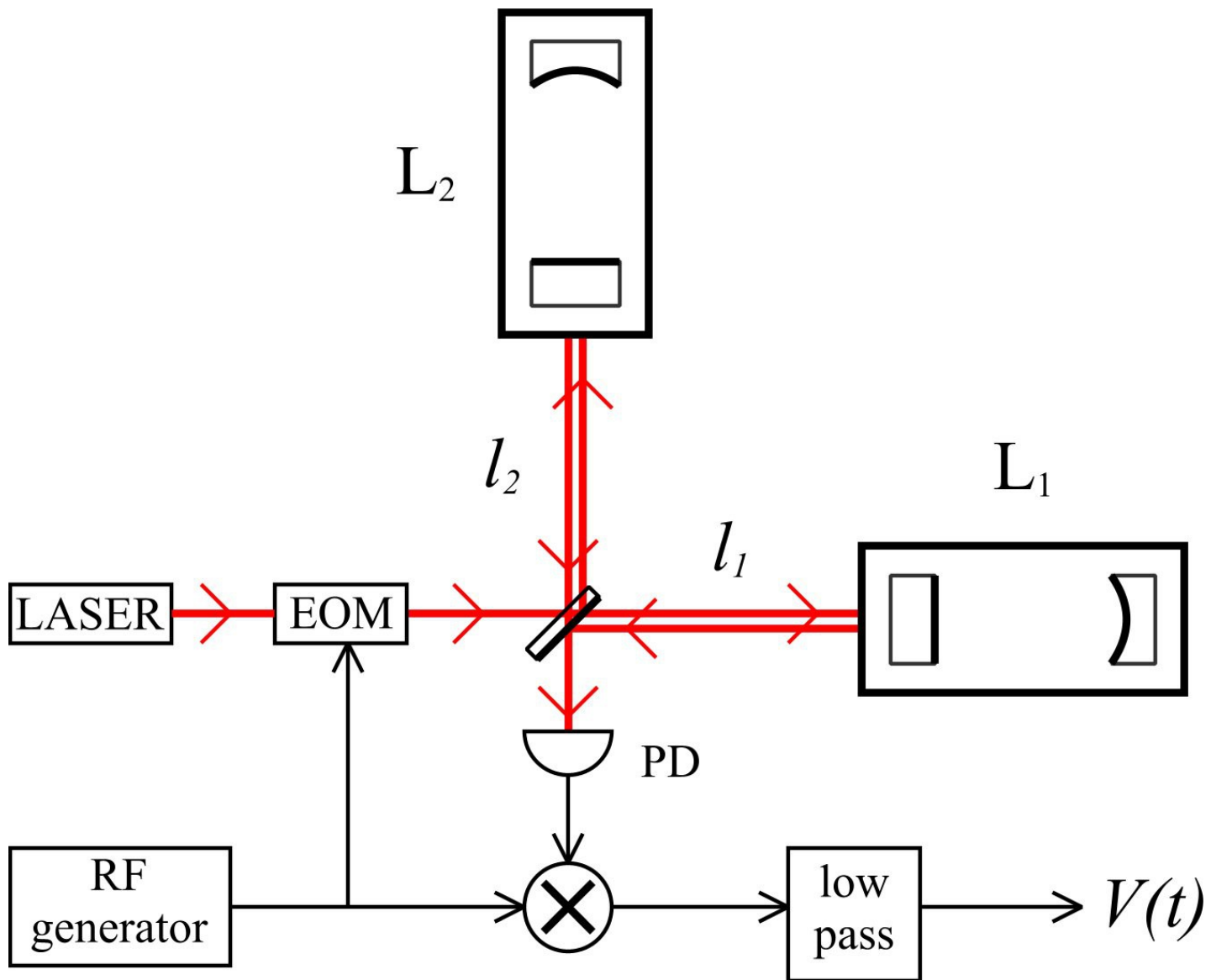
residual t-deformation  $\sim 10^{-10}/3 \cdot 10^5 = 10^{-16}$

GR effect  $\rightarrow (10^{-7} g_0 L / c^2) \cdot F \sim 5 \cdot 10^{-18}$



(Frasca effect: Sidereal periodicity in Virgo noise data)

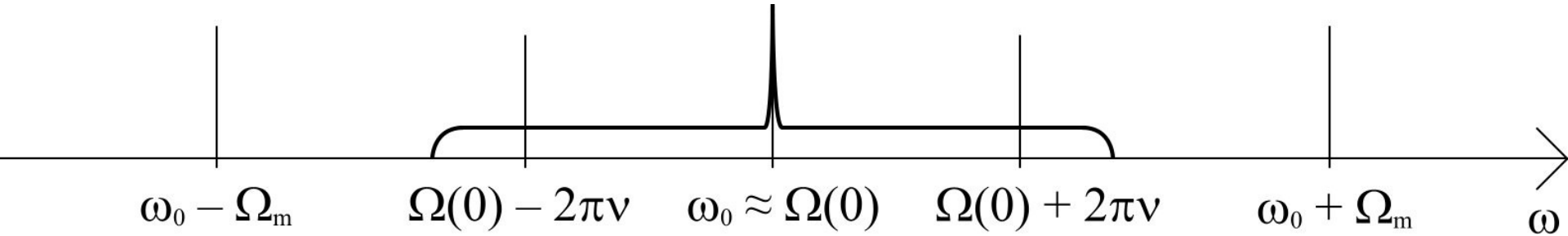
**the mean pseudo spectrum of the noise in the region 22 Hz**



Interferometer Optical Scheme (without recycling mirror)

analysis logic  $\rightarrow$  OS – optical system configuration  
 optical pump:  $E(t) = \text{Re}\{ E_1^*(t) \exp j\omega_0 t \}$ ,  $\omega_0$ - carrier

$$E_1^*(t) = [A^* + a^*(t)] \exp\{ j\beta \sin\Omega_m t \},$$



$$L \sim L_1 \neq L_2 \rightarrow \Delta L ; \quad \Omega(0) = \omega_0(1 + \xi),$$

- geo length variations

$$\xi = \delta l / L, \quad \nu = c / 2L$$

$$\Delta\xi = \xi_1 - \xi_2 = \Delta\xi_{\text{st}} + \Delta\xi_{\text{gw}} \quad \text{(quasi static variations + gw-signal frequencies)}$$

$$\text{OS output} \rightarrow E_2(t) = \langle E_1^*(t) \otimes g^*(t) \rangle ,$$

$$\langle \otimes \rangle \rightarrow \int E_1^*(\tau) g^*(t-\tau) d\tau ; \quad g(t) - \text{OS impulse characteristic}$$

$$\text{frequencies:} \quad \Omega(0) = \omega_0(1 + \xi) - \text{central mode}$$

$$G(0), G(\pm 1) \quad \Omega(0) = \omega_0(1 + \xi) \pm 2\pi\nu_{\text{FSR}} - \text{side modes}$$

transf. functions

$$G(\Omega_m) \quad \Omega(0) = \omega_0(1 + \xi) \pm \Omega_m - \text{sidebands}$$

$\Downarrow$

$$G^+(\Omega_m) = G(\Omega_m) + G(-\Omega_m) ; \quad G^-(\Omega_m) = G(\Omega_m) - G(-\Omega_m)$$

symmetric TF



OS Transfer Function (  $k = 0$  center ,  $k = \pm 1$  side modes)

$$G(\Omega(k)) \approx 1 - \exp j[\omega_0(2\Delta l/c) + \psi] \cdot \exp j[k (\pi/(1-r))(\Delta L/L)]$$

$$\delta l \sim \uparrow \uparrow \quad \uparrow \uparrow \sim \Delta \xi$$

$$\psi = \varphi_1(\Omega) - \varphi_2(\Omega) = \{2\pi/(1-r)\} (L/\lambda) \Delta \xi, \quad \Delta l = \langle \Delta l \rangle + \delta l$$

$$\text{“dark spot”} \rightarrow 4(\delta l/\lambda) + \psi_{\text{st}}/\pi = 0, \rightarrow \delta l_{\text{opt}} \approx (L/2(1-r)) \Delta \xi_{\text{st}}$$

$$\text{main point: } G(\Omega) \Rightarrow f(\Delta \xi_{\text{st}}, \delta l) \rightarrow \text{coupled values!}$$

$$\psi = \psi_{\text{st}} + \psi_{\text{gw}} \propto (\Delta \xi_{\text{st}} + \Delta \xi_{\text{gw}})$$

$$E_1(t) \sim [A + a(t)] \rightarrow a(t) = 0$$

$$\text{central mode regime: } E_1 \sim A$$

$$V_s(t) \approx 0, \quad V_c(t) \propto A \operatorname{Re} [G^*(0) j G^+(\Omega_m)]$$

$$V_c(t) \propto A [4(\delta l/\lambda) + (\psi_{st} + \psi_{gw})/\pi] \Omega_m(\Delta l/c),$$

$$\Delta l = \langle \Delta l \rangle + \delta l \Rightarrow \Delta \xi_{st} \quad \delta l = \delta l_{opt} = -[L/2(1-r)] \Delta \xi_{st}$$

$$V_c(t) \propto A \psi_{gw} \Omega_m (\langle \Delta l \rangle + \delta l)/c$$

$$\Downarrow$$

$$(\text{Frasca effect ?}) \quad h_{gw} \sim (1 + \xi \cos \Omega_{st}) \sin(\omega t + \theta)$$

neighbour mode regime :  $a^*(t) \neq 0$

$$V_s \propto \text{Re}[\langle a^*(t) \otimes g^*(t) \rangle_j G^+(\Omega_m)] ; \quad V_c \propto \{G^-(\Omega_m)\}$$

$$V(t) = V_c + j V_s ; \rightarrow \{\text{FSR filter}\} \rightarrow V^v(t) \rightarrow \sigma_v^2$$

Melissinos regime FSR amplitude modulation

$$\sigma_v^2 \propto [|G(1)|^2 + |G(-1)|^2] \cdot [|G^+(\Omega_m)|^2 + |G^-(\Omega_m)|^2]$$

$\delta l_{\text{opt}} \Downarrow$  dark spot

$$[4(\delta l/\lambda) + ((\psi_{\text{st}} + \psi_{\text{gw}})/\pi)^2 + [\Delta L/L(1-r)]^2 \rightarrow \text{no linear effect!}$$

But if  $\delta l \neq \delta l_{\text{opt}} \rightarrow \delta l = \delta l_{\text{opt}} + \delta l_{\text{er}} \leftarrow$  a small detuning

$$\sigma_v^2 \propto 4(\delta l/\lambda) (\psi_{\text{st}} + \psi_{\text{gw}})/\pi ; \quad [m = L/(1-r) \delta l_{\text{er}}]$$

Melissinos effect

## Interferometer with power recycling mirror.

Transfer Function  $G_I(\omega) = G_R(\omega) G(\omega)$

$\Downarrow$

recycling cavity TF

model: FP composed by

$M_R$  - (Michelson – FP arms cavity)

tuning parameters:

$$\Delta l = l_1 - l_2, \rightarrow \Delta l_R = l_{RB} + l_i$$

«geo amplitude modulation» of  
ITF output spectral components is kept  
but more powerful and with a new tuning

# Conclusions

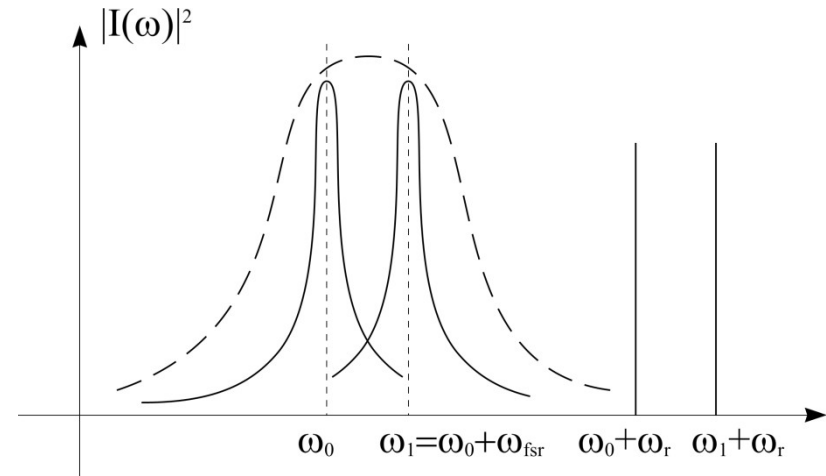
- Geophysical information from GW interferometers can be readout using compensation signal of circuits controlling the device operation regime.
  - besides due to parametric variations of FP resonance frequencies -
- Geo - information might be received through the amplitude modulation of the free spectral range frequency filtered at the main interferometer output.
- Noise spectral density at the main output also is modulated by very slow (quasi static) variations of the interferometer base, produced by geophysics.
- It was demonstrated that VIRGO can be used as a two coordinate very long base strain meter. However the quality of the data strongly depends on a number of operational regime brakes during the observational time.
- Idea of measuring relative angular variations of mirror's "plumb lines" for a sensing pure "gravity perturbations" up to now was not realized and requires a more detailed study.

.

**Radical method** for the linear parametric response reconstruction would be using the **two mode pump** at neighbor frequencies  $\omega_0$  and  $\omega_1 = \omega_0 + \omega_{\text{fsr}}$

“dark spot” condition is kept for the mode  $\omega_0$  ;

A residual part of the mode  $\omega_1$  interacting at photo diode with radio sideband  $\omega_1 + \omega_r$  will produce directly at the Pound-Drever mixer output a low frequency signal  $I(t)$  proportional to the optical length variations  $\Delta\xi$  .



$$I(t) \propto \frac{Const}{(1-r)^2} \left[ \left( \frac{2L}{\lambda} \right) \Delta\xi + \left( \frac{\Delta L}{L} \right) \right]^2$$

A possibility of application this method for a registering weak global geodynamical effects as well as detection of very low frequency GW is under the process in **SAI MSU**

Thanks for attention !