Advantages of NLO

- Less sensitivity to unphysical input scales, i.e. renormalization and factorization scales
- First level of prediction where normalization (and sometimes shape) can be taken seriously
- More physics
  - parton merging gives structure in jets
  - initial state radiation
  - more species of incoming partons
- Suppose I have a cross section σ calculated to NLO ($O(\alpha_s^n)$)
- Any remaining scale dependence is of one order higher ($O(\alpha_s^{n+1})$)
  - in fact, we know the scale dependent part of the $O(\alpha_s^{n+1})$ cross section before we perform the complete calculation, since the scale-dependent terms are explicit at the previous order

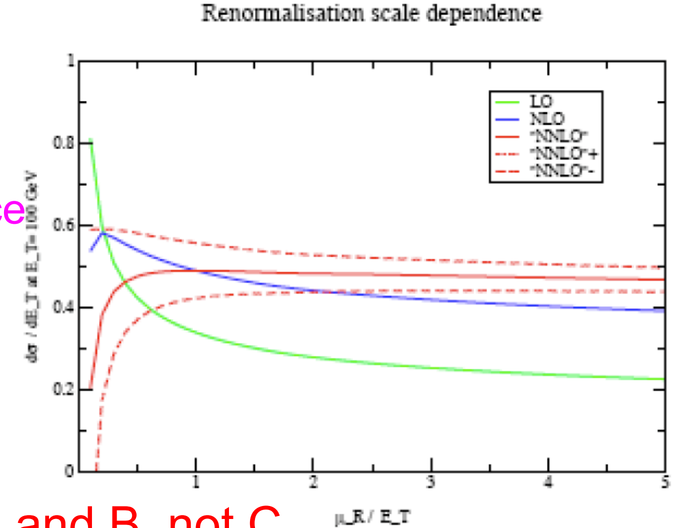
$$
\begin{aligned}
\frac{d\sigma}{dE_T} &= \alpha_s(\mu_R)^2 A \\
&+ \alpha_s(\mu_R)^3 (B + 2b_0 L A) \\
&+ \alpha_s(\mu_R)^4 (C + 3b_0 L B + (3b_0^2 L^2 + 2b_1 L) A)
\end{aligned}
$$

Inclusive jet prod at NNLO

with $L = \log(\mu_R/E_T)$ and b_i the known beta function coefficients. Note that L is a single log, unlike the double logs we saw with Sudakov factors

LO has monotonic scale dependence

non-monotonic at NLO



we know A and B, not C

Figure 11: Single jet inclusive distribution at $E_T =$ 100 GeV and $0.1 < |\eta| < 0.7$ at $\sqrt{s} = 1800$

The NNLO coefficient C is unknown. The curves show the guesses $C = 0$ (solid) and $C = \pm B^2/A$ (dashed).

Scale choices

- We know that we have two scales, μ_R and μ_F
- We know that they should be associated with the relevant scale in the hard scattering process
  - sometime this scale is evident, like m_W for W production, p_T^{jet} for inclusive jet production
  - but what if I have a process like W+jet(s)
    - there I have both m_W and p_T^{jet}, and these scales can be very different->very different answers
    - we'll see that for some cases, general scales like H_T may work best
- Often μ_R and μ_F are taken equal to each other, but the physics associated with each is a bit different, so they can be varied separately…as long as the ratio between the two scales is not too large (>2)
- For then, we would introduce a new log into the calculation, the log of the ratio of the two scales
- These logarithms would then have to be re-summed to restore precision to the measurement
- We don't want to have to do that

→ sum of transverse momenta of all objects in event

Scale uncertainties

- We try to estimate the uncertainty due to uncalculated higher order terms by varying μ_R, μ_F over some range, typically a factor of 2
- This is normally the best we can do, but we have to keep in mind that higher order corrections can arise from a number of other sources such as Sudakov effects, large color factors, large π^2 terms, the opening of new channels
- These contributions are not estimated by the variation of the scale logarithms and can be larger than the variation
  - for example, because of double logs for Sudakov compared to single logs for scale dependence

Why does the scale dependence have the shape it does?

- Write cross section indicating explicit scale-dependent terms
- First term (lowest order) in (3) leads to monotonically decreasing behavior as scale increases (the LO piece)
- Second term is negative for $\mu<p_T$, positive for $\mu>p_T$
- Third term is negative for factorization scale $M < p_T$
- Fourth term has same dependence as lowest order term
- Thus, lines one and four give contributions which decrease monotonically with increasing scale while lines two and three start out negative, reach zero when the scales are equal to p_T, and are positive for larger scales
- At NLO, result is a roughly parabolic behavior

Consider a large transverse momentum process such as the single jet inclusive cross section involving only massless partons. Furthermore, in order to simplify the notation, suppose that the transverse momentum is sufficiently large that only the quark distributions need be considered. In the following, a sum over quark flavors is implied. Schematically, one can write the lowest order cross section as

$$E\frac{d^3\sigma}{dp^3} \equiv \sigma = a^2(\mu)\,\hat{\sigma}_B \otimes q(M) \otimes q(M) \qquad (1)$$

where $a(\mu) = \alpha_s(\mu)/2\pi$ and the lowest order parton-parton scattering cross section is denoted by $\hat{\sigma}_B$. The renormalization and factorization scales are denoted by μ *and* M, respectively. In addition, various overall factors have been absorbed into the definition of $\hat{\sigma}_B$. The symbol $\otimes$ denotes a convolution defined as

$$f \otimes g = \int_x^1 \frac{dy}{y}\, f(\frac{x}{y})\, g(y). \qquad (2)$$

When one calculates the $\mathcal{O}(\alpha_s^3)$ contributions to the inclusive cross section, the result can be written as

$$\begin{aligned}
(1)\quad \sigma &= a^2(\mu)\,\hat{\sigma}_B \otimes q(M) \otimes q(M) \\
(2)\quad &+ 2a^3(\mu)\, b \ln(\mu/p_T)\hat{\sigma}_B \otimes q(M) \otimes q(M) \\
(3)\quad &+ 2a^3(\mu)\, \ln(p_T/M) P_{qq} \otimes \hat{\sigma}_B \otimes q(M) \otimes q(M) \\
(4)\quad &+ a^3(\mu)\, K \otimes q(M) \otimes q(M). \qquad (3)
\end{aligned}$$

In writing Eq. (3), specific logarithms associated with the running coupling and the scale dependence of the parton distributions have been explicitly displayed; the remaining higher order corrections have been collected in the function K in the last line of Eq. (3). The μ

Why does the scale dependence have the shape it does?

- Write cross section indicating explicit scale-dependent terms
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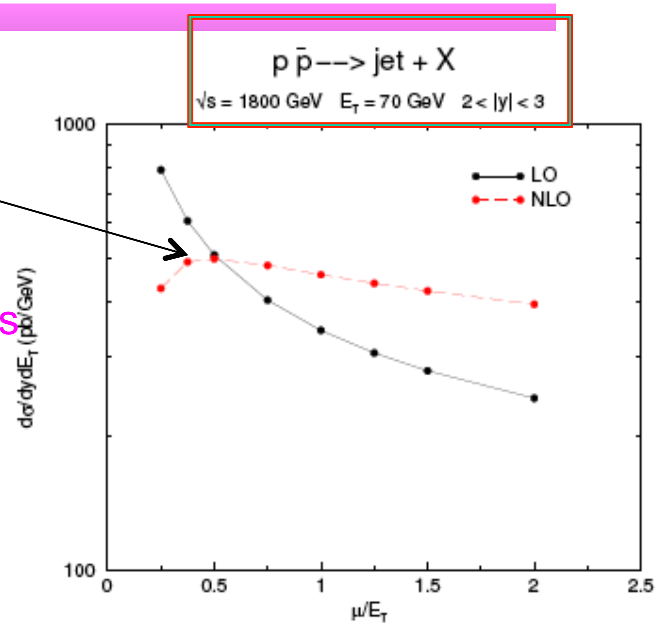
- At NLO, result is a roughly parabolic behavior

Note that NLO=LO for a scale of about $p_T/2$; for other scales NLO>LO, or NLO<LO



When one calculates the $\mathcal{O}(\alpha_s^3)$ contributions to the inclusive cross section, the result can be written as

$$
\begin{aligned}
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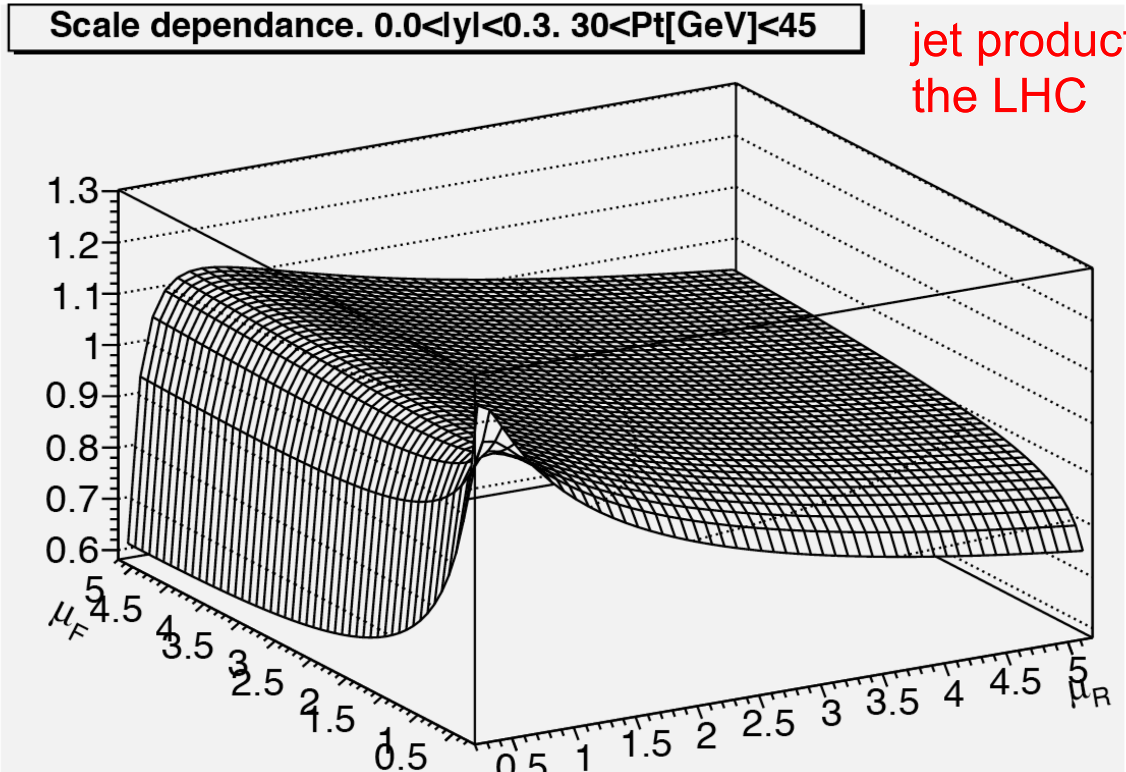
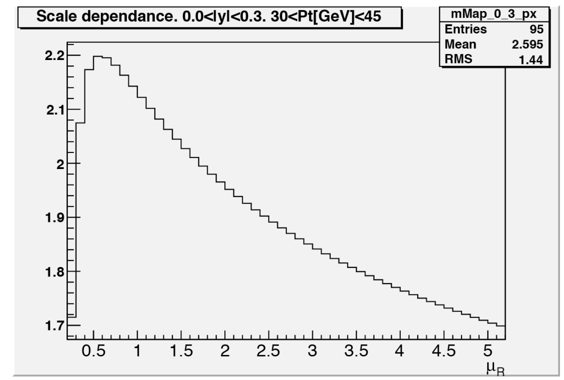
\end{aligned}
$$

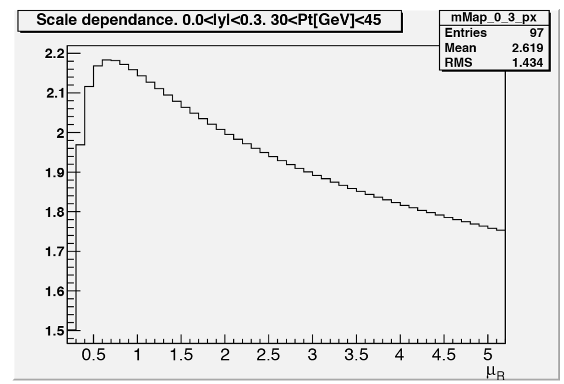
In writing Eq. (3), specific logarithms associated with the running coupling and the scale dependence of the parton distributions have been explicitly displayed; the remaining higher order corrections have been collected in the function K in the last line of Eq. (3). The μ

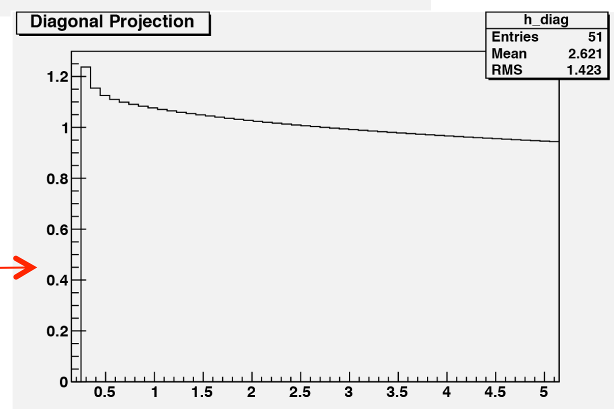
In 2-D parabola is a surface



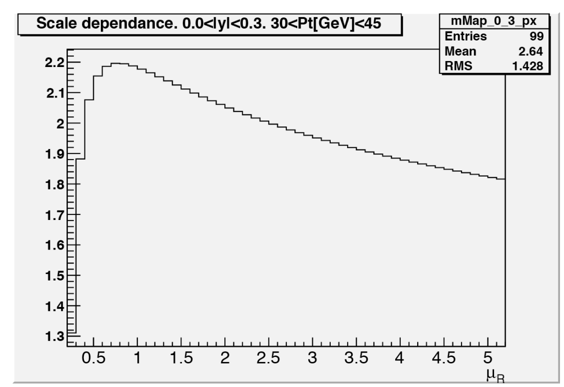
jet production at the LHC





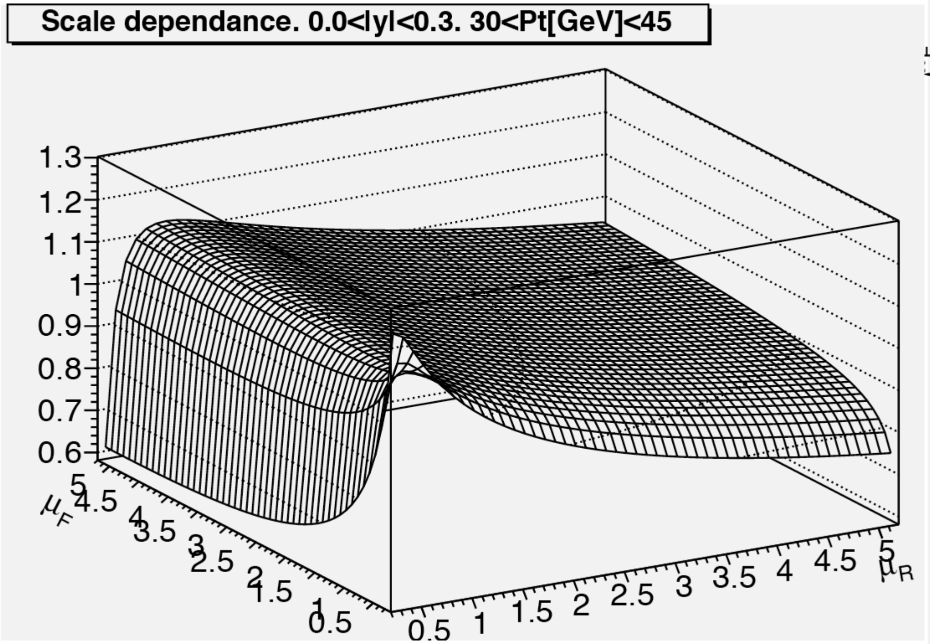


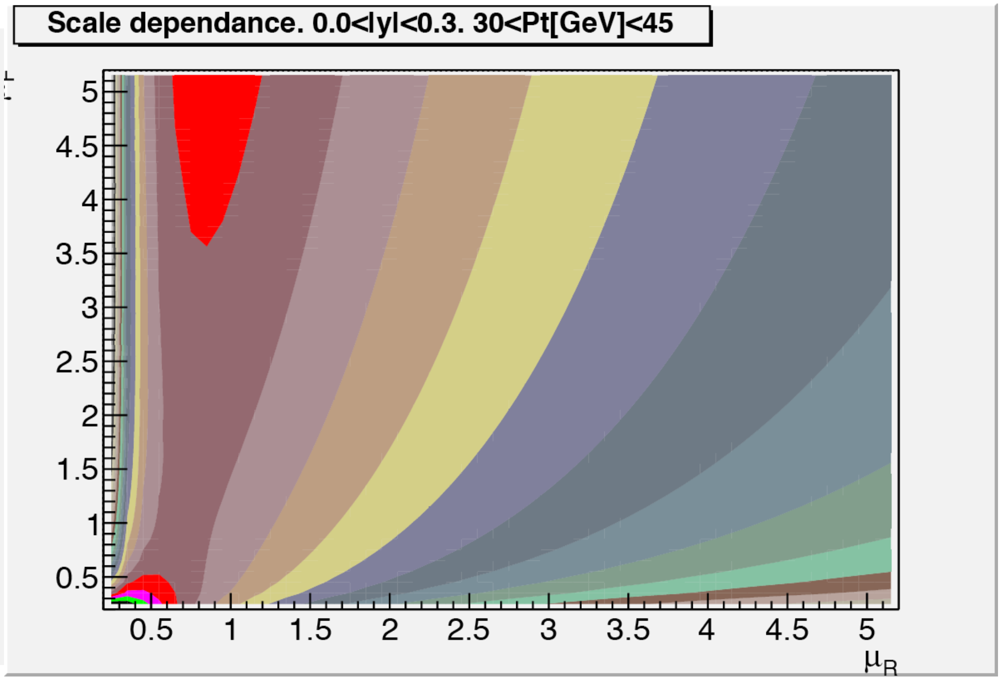
Various projections



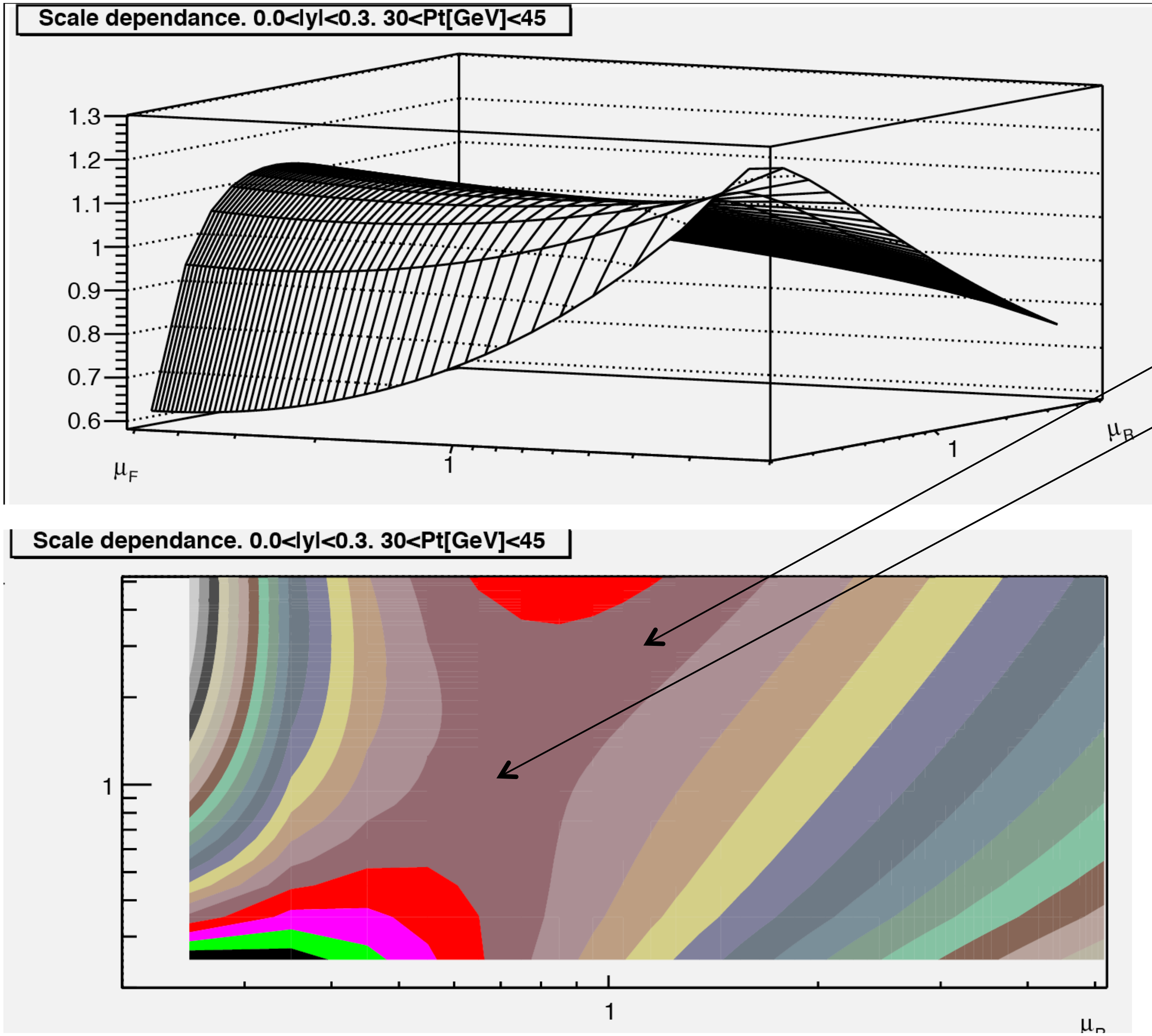
Useful to look at contour plots

Jet production at the LHC





It’s also useful to use a log-log scale



- …since perturbative QCD is logarithmic
- Note that there’s a saddle region, and a saddle point, where locally there is no slope for the cross section with respect to the two scales
- This is kind of the ‘golden point’ and typically around the expected scale (p_T^{jet} in this case)

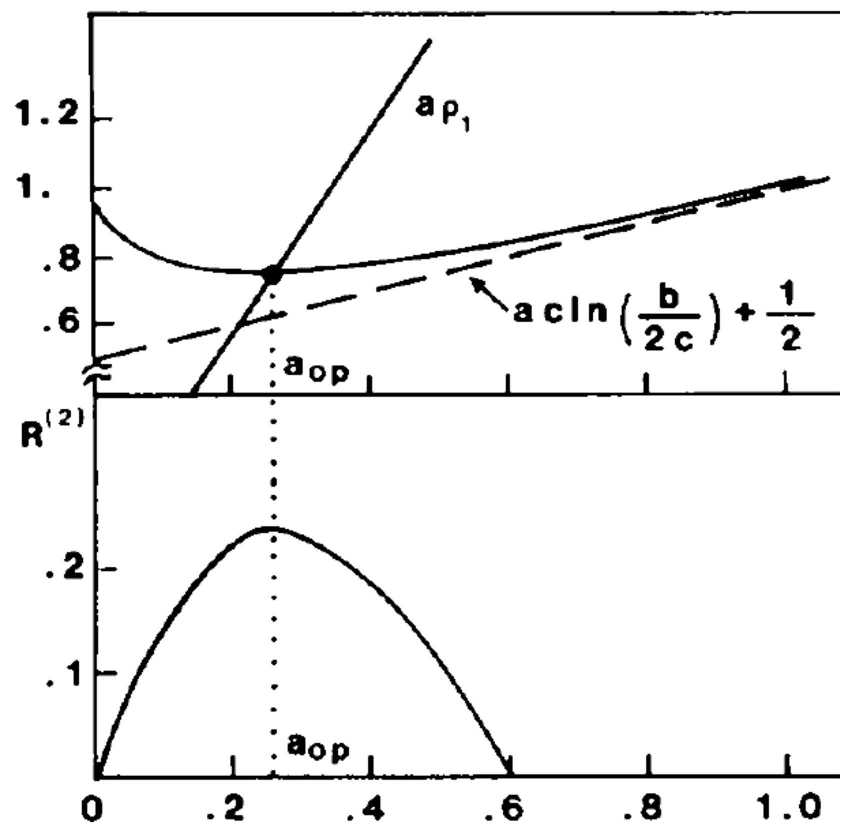
Aside: looking for saddle points

- Can find saddle point analytically by solving a transcendental equation

$$\tau + \frac{1}{2}\frac{c}{1+ca} = \rho_1$$

- …where ρ_1 is a dimensionless form of the jet cross section, and τ depends on the scale μ and on Λ
- Choosing the saddle point as the scale is called the PMS scheme (Principle of Minimal Sensitivity)

P. Aurenche et al. / Higher order QCD prediction

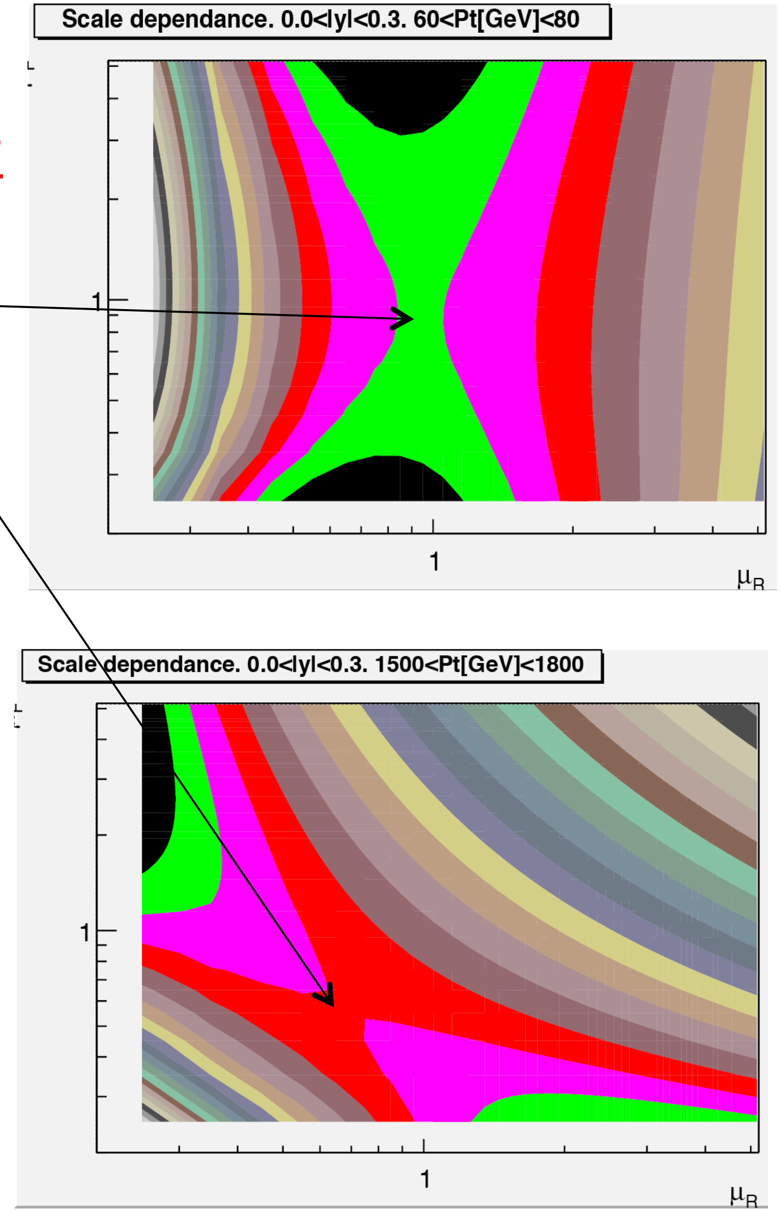


tion of Stevenson's equation for a_{op}; (b) plot of th
a function of a.

Scale choices

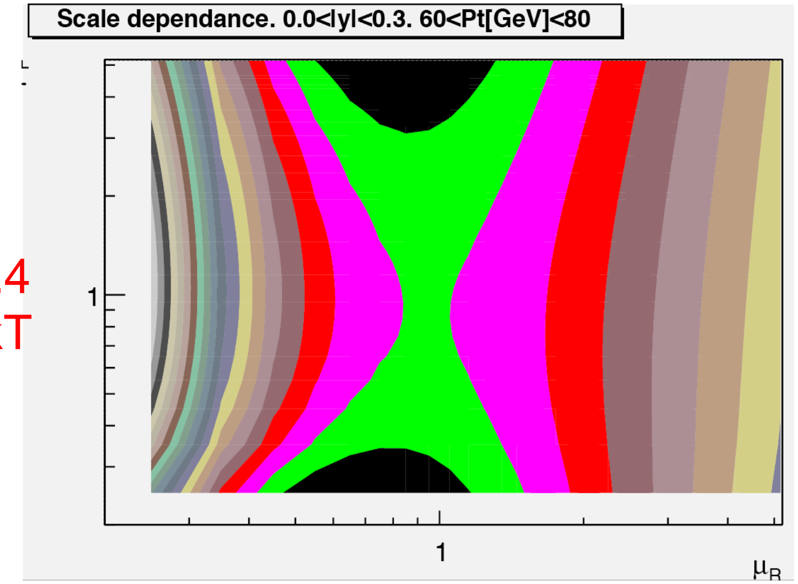
- Take inclusive jet production at the LHC
- We know that the scale should have something to do with the jet pT
- Canonical scale choice at the LHC is $\mu_r = \mu_f = 1.0 * p_T$
  - CDF used $0.5p_T$
  - CTEQ6.6/CT10 used this scale for determination of PDFs
  - CT10.1 uses p_T
  - (you can can see that the PDFs determined will depend on the scales used for the processes)
- Close to saddle point for low p_T
- But saddle point moves down for higher p_T (and the saddle region rotates)
- Don't know explicitly why but related to the kinematic convolutions shown on the previous slide
- Maybe a homework assignment for those of you who are energetic

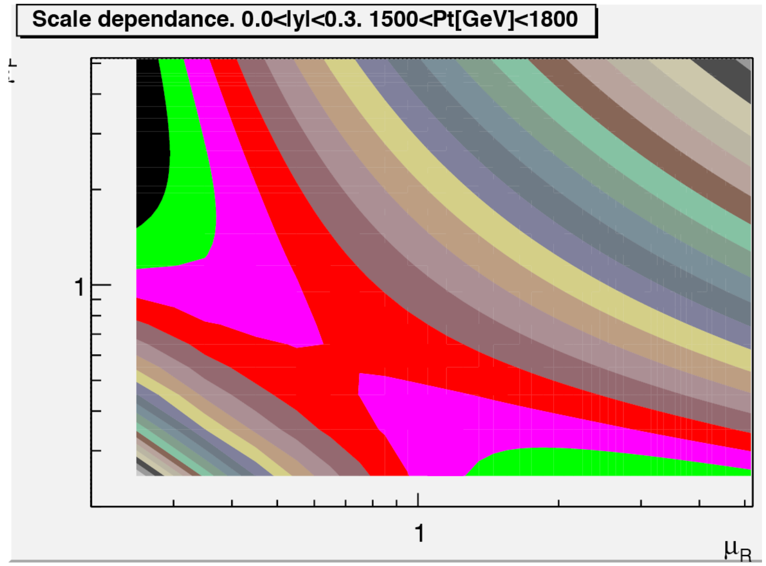
R=0.4
antikT



Scale dependence also depends on jet size; again see equation on previous page

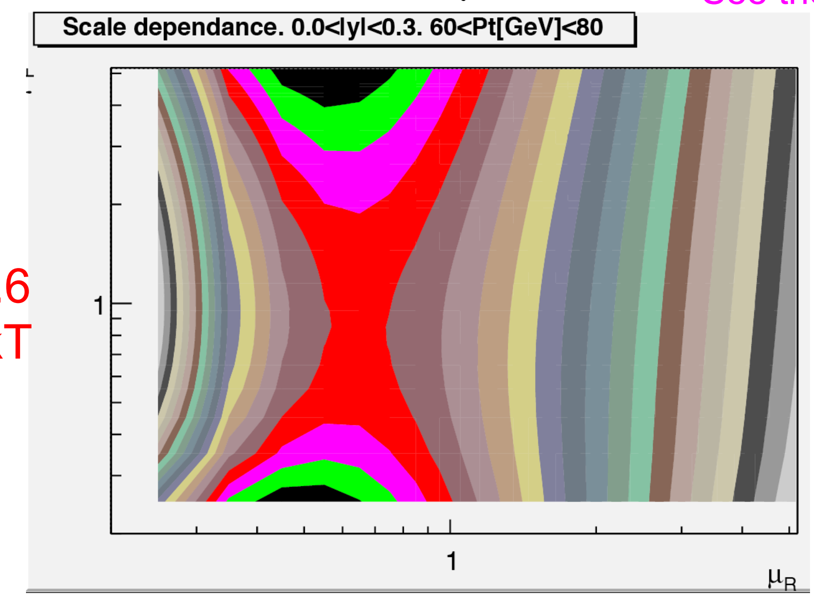
R=0.4
antikT

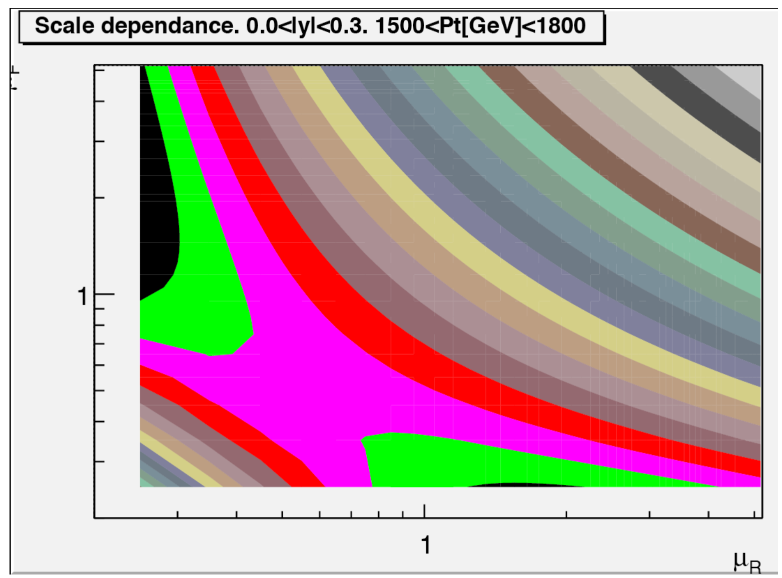




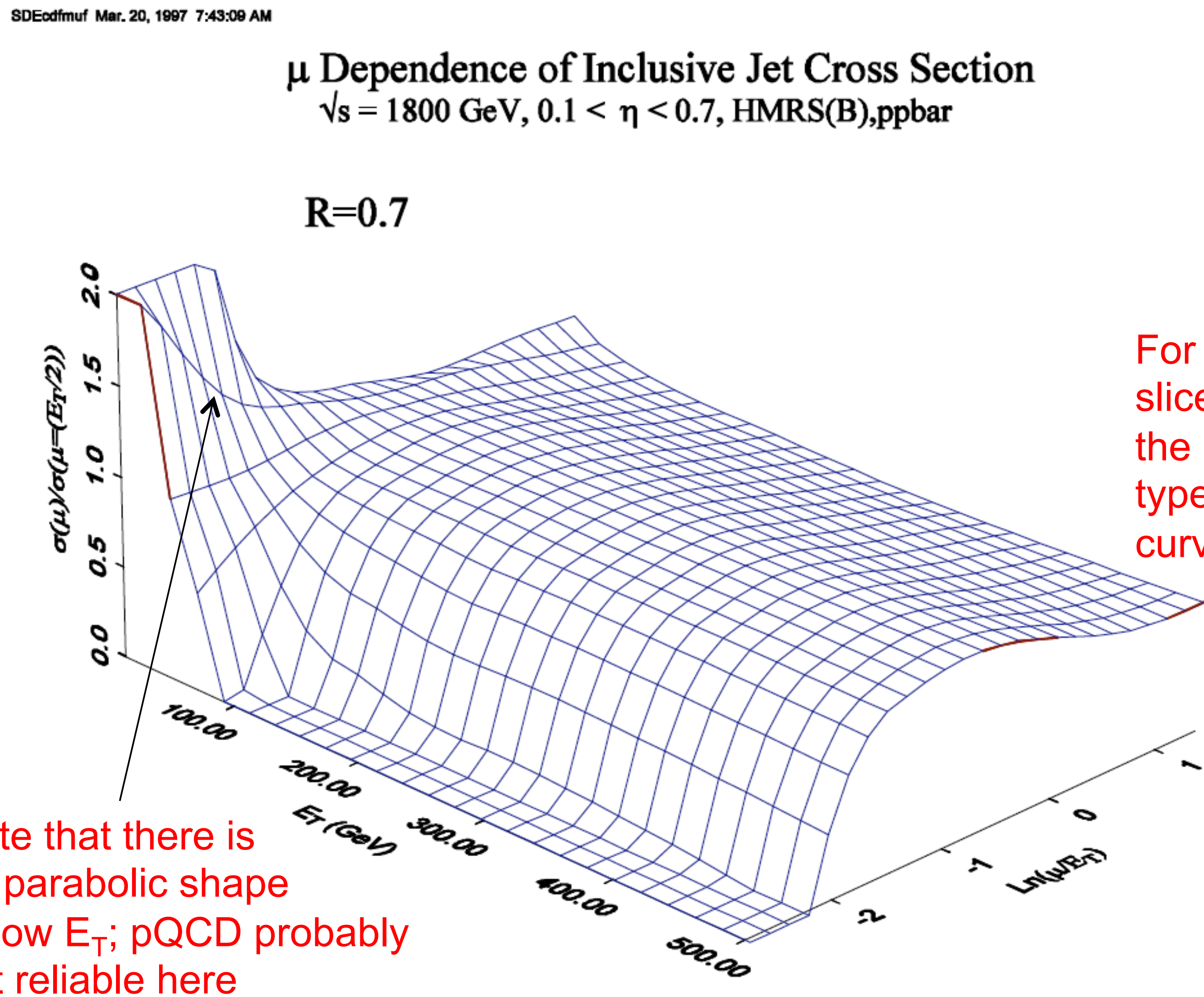
See the shift downwards for larger jet size

R=0.6
antikT





Predictions tend to be more reliable at higher E_T



For fixed E_T slices, note the parabolic type shape for the curve at high E_T

Note that there is no parabolic shape at low E_T; pQCD probably not reliable here

Back to W production to NLO

- In 4-dimensions, the contribution of the real diagrams can be written (ignoring diagrams with incoming gluons for simplicity)

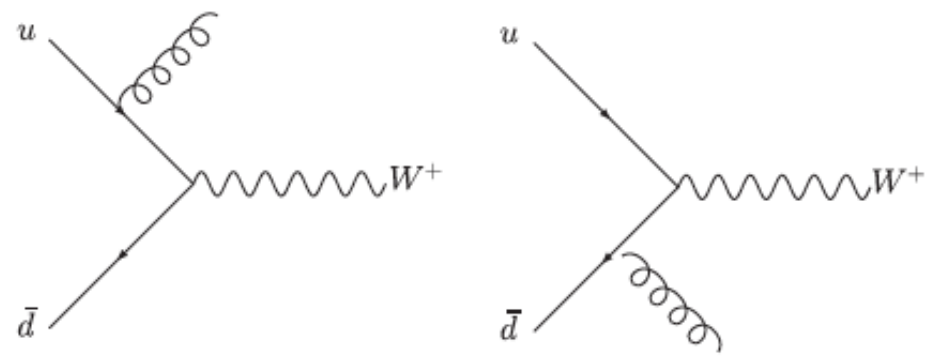
$$\left|M(u\bar{d} \to W^+ g\right|^2 \sim g^2 C_F \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2Q^2\hat{s}}{\hat{u}\hat{t}}\right]$$

$$\sim g^2 C_F \left[\left(\frac{1+z^2}{1-z}\right)\left(\frac{-\hat{s}}{\hat{t}} + \frac{-\hat{s}}{\hat{u}}\right) - 2\right]$$

  - where

$$z = \frac{Q^2}{s} \text{ and } \hat{s} + \hat{t} + \hat{u} = Q^2$$

- Note that the real diagrams contain collinear singularities, $\hat{u}$->0, $\hat{t}$->0, and soft singularities, z->1



…thanks to Keith Ellis for the next few slides

and don’t sweat the details; I just want you to see in general terms how a NLO calculation is carried out

Aside: dimensional regularization

- Suppose we have an integral of the form, typical of the integrals in a NLO calculation

$$I = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{\left(k^2 - m^2\right)^2}$$

- We get infinity if we integrate this in 4 dimensions, so go to 4-2ε dimensions

$$\int \frac{d^4 k}{(2\pi)^4} \to (\mu)^{2\varepsilon} \int \frac{d^{4-2\varepsilon} k}{(2\pi)^{4-2\varepsilon}} \to (\mu)^{2\varepsilon} \int \frac{d\Omega_{4-2\varepsilon}}{(2\pi)^{4-2\varepsilon}} \int dk_E k_E^{\;3-2\varepsilon}$$

$$\int \frac{d\Omega_{4-2\varepsilon}}{(2\pi)^{4-2\varepsilon}} = \frac{2}{(4\pi)^{2-\varepsilon}} \frac{1}{\Gamma(2-\varepsilon)}$$

$$(\mu)^{2\varepsilon} \int_0^\infty dk_E \frac{k_E^{\;3-2\varepsilon}}{\left(k_E^{\;2} + m^2\right)^2} = \frac{(\mu)^{2\varepsilon}}{2(m)^{2\varepsilon}} \int_0^1 dz z^{1-\varepsilon} (1-z)^{\varepsilon - 1} = \frac{1}{2}\left(\frac{\mu}{m}\right)^{2\varepsilon} \frac{\Gamma(\varepsilon)\Gamma(2-\varepsilon)}{\Gamma(2)}$$

- Using

$$\Gamma(1+z) = z\Gamma(z); \Gamma'(1) = -\gamma_E = -0.5772...$$

Dimensional regularization, continued

- Find

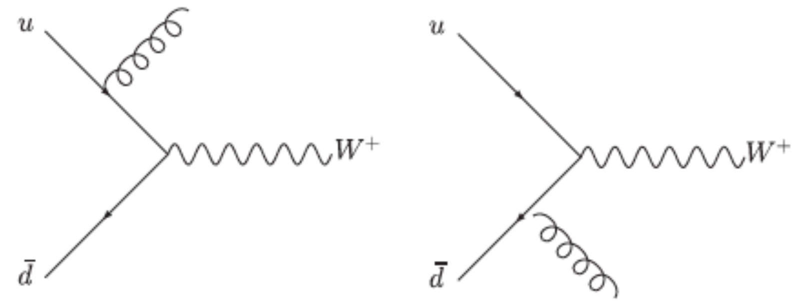
$$I = \frac{\Gamma(\varepsilon)}{(4\pi)^{2-\varepsilon}}\left(\frac{\mu}{m}\right)^{2\varepsilon} \underset{\varepsilon\text{->}0}{\longrightarrow} \frac{1}{(4\pi)^2}\left[+\frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) + 2\ln\left(\frac{\mu}{m}\right) + O(\varepsilon)\right]$$

  - singular bits, plus finite bits as ε->0, plus log singularity as m->0

- Define $\overline{\text{MS}}$ scheme: subtract (absorb) 1/ε pole, γ_E, and ln(4π) bits

Now do the dimension trick for the real part

- Problem: if I work in 4 dimensions, I get divergences
- Solution: working in 4-2ε dimensions, to control the divergences (dimensional reduction)



$$\sigma_{real} = \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c_\Gamma \left[\left(\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} - \frac{\pi^2}{3}\right)\delta(1-z) - \frac{2}{\varepsilon} P_{qq}(z) - 2(1-z) + 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{1-z}\ln z\right]$$

- with

$$c_\Gamma = \frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)}$$

"+ distribution"

$$\left(\frac{\log(1-z)}{1-z}\right)_+ \equiv \lim_{\beta\to 0}\left\{\frac{\log(1-z)}{1-z}\theta(1-z-\beta) + \frac{1}{2}\log^2(\beta)\delta(1-z-\beta)\right\}$$

We get 1/ε terms from individual soft and collinear singularities

We get 1/ε^2 terms for overlapping IR singularities.

Ditto for the virtual part

$$\sigma_{virt} = \delta(1-z)\left[1 + \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c'_\Gamma \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 6 + \pi^2\right)\right]$$

from soft and collinear bits

- where

$$c'_\Gamma = c_\Gamma + O(\varepsilon^3)$$

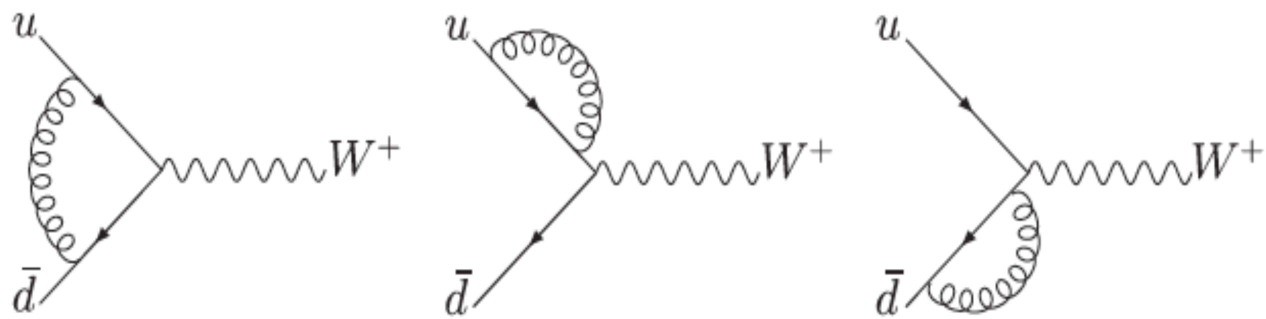


Figure 14. Virtual diagrams included in the next-to-leading order corrections to the Drell–Yan production of a W at hadron colliders.

We also get UV divergences when the loop momenta go off to infinity. The summation of these singularities leads to the running of the strong couplings, i.e. we define the sum of all such contributions (scales $>\mu_{UV}$) as the physical renormalized coupling, α_s.

Now add real and virtual

$$\sigma_{real+virt} = \frac{\alpha_s}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\varepsilon} c_\Gamma \left[\left(\frac{2\pi^2}{3} - 6\right)\delta(1-z) - \frac{2}{\varepsilon} P_{qq}(z) - 2(1-z) + 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{1-z}\ln z\right]$$

- Notice that the ε^2 terms cancel
- The divergences that are proportional to the branching probabilities are universal
- We can factorize them into the parton distributions, performing mass factorization by subtracting the counter-term (MSbar scheme)

$$2\frac{\alpha_s}{2\pi} C_F \left[\frac{-c_\Gamma}{\varepsilon} P_{qq}(z) - (1-z) + \delta(1-z)\right]$$

- To get

$$\hat{\sigma}_{real+virt} = \frac{\alpha_s}{2\pi} C_F \left[\left(\frac{2\pi^2}{3} - 8\right)\delta(1-z) + 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{1-z}\ln z + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2}\right]$$

- Plus a similar correction for incoming gluons
- That works for the total cross section, but we need differential distributions for comparisons to data, so we need a general subtraction procedure at NLO, using Monte Carlo techniques

In general

- That works for the total cross section, but we need differential distributions for comparisons to data, so we need a general subtraction procedure at NLO, using Monte Carlo techniques
- For incoming partons a and b, producing m outgoing partons

$$\sigma_{ab} = \sigma_{ab}^{LO} + \sigma_{ab}^{NLO}$$

$$\sigma_{ab}^{LO} = \int_m d\sigma_{ab}^{Born}$$

$$\sigma_{ab}^{NLO} = \int_{m+1} d\sigma_{ab}^{real} + \int_m d\sigma_{ab}^{virt}$$

the singular parts of the matrix elements for real emission, corresponding to soft and collinear emission, can be isolated in a process independent manner; of course it gets a lot more complicated for large m

- It's too difficult to do this integral, so
- …we have to construct a series of counter-terms

$$d\sigma_{ct} = \sum_{ct} \int_m d\sigma_B \otimes \int_1 dV_{ct}$$

- Where σ_B denotes the appropriate color and spin projection of the Born level cross section, and the counter-terms are independent of the details of the process under consideration

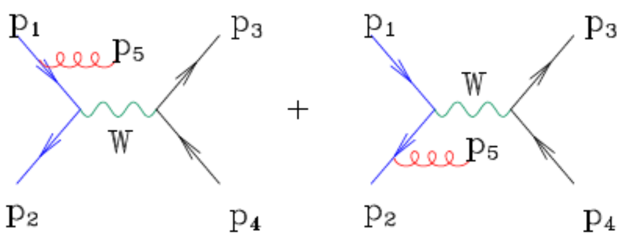
Subtractions

- The counter-terms provide a local approximation for the real emission process, describing the amplitude in the soft and collinear limits
  - the $1/\varepsilon$ and $1/\varepsilon^2$ poles that we were able to explicitly cancel when we were calculating an inclusive cross section
- These counter-terms cancel all non-integrable singularities in $d\sigma^{real}$, so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} \left[d\sigma_{ab}^{real} - d\sigma_{ab}^{ct} \right] + \int_{m+1} d\sigma_{ab}^{ct} + \int_{m} d\sigma_{ab}^{virt}$$

- The phase space integration in the first term can now be performed numerically in 4 dimensions
- The integral in the 2nd term can be done easily and analytically

Consider matrix element counter-event for W production



real corrections to W production at NLO

eikonal factor; an approximation to the full matrix element valid when the gluon is soft (we saw this before)

- In soft limit (p_5->0), we have

$$\left|M_1(p_1,p_2,p_3,p_4,p_5)\right|^2 = g^2 C_F \frac{p_1 \bullet p_2}{p_1 \bullet p_5 \, p_2 \bullet p_5} \left|M_0(p_1,p_2,p_3,p_4)\right|^2$$

- The eikonal factor can be associated with radiation from a given leg by partial fractioning

$$\frac{p_1 \bullet p_2}{p_1 \bullet p_5 \, p_2 \bullet p_5} = \left[\frac{p_1 \bullet p_2}{p_1 \bullet p_5 + p_2 \bullet p_5}\right]\left[\frac{1}{p_1 \bullet p_5} + \frac{1}{p_2 \bullet p_5}\right]$$

- Including the collinear contributions, singular as $p_1 \cdot p_5$->0, the matrix element for the counter-event has the structure

$$\left|M_1(p_1,p_2,p_3,p_4,p_5)\right|^2 = \frac{g^2}{x_a p_1 \bullet p_5} \hat{P}_{qq}(x_a) \left|M_0(p_1,p_2,p_3,p_4)\right|^2$$

- where

$$1 - x_a = \frac{p_1 \bullet p_5 + p_2 \bullet p_5}{p_1 \bullet p_2} \qquad \hat{P}_{qq}(x_a) = C_F \frac{1+x^2}{1-x}$$

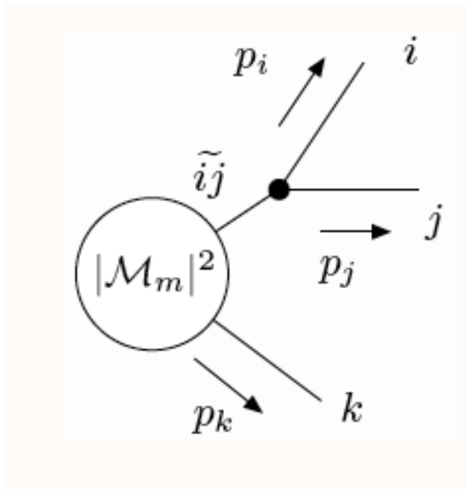
Making an counter-event

- For event $q(p_1) + \overline{q}(p_2) \rightarrow W^+(\nu(p_3) + e^+(p_4)) + g(p_5)$
  - with $p_1 + p_2 = \sum_{i=1}^{5} p_i$
- Generate a counter-event $q(x_a p_1) + \overline{q}(p_2) \rightarrow W^+(\nu(\tilde{p}_3) + e^+(\tilde{p}_4))$
  - with $x_a p_1 + p_2 = \sum_{i=3}^{4} \tilde{p}_i; 1 - x_a = \left(p_1 \bullet p_5 + p_2 \bullet p_5\right) / p_1 \bullet p_2$
- Perform a Lorentz transformation on all j final state momenta $\tilde{p}_j = \Lambda_\nu^\mu p_j^n, j = 3,4$
  - such that $\tilde{p}_j^\mu \rightarrow p_j^\mu$
  - for p_5 collinear or soft
- The longitudinal momentum of p_5 is absorbed by re-scaling with x
- The other components of the momentum p_5 are absorbed by the Lorentz transformation
- A lot of transformations done to get the momenta to work out right

Catani-Seymour dipoles

- The case of constructing counter-terms for W production is particularly simple since the color flow at Born level is trivial
  - only 1 possible spectator
- For more complex final states, have to find Catani-Seymour dipoles
- In the Catani-Seymour approach, the additional soft or collinear parton is emitted from an emitter-spectator pair (called a dipole)
- The emitter and spectator can each be in either the initial or final state, so 4 possible combinations
  - II, IF,FI,FF
- Note: an alternative technique, called phase space slicing, involves using a simpler version of the matrix element in the soft and collinear limits
  - this simpler version of the matrix element can be easily evaluated, and as long as the soft and collinear limits are appropriately chosen, the result is accurate

- The dipole terms describe the limits 3->2 partons
  $$\{i,j,k\} \rightarrow \{\widetilde{ij},\tilde{k}\}$$
- The spectator k ensures 4-momentum conservation and on-mass-shell conditions
  $$p_i + p_j + p_k = p_{\widetilde{ij}} + p_{\tilde{k}}$$
- On mass-shell condition allows the factorization of phase space needed for this calculation
- Emitter and spectator always color-connected

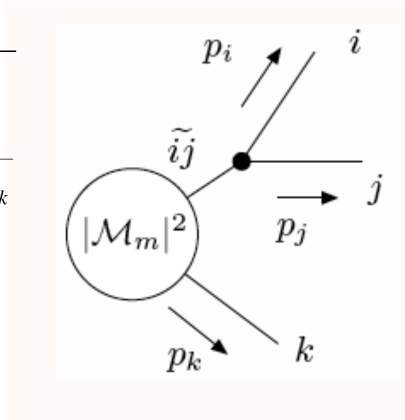


Example: Final-final CS dipole

- The branching shown can be characterized by Lorentz invariant variables

$$y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$$

$$\tilde{z}_i = 1 - \tilde{z}_j = \frac{p_i p_k}{p_i p_k + p_j p_k}$$



- The factorized form of the fully differential (m+1) parton cross section that exactly reproduces the corresponding soft and collinear emissions of the real-emission process is

$$d\hat{\sigma}_{m+1} = d\hat{\sigma}_m \sum_{ij} \sum_{k \neq ij} \frac{dy_{ij,k}}{y_{ij,k}} d\hat{z}_i \frac{d\phi_i}{2\pi} \frac{\alpha_s}{2\pi} \frac{1}{N_{ij}^{spec}} \left(1 - y_{ij,k}\right) \left\langle V_{ij,k}\left(\tilde{z}_i, y_{ij,k}\right)\right\rangle$$

1,2 depending on
possible spectators

- The spin-averaged splitting kernels <V$_{ij,k}$> for the branchings q->qg,g->gg, g->qqbar are

$$\left\langle V_{q_i g_j k}\left(\tilde{z}_i, y_{ij,k}\right)\right\rangle = C_F \left[\frac{2}{1 - \tilde{z}_i + \tilde{z}_i y_{ij,k}} - \left(1 + \tilde{z}_i\right)\right]$$

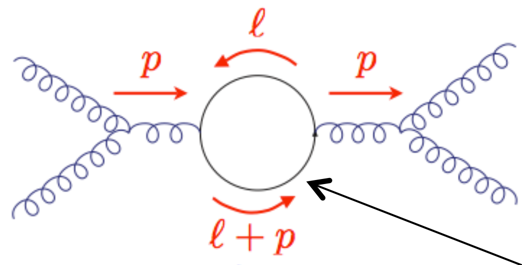
$$\left\langle V_{g_i g_j k}\left(\tilde{z}_i, y_{ij,k}\right)\right\rangle = 2C_A \left[\frac{1}{1 - \tilde{z}_i + \tilde{z}_i y_{ij,k}} + \frac{1}{\tilde{z}_i + y_{ij,k} - \tilde{z}_i y_{ij,k}} - 2 + \tilde{z}_i\left(1 - \tilde{z}_i\right)\right]$$

$$\left\langle V_{q_i g_j, k}\left(\tilde{z}_i, y_{ij,k}\right)\right\rangle = T_R\left[1 - 2\tilde{z}_i(1 - \tilde{z}_i)\right]$$

Note that these terms look a lot like parton shower branchings

UV singularities

- What to do about UV singularities?
- Consider loop correction to dijet production?
- Have to integrate over loop momentum
- Problems result for large loop momenta; this is called an ultraviolet singularity
- Use dimensional regularization; 4->4-2ε
  - here, though, we would like ε to be +; for IR divergences would like it to be -
- QCD is a renormalizable theory which means that this singularity can be absorbed into the running of $\alpha_s(Q^2)$
- μ is the renormalization scale; in 1st lecture we switched to using Λ



$$\int \frac{d^4\ell}{(2\pi)^4} \frac{1}{\ell^2(\ell+p)^2}$$

2 propagators mom^2 for each

$$\int \frac{d^4\ell}{(2\pi)^4} \frac{1}{\ell^2(\ell+p)^2} \sim \frac{1}{(2\pi)^4} \int \frac{|\ell|^3 d|\ell|}{(\ell^2)^2} \sim \log(|\ell|)$$

$$\int \frac{d^{4-2\epsilon}\ell}{(2\pi)^{4-2\epsilon}} \frac{1}{\ell^2(\ell+p)^2} \sim \frac{1}{(2\pi)^{4-2\epsilon}} (p^2)^{-\epsilon} \int \frac{d|\ell|}{|\ell|^{1+2\epsilon}} \sim \frac{(p^2)^{-\epsilon}}{\epsilon}$$

this must be the factor, by dimension counting

for ϵ>0, i.e. less than 4 dim.

Quantum Chromodynamics - John Campbell -

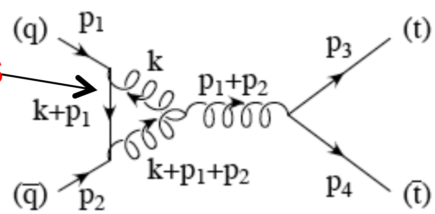
$$\frac{(p^2)^{-\epsilon}}{\epsilon} \longrightarrow \frac{(p^2/\mu^2)^{-\epsilon}}{\epsilon} = \frac{1}{\epsilon} - \log(p^2/\mu^2)$$

…from John Campbell's Fermi SS lectures

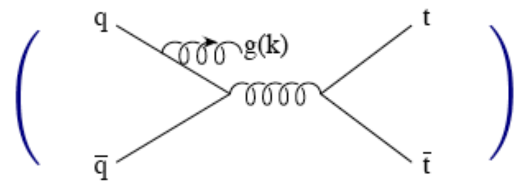
Another example: tt production

Example. Consider the vertex correction:

mom^2 for 3 legs



$\leftrightarrow$



the (unpolarized) amplitude associated to this diagram is of the form:

$$\mathcal{A} \propto \int \frac{d^dk}{(2\pi)^d} \frac{\bar{v}(p_2)\gamma_\rho(\not{k}+\not{p}_1)\gamma_\nu u(p_1)}{k^2(k+p_1)^2(k+p_1+p_2)^2} \frac{\bar{u}(p_3)\gamma_\mu v(p_4)}{(p_1+p_2)^2} V^{\mu\nu\rho}(-p_1-p_2,-k,k+p_1+p_2$$

and depends on the following scalar/tensor integrals:

so it's the tensor term in the numerator that causes the divergence

$$C_0, C_1^\mu, C_2^{\mu\nu}(p_1,p_2) = \int \frac{d^dk}{(2\pi)^d} \frac{1, k^\mu, k^\mu k^\nu}{k^2(k+p_1)^2(k+p_1+p_2)^2} = \int \frac{d^dk}{(2\pi)^d} \frac{1, k^\mu, k^\mu k^\nu}{D_3(p_1,p_2)}$$

$$\frac{\text{mom}^6}{\text{mom}^6}$$

$C_2^{\mu\nu}$ is UV divergent, while all of them are IR divergent, as can be easily recognized by simple power counting and observing that

$$D_3(p_1,p_2) \stackrel{k\to k-p_1}{\longrightarrow} k^2(k-p_1)^2(k+p_2)^2 \stackrel{k^2\simeq 0}{\longrightarrow} k^2(k\cdot p_1)(k\cdot p_2)$$

- $k^0 \to 0$: soft divergence;
- $k\cdot p_1 \to 0$ or $k\cdot p_2 \to 0$: collinear divergence.

Using standard d-dimensional integrals:

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 - \Delta)^n} = i \frac{(-1)^n}{(4\pi)^{d/2}} \frac{\Gamma(n - d/2)}{\Gamma(n)} \frac{1}{\Delta^{n-d/2}}$$

and upon integration over the Feynman parameters one gets (including couplings and color factor):

$$\frac{\alpha_s}{4\pi} \left(\frac{4\pi\mu}{\hat{s}}\right)^\epsilon \frac{N}{2} \Gamma(1+\epsilon) \left(-\frac{4}{\epsilon_{IR}} + \frac{3}{\epsilon_{UV}} - 2\right) \mathcal{A}_0(q\bar{q} \to t\bar{t})$$

NLO calculations

- Programs that do NLO calculations, such as MCFM, are parton-level Monte Carlo generators in which (weighted) events and counter-events are generated
  - for complicated processes, such as W + 2 jets, there can be many counter-events (24), corresponding to the Catani-Seymour subtraction terms, for each event (other codes have calculated W+n jets at NLO, with n up to 5, so even more; see discussion of Blackhat+Sherpa)
  - only the sum of all events (events + counter-events) is meaningful, since many positive and negative weights need to cancel against each other; if too few events are generated, or if the binning is too small, can have negative results
  - a great deal of progress has been made in recent years towards the inclusion of NLO calculations into parton shower Monte Carlos like
    - MC@NLO
    - Powheg
    - Sherpa

Thomas Binoth 1965-2010

- This accord should make the kinds of discussion we're having here easier (in the future)
- Binoth Les Houches Accord

ABSTRACT: Many highly developed Monte Carlo tools for the evaluation of cross sections based on tree matrix elements exist and are used by experimental collaborations in high energy physics. As the evaluation of one-loop matrix elements has recently been undergoing enormous progress, the combination of one-loop matrix elements with existing Monte Carlo tools is on the horizon. This would lead to phenomenological predictions at the next-to-leading order level. This note summarises the discussion of the next-to-leading order multi-leg (NLM) working group on this issue which has been taking place during the workshop on Physics at TeV colliders at Les Houches, France, in June 2009. The result is a proposal for a standard interface between Monte Carlo tools and one-loop matrix element programs.

Dedicated to the memory of, and in tribute to, Thomas Binoth, who led the effort to develop this proposal for Les Houches 2009. Thomas led the discussions, set up the subgroups, collected the contributions, and wrote and edited this paper. He made a promise that the paper would be on the arXiv the first week of January, and we are faithfully fulfilling his promise. In his honor, we would like to call this the Binoth Les Houches Accord.
The body of the paper is unchanged from the last version that can be found on his webpage `http://www.ph.ed.ac.uk/~binoth/NLOLHA_CURRENT_VERSION.pdf`

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A proposal for a standard interface between Monte Carlo tools and one-loop programs

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Thomas was a long-time friend for all at LAPP, and for me.

MCFM

- Many processes available at LO and NLO
  - note these are partonic level only
- Option for ROOT output (see later)
- mcfm.fnal.gov

$p\bar{p} \to W^{\pm}/Z$	$p\bar{p} \to W^{+} + W^{-}$
$p\bar{p} \to W^{\pm} + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^{\pm} + \gamma$	$p\bar{p} \to W^{\pm}/Z + H$
$p\bar{p} \to W^{\pm} + g^{\star}\,(\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^{\pm}/Z + 1$ jet	$p\bar{p} \to W^{\pm}/Z + 2$ jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H + 1$ jet (2 jets now)
$p\bar{p}(VV) \to H + 2$ jets	$p\bar{p} \to t + X$
$pp \to t + W$	

State of the art

Relative order	2->1	2->2	2->3	2->4	2->5	2->6
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		*NNLO*	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO
α_s^6						NLO

- LO: well under control, even for multiparticle final states
- NLO: well understood for 2->1, 2->2, 2->3, 2->4 (W/Z+3 jets, ttbb,WWbb,tttt,…); 2->5 (W+4 jets) and even 2->5=6 (W+5 jets)
  - for W+4 jets, the complaint is that the tree level, not the virtual, calculations are causing most of the difficulties (working with all of the Catani-Seymour terms)
- NNLO: known for inclusive and exclusive 2->1 (i.e. Higgs, Drell-Yan); work on 2->2 (dijet, Z/Higgs + 1 jet)

An experimenter's wishlist

Run II Monte Carlo Workshop

Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		single top
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

Realistic wishlist

- Was developed at Les Houches in 2005, and expanded in 2007 and 2009
- Calculations that are important for the LHC AND do-able in finite time
- In 2009, we added tttt, Wbbj, W/Z+4j plus an extra column for each process indicating the level of precision required by the experiments
  - to see for example if EW corrections may need to be calculated
- In order to be most useful, decays for final state particles (t,W,H) need to be provided in the codes as well

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [4, 5]; Campbell/Ellis/Zanderighi [6]. ZZjet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [7]
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [8]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [9, 10]
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4. $pp \to t\bar{t}b\bar{b}$	relevant for $t\bar{t}H$ computed by Bredenstein/Denner/Dittmaier/Pozzorini [14, 15] and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [16]
5. $pp \to V$+3jets	calculated by the Blackhat/Sherpa [17] and Rocket [18] collaborations
Calculations remaining from Les Houches 2005	
6. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$ computed by Bevilacqua/Czakon/Papadopoulos/Worek [19]
7. $pp \to VV\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
8. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [20–22]
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	$q\bar{q}$ channel calculated by Golem collaboration [23]
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10. $pp \to V$+4 jets	top pair production, various new physics signatures
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13. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
14. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
15. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
16. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Table 1: The updated experimenter's wishlist for LHC processes

Realistic wishlist

- Until recently, W+4 jets was calculated only to leading color
- subleading color terms (suppressed by $1/N_c$) are only a few percent, but are hardest to calculate

…we saw earlier for Wgg color-suppressed terms for the real part

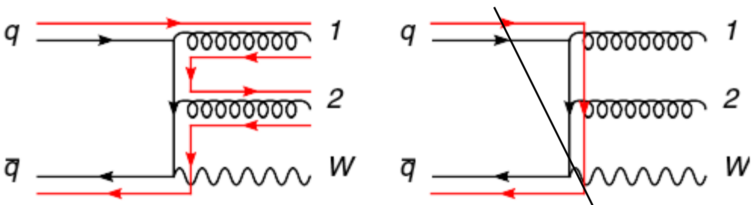

Figure 12. Two examples of colour flow in a $W+2$ jet event, shown in red. In the left-hand diagram, a leading colour flow is shown. The right-hand diagram depicts the sub-leading colour flow resulting from interference.

$$|\mathcal{M}^{q\bar{q}\to Wgg}|^2 \xrightarrow{\text{soft}} \frac{C_F N^2}{2}\left[[q\ p_2]+[p_2\ \bar{q}]-\frac{1}{N^2}[q\ \bar{q}]\right]\mathcal{M}^{q\bar{q}\to Wg}$$

…in future calcs may be best to approximate virtual subleading color terms

Realistic wishlist

- With the recent calculation of tttt, all processes on the wishlist have been calculated
- The wishlist has been retired since new techniques allow for the semi-automatic generation of new (reasonable) NLO cross sections

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Table 1: The updated experimenter's wishlist for LHC processes

Realistic wishlist

- 4 top final state

arXiv:1206.3064v1 [hep-ph] 14 Jun 2012

Constraining BSM Physics at the LHC: Four top final states with NLO accuracy in perturbative QCD

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E-mail: bevilacqua@physik.rwth-aachen.de, worek@physik.uni-wuppertal.de

Abstract: Many theories, from Supersymmetry to models of Strong Electroweak Symmetry Breaking, look at the production of four top quarks as an interesting channel to evidentiate signals of new physics beyond the Standard Model. The production of four-top final states requires large partonic energies, above the $4m_t$ threshold, that are available at the CERN Large Hadron Collider and will become more and more accessible with increasing energy and luminosity of the proton beams. A good theoretical control on the Standard Model background is a fundamental prerequisite for a correct interpretation of the possible signals of new physics that may arise in this channel. In this paper we report on the calculation of the next-to-leading order QCD corrections to the Standard Model process $pp \to t\bar{t}t\bar{t} + X$. As it is customary for such studies, we present results for both integrated and differential cross sections. A judicious choice of a dynamical scale allows us to obtain nearly constant $\mathcal{K}$-factors in most distributions.

Keywords: NLO Computations, Heavy Quark Physics, Standard Model, Beyond Standard Model

WUB/12-12, TTK-12-22

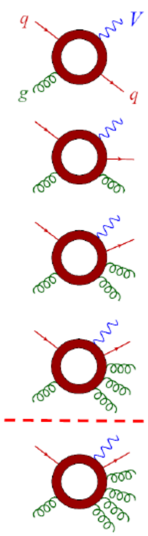
Realistic wishlist

- There's a limit as to how far Feynman diagram techniques can take you

One-loop QCD amplitudes via Feynman diagrams

For $V + n$ jets (maximum number of external gluons only)

# of jets	# 1-loop Feynman diagrams
1	11
2	110
3	1,253
4	16,648
5	256,265



Motivates "on-shell" methods, which exploit unitarity to reduce loop amplitudes to products of tree amplitudes

L. Dixon (N)NLO Future ECT* Trento 27 Sept 2010 18

- Basically everything from 5-12 has been done with on-shell methods
- See Lance's talk at Trento for example

indico.cern.ch/conferenceDisplay.py?confId=93790

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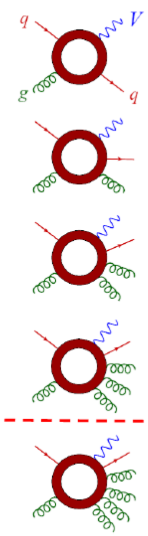
For $V + n$ jets (maximum number of external gluons only)



# of jets	# 1-loop Feynman diagrams
1	11
2	110
3	1,253
4	16,648
5	256,265

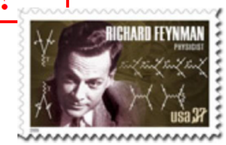
Motivates "on-shell" methods, which exploit unitarity to reduce loop amplitudes to products of tree amplitudes

L. Dixon (N)NLO Future ECT* Trento 27 Sept 2010 18

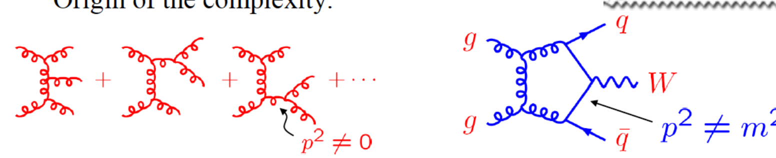
- Basically everything from 5-12 has been done with on-shell methods
- See Lance's talk at Trento for example

indico.cern.ch/conferenceDisplay.py?confId=93790

Why are Feynman diagrams clumsy for high loop or multiplicity processes?



- Vertices and propagators involve gauge-dependent off-shell states. Origin of the complexity.



- To get at root cause of the trouble we must rewrite perturbative quantum field theory.

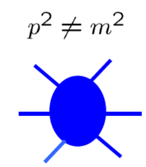
- **All steps should be in terms of gauge invariant on-shell states. $p^2 = m^2$ On shell formalism.**
- **Radical rewrite of gauge theory needed.**

10

Off-shell Formalisms

In graduate school you learned that scattering amplitudes need to be calculated using unphysical gauge dependent quantities: off-shell Green functions

Standard machinery:

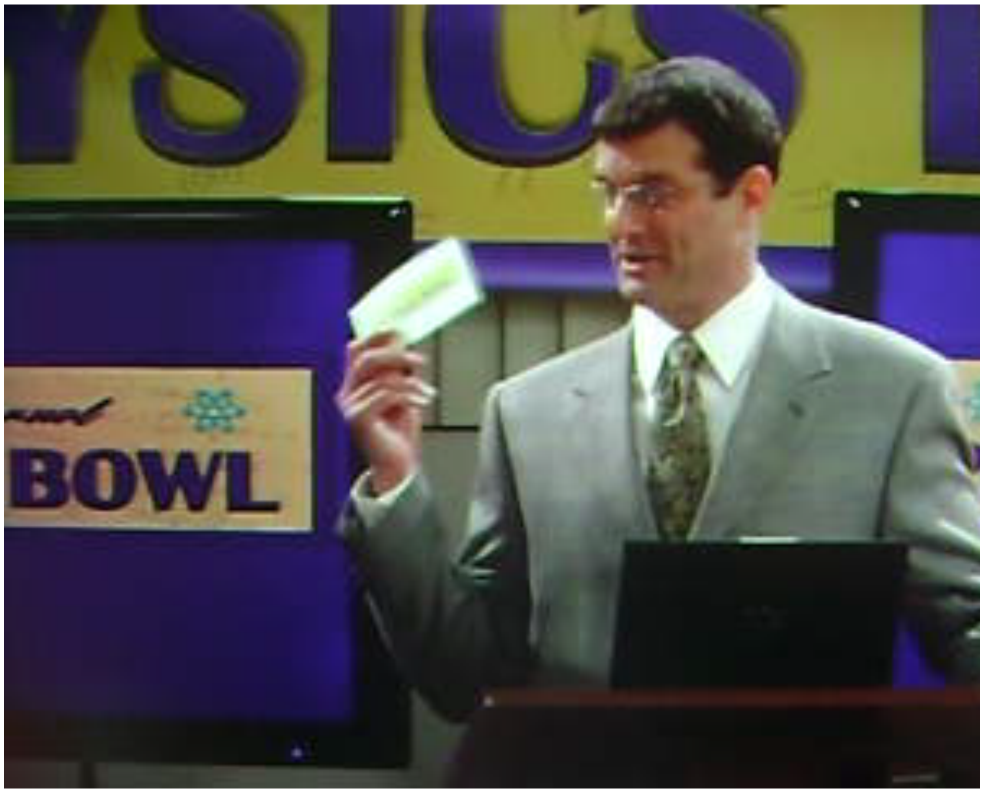


- **Fadeev-Popov procedure for gauge fixing.**
- **Taylor-Slavnov Identities.**
- **BRST.**
- **Gauge fixed Feynman rules.**
- **Batalin-Fradkin-Vilkovisky quantization for gravity.**
- **Off-shell constrained superspaces.**

We won't need any of this. We will reformulate perturbative quantum field theory in terms of on-shell quantities.

11

If all else fails…

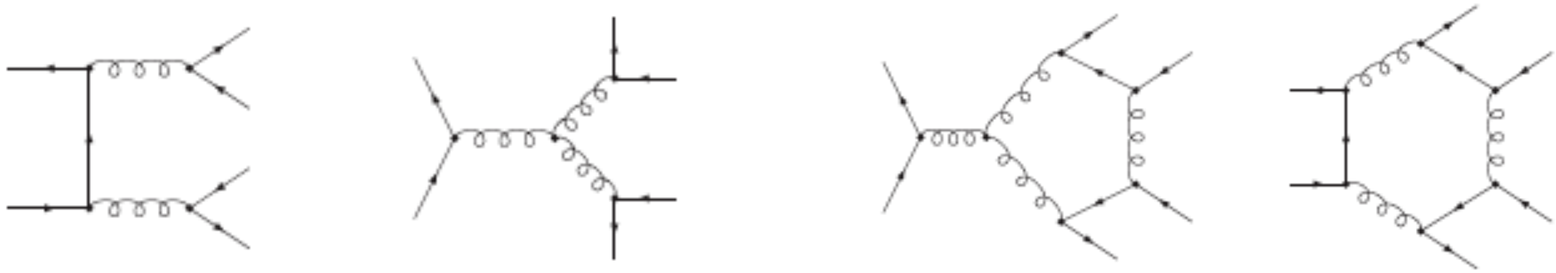


Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [4, 5]; Campbell/Ellis/Zanderighi [6]. ZZjet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [7]
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [8]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [9, 10]
3. $pp \to VVV$	ZZZ completed by Lazopoulos/Melnikov/Petriello [11] and WWZ by Hankele/Zeppenfeld [12] (see also Binoth/Ossola/Papadopoulos/Pittau [13])
4. $pp \to t\bar{t}b\bar{b}$	relevant for $t\bar{t}H$ computed by Bredenstein/Denner/Dittmaier/Pozzorini [14, 15] and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [16]
5. $pp \to V$+3jets	calculated by the Blackhat/Sherpa [17] and Rocket [18] collaborations
Calculations remaining from Les Houches 2005	
6. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$ computed by Bevilacqua/Czakon/Papadopoulos/Worek [19]
7. $pp \to VV\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
8. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [20–22]
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	$q\bar{q}$ channel calculated by Golem collaboration [23]
NLO calculations added to list in 2009	
10. $pp \to V$+4 jets	top pair production, various new physics signatures
11. $pp \to Wb\bar{b}j$	top, new physics signatures
12. $pp \to t\bar{t}t\bar{t}$	various new physics signatures
Calculations beyond NLO added in 2007	
13. $gg \to W^*W^*\ \mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
14. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
15. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
16. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Table 1: The updated experimenter's wishlist for LHC processes

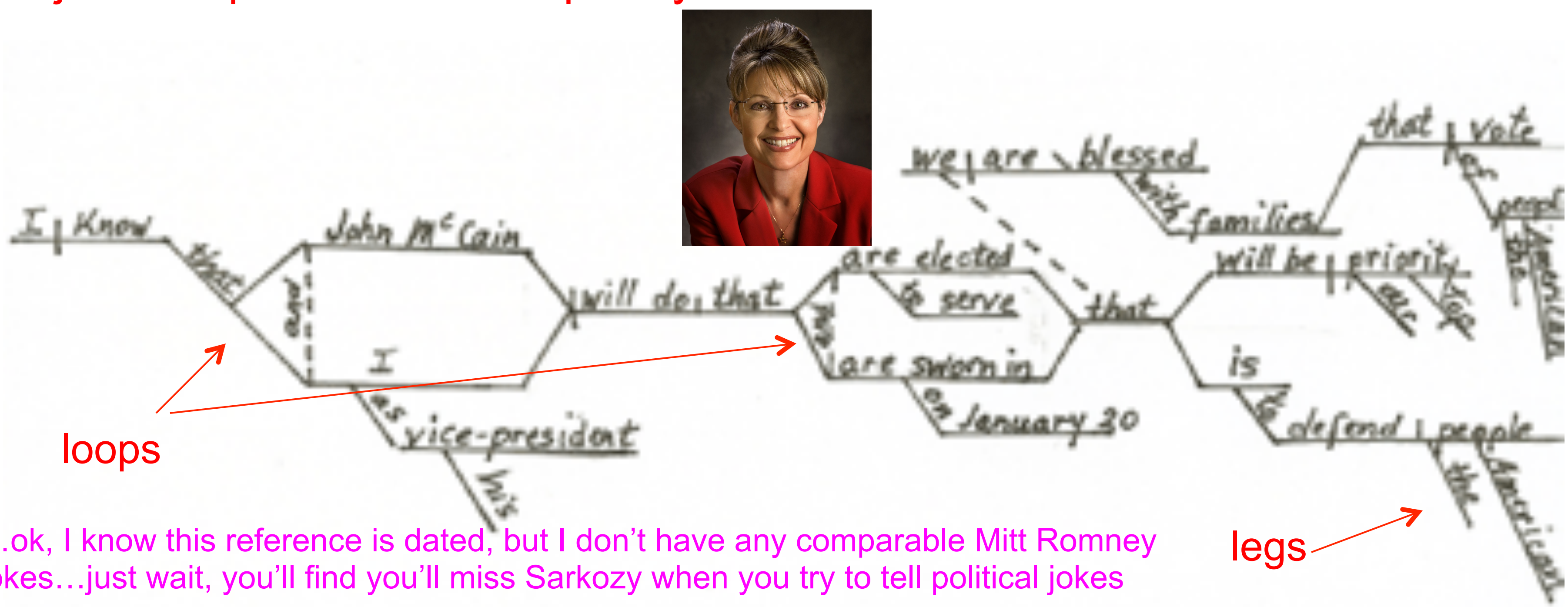
Loops and legs

2->4 is very impressive



but just compare to the complexity of the sentences that Sarah Palin uses

.



loops

legs

…ok, I know this reference is dated, but I don't have any comparable Mitt Romney jokes…just wait, you'll find you'll miss Sarkozy when you try to tell political jokes

What's next for the Les Houches NLO wishlist?

- Nothing: as I said, it's being retired
- It's being replaced by a NNLO wishlist plus a wishlist for EW corrections for hard processes

Below we construct a table of calculations needed at the LHC, and which are feasible within the next few years. Certainly, results for inclusive cross sections at NNLO will be easier to achieve than differential distributions, but most groups are working towards a partonic Monte Carlo program capable of producing fully differential distributions for measured observables.

- $t\bar{t}$ production: qqbar->ttbar at NNLO now finished (arXiv:1205.5201)
  needed for accurate background estimates, top mass measurement, top quark asymmetry (which is zero at tree level, so NLO is the leading non-vanishing order for this observable, and a discrepancy of theory predictions with Tevatron data needs to be understood). Several groups are already well on the way to complete NNLO results for $t\bar{t}$ production [84, 85, 86, 87].
- W^+W^- production:
  importand background to Higgs search. At the LHC, $gg \to WW$ is the dominant subprocess, but $gg \to WW$ is a loop-induced process, such that two loops need to be calculated to get a reliable estimate of the cross section. Advances towards the full two-loop result are reported in [88, 89].
- inclusive jet/dijet production:
  NNLO parton distribution function (PDF) fits are starting to become the norm for predictions and comparisons at the LHC. Paramount in these global fits is the use of inclusive jet production to tie down the behavior of the gluon distribution, especially at high x. However, while the other essential processes used in the global fitting are known to NNLO, the inclusive jet production cross section is only known at NLO. Thus, it is crucial for precision predictions for the LHC for the NNLO corrections for this process to be calculated, and to be available for inclusion in the global PDF fits. First results for the real-virtual and double real corrections to gluon scattering can be found in [90, 91].

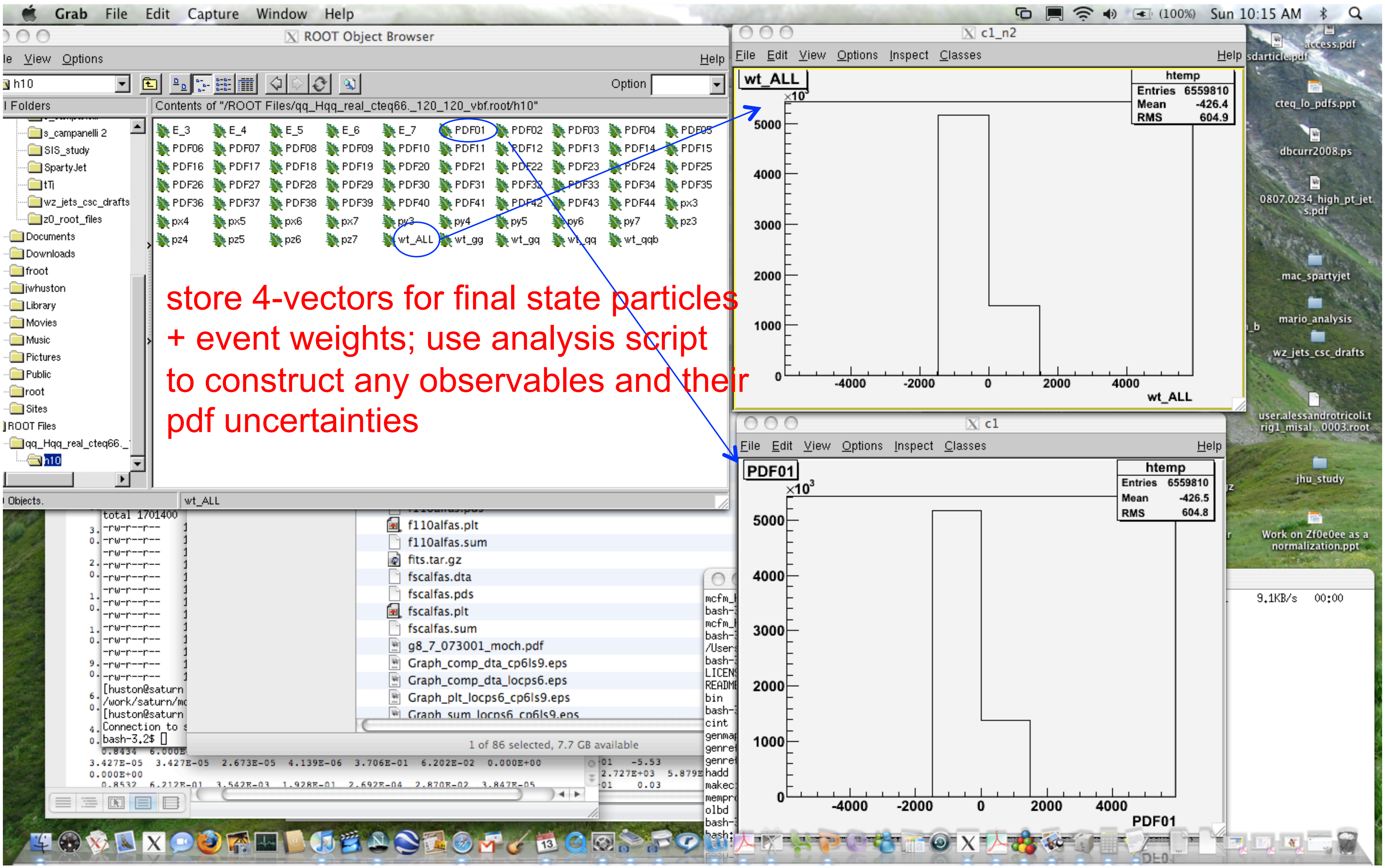
NNLO wishlist: continued

- V+1 jet production:
  $W/Z/\gamma$ + jet production form the signal channels (and backgrounds) for many key physics processes, for both SM and BSM. In addition, they also serve as calibration tools for the jet energy scale and for the crucial understanding of the missing transverse energy resolution. The two-loop amplitudes for this process are known [92, 93], therefore it can be calculated once the parts involving unresolved real radiation are available.
- V+γ production:
  important signal/background processes for Higgs and New Physics searches. The two-loop helicity amplitudes for $q\bar{q} \to W^{\pm}\gamma$ and $q\bar{q} \to Z^{0}\gamma$ recently have become available [94].
- Higgs+1 jet production:
  As mentioned previously, events in many of the experimental Higgs analyses are separated by the number of additional jets accompanying the Higgs boson. In many searches, the Higgs + 0 jet and Higgs + 1 jet bins contribute approximately equally to the sensitivity. It is thus necessary to have the same theoretical accuracy for the Higgs + 1 jet cross section as already exists for the inclusive Higgs cross section, i.e. NNLO. The two-Loop QCD Corrections to the Helicity Amplitudes for $H \to 3$ partons are already available [95].

Editorial Comment

- Once we have the calculations, how do we (experimentalists) use them?
- If a theoretical calculation is done, but it can not be used by any experimentalists, does it make a sound?
- We need public programs and/or public ntuples
- Oftentimes, the program is too complex to be run by non-authors
- In that case, ROOT ntuples may be the best solution

MCFM has ROOT output built in;
standard Les Houches format has been developed



store 4-vectors for final state particles + event weights; use analysis script to construct any observables and their pdf uncertainties

NLO with BlackHat+Sherpa

NLO cross section

Born — loop: lc and fmlc — real

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

vsub

BlackHat

so this is not Sherpa the parton shower, but Sherpa used as a (very efficient) fixed order matrix element generator

Sherpa

How it’s put together

NLO with BlackHat+Sherpa

NLO cross section

Born — loop: lc and fmlc — real

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

vsub

for W+3 jets,
W+3 parton tree-level
matrix elements

the dipole subtraction terms evaluated in n-body phase space; to make matters more complex, vsub can be either + or -, compensated by other terms in the total cross section; note the sum over all quarks and antiquarks; makes matters more complex when coming to scale uncertainties

all of the real emission terms, (W+4 partons for W + 3 jets), modified by the dipole subtraction terms; divergences are gone

all of the virtual terms, both leading color and full-minus-leading color; the latter is typically a few % effect, but much of the complexity of the calculation

How it’s put together

NLO with BlackHat+Sherpa

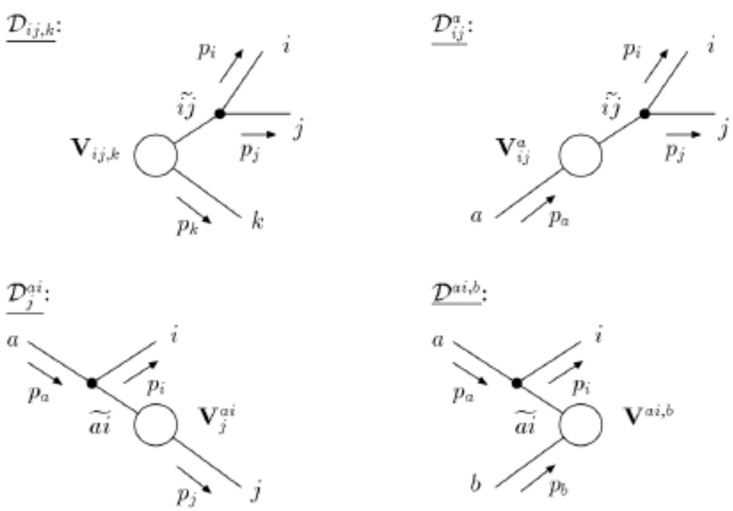
NLO cross section

Born — loop: lc and fmlc — real

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

vsub

possible Catani-Seymour dipoles, for FF, FI,IF and II situations



the dipole subtraction terms evaluated in n-body phase space; to make matters more complex, vsub can be either + or -, compensated by other terms in the total cross section; note the sum over all quarks and antiquarks

all of the real emission terms, (W+4 partons for W + 3 jets), modified by the dipole subtraction terms; divergences are gone

many counterevents due to C-S dipoles that are correlated; have to use special weights/procedures to get correct statistical error

note the need for a 3rd parton, the ‘spectator’; in the soft limit, it’s the color partner

ROOT ntuples

- More complex to use than MCFM
  - no manual for example
  - and you don't produce the events yourself
- ntuples produced separately by Blackhat + Sherpa for →
  - so TB's of disk space
- No jet clustering has been performed; that's up to the user
  - a difference from MCFM, where the program has to be re-run for each jet size/algorithm
- What algorithms/jet sizes that can be run depends on how the files were generated
  - i.e. whether the right counter-events are present
- For the files on the right at 7 TeV (for W^+ + 3 jets), one can use kT, antikT, siscone (f=0.75) for jet sizes of 0.4, 0.5, 0.6 and 0.7

- bornLO (stands alone for pure LO comparisons; not to be added with other contributions below)
  - 20 files, 5M events/file, 780 MB/file
- Born
  - 18 files, 5M events/file, 750 MB/file
- loop-lc (leading color loop corrections)
  - 398 files, 100K events/file, 19 MB/file
- loop-fmlc (needed for full color loop corrections)
  - 399 files, 15K events/file, 3 MB/file
- real (real emission terms)
  - 169 files, 2.5 M event/file, 5 GB/file
- vsub (subtraction terms)
  - 18 files, 10M events/file, 2.8 GB/file

Jet Clustering

- For jet clustering, we use SpartyJet, and store the jet results in SJ ntuples
  - and they tend to be big since we store the results for multiple jet algorithms/ sizes
- Then we friend the Blackhat +Sherpa ntuples with the SpartyJet ntuples producing analysis ntuples (histograms with cuts) for each of the event categories
- Add all event category histograms together to get the plots of relevant physical observables

http://projects.hepforge.org/spartyjet/

arXiv:1201.3617 (manual)

SpartyJet is a set of software tools for jet finding and analysis, built around the FastJet library of jet algorithms. SpartyJet provides four key extensions to FastJet: a simple Python interface to most FastJet features, a powerful framework for building up modular analyses, extensive input file handling capabilities, and a graphical browser for viewing analysis output and creating new on-the-fly analyses.

Branches in ntuple

branch name	type	Notes
id	I	id of the event. Real events and their associated counterterms share the same id. This allows for the correct treatment of statistical errors.
nparticle	I	number of particles in the final state
px	F[nparticle]	array of the x components of the final state particles
py	F[nparticle]	array of the y components of the final state particles
pz	F[nparticle]	array of the z components of the final state particles
E	F[nparticle]	array of the energy components of the final state particles
alphas	D	$alpha_s$ value used for this event
kf	I	PDG codes of the final state particles
weight	D	weight of the event
weight2	D	weight of the event to be used to treat the statistical errors correctly in the real part
me_wgt	D	matrix element weight, the same as weight but without pdf factors
me_wgt2	D	matrix element weight, the same as weight2 but without pdf factors
x1	D	fraction of the hadron momentum carried by the first incoming parton
x2	D	fraction of the hadron momentum carried by the second incoming parton
x1p	D	second momentum fraction used in the integrated real part
x2p	D	second momentum fraction used in the integrated real part
id1	I	PDG code of the firt incoming parton
id2	I	PDG code of the second incoming parton
fac_scale	D	factorization scale used
ren_scale	D	renormalization scale used
nuwgt	I	number of additional weights
usr_wgts	D[nuwgt]	additional weights needed to change the scale

Reweighting

can reweight each event to new

- -PDF
- -factorization scale
- -renormalization scale
- -α_s (tied to the relevant PDFs)

based on weights stored in ntuple (and linking with LHAPDF)

so, for example, the events were generated with CTEQ6, and were re-weighted to CTEQ6.6

2.1 Born and real contributions

The new weight is given by

$$w = \mathbf{me\_wgt2} \cdot f(\mathbf{id1}, \mathbf{x1}, \mu_F) F(\mathbf{id2}, \mathbf{x2}, \mu_F) \frac{\alpha_s(\mu_R)^n}{(\mathbf{alphas})^n} \quad (1)$$

with μ_F the new factorization scale, μ_R the new factorization scale, f the new PDF, α_s the correponding running coupling and n the number of strong coupling (the number of jets n_j for the born contribution and $n_j + 1$ for the real contribution). If the factorization scale is not changed, one can simplify the computation (and save the pdf function call):

$$w = \mathbf{weight2} \frac{\alpha_s(\mu_R)^n}{(\mathbf{alphas})^n} \quad (2)$$

2.2 Virtual contribution

The virtal contribution is treated like the real and born contribution, but the matrix element has a dependence on the renormalization scale parametrized using the additional weights **usr_wgts**.

$$w = m \cdot f(\mathbf{id1}, \mathbf{x1}, \mu_F) F(\mathbf{id2}, \mathbf{x2}, \mu_F) \frac{\alpha_s(\mu_R)^n}{(\mathbf{alphas})^n} \quad (3)$$

$$m = \mathbf{me\_wgt2} + l\,\mathbf{usr\_wgts}[0] + \frac{l^2}{2}\mathbf{usr\_wgts}[1] \quad (4)$$

$$l = \log\left(\frac{\mu_R^2}{\mathbf{ren\_scale}^2}\right) \quad (5)$$

Reweighting, cont.

2.3 Integrated subtraction

The computation of the new weight for the integrated subtraction is the most complicated. The ROOT file has 16 additional weights to make this possible.

$$w = m\frac{\alpha_s(\mu_R)^n}{(\mathbf{alphas})^n} \tag{6}$$

$$m = \mathbf{me\_wgt2} \cdot f(\mathbf{id1}, \mathbf{x1}, \mu_F) f(\mathbf{id2}, \mathbf{x2}, \mu_F) \tag{7}$$

$$+ \left(f_a^1\omega_1 + f_a^2\omega_2 + f_a^3\omega_3 + f_a^4\omega_4\right) F_b(x_b) \tag{8}$$

$$+ \left(F_b^1\omega_5 + F_b^2\omega_6 + F_b^3\omega_7 + F_b^4\omega_8\right) f_a(x_a) \tag{9}$$

$$\omega_i = \mathbf{usr\_wgts}[i-1] + \mathbf{usr\_wgts}[i+7] \log\left(\frac{\mu_R^2}{\mathbf{ren\_scale}^2}\right) \tag{10}$$

complex: carry both single and double logs

9

where

$$f_a^1 = \begin{cases} a = \text{quark} & : \quad f_a(x_a, \mu_F) \\ a = \text{gluon} & : \quad \sum_{\text{quarks}} f_q(x_a, \mu_F) \end{cases} \tag{11}$$

$$f_a^2 = \begin{cases} a = \text{quark} & : \quad \frac{f_a(x_a/x_a', \mu_F)}{x_a'} \\ a = \text{gluon} & : \quad \sum_{\text{quarks}} \frac{f_q(x_a/x_a', \mu_F)}{x_a'} \end{cases} \tag{12}$$

$$f_a^3 = f_g(x_a, \mu_F) \tag{13}$$

$$f_a^4 = \frac{f_g(x_a/x_a', \mu_F)}{x_a'} \tag{14}$$

we run into the sum over quarks and antiquarks again

and $n = n_j + 1$.

PDF Errors

Better than what is done in MCFM (as far as disk space is concerned); PDF errors are generated on-the-fly through calls to LHAPDF. But then don't store information for individual eigenvectors.

```
void BlackhatAnalysis::GetPdfErrors(const std::vector<Double_t> x,
                                    const Double_t f_c,
                                    const std::vector<int> flav,
                                    Double_t Q,
                                    bool shiftUp,
                                    Double_t &delta)
{
    Double_t f_p,f_m;
    // Loop over all eigenvectors
    for(int e=1;e<=m_nEigen;e++)
    {
        LHAPDF::initPDF(2, 2*e-1); // init positive shift pdf
        LHAPDF::initPDF(3, 2*e); // init negative shift pdf
        //std::cout << "Eigenvector " << e << std::endl;
        f_p = LHAPDF::xfx(2,x[0],Q,flav[0])/x[0]*LHAPDF::xfx(2,x[1],Q,flav[1])/x[1];
        f_m = LHAPDF::xfx(3,x[0],Q,flav[0])/x[0]*LHAPDF::xfx(3,x[1],Q,flav[1])/x[1];
        if(shiftUp) // if positive pdf shift
            delta += pow(std::max(std::max(f_p-f_c,f_m-f_c),0.0),2);
        else // if negative pdf shift
            delta += pow(std::max(std::max(f_c-f_p,f_c-f_m),0.0),2);
    }
    delta = sqrt(delta);
    if(!shiftUp) delta *= -1.0;
    //std::cout << "Total delta: " << delta << std::endl;
}
```

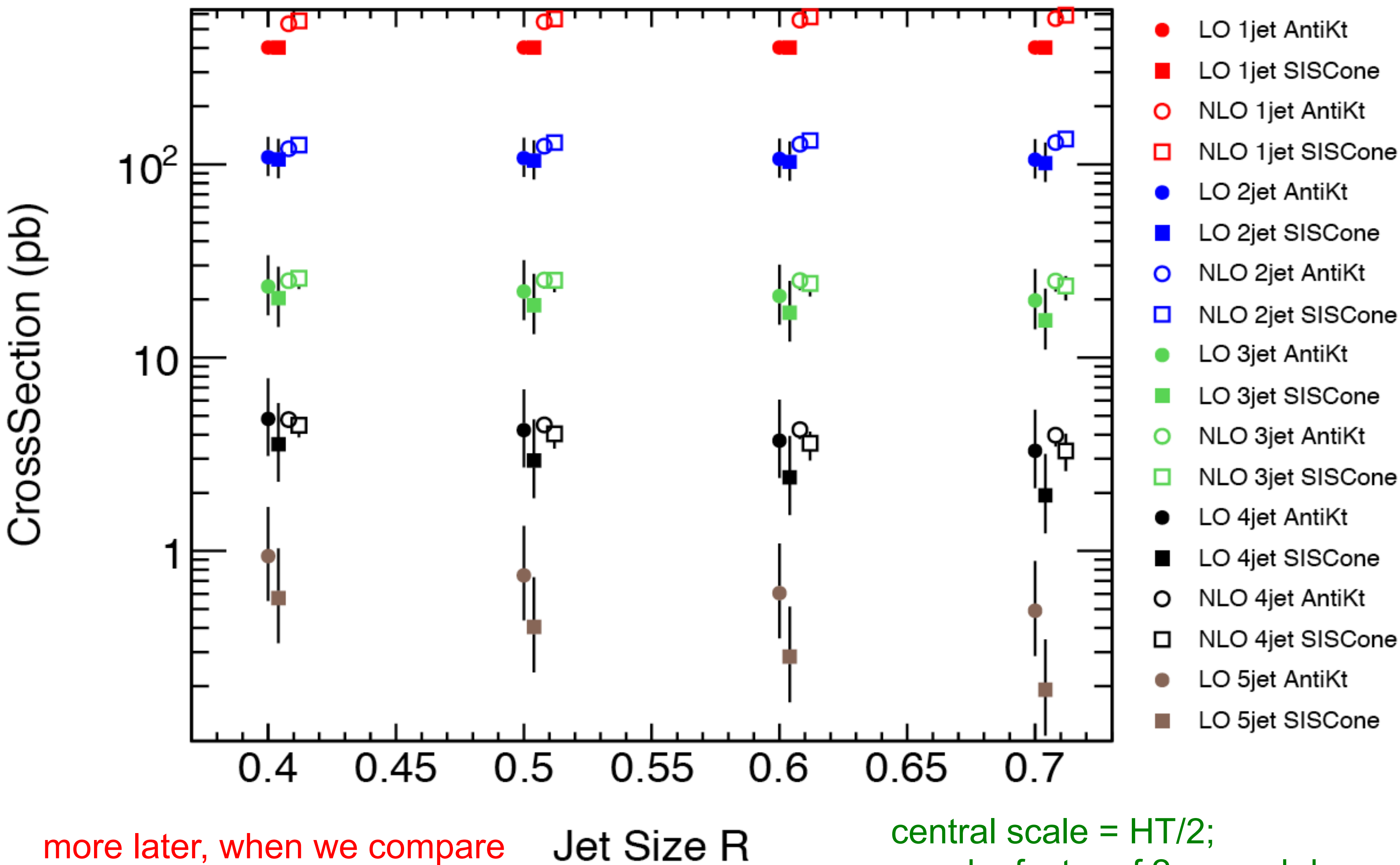
$$\Delta X^{+}_{\max} = \sqrt{\sum_{i=1}^{N}[\max(X_i^{+} - X_0, X_i^{-} - X_0, 0)]^2},$$

$$\Delta X^{-}_{\max} = \sqrt{\sum_{i=1}^{N}[\max(X_0 - X_i^{+}, X_0 - X_i^{-}, 0)]^2}.$$

Logistics

- So total file disk space is quite large, multi-TB (and there are many events to be processed)
  - I bought a 20TB disk specifically for this purpose
- But they're divided into few GB files (Blackhat+SJ)
- So we can make our analysis parallel using 200-250 nodes at MSU
  - we've agreed not to take up more than 50% of the nodes at any one time
- With all of the jet algorithms, scale choices, histograms that I've been using ~3 weeks running time
- A slimmer set can finish within a week

Look at jet size, algorithm dependences; scale uncertainty



more later, when we compare to LHC data

central scale = HT/2;
vary by factor of 2 up and down

K-factors

- Often we work at LO by necessity (LO parton shower Monte Carlos), but would like to know the impact of NLO corrections
- K-factors (NLO/LO) can be a useful short-hand for this information
- But caveat emptor; the value of the K-factor depends on a number of things
  - PDFs used at LO and NLO
  - scale(s) at which the cross sections are evaluated
- And often the NLO corrections result in a shape change, so that one K-factor is not sufficient to modify the LO cross sections