Hadronic contributions to $(g-2)_\mu$ and lattice QCD

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Motivation

- Significant disagreement between experiment and SM: (Davier et al '11, E821 '06, PDG '10)

$$\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = 28.7(8.0) \times 10^{-10} \qquad [3.6\,\sigma]$$

w/ $(\delta a_\mu^{\rm SM} = 4.9 \times 10^{-10}) \quad \simeq \quad (\delta a_\mu^{\rm exp} = 6.3 \times 10^{-10})$

- $g-2$/EDM & E821 expect $\delta a_\mu^{\rm exp}/4$
  $\Rightarrow$ potentially very large signal for BSM physics . . .
  . . . but theory has to follow

process	$a_\mu^{\rm SM} \times 10^{10}$	$\delta a_\mu^{\rm SM} \times 10^{10}$
QED (leptons)	11658471.809	0.015
HVP (LO)	692.3	4.2
EW	15.4	0.2
HLbyL	10.5	2.6
HVP (NLO)	-9.79	0.09

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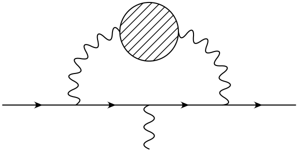
$\Rightarrow$ $\delta a_\mu^{\rm SM}$ dominated by HVP (LO) and HLbyL

$\rightarrow$ both require precise computation of nonperturbative QCD effects

$\rightarrow$ opportunity and challenge for lattice QCD (LQCD)

Hadronic contributions: introduction



LO hadronic vacuum polarization (HVP) $= O(\alpha^2)$

- Obtained from measurement of $e^+e^- \to$ hadrons and $\tau \to \nu_\tau +$ hadrons using dispersion relations
- $\delta a_\mu^{\mathrm{HVP,LO}}$: 0.6% (current) $\to$ 0.3% (in 3-5 years)

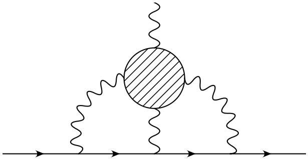
$\Rightarrow$ LQCD not competitive in short term ...
... but worth pursuing as cross check and in long term

Hadronic light-by-light (HLbyL) $= O(\alpha^3)$

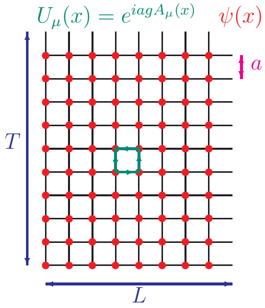
- Cannot be obtained from experiment
- Currently computed with models (χPT+, MHA, ENJL, AdS-QCD, Schwinger-dyson, ...)

$\to$ Glasgow concensus: $10.5(2.6) \times 10^{-10}$ [25%]
(Prades et al '09)

- reasonable, but error is a guesstimate
- LQCD can really help, but very challenging

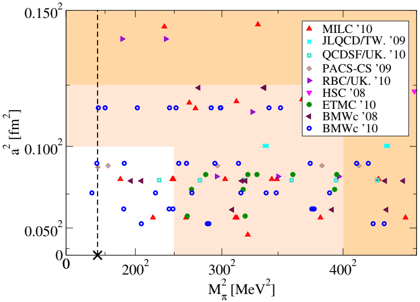


Lattice QCD on a page



- Lattice gauge theory $\rightarrow$ mathematically sound definition of NP QCD
- Large but finite # of dof's $\rightarrow$ evaluate path integral stochastically
- NOT A MODEL: LQCD is QCD when $m_q \rightarrow m_q^{\text{ph}}$, $a \rightarrow 0$, $L \rightarrow \infty$ and stats $\rightarrow \infty$
- Huge challenge: take these limits w/ fully controlled systematics (in particular $m_{ud} \rightarrow m_{ud}^{\text{ph}}$)

- Challenge has been met in last couple of years . . .
- . . . for simple quantities (at most 1 hadron in $i \rightarrow f$ state) . . .
- . . . thanks to important theoretical and algorithmic advances as well as PFlop/s supercomputers



Compute directly in Euclidean spacetime

$$\Pi_{\mu\nu}(q) \quad = $$

$$= \int d^4x\, e^{iQ\cdot x} \langle J_\mu^{\rm EM}(x) J_\nu^{\rm EM}(0)\rangle$$

$$= \left(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right) \Pi(Q^2)$$

Then (Lautrup et al '69, Blum '02)

$$a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, w(Q^2/m_\mu^2) \hat{\Pi}(Q^2)$$

w/ $\hat{\Pi}(Q^2) \equiv \left[\Pi(Q^2) - \Pi(0)\right]$ and $w(Q^2/m_\mu^2)$ known function which heavily weighs $Q^2 \lesssim m_\mu^2$

- $\Rightarrow$ dominated by very low energies
- $\Rightarrow$ challenge for lattice

HVP challenges for LQCD

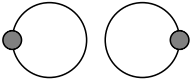
- In $L^3 \times T$, momenta are quantized (e.g. periodic BCs)

$$Q_\mu = 2\pi \left(n_0/T, \cdots, n_3/L\right)$$

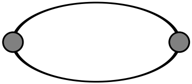
 Even for $T \sim L \lesssim 6\,\mathrm{fm} \Rightarrow Q^2_{\rm min} \gtrsim (200\,\mathrm{MeV})^2 > m_\mu^2$
 $\rightarrow$ fix by using "twisted" BCs (Sachrajda et al '05, Della Morte et al '10)

- Only one calculation of $\Pi_{\mu\nu}(Q)$ has $M_\pi \searrow 170\,\mathrm{MeV}$ (Boyle et al '11)
 $\rightarrow$ fix by using simulations at $M_\pi^{\rm ph}$

- No LQCD calculation of $\Pi_{\mu\nu}(Q)$ has reliably evaluated computationally demanding



 $\rightarrow$ $SU(3)$ and Zweig suppressed, but can be $\sim 10\%$ (Della Morte et al '10) of



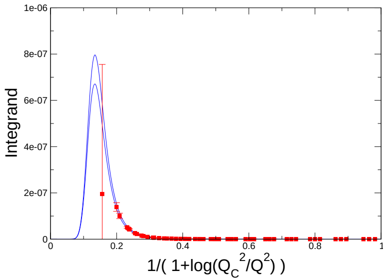
HVP challenges: example

- Boyle et al '11, $a = 0.086\,\mathrm{fm}$, $M_\pi \simeq 290\,\mathrm{MeV}$, $L^3 \times T = (2.8^3 \times 5.6)\,\mathrm{fm}^4$
- Fit to model

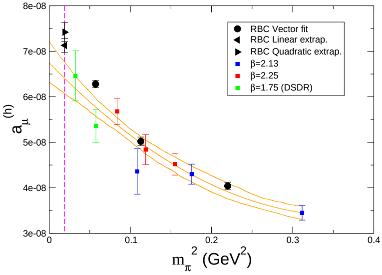
$$\Pi(Q^2) = A + \frac{F_1^2}{Q^2 + M_1^2} + \frac{F_2^2}{Q^2 + M_2^2}$$

- Not much data in relevant region

$\Rightarrow$ significant model-dependence
$\rightarrow$ reduced in future w/ results at lower Q^2 and new parametrizations (Aubin et al '12)



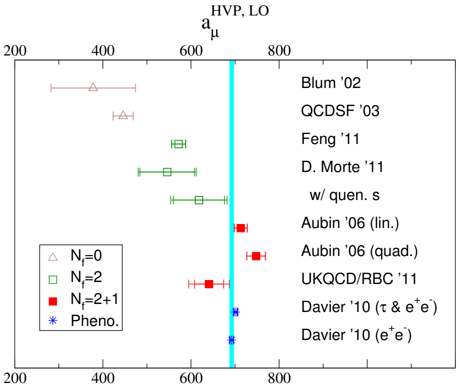
[Plotted as function of $t = \left(1 + \ln(Q_c^2/Q^2)\right)^{-1}$ w/ $Q_c^2 = 11\,\mathrm{GeV}^2$]



- Integrate fit $0 \rightarrow Q_c^2$ and 3-loop PT (Chetyrkin et al '96) w/ $m_u = m_d = 0$ and $m_s = m_s^{\mathrm{lat}}$ $Q_c^2 \rightarrow \infty$
- m_{ud} dependence modeled by quadratic polynomial in M_π^2

$\Rightarrow$ also some model-dependence
$\rightarrow$ reduced in future w/ simulations at M_π^{ph}

HVP from LQCD: summary



- Only results with $N_f = 2 + 1$ (or quenched s) should be compared to phenomenology
- No LQCD result has a complete systematic error estimate
- Only Feng et al '11 attempt to estimate quark-disconnected contractions numerically

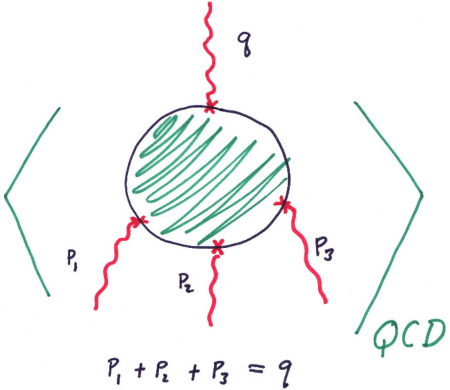
$\Rightarrow$ current LQCD error $\gtrsim 10\%$

(Della Morte et al '10)

- May reduce to few % in 3-5 years

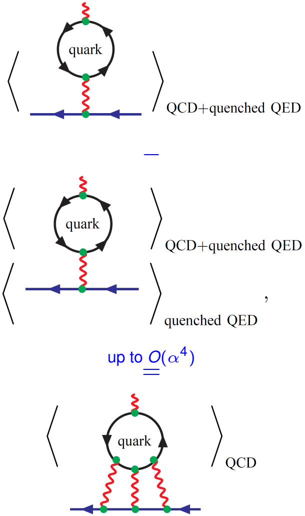
$\Rightarrow$ not competitive w/ phenomenology for a while

HLbyL from LQCD: conventional approach



- Correlator of 4 EM currents $\Pi^{\mu\nu\rho\sigma}(q, p_1, p_2)$
- 2 loop momenta & q
- Compute for all possible p_1 & p_2 ($O(V_4^2)$) and up to 256 index combinations ...
- ... for several q to allow $q \to 0$ ...
- ... and for several multiply-disconnected contractions ...
- ... and fit and plug into 2-loop QED integrals!!

HLbyL from LQCD: new approach



Blum et al hep-lat/0509124, PoS LATTICE2008 '08

- Compute and attach "by hand" $\langle J_\nu^{\rm EM}(y) J_\mu^{\rm EM}(x)\rangle$ to $\langle\mu(p)J_\alpha(z)\mu^\dagger(p')\rangle$ with γ line $D_{\alpha\nu}(z,y)$
- Integrate over y & z and sum over ν & α

→ q coupled to g and to quenched γ

→ μ coupled to quenched γ only

- Average over combined gluon & photon configurations
- Subtraction term is product of separate averages of q loop and μ line
- Difference is HLbyL up to $O(\alpha^4)$ corrections
- Gauge configurations identical in both
  $\Rightarrow$ high correlation should allow isolating $O(\alpha^2)$-suppressed difference
- Of course, need quark-disconnected contributions

. . .

New approach: QED test

Chowdhury, PhD Thesis, U. Conn '09

- Quenched QED on $16^3 \times 32 \times 8$ lattice
- $m_\mu / m_e = 40$, $e = 1$
- Stat. error only

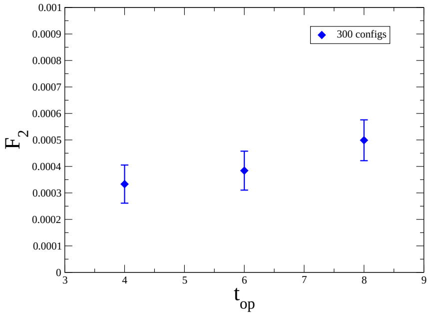
$$F_2(q^2_{\rm min}) = (3.96 \pm 0.70) \times 10^{-4}$$

- $F_2(q^2_{\rm min}) = (1.19 \pm 0.32) \times 10^{-4}$ on 24^3 volume

$\Rightarrow$ large finite-V effects

- $(\alpha/\pi)^3 = 1.63 \times 10^{-5}$ which is $\sim 1/10$ LQCD results
- Signal found at $q^2_{\rm min}$ in preliminary QCD + QED calculation w/ unphysically large charge and masses

$\rightarrow$ very promising, but still a long way to go



Conclusion

- New $(g-2)_\mu$ experiments are expected to reduce error by 4
- If central values remain the same and $\delta a_\mu^{\rm SM}$ is halved
  $\Rightarrow$ a 10 σ deviation from the SM!
- Dominant sources of theory uncertainties are $a_\mu^{\rm HVP,LO}$ and $a_\mu^{\rm HLbyL}$

$\Rightarrow$ LQCD calculations will be very helpful for reducing these uncertainties

- $\delta a_\mu^{\rm HVP,LO}$ using $e^+e^-, \tau \to \mathrm{hadrons}$ should be reduced by 1/2 in 3-5 years
  $\to$ LQCD can serve as cross-check (and be competitive in long term)
- It should be possible to reduce $\delta a_\mu^{\rm HLbyL}$ by 1/2 in $\sim$ 5 years using LQCD, models and experiment for $\pi\gamma^*\gamma$ (KLOE)
  $\to$ LQCD is vital here

$\Rightarrow$ The theory should follow and we all look forward to compare its predictions w/ g-2/EDM and E989 measurements