Status of the Muon g-2
Hadronic Vacuum Polarization

Michel Davier (LAL – Orsay)

- HVP in the muon magnetic anomaly
- revisited τ spectral functions: Belle + updated corrections
- ee spectral functions after KLOE and BABAR
- new evaluation using BABAR multihadronic data
- discussion and perspectives
- new precise value for $\alpha(M_Z)$

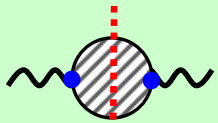
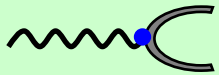
Hadronic Vacuum Polarization and Muon $(g-2)_\mu$

Dominant uncertainty from lowest-order HVP piece

Cannot be calculated from QCD (low mass scale), but one can use experimental data on e+e−→hadrons cross section (Bouchiat-Michel 1961)

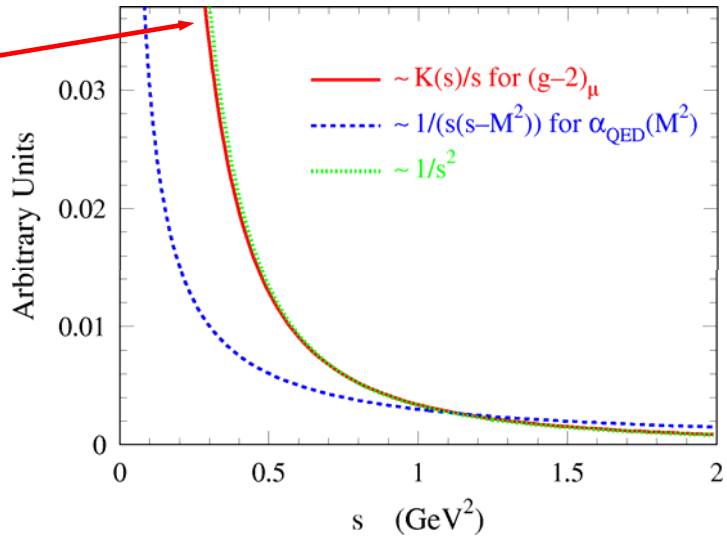
Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

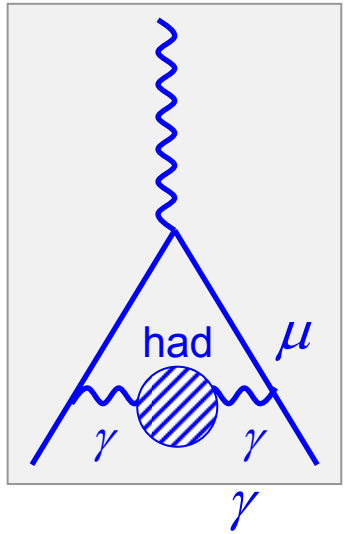
$$12\pi\,\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0[e^+e^-\to\mathrm{hadrons}(\gamma)]}{\sigma_{pt}} \equiv R(s)$$

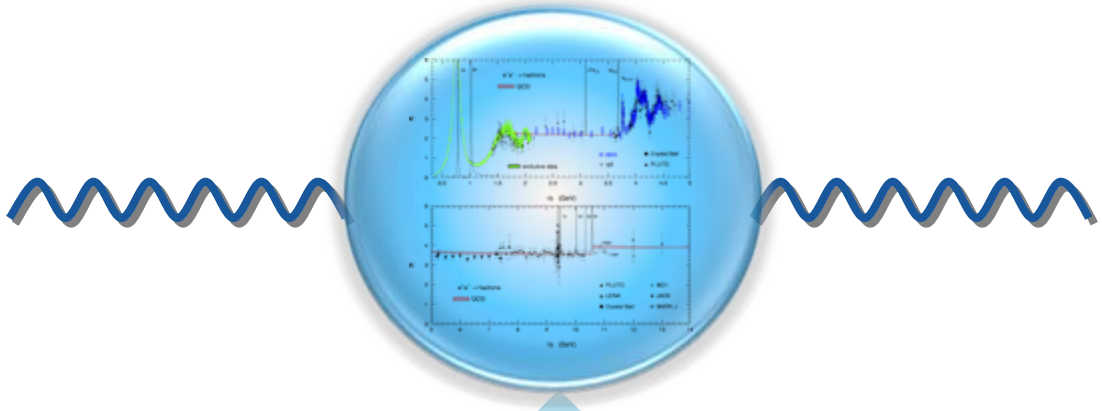
Im[] ∝ |  hadrons |²

$$a_\mu^{\mathrm{had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\,\frac{K(s)}{s}\,R(s)$$

Dispersion relation



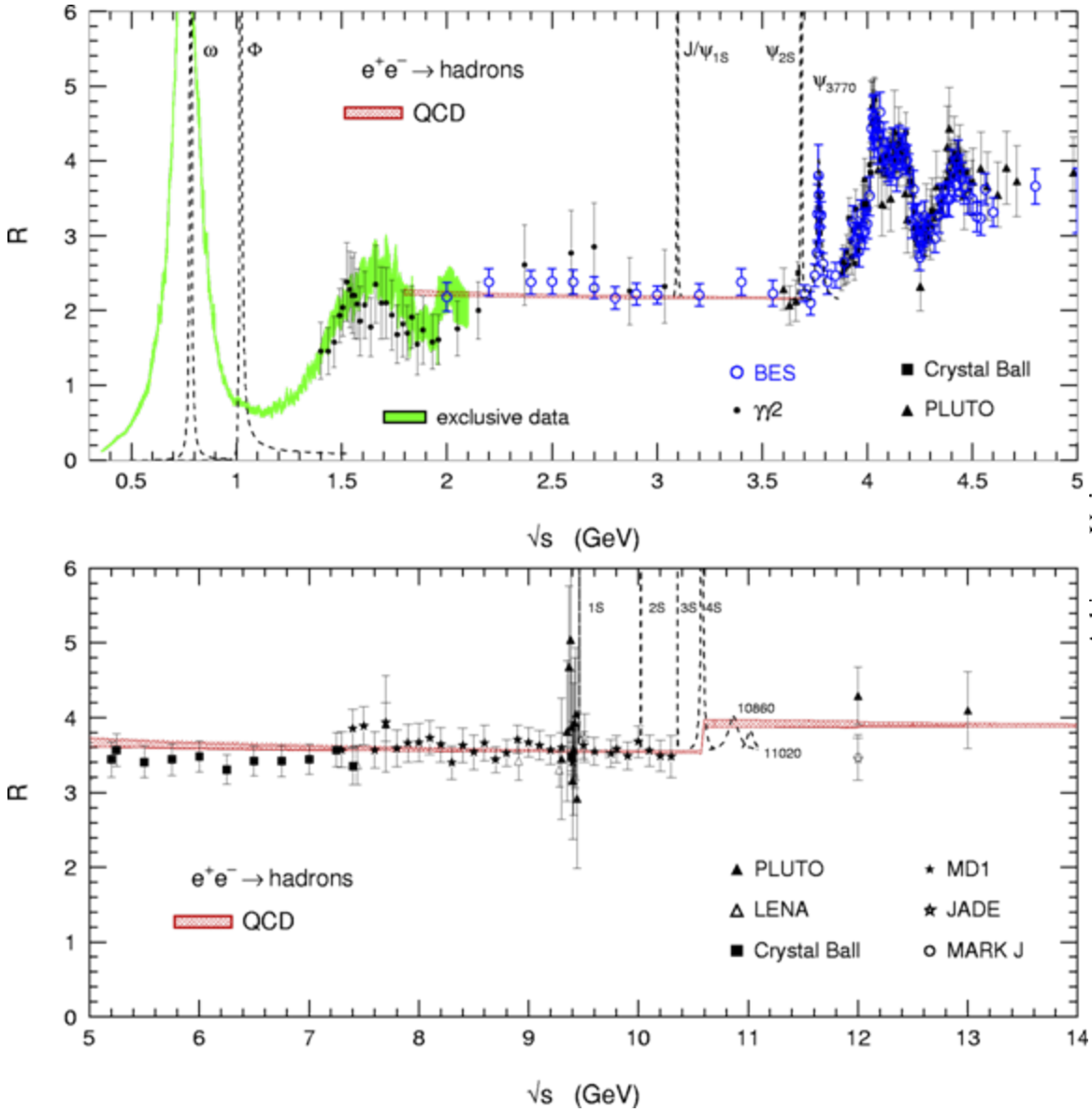




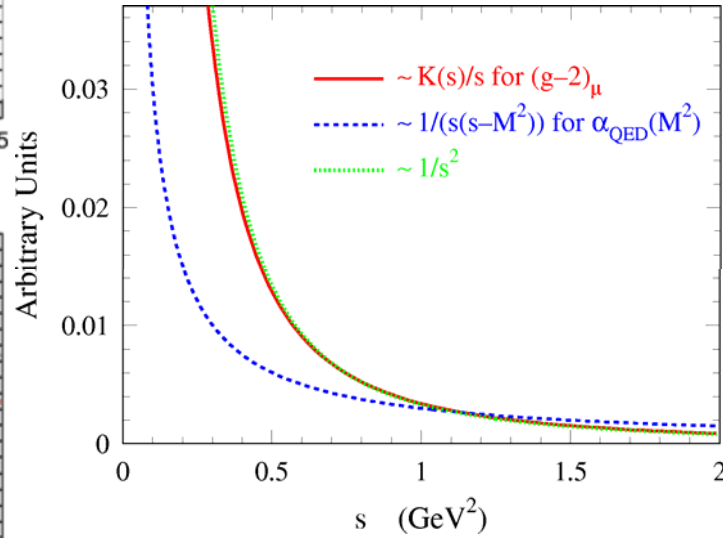
$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{m_{\pi^0}^2}^{\infty} ds \; \frac{K(s)}{s} \; R(s)$$

Huge 20-years effort by experimentalists and theorists to reduce error on lowest-order hadronic part

- **Improved** e^+e^- **cross section data** from Novisibirsk (Russia)
- More use of **perturbative QCD**
- Technique of "**radiative return**" allows to use data from Φ and *B* factories
- Isospin symmetry allows us to also use τ **hadronic spectral functions**



$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \quad \frac{K(s)}{s} \quad R(s)$$



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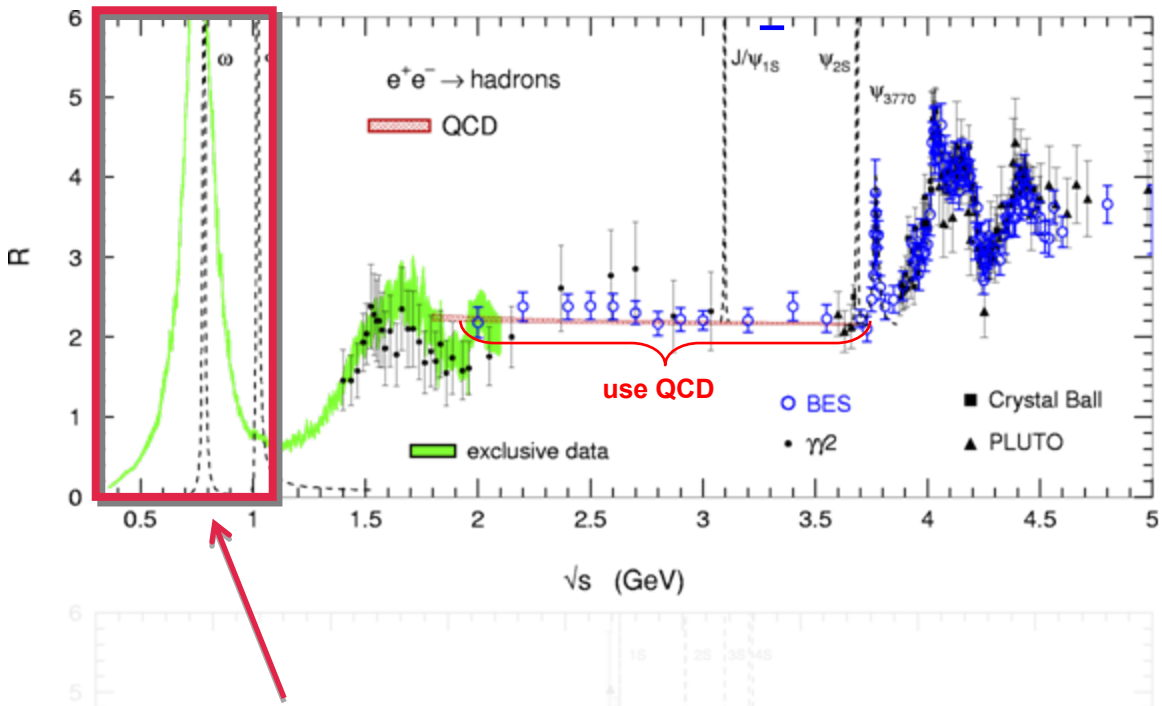
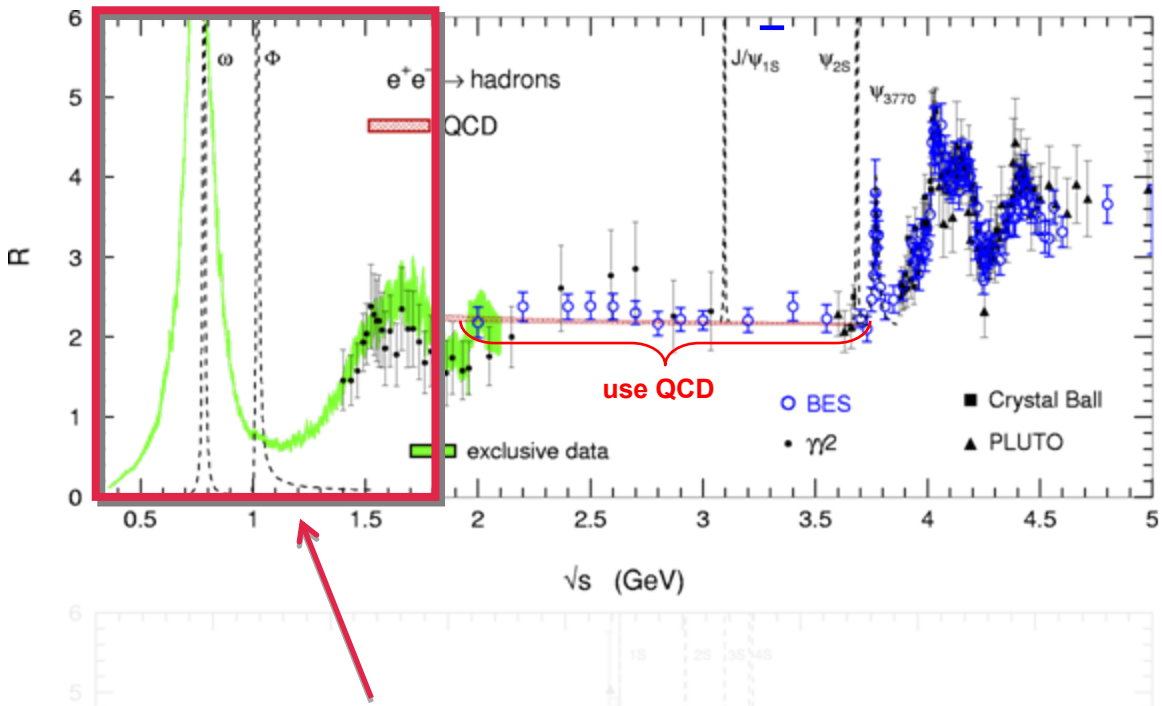


$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

Due to the strongly decaying integration kernel, 73% of dispersion integral stems from $\pi^+\pi^-$ channel, which must be obtained from experiment (dominant contribution to the final uncertainty)

Plot not fully up to date – for illustration only



$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

At low energy, the inclusive hadronic cross section is obtained by summing up to 26 exclusively measured final states, and by estimating unmeasured modes using isospin symmetry

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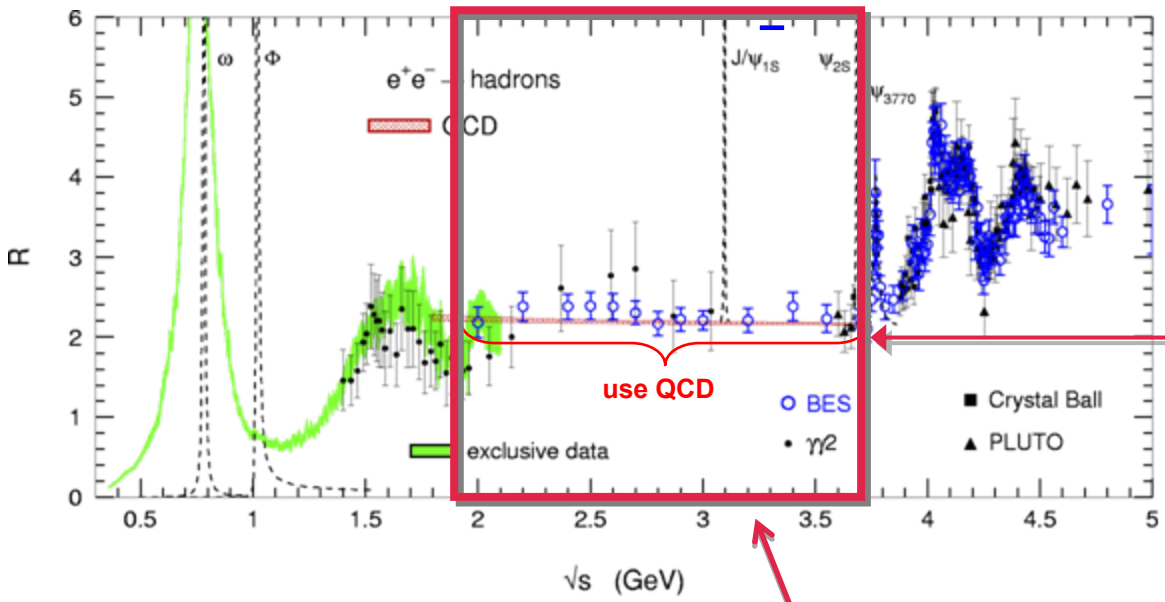
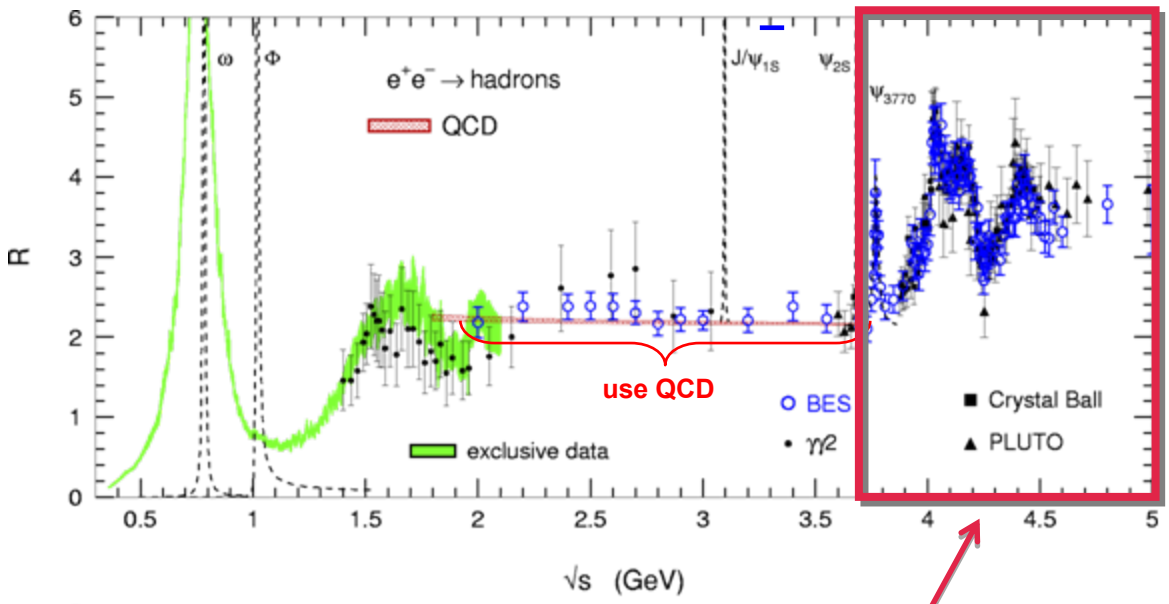


$$a_\mu^{had,LO} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

Agreement between Data (BES) and pQCD (within *correlated* systematic errors)

Perturbative QCD can be used away from the quark thresholds in the continuum region

Plot not fully up to date – for illustration only

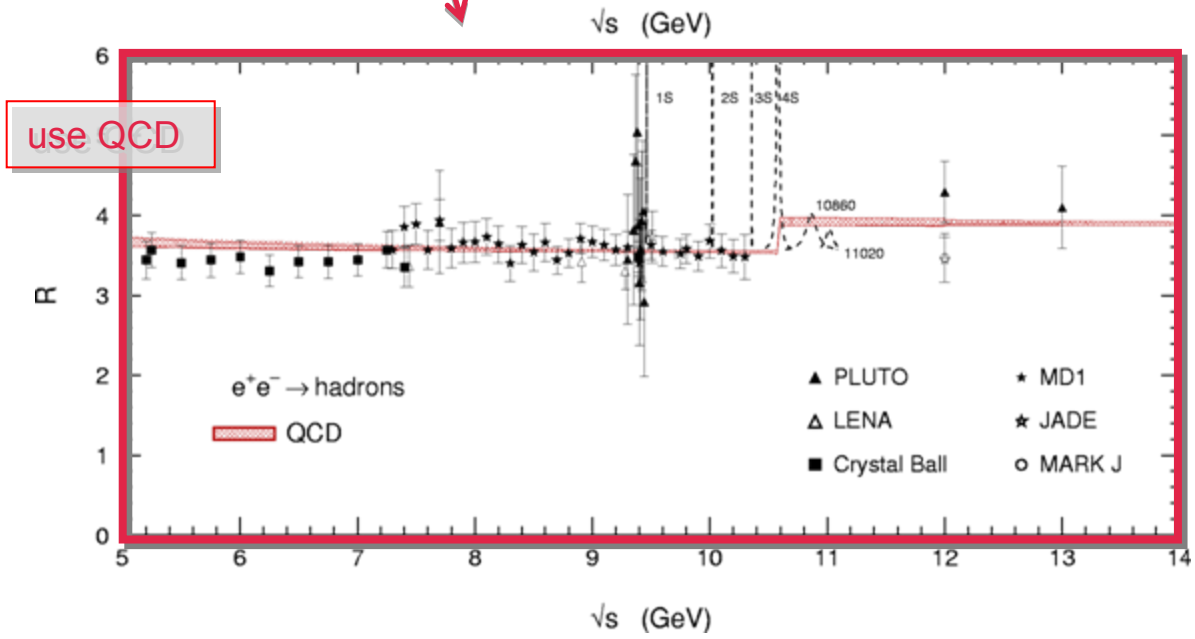


$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

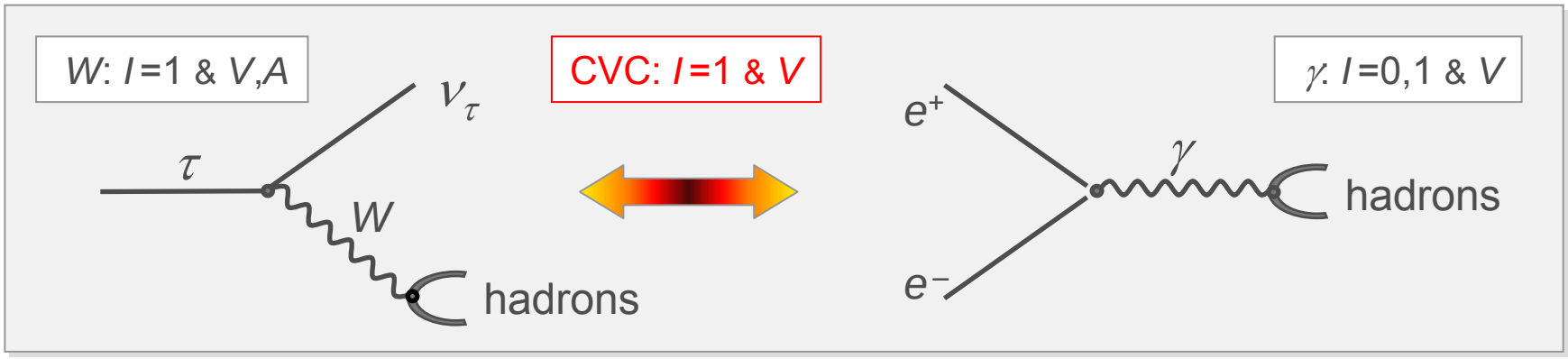
Experimental data must be used in the charm anti-charm resonance region

$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

Perturbative QCD can be used beyond the charm



The Role of τ Data through CVC – SU(2)



Hadronic physics factorizes (spectral Functions)

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching fractions — mass spectrum — kinematic factor (PS)

R. Alemany, MD, A. Höcker, EPJC 1998

SU(2) breaking corrections

Corrections for SU(2) breaking applied to τ data for dominant $\pi^-\pi^+$ contrib.:

- Electroweak radiative corrections:
  - dominant contribution from short distance correction S_{EW}
  - subleading corrections (small)
  - long distance radiative correction $G_{EM}(s)$
- Charged/neutral mass splitting:
  - $m_{\pi^-} \neq m_{\pi^0}$ leads to phase space (cross sec.) and width (FF) corrections
  - ρ-ω mixing (EM $\omega \to \pi^-\pi^+$ decay) corrected using FF model
  - $m_{\rho^-} \neq m_{\rho^0}$ *** and $\Gamma_{\rho^-} \neq \Gamma_{\rho^0}$ ***
- Electromagnetic decays: $\rho \to \pi\pi\gamma$ ***, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to l^+l^-$
- Quark mass difference $m_u \neq m_d$ (negligible)

Marciano-Sirlin' 88

Braaten-Li' 90

Cirigliano-Ecker-Neufeld' 02
Lopez Castro et al.' 06

Alemany-Davier-Höcker' 97, Czyż-Kühn' 01

Flores-Baez-Lopez Castro' 08
Davier et al.'09

Situation 2009-10

- including BaBar 2π results in the e+e– combination + estimate of hadronic LBL contribution (Prades-de Rafael-Vainhstein, 2009) yields

$$a_\mu^{SM}[e+e-] = (11\,659\,183.4 \pm 4.1 \pm 2.6 \pm 0.2)\ 10^{-10}$$

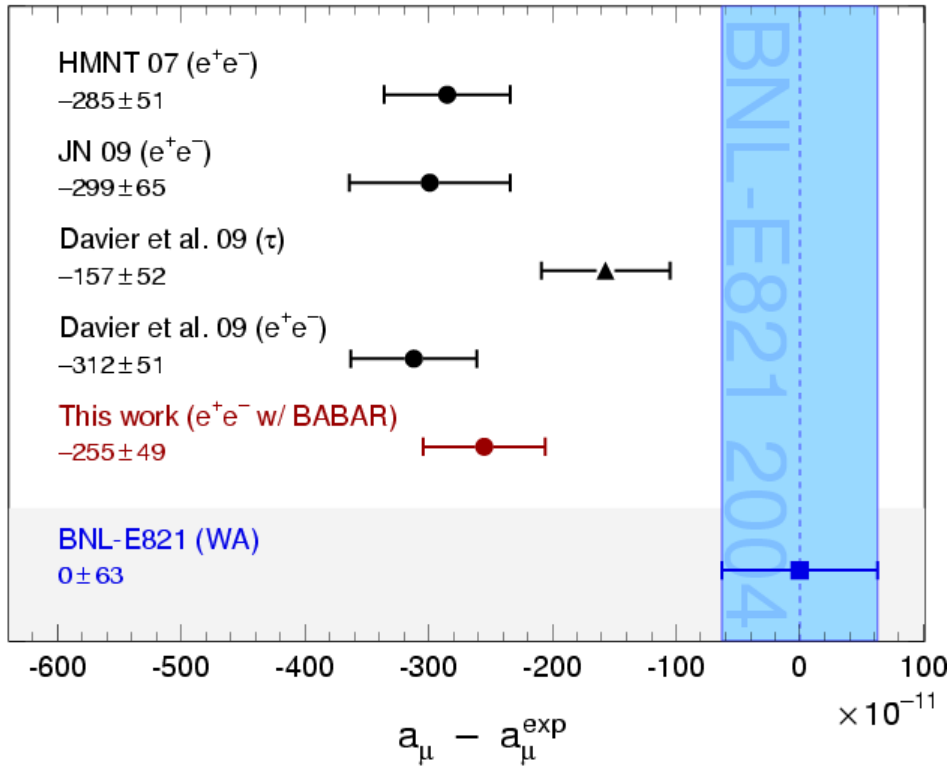
HVP LBL EW (±4.9)

- E-821 updated result 11 659 208.9 ± 6.3

- deviation (ee) 25.5 ± 8.0 (3.2 σ)

- updated τ analysis +Belle +revisited IB corrections

- deviation (τ) 15.7 ± 8.2 (1.9 σ)



Recent data and analyses

- $\tau \to \pi\, \pi^0\, \nu_\tau$ data from Belle PRD 78 (2008) 072006

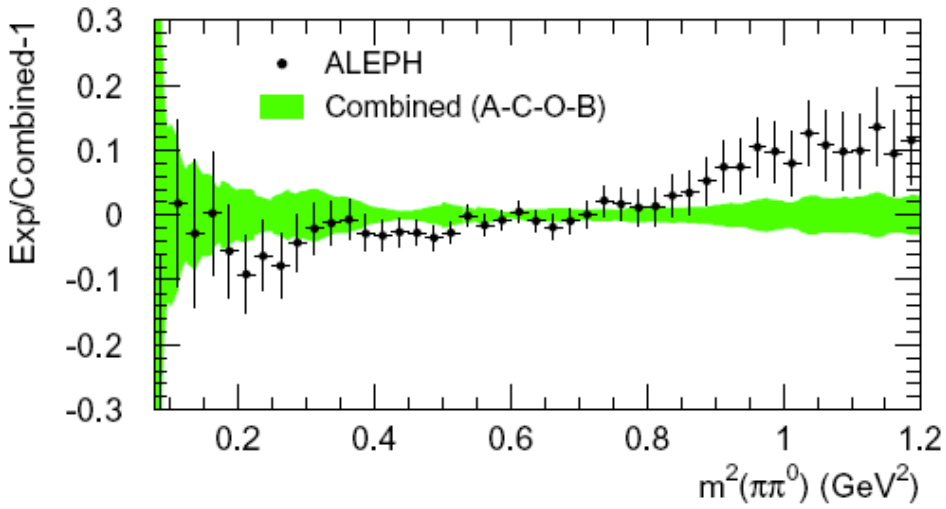
- $e^+\, e^- \to \pi^+\, \pi^-$ data
  - KLOE PLB 670 (2009) 285
    arXiv:1006.5313
  - BaBar arXiv:0908.3589v1 (PRL)

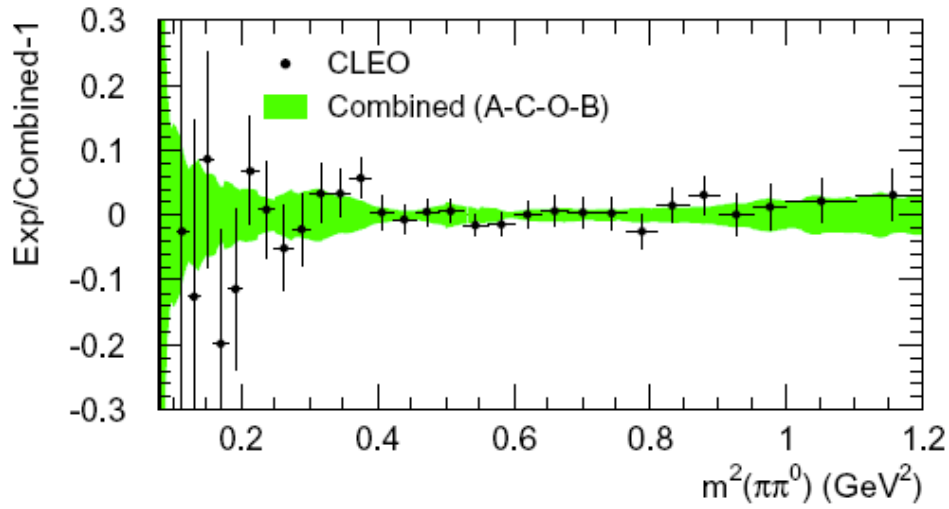
- updated τ-based analysis arXiv:0906.5443v3 (EPJC)
  MD, A. Hoecker, G. Lopez Castro, B. Malaescu, X.H.Mo, G. Toledo Sanchez, P. Wang, C.Z. Yuan, Z. Zhang

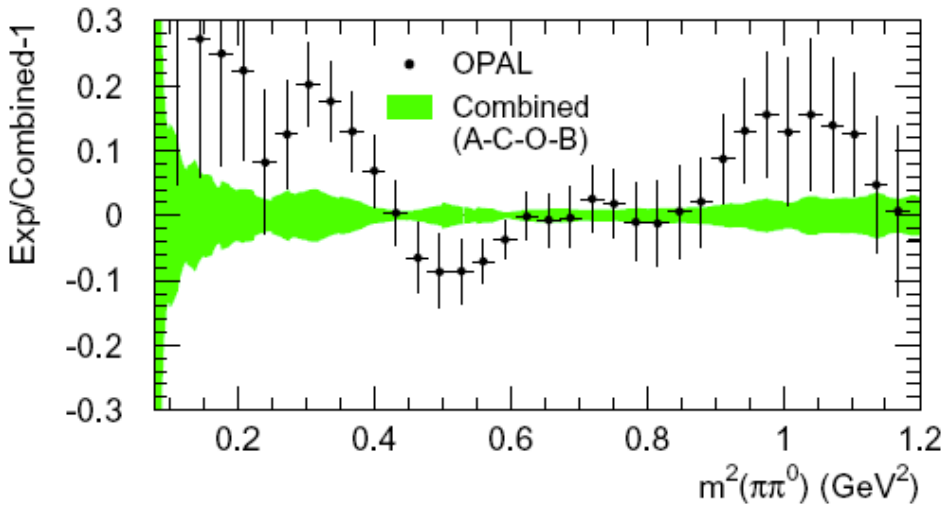
- updated ee-based analysis arXiv:0908.4300v2 (EPJC)
  MD, A. Hoecker, B. Malaescu, C.Z. Yuan, Z. Zhang

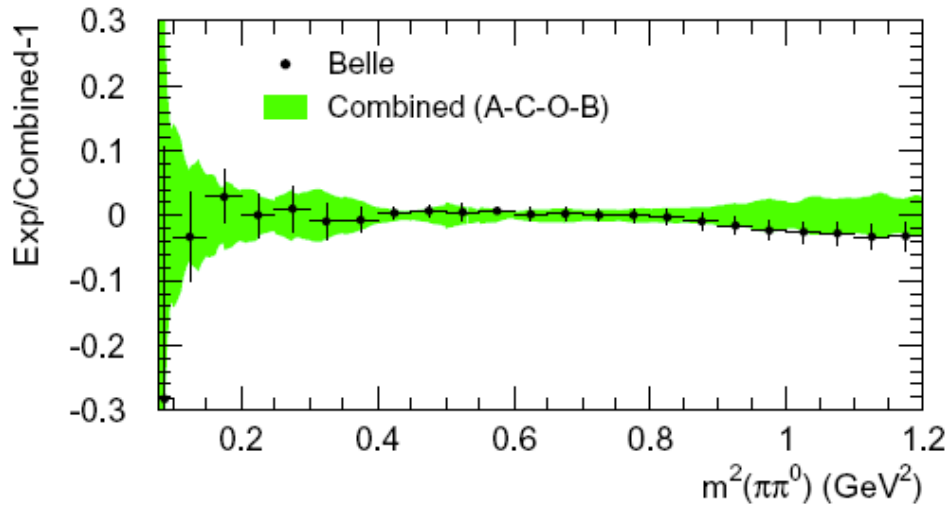
- complete re-evaluation arXiv:1010.4180v2 (EPJC)
  MD, A. Hoecker, B. Malaescu, Z. Zhang

Revisited Analysis using τ Data: including Belle







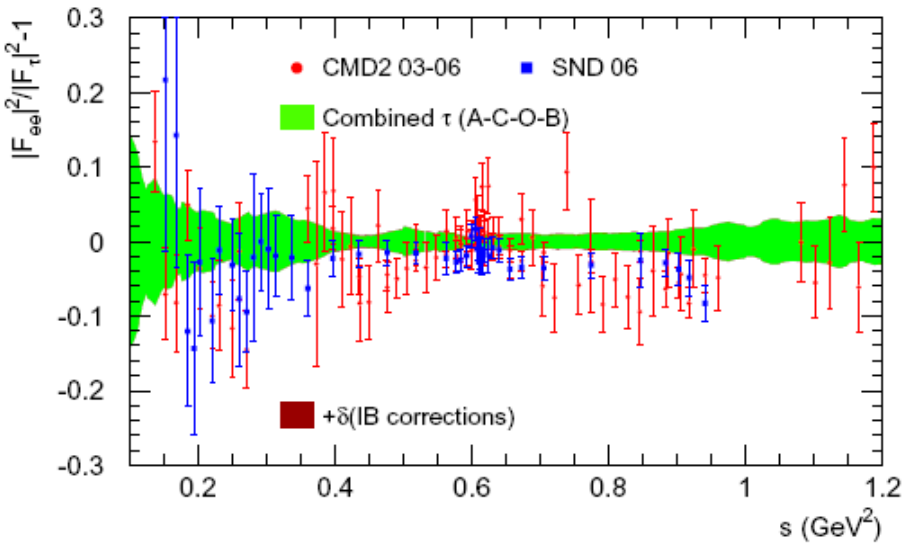


Test of the spectral function shapes from different experiments: WA BR used

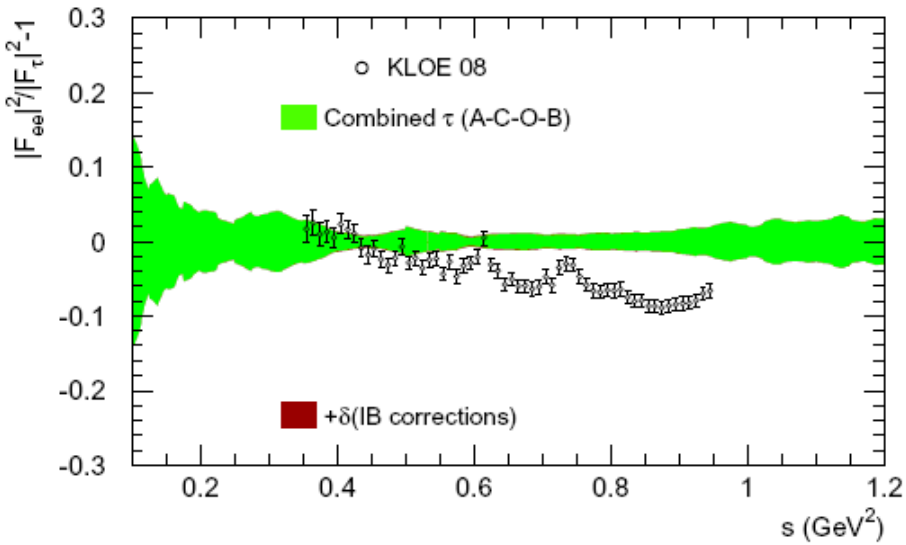
Comparison of ee and τ Data Revisited (1)

Relative comparison of IB-corrected τ and ee spectral functions (τ green band)

CMD-2, SND



KLOE



⇒ better agreement than before with CMD2-SND

⇒ strong disagreement with KLOE : slope…

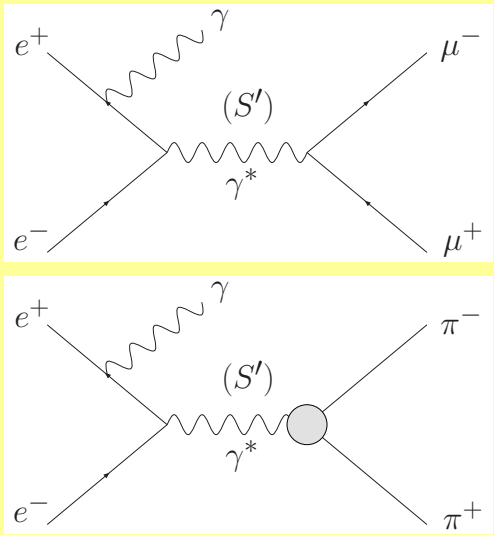
The BaBar Analysis

$e^+ e^- \to \mu^+ \mu^- \gamma\,(\gamma)$ and $\pi^+ \pi^- \gamma\,(\gamma)$ measured simultaneously

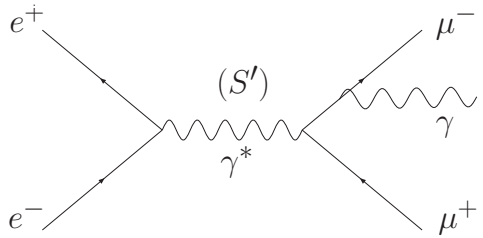
$$x = 2E^*_\gamma / \sqrt{s}$$

$$s' = s(1-x)$$

ISR

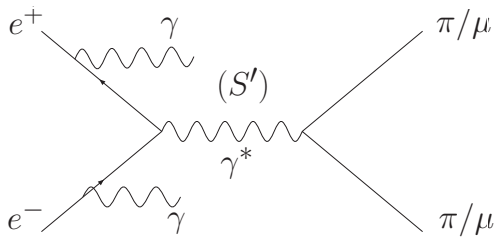


FSR

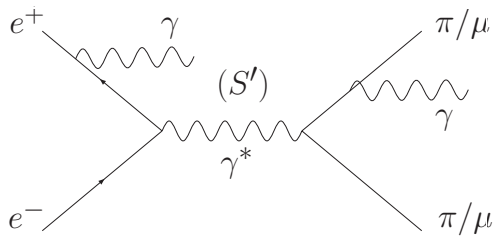


LO FSR negligible for $\pi\pi$ at s~$(10.6\ \mathrm{GeV})^2$

ISR + add. ISR



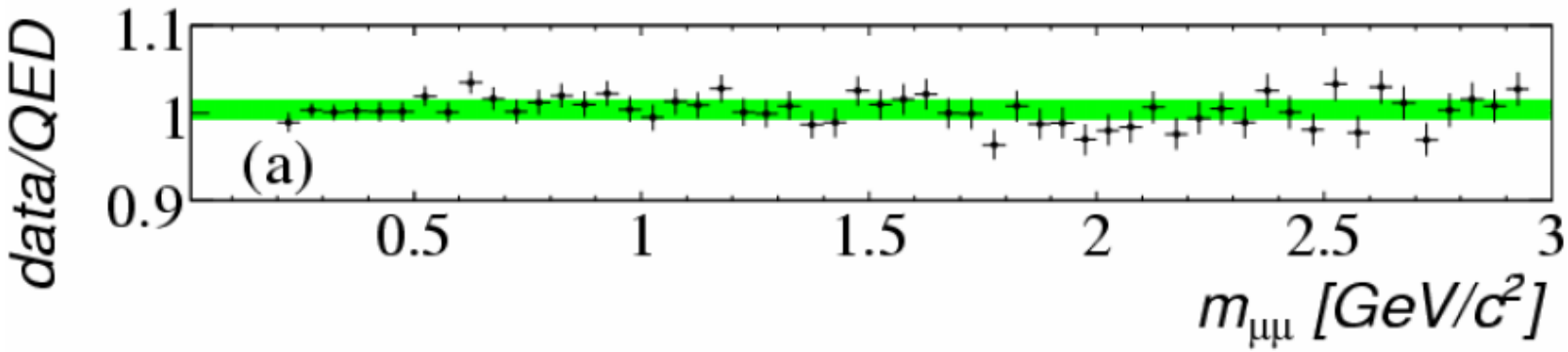
ISR + add. FSR



QED Test with μμγ sample

- absolute comparison of μμ mass spectra in data and in simulation
- simulation corrected for data/MC efficiencies
- AfkQed corrected for incomplete NLO using Phokhara
- strong test (ISR probability drops out for ππ)

BaBar



$$\frac{\sigma^{data}_{\mu\mu\gamma(\gamma)}}{\sigma^{NLO\ QED}_{\mu\mu\gamma(\gamma)}} = 1 + (4.0 \pm 1.9 \pm 5.5 \pm 9.4)\ 10^{-3}$$

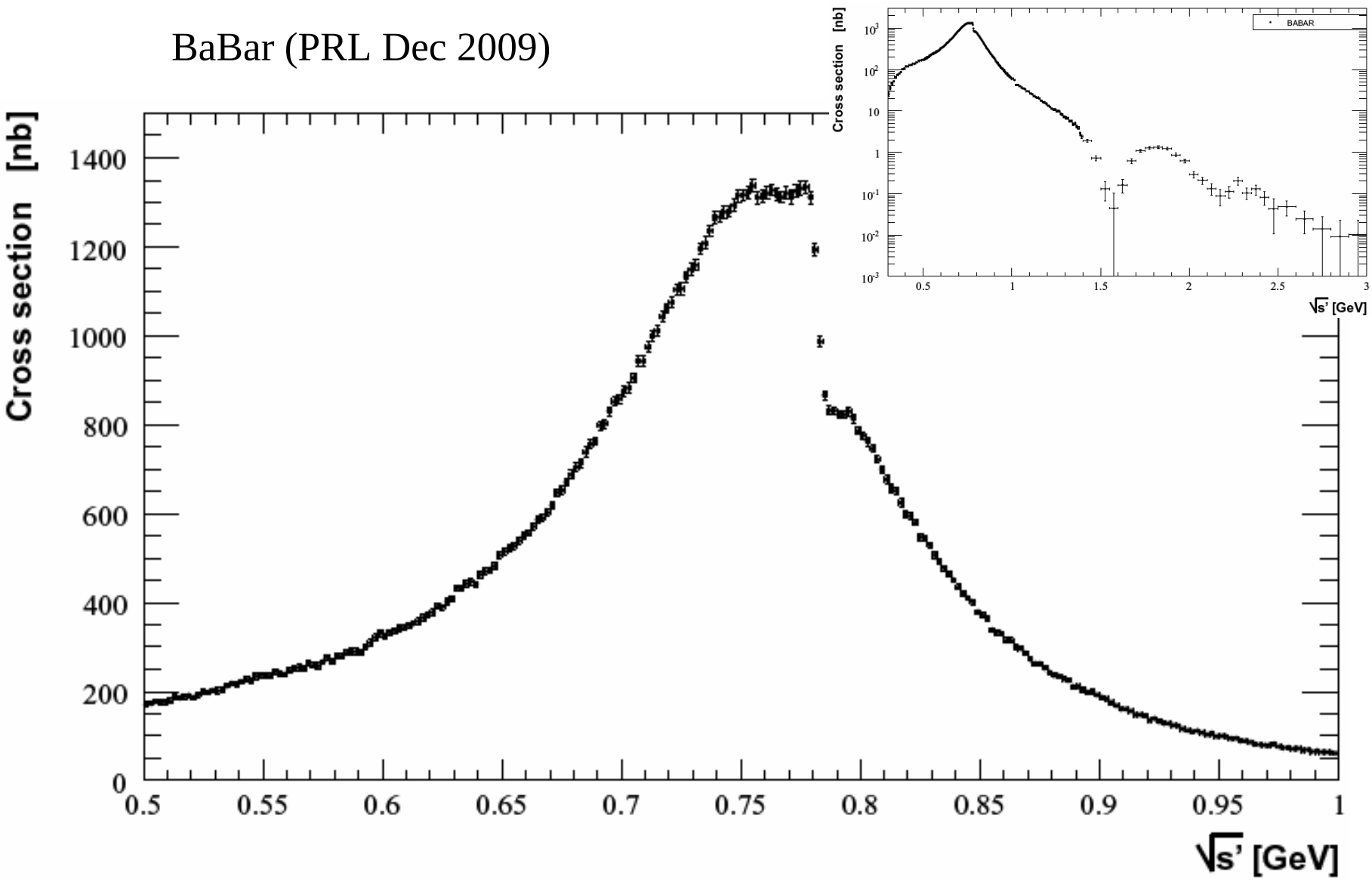
(0.2 – 3 GeV)

ISR γ efficiency 3.4 syst.
trig/track/PID 4.0

BaBar ee luminosity

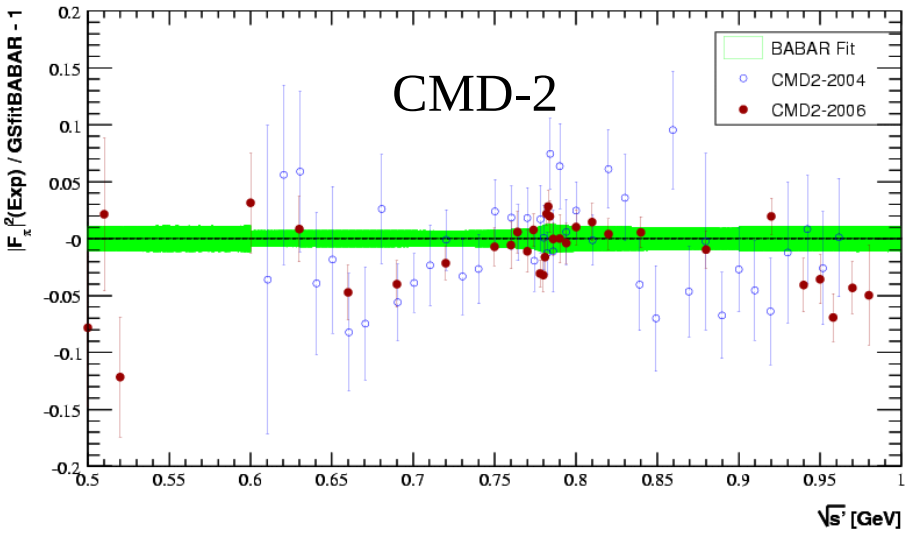
Data on $e^+e^- \to \pi^+ \pi^-$

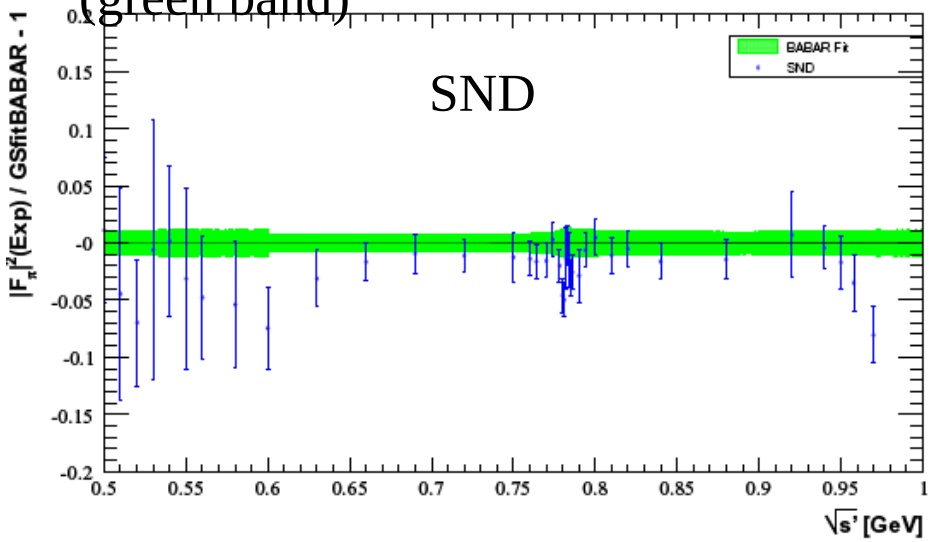
BaBar (PRL Dec 2009)

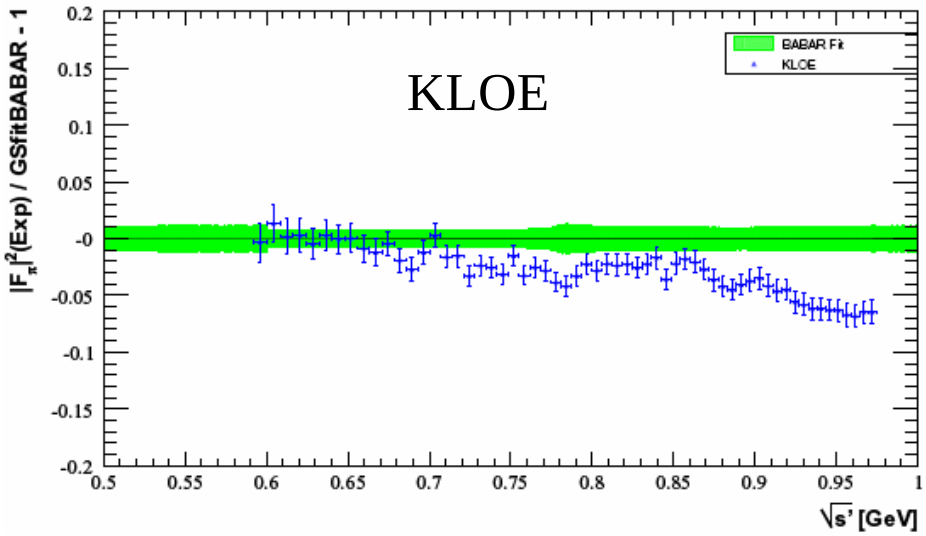


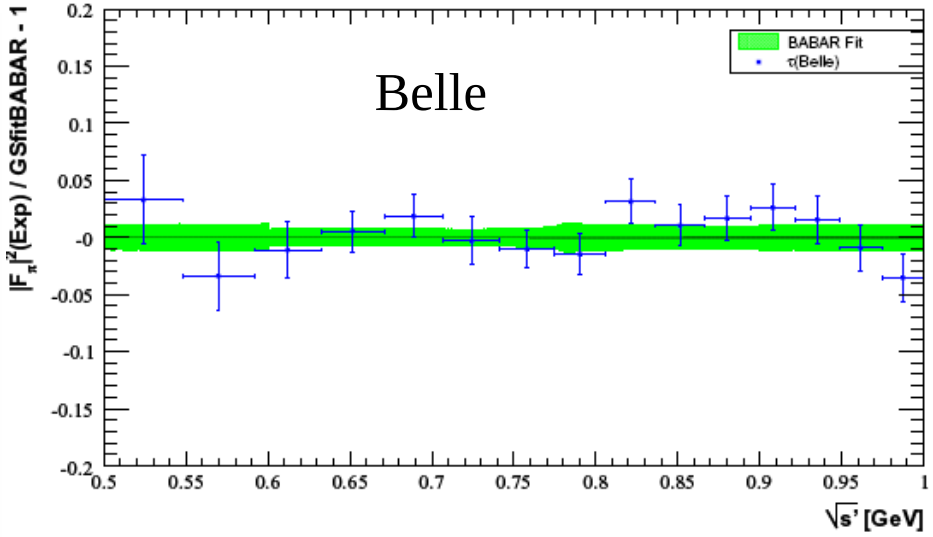
BaBar vs.other ee data (0.5-1.0 GeV)

direct relative comparison of cross sections with BaBar fit (stat + syst errors included) (green band)









Combining and integrating data

New analysis: MD, A. Hoecker, B. Malaescu, Z. Zhang (DHMZ Oct.2010)

The integration of data points belonging to different experiments, with different within-experiment and inter-experiment correlated systematic errorssss, and with different data densities requires a **careful treatment**

It is mandatory to test the accurateness of the integration procedure in terms of central value and error using representative models with known truth.

DHMZ approach (HVPTools):

- Quadratic interpolation of the data points/bins for each experiment
- Local weighted average between interpolations performed in infinitesimal bins
- Full covariance matrices: correlations between data points of an experiment (systematic errors), between experiments and channels (VP, luminosity, …)
- Consistent error propagation using pseudo experiments
- Possible bias tested in 2π channel using a GS model: negligible for quadratic interpolation, but not for linear model (trapezoidal rule)

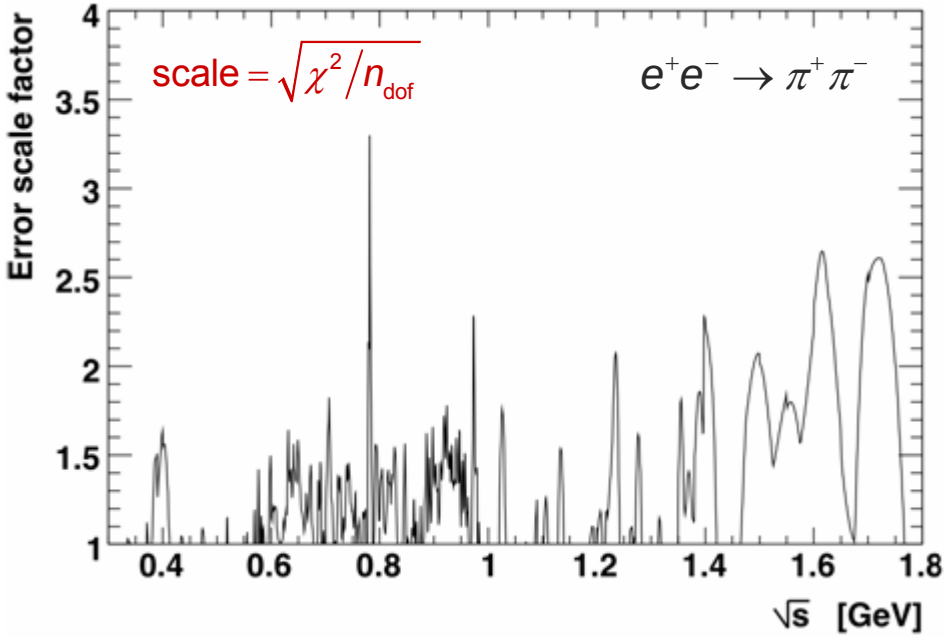
Combining and integrating data

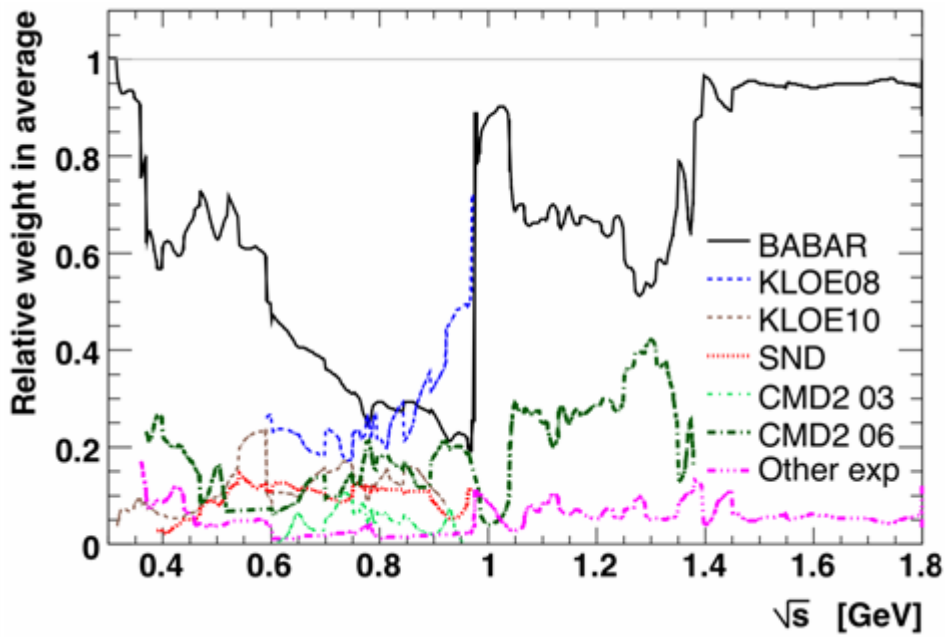
Incompatibilities between data points lead to error rescaling

Performed using PDG prescription

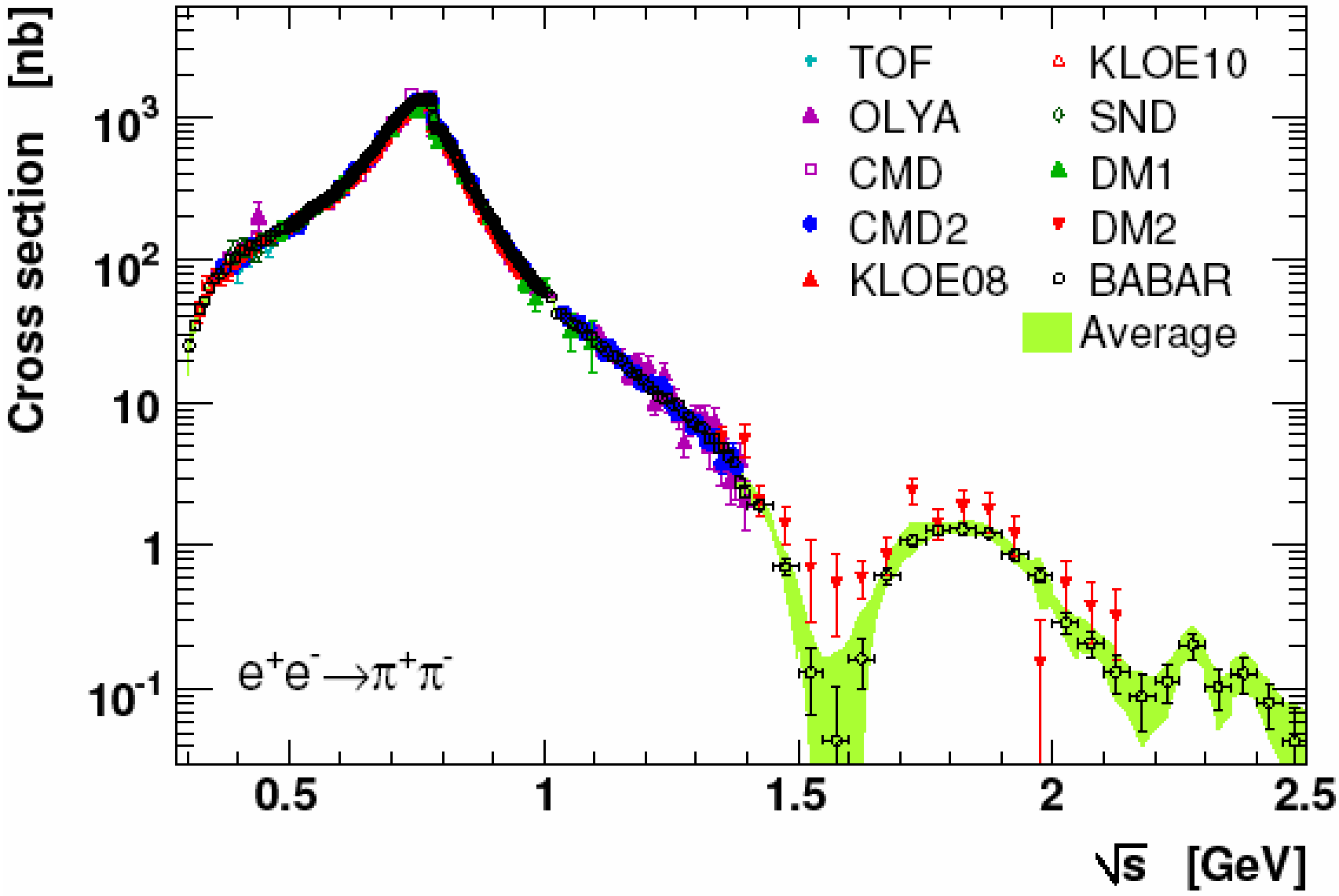
Weights of experiments in average versus mass

BABAR dominates everywhere, except for KLOE 08 between 0.8 and 0.93 GeV

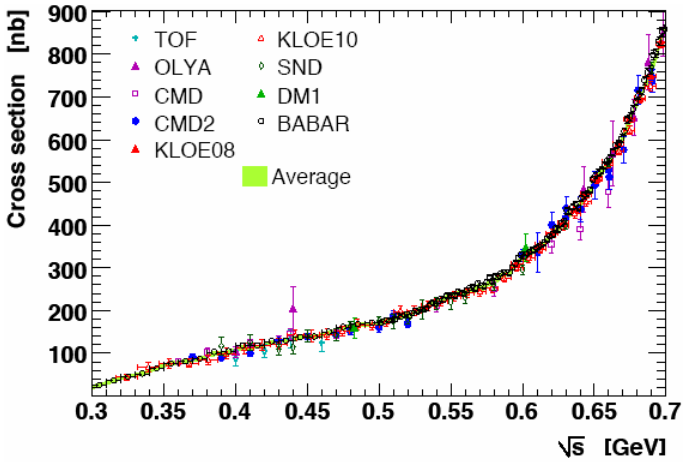




2π combined


Cross section [nb]
10^3
10^2
10
1
10^{-1}
TOF
OLYA
CMD
CMD2
KLOE08
KLOE10
SND
DM1
DM2
BABAR
Average
$e^+e^- \to \pi^+\pi^-$
0.5
1
1.5
2
2.5
$\sqrt{s}$ [GeV]

2π combined


Cross section [nb]
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CMD2
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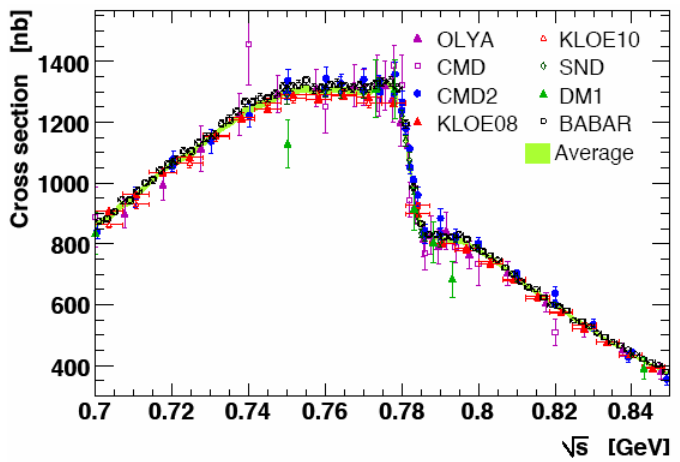
SND
DM1
BABAR
Average


Cross section [nb]
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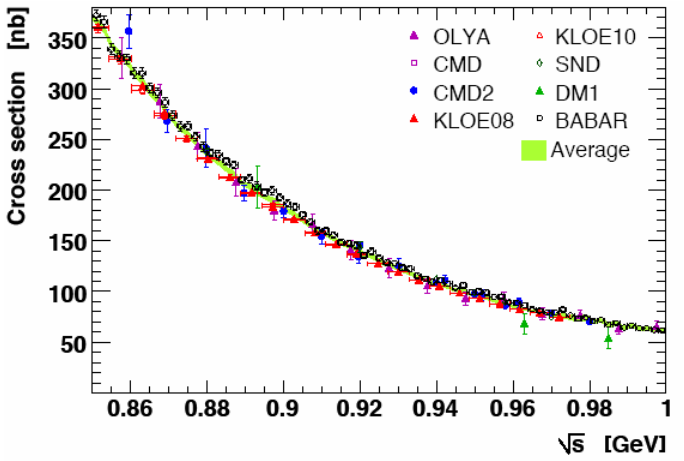
BABAR
Average


Cross section [nb]
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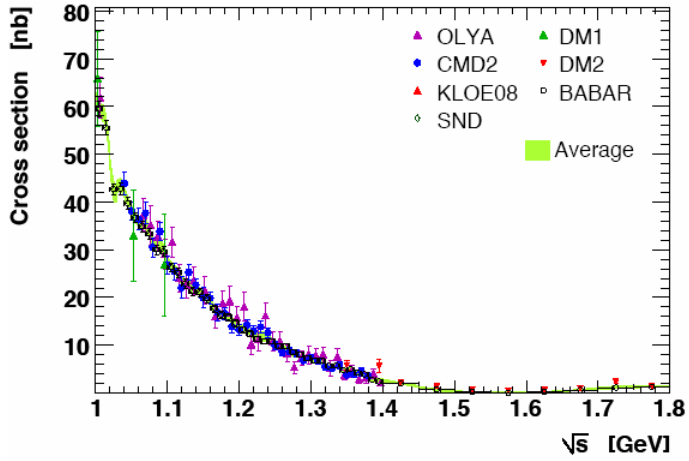
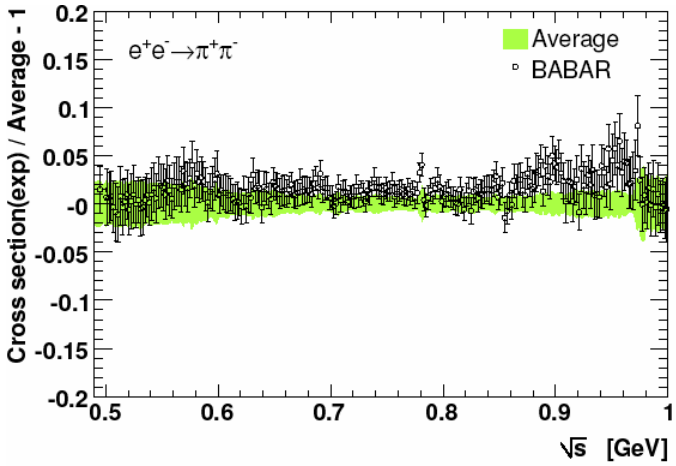
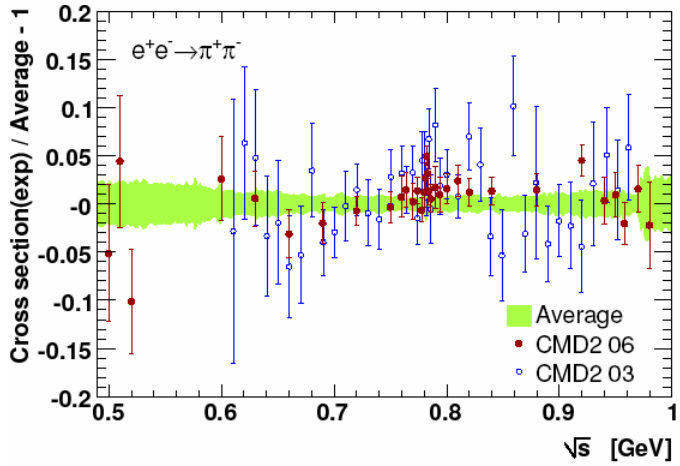
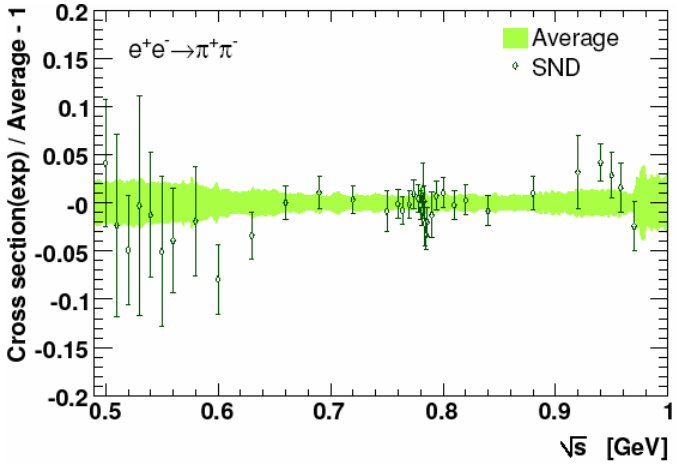
BABAR
Average

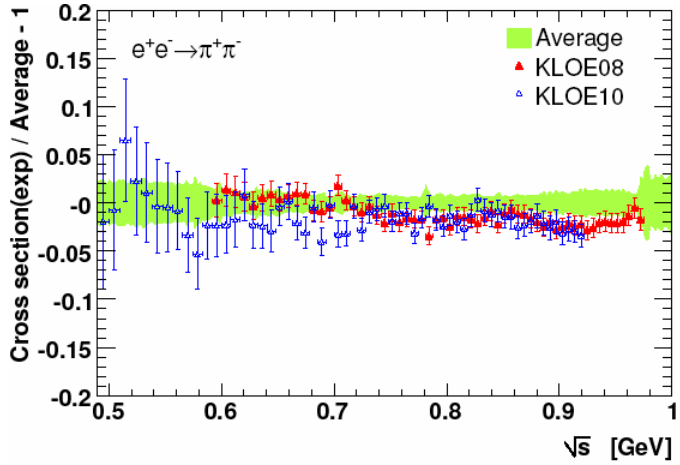

Cross section [nb]
√s [GeV]
OLYA
CMD2
KLOE08
SND
DM1
DM2
BABAR
Average

2π experiments compared to combined



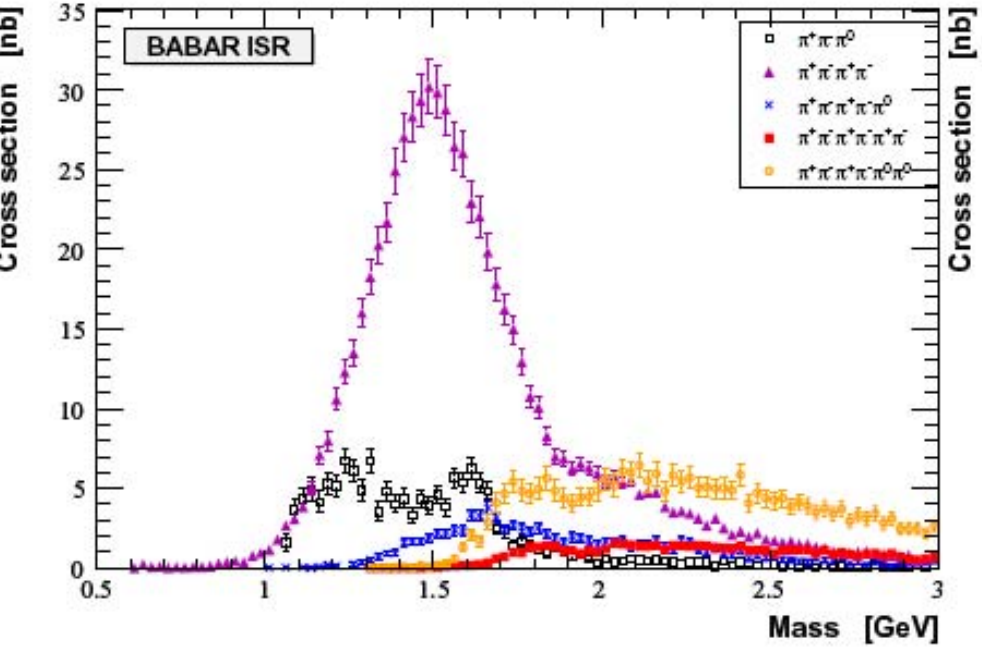


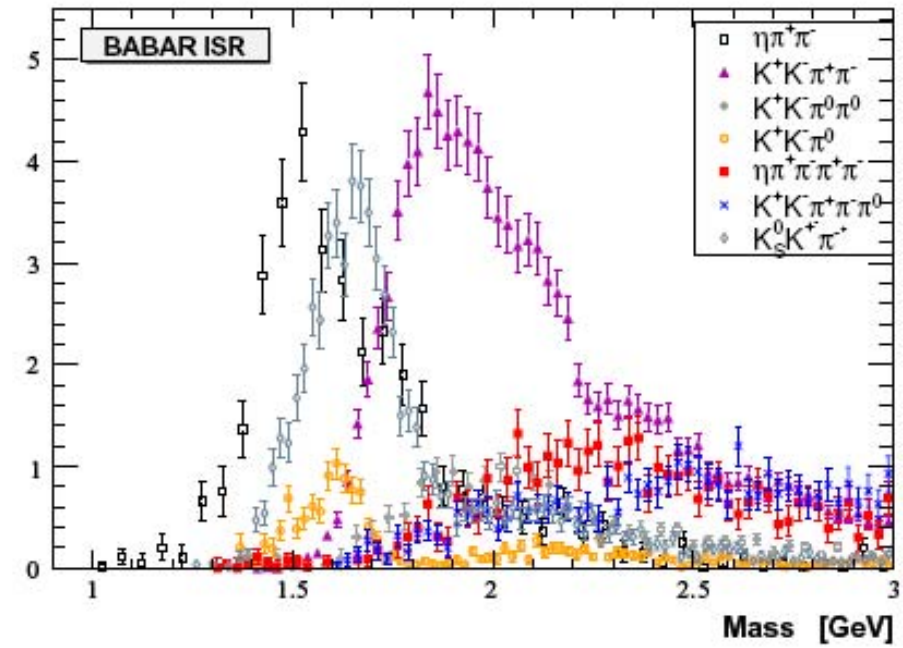




Multihadronic channels from BABAR

Statistical + systematic errors

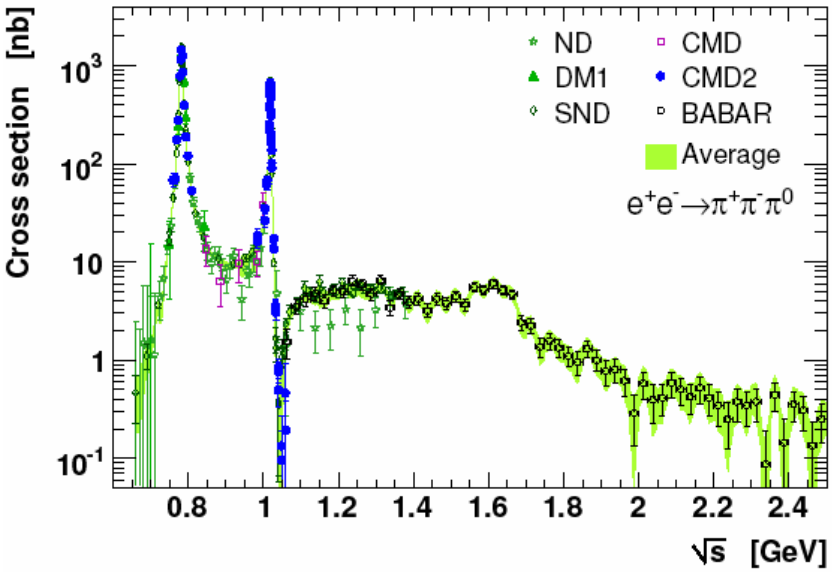


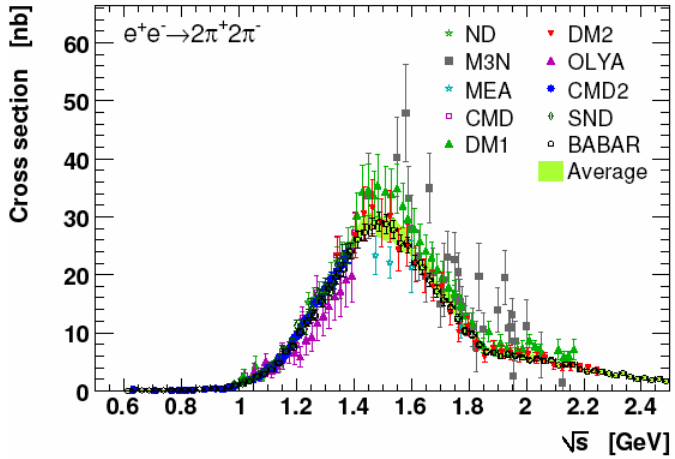


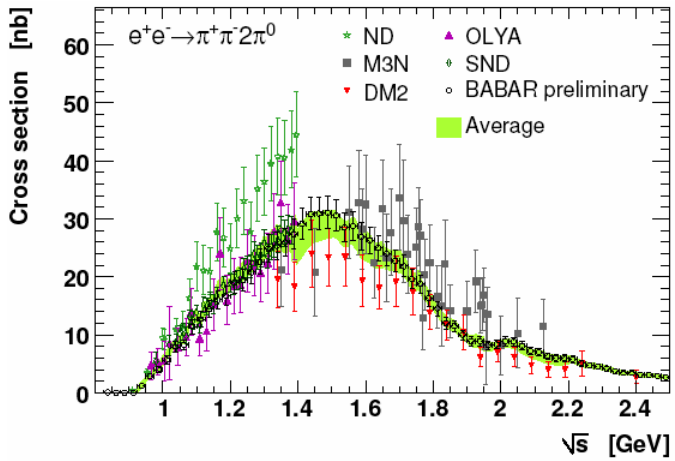
Still more channels under analysis: K^+K^-, $KK\pi\pi$ with K^0, $\pi^+\pi^-2\pi^0$

Also for each channel: dynamics (resonance production) has been studied
$\Rightarrow$ important to estimate missing channels through isospin relations

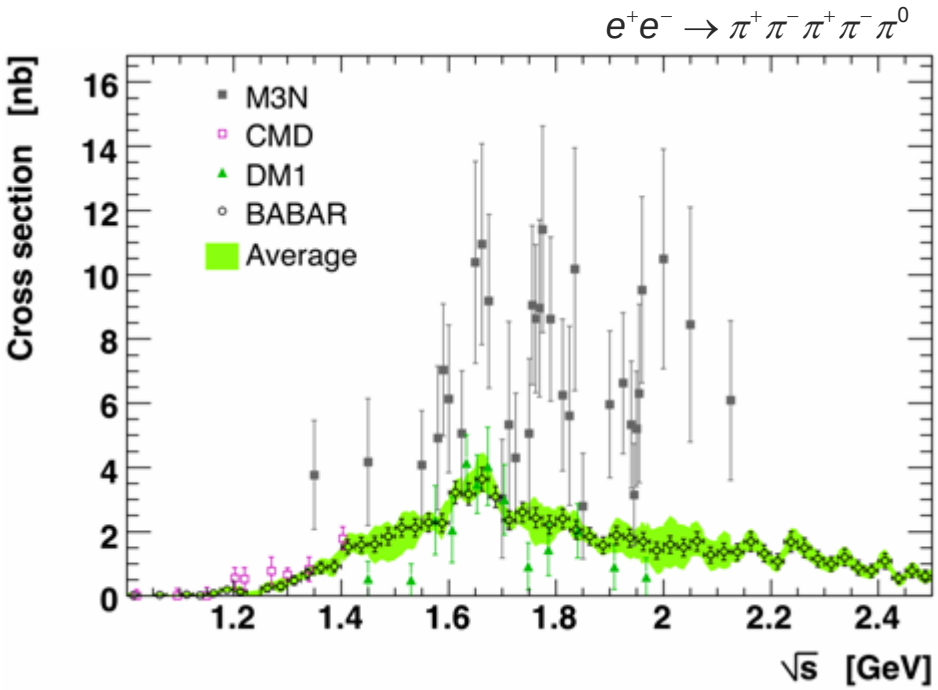
3π, 4π, $2\pi2\pi^0$ combined

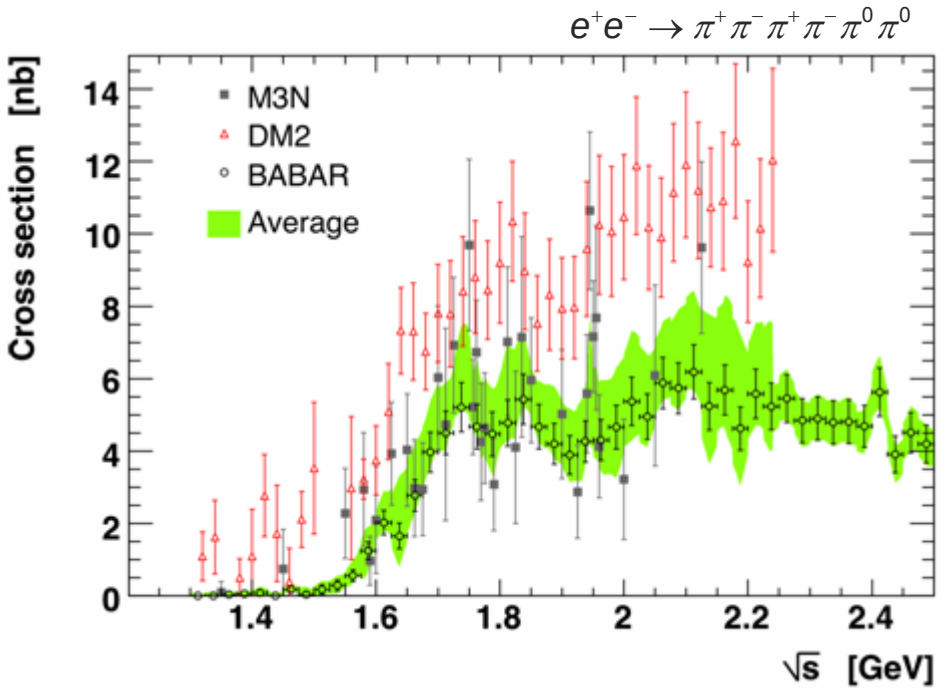






Some changes





In several cases, earlier poor measurements have lead to overestimated cross sections

Results

Channel	a_μ^{had} (e^+e^-)	a_μ^{had} (τ)
$\pi^0\gamma$	$4.4 \pm 0.1_{stat} \pm 0.2_{syst}$	
$\pi^+\pi^-$	$507.8 \pm 1.2 \pm 2.6$	$515.2 \pm 3.0 \pm 1.9_{IB}$
$\pi^+\pi^-\pi^0$	$46.0 \pm 0.4 \pm 1.4$	
$2\pi^+2\pi^-$	$13.4 \pm 0.1 \pm 0.5$	$11.9 \pm 1.4 \pm 0.3_{IB}$
$\pi^+\pi^-2\pi^0$	$18.0 \pm 0.1 \pm 1.2$	$21.2 \pm 1.3 \pm 0.5_{IB}$
KK	$34.6 \pm 0.3 \pm 0.9$	
*KKn*π	$3.7 \pm 0.1 \pm 0.4$	
Other exclusive modes	$6.0 \pm 0.3 \pm 0.3$	
Charm region (3.7 – 5 GeV)	$7.3 \pm 0.1 \pm 0.3$	
J/ψ, ψ (2S)	7.9 ± 0.2	
R [QCD]	$43.3 \pm 0.3_{theo}$	*Error includes size of 4-loop term + FOPT/CIPT ambiguity*
Sum	$692.3 \pm 1.4 \pm 3.9$	

$\times 10^{-10}$

Adding all (28) Contributions Together

Hadronic LO term:

$$a_\mu^{\text{had,LO}}[e^+e^-] = (692.3 \pm 4.2_{ee+\text{QCD}}) \times 10^{-10}$$

$$a_\mu^{\text{had,LO}}[\tau] = (701.5 \pm 3.5_{\tau} \pm 1.9_{\text{SU(2)}} \pm 2.4_{ee+\text{QCD}}) \times 10^{-10}$$

Davier *et al.*, EPJ C 71, 1515 (2011)

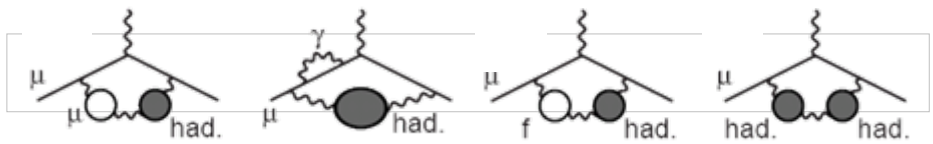
Hadronic NLO terms: 1.8 σ

Vacuum polarization (1-loop) + additional photon or VP insertion

•Computed akin to LO part via dispersion integral with modified kernel function

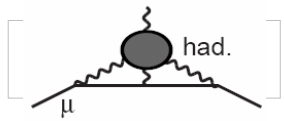
$$a_\mu^{\text{had,NLO}} = -9.8(0.1) \times 10^{-10}$$

Hagiwara *et al.* 2010 (and others)



Light-by-light scattering

- Dispersion relation approach not possible (4-point function)
- Model-dependent calculations



$$a_\mu^{\text{had,LBL}} = +10.5(2.6) \times 10^{-10}$$

Prades-deRafael-Vainshtein (and others)

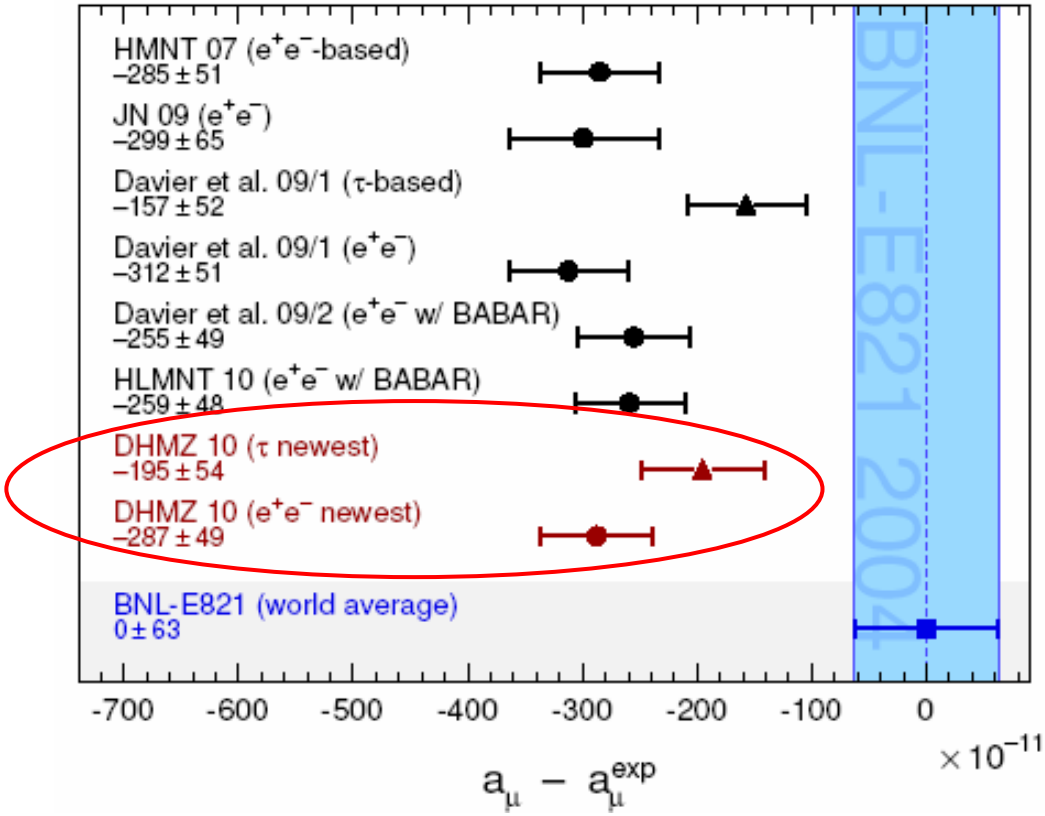
Jansen *et al.*, arXiv:1103.4818

- Lattice results may be in reach (LO HVP (*u+d* quarks only) computed to 3% accuracy)

Full a_μ^{SM}

$$a_\mu^{SM}(ee) = (11\,659\,180.2 \pm 4.2_{\text{had LO}} \pm 2.6_{\text{NLO}} \pm 0.2_{\text{QED+EW}})\ 10^{-10}$$

DHMZ 2010



Observed Difference with Experiment:

$$a_\mu^{exp} - a_\mu^{SM} = (29.6 \pm 8.1) \times 10^{-10}$$

➡ 3.6 ”standard deviations“

τ-based (2π, 4π, $2\pi 2\pi^0$) estimate also decreased (multihadronic modes) ⇒ 2.4σ

Summarizing the changes

The Tau-2010 DHMZ result is -3.2×10^{-10} smaller than that of 2009

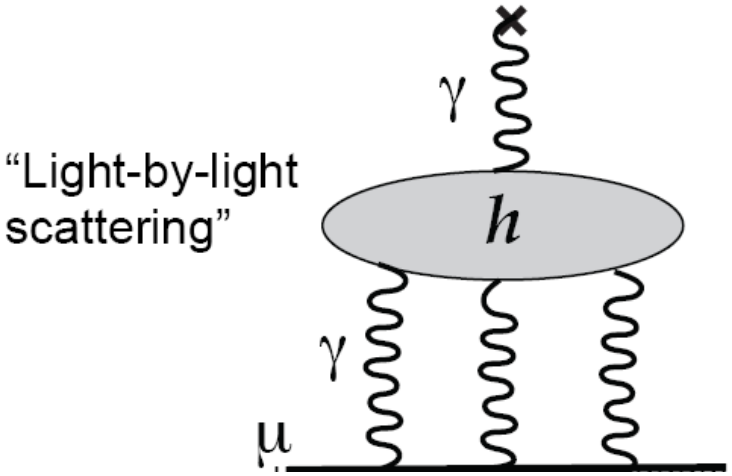
Origins of main changes:

$\pi^+\pi^-$	–0.6	New KLOE 2010 data
$\pi^+\pi^-\pi^0\pi^0$	+0.4	Use preliminary BABAR data
Other exclusive modes and resonances	–2.5	New BABAR data, improved isospin constraints on unknown modes
QCD	–0.5	4-loop pQCD coefficient

Many modes also have been computed for the first time with HVPTools

featuring a more precise interpolation, and better error propagation than our previous software

A possible show-stopper: HLBL contribution



- cannot be obtained from data
- need models
- eventful track record: sign, magnitude
- identified contributions:
  - meson poles (π^0, η, η')
  - meson loops
  - quark loops
- several groups $\Rightarrow$ consistent results

- "consensus" (de Rafael, Prades, Vainhstein)
  $+(10.5 \pm 2.6)\ 10^{-10}$
- estimated error more controversial (Nyffeler)
- beware of double counting

Running of α_{QED}

Same principle as for $g-2$: energy-dependent vacuum polarisation effects screen the bare electromagnetic coupling. Leptonic contributions computed via QED, hadronic contributions obtained from dispersion relation:

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)} \quad \text{with:} \quad \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha_{\text{had}}(s) = -4\pi\alpha\,\text{Re}\left[\Pi_\gamma(s) - \Pi_\gamma(0)\right]$$

$$\Delta\alpha_{\text{lep}}^{\text{3-loop}}(M_Z^2) = 0.031497686$$

Steinhauser, hep-ph/9803313 (1998)

$$\Delta\alpha_{\text{had}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\int_{m_{\pi^0}^2}^{\infty} \frac{R(s)}{s\left(s-M_Z^2\right)-i\varepsilon}\,ds = 0.02768(22)_{\text{had (5)}} - 0.000073(2)_{\text{top}}$$

Integration kernel more "democratic" than for $g-2$
(influence of tau data less pronounced)

New evaluation of $\alpha(M_Z)$

Also the hadronic contribution to $\alpha_{QED}(M_Z)$ has been re-evaluated:

$$\Delta\alpha_{had}^{(5)} = (275.7 \pm 0.2_{stat} \pm 0.9_{syst} \pm 0.5_{QCD} (\pm 1.0_{tot})) \, 10^{-4}$$

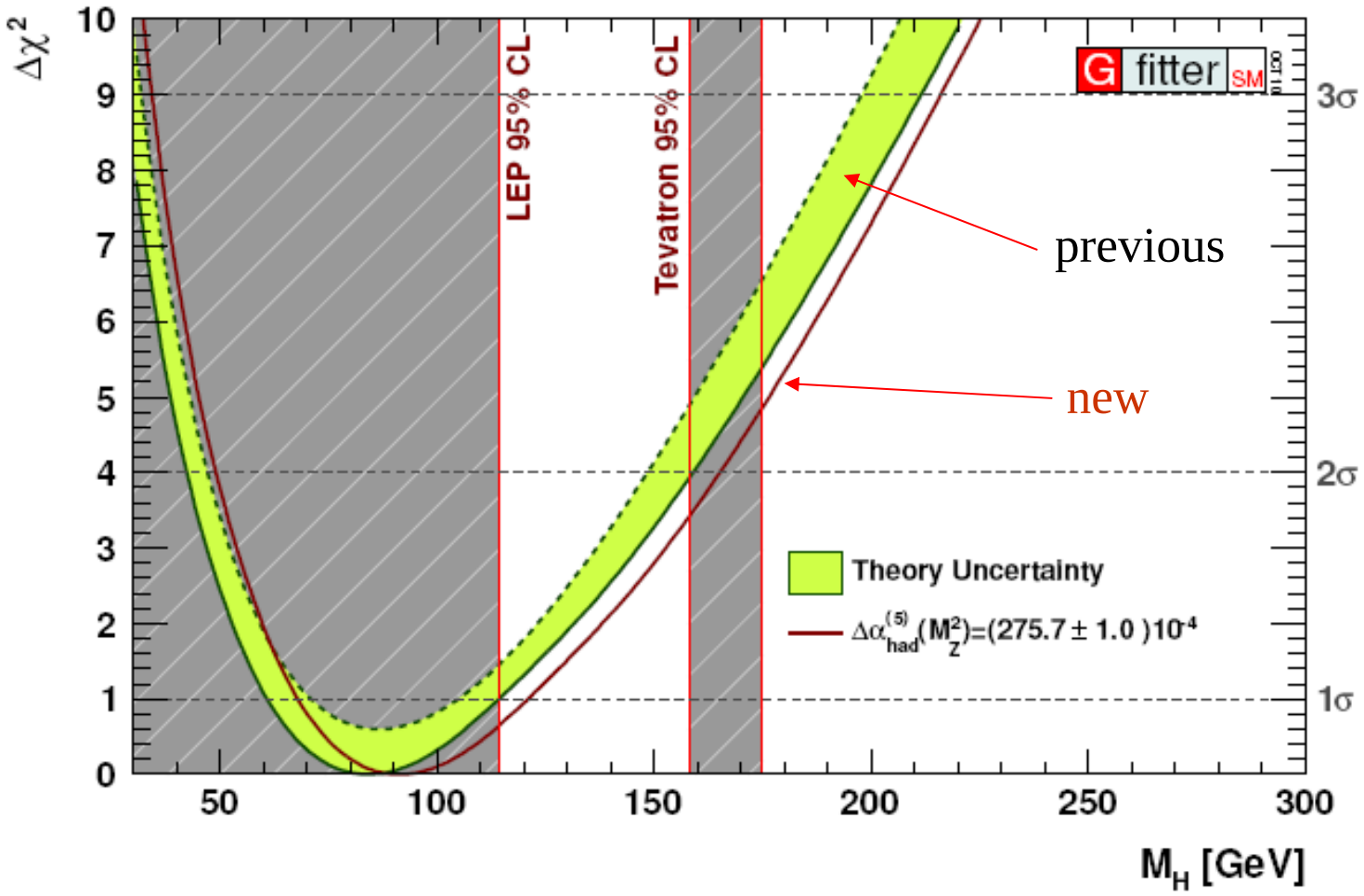
$$\alpha^{-1}(M_Z) = 128.952 \pm 0.014$$

The better precision of the DHMZ value with respect to HMNT ($\pm$ 1.5) is because of the use of QCD instead of BES data between 1.8 and 3.7 GeV

(HMNT employs QCD central values, but with BES errors)

Due to the –40% correlation between $\Delta\alpha_{had}(M_Z)$ and M_H in the global electroweak fit, the change in the central value increases M_H and reduces tension between fit and direct searches !

Global EW fit with new value of $\alpha(M_Z)$



$\Rightarrow$ (indirect) M_H increased from $(84 \pm {}^{30}_{23})$ GeV to $(91 \pm {}^{30}_{23})$ GeV
reduces tension with direct-search lower limit

Conclusions and Perspectives (1)

Recent progress in evaluating hadronic contribution to muon (g–2)

- better data to compute dispersion integral
  2π (CMD2, SND, KLOE, BABAR) multihadrons (BABAR)
- multihadronic dynamics (BABAR) to estimate un-measured modes
- still some disagreement for 2π (KLOE/BABAR), but reduced
- better τ data (ALEPH, CLEO, BELLE)
- better estimates of isospin-breaking corrections to τ
- ee/τ disagreement reduced (+ BABAR), still problem with KLOE
- KLOE/(BABAR-τ) discrepancy must be understood

Conclusions and Perspectives (2)

- most recent/complete evaluation (DHMZ) increases SM/exp discrepancy on a_μ to 3.6σ

- progress will continue on LO HVP contribution (VEPP-2000)

- NLO contribution from hadronic light-by-light still a weak spot will soon limit the SM prediction uncertainty
  $\Rightarrow$ more work needed

- SM uncertainty (4.9) smaller than present experimental error (6.3)
  $\Rightarrow$ need a new experiment (FNAL, JPARC)

- new evaluation of $\alpha(M_Z)$ is more precise and increases (indirect) Higgs mass by 12 GeV

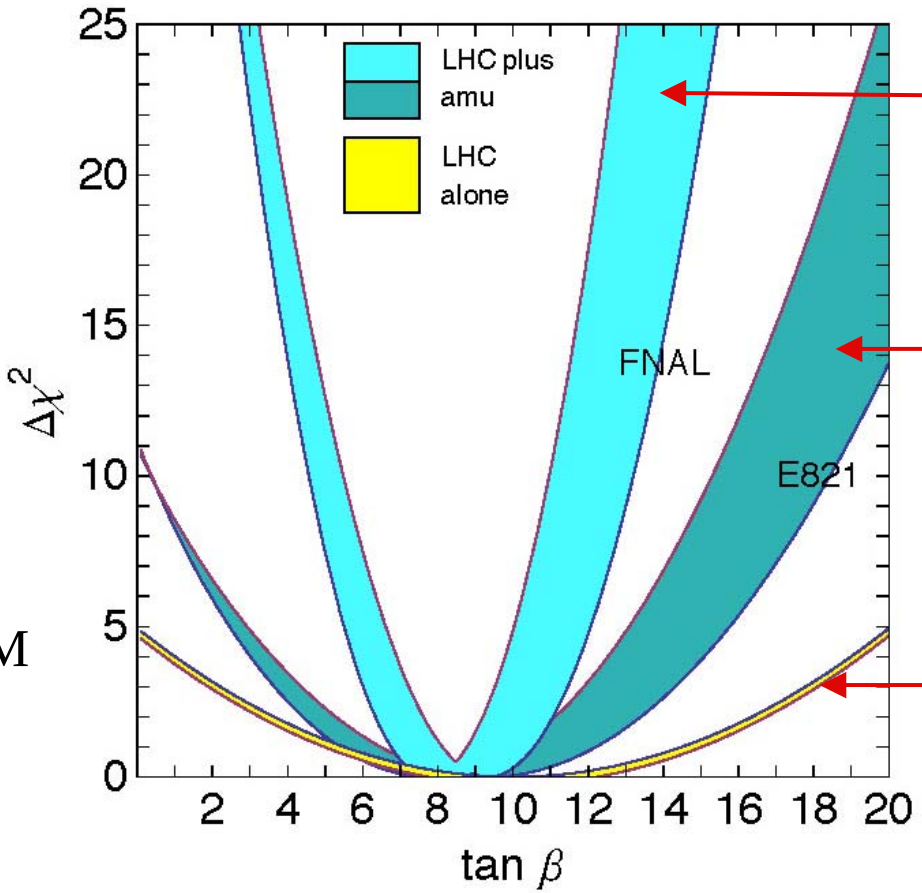
Backup Slides

Impact on new physics : ex. SUSY

LHC: direct search for SUSY partners

difficult to measure couplings and disentangle between models (ILC)

g-2 measurement + theory prediction: sensitivity to couplings



Future g-2

Present g-2

LHC (sfitter)

taking one MSSM parameter point (D. Stöckinger)

The E-821 a_μ Measurement at BNL Updated

a_μ measured from a ratio of frequencies

$\omega_a = \omega_{precession} - \omega_{cyclotron}$ $\omega_{precession} = \omega_L + \omega_T$ $\omega_a = a_\mu \frac{eB}{m_\mu}$

$$a_\mu = \frac{\omega_a}{\omega_L - \omega_a} = \frac{\omega_a/\widetilde{\omega}_p}{\omega_L/\widetilde{\omega}_p - \omega_a/\widetilde{\omega}_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

$\lambda = \omega_L / \omega_p = \mu_\mu / \mu_p$ from muonium hyperfine splitting

value used by E-821 3.18334539(10)
new value 3.183345137(85) Mohr et al., RMP 80 (2008) 633

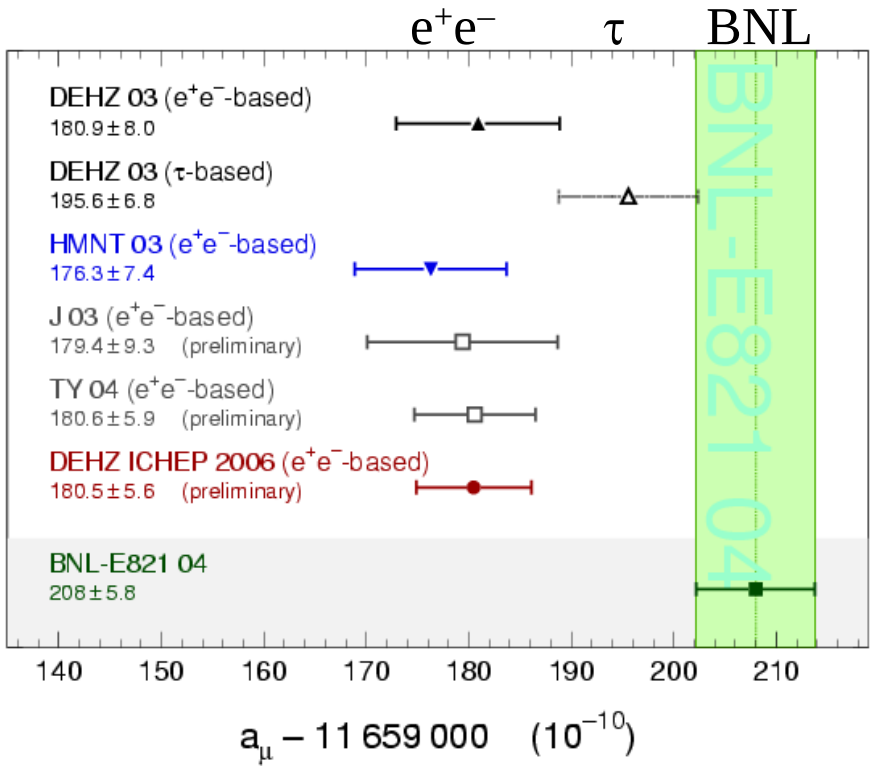
$\Rightarrow$ change in a_μ ($+0.92\ 10^{-10}$)
(review in RPP2009 (Höcker-Marciano)

$a_\mu^{exp} = (11\ 659\ 208.9 \pm 5.4 \pm 3.3)\ 10^{-10}$ updated
(± 6.3) (0.54 ppm)

Situation at ICHEP'06 / 08

$$a_\mu^{had}[ee] = (690.9 \pm 4.4) \times 10^{-10}$$

$$a_\mu[ee] = (11\,659\,180.5 \pm 4.4_{had} \pm 3.5_{LBL} \pm 0.2_{QED+EW}) \times 10^{-10}$$



Hadronic HO	$-(9.8 \pm 0.1) \times 10^{-10}$
Hadronic LBL	$+(12.0 \pm 3.5) \times 10^{-10}$
Electroweak	$(15.4 \pm 0.2) \times 10^{-10}$
QED	$(11\,658\,471.9 \pm 0.1) \times 10^{-10}$

Knecht-Nyffeler (2002), Melnikov-Vainhstein (2003)

Davier-Marciano (2004)

Kinoshita-Nio (2006)

Observed Difference with BNL using e^+e^-:

$$a_\mu[\text{exp}] - a_\mu[\text{SM}] = (27.5 \pm 8.4) \times 10^{-10}$$

➡ 3.3 „standard deviations"

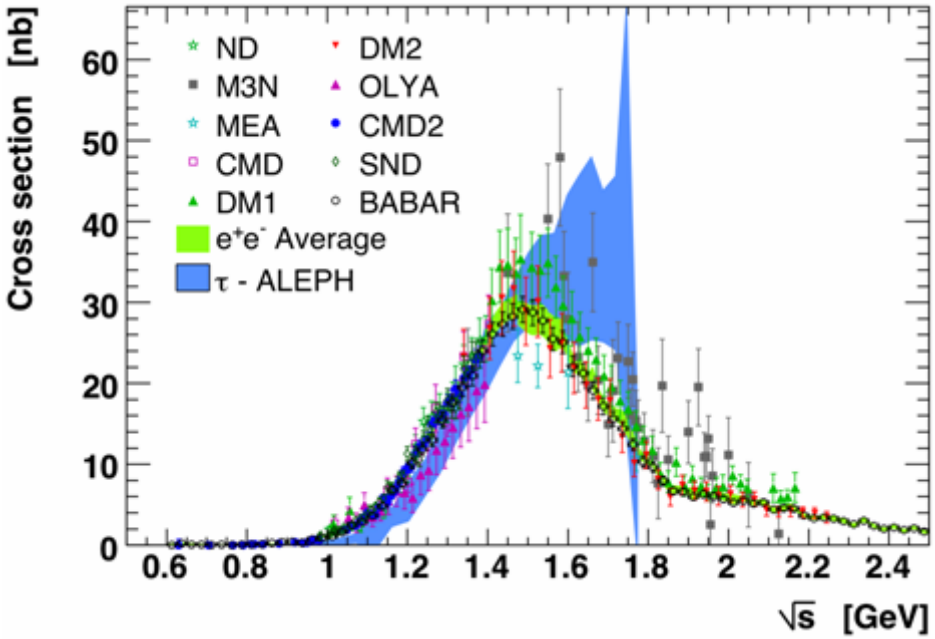
But estimate using τ data consistent with E-821 !

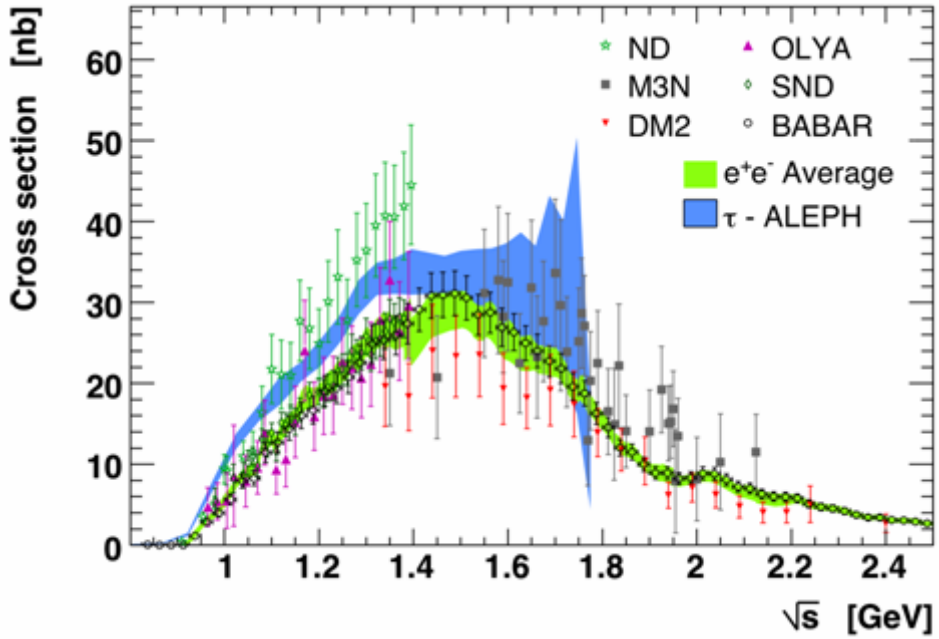
Multihadron channels: 4-pion (ee and τ)

Recall 4π isospin relations:

$$\sigma^{(I=1)}_{2\pi^+2\pi^-} = 2 \cdot \frac{4\pi\alpha^2}{s} \upsilon_{\pi^- 3\pi^0 \nu_\tau}$$

$$\sigma^{(I=1)}_{\pi^+\pi^- 2\pi^0} = \frac{4\pi\alpha^2}{s} \left(\upsilon_{2\pi^-\pi^+\pi^0\nu_\tau} - \upsilon_{\pi^- 3\pi^0 \nu_\tau} \right)$$



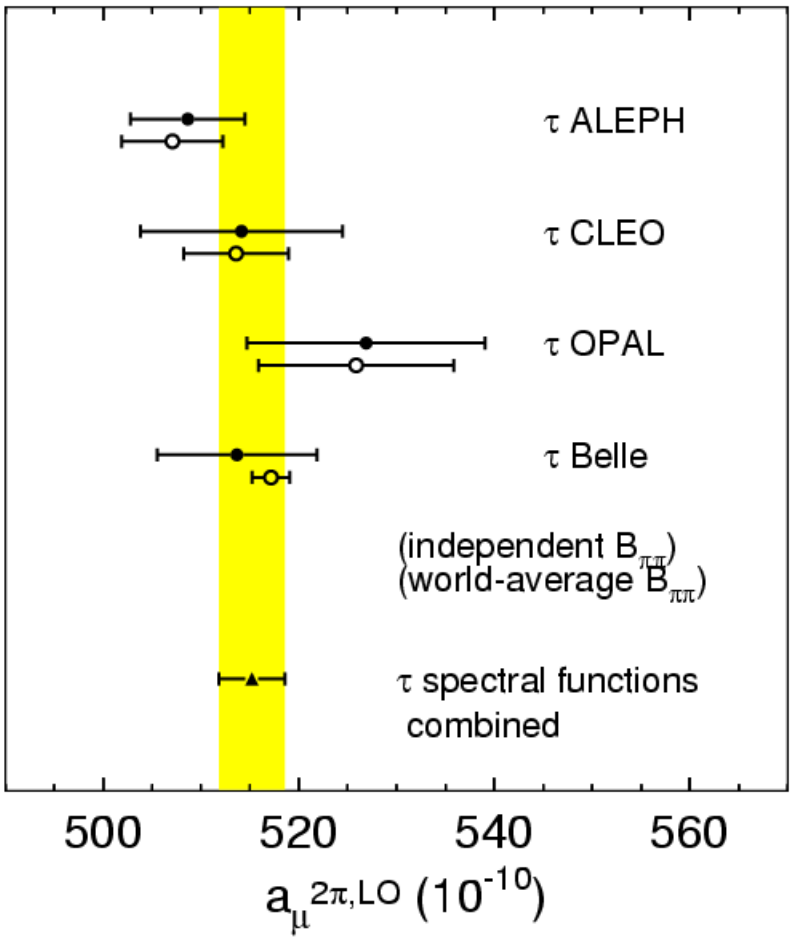


Revisited Analysis τ Data: new IB corrections

Source	$\Delta a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model
$S_{\rm EW}$	-12.21 ± 0.15	
$G_{\rm EM}$	-1.92 ± 0.90	
FSR	$+4.67 \pm 0.47$	
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64
Total	-16.07 ± 1.22	-16.70 ± 1.23
	-16.07 ± 1.85	

⇐ large change since DEHZ (2003)

Consistency of τ Data: Dispersion Integrals



- using BR from each experiment makes results independent from each other
- consistent results (disagreement with Benayoun et al. arXiv:09075603v1)

- using WA BR checks consistency for the spectral function shapes

- WA BR + combined spectral function ⇒

$a_\mu^{2\pi,LO}=(515.2\pm2.0_{exp}\pm0.9_{Be}\pm2.1_{B\pi\pi}\pm1.6_{IB})\ 10^{-10}$

- 0.7% precision