

Strong Coupling Constant from First Principles Konstantin Petrov

# $\Lambda_{\bar{M} S}$ and $\alpha_{s}$ from first principles 

Konstantin Petrov (LAL/ORSAY)

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B. Blossier, Ph. Boucaud, M. Brinet, F. De Soto, X. Du, V. Morenas, O. Pène, K. Petrov, J. Rodríguez-Quintero


## Introduction

- number of parameters of QCD is small
- quark mass for every species
- and a scale $\Lambda$
- or, equivalently, strong coupling constant $\alpha_{s}$
- once these are fixed, we can calculate everything else


## Introduction

- experimental determination error is dominated by the QCD error
- and it is important to improve on that
- as it matters for LHC crossection studies
- and exploring new physics
- it has to be done non-perturbatively
- with full control over systematic errors


## Lattice

- involves a lot of heavy machinery
- such as supercomputers, various formulations
- continuum, chiral and infinite volume limits
- and very exciting talks describing them
- but is a well-defined theory in discrete space-time
- consists of generating snap-shots of reality
- guided by (discretized) QCD Lagrangian
- and looking at each very carefully
- then, averaging


## Basics

- $\bar{M} S$ is not accessible on the lattice
- so we will perform calculation in so-called Taylor scheme
- and then convert to $\bar{M} S$

$$
\begin{equation*}
\alpha_{T}\left(\mu^{2}\right) \equiv \frac{g_{T}^{2}\left(\mu^{2}\right)}{4 \pi}=\lim _{\Lambda \rightarrow \infty} \frac{g_{0}^{2}\left(\Lambda^{2}\right)}{4 \pi} G\left(\mu^{2}, \Lambda^{2}\right) F^{2}\left(\mu^{2}, \Lambda^{2}\right), \tag{1}
\end{equation*}
$$

where $F$ and $G$ stand for the ghost and gluon dressing functions

## Propagators and Green Functions

$$
\begin{equation*}
A_{\mu}(x+\hat{\mu} / 2)=\frac{U_{\mu}(x)-U_{\mu}^{\dagger}(x)}{2 i a g_{0}}-\frac{1}{3} \operatorname{Tr}\left(\frac{U_{\mu}(x)-U_{\mu}^{\dagger}(x)}{2 \operatorname{iag}_{0}}\right) \tag{2}
\end{equation*}
$$

The 2-point Green functions is computed in momentum space by

$$
\begin{align*}
& \left(G^{(2)}\right)_{\mu_{1} \mu_{2}}^{a_{1} a_{2}}(p)=\left\langle A_{\mu_{1}}^{a_{1}}(p) A_{\mu_{2}}^{a_{2}}(-p)\right\rangle  \tag{3}\\
& \left(F^{(2)}\right)^{a b}(x-y) \equiv\left\langle\left(M^{-1}\right)_{x y}^{a b}\right\rangle \tag{4}
\end{align*}
$$

as the inverse of the Faddeev-Popov operator, that is written as the lattice divergence,

$$
\begin{equation*}
M(U)=-\frac{1}{N} \nabla \cdot \widetilde{D}(U) \tag{5}
\end{equation*}
$$

## Technical Details

- So we calculate propagators on all our ensembles of "reality"
- Remove discretization artefacts which are not rotation-invariant
- then remove the invariant ones


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- So we calculate propagators on all our ensembles of "reality"
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- then remove the invariant ones
- now, back to QCD.


## Lattice and Perturbation theory: an unlikely friendship

$$
\begin{align*}
\alpha_{T}\left(\mu^{2}\right) & =\alpha_{T}^{\text {pert }}\left(\mu^{2}\right)\left(1+\frac{9}{\mu^{2}} R\left(\alpha_{T}^{\text {pert }}\left(\mu^{2}\right), \alpha_{T}^{\text {pert }}\left(q_{0}^{2}\right)\right)\right. \\
& \left.\times\left(\frac{\alpha_{T}^{\text {pert }}\left(\mu^{2}\right)}{\alpha_{T}^{\text {pert }}\left(q_{0}^{2}\right)}\right)^{1-\gamma_{0}^{A^{2}} / \beta_{0}} \frac{g_{T}^{2}\left(q_{0}^{2}\right)\left\langle A^{2}\right\rangle_{R, q_{0}^{2}}}{4\left(N_{C}^{2}-1\right)}\right), \tag{6}
\end{align*}
$$

- derived from the OPE description of ghost and gluon dressing function
- $\gamma_{0}^{A^{2}}$ is calculated by perturbation theory (Gracey, Chetyrkin)
- $N_{f}=4,1-\gamma_{0}^{A^{2}} / \beta_{0}=27 / 100$,


## Further Perturbations

$$
\begin{align*}
R\left(\alpha, \alpha_{0}\right) & =\left(1+1.18692 \alpha+1.45026 \alpha^{2}+2.44980 \alpha^{3}\right) \\
& \times\left(1-0.54994 \alpha_{0}-0.13349 \alpha_{0}^{2}-0.10955 \alpha_{0}^{3}\right) \\
\alpha_{T}^{\text {pert }}\left(\mu^{2}\right) & =\frac{4 \pi}{\beta_{0} t}\left(1-\frac{\beta_{1}}{\beta_{0}^{2}} \frac{\log (t)}{t}+\frac{\beta_{1}^{2}}{\beta_{0}^{4}} \frac{1}{t^{2}}\left(\left(\log (t)-\frac{1}{2}\right)^{2}\right.\right. \\
& \left.+\frac{\beta_{2} \beta_{0}}{\beta_{1}^{2}}-\frac{5}{4}\right)+\frac{1}{\left(\beta_{0} t\right)^{3}}\left(\frac{\beta_{3}}{2 \beta_{0}}+\frac{1}{2}\left(\frac{\beta_{1}}{\beta_{0}}\right)^{3}\right. \\
& \times\left(-2 \log ^{3}(t)+5 \log ^{2}(t)\right. \\
& \left.\left.\left.+\left(4-6 \frac{\beta_{2} \beta_{0}}{\beta_{1}^{2}}\right) \log (t)-1\right)\right)\right) \tag{7}
\end{align*}
$$

with $t=\ln \frac{\mu^{2}}{\Lambda_{T}^{2}}$

## Fitting and Condensing

- So now we can compare prediction of OPE with lattice data
- and fit using two coefficients
- $g^{2}\left\langle A^{2}\right\rangle$, the Landau gluon condensate
- and $\Lambda_{T}$, the $\Lambda_{\mathrm{QCD}}$ parameter in Taylor scheme


## Plots: Plateau of Saclay



## Plots: Taylor scheme




## Results

- First results with non-perturbative charm $(2+1+1) \mathrm{F}$
- Systematic errors controlled at all steps
- Both on lattice and on perturbative side
- and the only thing you have to remember from this...


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