

Lattice computation of $K \rightarrow \pi\pi$ amplitudes

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- Background: kaon decay, kaon mixing and CP violation
- Computation of the $\Delta I = 3/2$ amplitude
 - Challenge and setup of the computation
 - Bare matrix elements
 - Phase shift
 - Renormalization
 - Physical implications
- Toward a computation of the $K \rightarrow (\pi\pi)_{I=0}$ amplitude

Based on work done within the RBC-UKQCD collaborations

■ *K to $\pi\pi$ Decay amplitudes from Lattice QCD.*

T. Blum, P. A. Boyle, N. H. Christ, N.G. , E. Goode, T. Izubuchi, C. Lehner, Q. Liu, R. D. Mawhinney, C. T. Sachrajda, A. Soni, C. Sturm, H. Yin, R. Zhou.
Phys.Rev.D., 2011.

■ *Opening the Rome-Southampton window for operator mixing matrices.*

R. Arthur, P. A. Boyle, N.G. , C. Kelly, A. T. Lytle .
Phys.Rev.D., 2011.

■ *The $K \rightarrow (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD.*

T. Blum, P. A. Boyle, N. H. Christ, N.G. , E. Goode, T. Izubuchi, C. Jung, C. Kelly, C. Lehner, M. Lightman, Q. Liu, A. T. Lytle, R. D. Mawhinney, C. T. Sachrajda, A. Soni, C. Sturm.
PRL, 2012.

Background: Kaon decays and CP violation

- First discovery of CP violation was made in kaon system in 1964 (Christenson, Cronin, Fitch and Turlay)
- Noble prize in 1980 (Cronin and Fitch)
- Very nice measurements of both direct and indirect CP violation [PDG '10]

$$\left\{ \begin{array}{l} \text{Re} \left(\frac{\epsilon'}{\epsilon} \right) = (1.65 \pm 0.26) \times 10^{-3} \\ |\epsilon| = (2.228 \pm 0.011) \times 10^{-3} \end{array} \right.$$

- Still lacking a quantitative theoretical description
- Theoretically:

Relate indirect CP violation parameter (ϵ) to neutral kaon mixing (B_K)

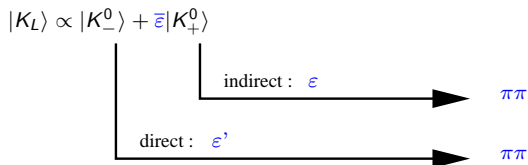
Still lacking a quantitative description of direct CP violation (ϵ')

Background: Kaon decays and CP violation

Flavour eigenstates $\left(\begin{array}{l} K^0 = \bar{s}\gamma_5 d \\ \bar{K}^0 = \bar{d}\gamma_5 s \end{array} \right) \neq$ CP eigenstates $|K_{\pm}^0\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle \mp |\bar{K}^0\rangle\}$

They are mixed in the physical eigenstates $\begin{cases} |K_L\rangle & \sim |K_-^0\rangle + \bar{\varepsilon}|K_+^0\rangle \\ |K_S\rangle & \sim |K_+^0\rangle + \bar{\varepsilon}|K_-^0\rangle \end{cases}$

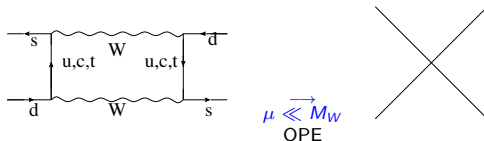
Direct and indirect CP violation in $K \rightarrow \pi\pi$



Experimentally [PDG '10] $\begin{cases} \text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) & = (1.65 \pm 0.26) \times 10^{-3} \\ |\varepsilon| & = (2.228 \pm 0.011) \times 10^{-3} \end{cases}$

Neutral kaon mixing

In the SM $K^0 - \bar{K}^0$ mixing dominated by box diagrams with W exchange, e.g.

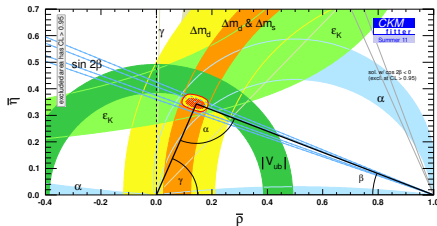


Factorise the non-perturbative contribution into

$$\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle = \frac{8}{3} F_K^2 M_K^2 B_K(\mu) \quad \text{w/} \quad \mathcal{O}_{LL}^{\Delta S=2} = (\bar{s} \gamma_\mu (1 - \gamma_5) d) (\bar{s} \gamma^\mu (1 - \gamma_5) d)$$

Related to ε via CKM parameters, schematically

$$\varepsilon \sim \text{known factors} \times V_{CKM} \times C(\mu) \times B_K(\mu)$$



[CKMfitter'11]

$K \rightarrow \pi\pi$ amplitudes

Two isospin channels: $\Delta I = 1/2$ and $\Delta I = 3/2$

$$K \rightarrow (\pi\pi)_{I=0,2}$$

Corresponding amplitudes defined as

$$A[K \rightarrow (\pi\pi)_I] = A_I \exp(i\delta_I) \quad /w \ I = 0, 2 \quad \delta = \text{strong phases}$$

$\Delta I = 1/2$ rule

$$\omega = \frac{\text{Re}A_2}{\text{Re}A_0} \sim 1/22 \quad (\text{experimental number})$$

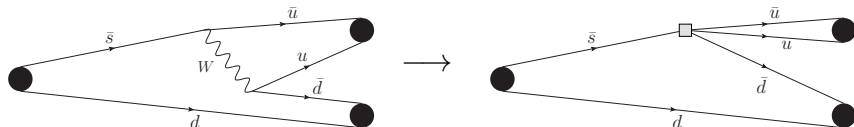
Amplitudes are related to the parameters of CP violation $\varepsilon, \varepsilon'$ via

$$\text{Re} \left[\frac{\varepsilon'}{\varepsilon} \right] = \frac{\omega}{\sqrt{2}|\varepsilon|} \left[\frac{\text{Im}(A_2)}{\text{Re}(A_2)} - \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right]$$

$$\varepsilon' = \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}} \left[\frac{\text{Im}(A_2)}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right]$$

Overview of the computation

Operator Product expansion



Describe $K \rightarrow (\pi\pi)_{I=0,2}$ with an effective Hamiltonian

$$H^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{10} (V_{ud} V_{us}^* Z_i(\mu) - V_{td} V_{ts}^* Y_i(\mu)) Q_i(\mu) \right\}$$

Short distance effects factorized in the Wilson coefficients y_i, z_i

Long distance effects factorized in the matrix elements

$$\langle \pi\pi | Q_i | K \rangle \longrightarrow \text{Lattice}$$

See eg [\[Norman Christ @ Kaon'09\]](#) for an overview of different strategies.

4-quark operators (II)

Current-Current

$$Q_2 = (\bar{s}u)_{V-A}(\bar{u}d)_{V-A} \quad Q_1 = \text{color mixed}$$

QCD penguins

$$Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V-A} \quad Q_4 = \text{color mixed}$$

$$Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V+A} \quad Q_6 = \text{color mixed}$$

EW penguins

$$Q_7 = \frac{3}{2}(\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}q)_{V+A} \quad Q_8 = \text{color mixed}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}q)_{V-A} \quad Q_{10} = \text{color mixed}$$

A challenge !

Many obstacles:

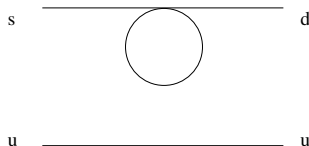
- Final state with two pions
- Many operators that mix under renormalisation
- Require the evaluation of disconnected graphs (numerically difficult)

Moreover, using a chiral discretisation of the Dirac operator is probably unavoidable.

Plus the usual difficulties: light dynamical quarks, large volume, . . .

Isospin channels

- A priori 10 four-quark operators
- 7 are linearly independent
- Only 3 of these operators contribute to the $\Delta I = 3/2$ channel
 - A tree-level operator
 - 2 electroweak penguins
- No disconnect graphs (which are hard to compute) contribute to the $\Delta I = 3/2$ channel



$\Rightarrow A_0$ is much more challenging than A_2

First problem: $|\pi\pi\rangle$ two hadrons in the final state

In a infinite volume, the physical two-pion state cannot be obtained from the Euclidean one

[Maiani & Testa '90] \longrightarrow no-go theorem

Solution: Lellouch-Lüscher method [Lellouch Lüscher '00]

Physical matrix element obtained from the finite-volume Euclidean amplitude and the derivative of the phase shift

New problem(s): simulating the physical kinematic requires a very large (physical) volume

- Large volume and low masses \longrightarrow numerically very expensive
- Have to use a coarse lattice \longrightarrow cutoff $a^{-1} \sim \Lambda_{\text{QCD}}$

Remember the Rome-Southampton window $\Lambda_{\text{QCD}} \ll \mu \ll a^{-1}$

\longrightarrow How do we renormalize the matrix elements ?

\longrightarrow We use a step-scaling matrix \Leftrightarrow universal (continuum) running matrix

Another difficulty

- With Periodic B.C., the 2-pion ground state corresponds to each pion being at rest
- For the first 2-pion state to have the energy $= m_K$ we need $L \sim 6$ fm
- Solution: combine
 - Wigner-Eckart theorem (Exact up to isospin symmetry breaking !)

$$\langle \pi^+(p_1) \pi^0(p_2) | O_{\Delta I_Z=1/2}^{\Delta I=3/2} | K^+ \rangle = 3/2 \langle \pi^+(p_1) \pi^+(p_2) | O_{\Delta I_Z=3/2}^{\Delta I=3/2} | K^+ \rangle$$

and then compute the unphysical process $K^+ \rightarrow \pi^+ \pi^+$

- Use Anti-periodic B.C. to eliminate the unwanted (wrong-kinematic) ground state
- It is enough to use antiPBC in the valence sector [Sachrajda & Villadoro PLB 2005, hep-lat 0411033]
- Works only for the $\Delta I = 3/2$ part

Only 3 operators: a (27, 1) and two (8, 8)

■ The physical ones:

$$Q_1 = (\bar{s}_\alpha \gamma_\mu^L d_\alpha) [(\bar{u}_\beta \gamma_\mu^L u_\beta) - (\bar{d}_\beta \gamma_\mu^L d_\beta)] + (\bar{s}_\alpha \gamma_\mu^L u_\alpha)(\bar{u}_\beta \gamma_\mu^L d_\beta)$$

$$Q_7 = (\bar{s}_\alpha \gamma_\mu^L d_\alpha) [(\bar{u}_\beta \gamma_\mu^R u_\beta) - (\bar{s}_\beta \gamma_\mu^R s_\beta)] + (\bar{s}_\alpha \gamma_\mu^L u_\alpha)(\bar{u}_\beta \gamma_\mu^R d_\beta)$$

$$Q_8 = (\bar{s}_\alpha \gamma_\mu^L d_\beta) [(\bar{u}_\beta \gamma_\mu^R u_\alpha) - (\bar{s}_\beta \gamma_\mu^R s_\alpha)] + (\bar{s}_\alpha \gamma_\mu^L u_\beta)(\bar{u}_\beta \gamma_\mu^R d_\alpha)$$

■ In the Wigner-Eckart basis

$$Q_{(27,1)} = (\bar{s}_\alpha \gamma_\mu^L d_\alpha)(\bar{u}_\beta \gamma_\mu^L d_\beta)$$

$$Q_{(8,8)} = (\bar{s}_\alpha \gamma_\mu^L d_\alpha)(\bar{u}_\beta \gamma_\mu^R d_\beta)$$

$$Q_{(8,8), \text{mix}} = (\bar{s}_\alpha \gamma_\mu^L d_\beta)(\bar{u}_\beta \gamma_\mu^R d_\alpha)$$

Simulation details

Use Domain-Wall fermions :

- Good chiral-flavour properties
- Numerically expensive

With $n_f = 2 + 1$ dynamical quarks :

- The light quarks are such that the lightest pion mass is $m_\pi \sim 170$ MeV for the unitary sector and $m_\pi \sim 140$ MeV for the partially quenched one
- The strange is close to its physical value

In order to avoid large finite volume effect, we have $V \sim (4.6 \text{ fm})^3$

Price to pay: coarse lattice : $a \sim 0.145 \text{ fm} \quad \leftrightarrow \quad a^{-1} \sim 1.364 \text{ GeV}$

So far only one lattice spacing

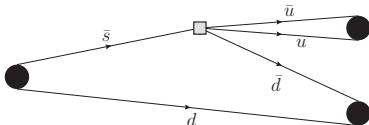
\Rightarrow Chiral dynamical fermions, light pion, large volume and coarse lattice

Extraction of the bare matrix elements

Compute a correlator

$$C_{K\pi\pi}^i = \langle 0 | J_{\pi\pi}(t_{\pi\pi}) Q_i(t_Q) J_K^\dagger(t_K) | 0 \rangle$$

$$\longrightarrow e^{-m_K(t_Q - t_K)} e^{-E_{\pi\pi}(t_{\pi\pi} - t_Q)} \langle 0 | J_{\pi\pi}(0) | \pi\pi \rangle \langle \pi\pi | Q_i(0) | K \rangle \langle K | J_K^\dagger(0) | 0 \rangle$$

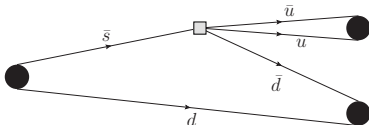


Extraction of the bare matrix elements

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Needs also

$$C_K(t) = \langle 0 | J_K(t) J_K^\dagger(0) | 0 \rangle \longrightarrow |\langle K | J_K^\dagger(0) | 0 \rangle|^2 e^{-m_K t}$$

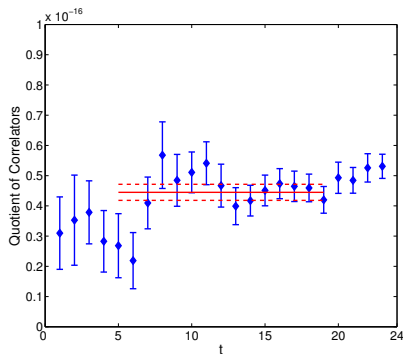
$$C_{\pi\pi}(t) = \langle 0 | J_{\pi\pi}(t) J_{\pi\pi}^\dagger(0) | 0 \rangle \longrightarrow |\langle 0 | J_{\pi\pi}(0) | \pi\pi \rangle|^2 e^{-E_{\pi\pi} t}$$

And compute the ratios

$$R(t_Q) \equiv \frac{C_{K\pi\pi}(t_K, t_Q, t_{\pi\pi})}{C_K(t_Q - t_K) C_{\pi\pi}(t_{\pi\pi} - t_Q)} \longrightarrow \frac{\langle \pi\pi | Q_i | K \rangle}{\langle 0 | J_{\pi\pi}(0) | \pi\pi \rangle \langle K | J_K^\dagger(0) | 0 \rangle}$$

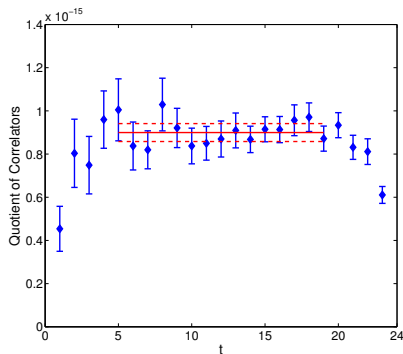
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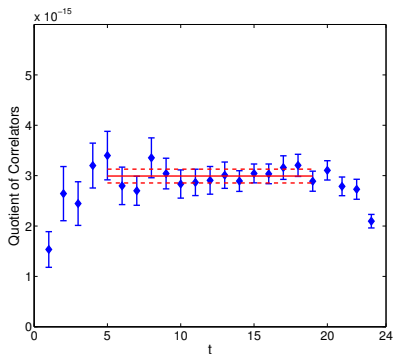
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Extraction of the bare matrix elements

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- With our boundary conditions we “give” to the pions the momentum $|\mathbf{p}| = \sqrt{2}\pi/L$
- We can compute $E_{\pi\pi} = 2E_\pi + \Delta E$ from a 2-point function $C_{\pi\pi}$
- Define the momentum k_π of each pion in the 2-pion state from the relation dispersion

$$E_{\pi\pi} = 2\sqrt{m_\pi^2 + k_\pi^2}$$

In MeV

m_π	m_K	$E_{\pi,2}$	$E_{\pi\pi,0}$	$E_{\pi\pi,2}$	$m_K - E_{\pi\pi,2}$
142.9(1.1)	511.3(3.9)	238.8(2.4)	288.0(2.2)	492.6(5.5)	18.7(4.8)

The subscripts 0, 2 denote $p = 0$ and $p = \sqrt{2}\pi/L$ respectively, where $p = |\mathbf{p}|$

The phase shift

The finite-volume matrix elements computed on the lattice \mathcal{M}_i are related to the corresponding infinite-volume ones \mathcal{A}_i by the Lellouch-Lüscher factor [Lellouch Lüscher '00, Lin et al '01]

$$\mathcal{A}_i = \left[\frac{\sqrt{\nu/4}}{\pi q_\pi} \sqrt{\frac{\partial \phi}{\partial q_\pi} + \frac{\partial \delta}{\partial q_\pi}} \right] \frac{1}{\sqrt{\nu}} \sqrt{m_K} E_{\pi\pi} \mathcal{M}_i$$

where :

$[\]$ is the LL factor

δ is the s -wave phase shift

ν is a factor counting the free-field degenerate states

$q_\pi = k_\pi L/2\pi$ where k_π is the pion momentum

ϕ is a kinematic function defined in [Lellouch Lüscher '00]

Once $E_{\pi\pi}$ has been measured and q_π determined, δ can be calculated using the Lüscher quantization condition [Lüscher 1990]

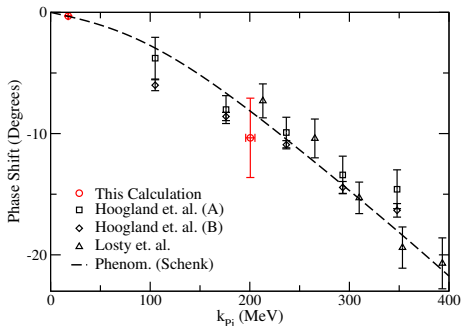
$$n\pi = \delta(k_\pi) + \phi(q_\pi).$$

The phase shift

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⇒ have to compute $\partial \delta / \partial q_\pi$



Renormalization of 4-quark operators

- Once the amplitudes $\mathcal{A}_i^{\text{lat}}$ have been obtained on the lattice, the physical decay amplitude \mathcal{A}_2 is obtained via

$$A_2^{\text{phys}} = a^{-3} \sqrt{\frac{3}{2}} G_F V_{ud} V_{us}^* \sum_{i,j} C_i(\mu) Z_{ij}(\mu, a) A_j^{\text{lat}}(a) \quad i, j = 1, 7, 8$$

- Have to compute the renormalization matrix Z
⇒ Better use a non perturbative scheme, like RI-MOM or S.F.
- Mixing pattern given by the $SU(3)_L \otimes SU(3)_R$ decomposition of the operators
- Motivation to work with a chiral action: the mixing pattern is the continuum one

Renormalisation pattern

- Q_1 renormalises multiplicatively
- Q_7 and Q_8 mix together

Renormalization of 4-quark operators

- Wilson coefficients are computed in perturbation theory (e.g. $\overline{\text{MS}}$) and Z non-perturbatively in a lattice (MOM) scheme, so in practice:

$$C(\mu) Z(\mu, a) A^{\text{lat}}(a) = \underbrace{C(\mu) R^{\overline{\text{MS}} \leftarrow \text{MOM}}(\mu)}_{\text{perturbative } \mu} \underbrace{U^{\text{MOM}}(\mu, \mu_0)}_{\text{running}} \underbrace{Z^{\text{MOM}}(\mu_0, a) A^{\text{lat}}(a)}_{\text{low energy } \mu_0}$$

- Window problem: in RI-MOM, to compute $Z^{\text{MOM}}(\mu, a)$, we need

$$\Lambda_{\text{QCD}} \ll \mu \ll \pi a^{-1}$$

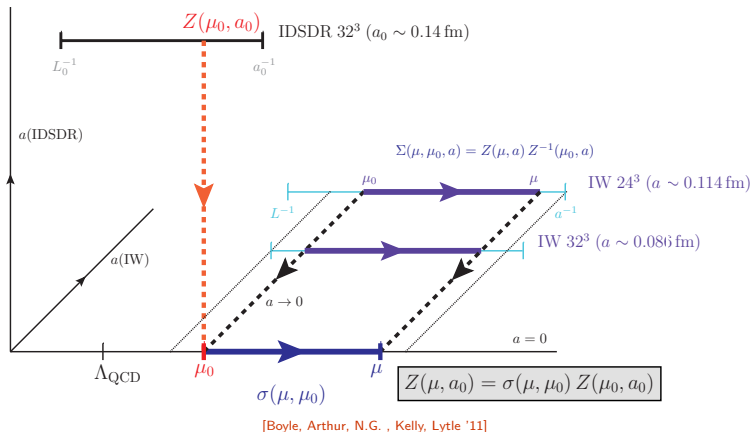
- Problem: in our computation, $a^{-1} \sim 1.35 \text{ GeV}$
- Solution: compute the running non-perturbatively on finer lattices and extrapolate to the continuum

$$\lim_{a_1 \rightarrow 0} \underbrace{\left[Z(\mu_1, a_1) Z^{-1}(\mu_0, a_1) \right]}_{\text{fine lattice}} \times \underbrace{Z(\mu_0, a_0)}_{\text{coarse lattice}} = Z(\mu_1, a_0)$$

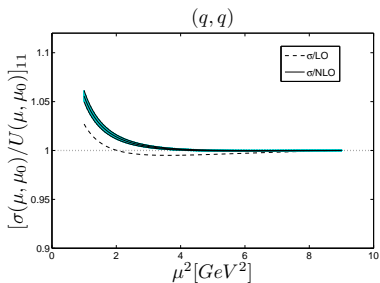
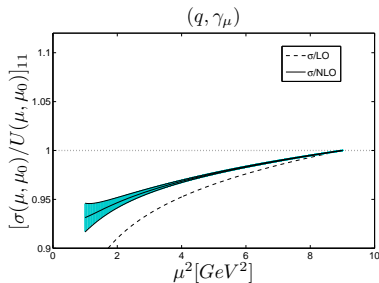
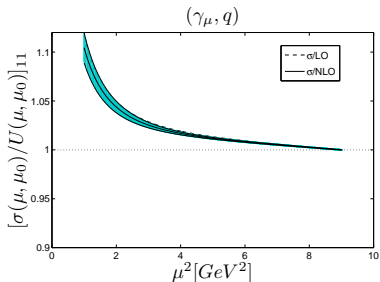
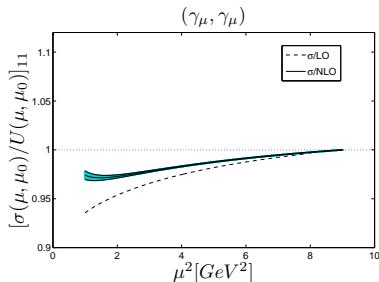
[Boyle, Arthur, N.G. , Kelly, Lytle '11]

Strategy to compute the non-perturbative running

Note that we need an overlap between the two set of lattices, since the “matching scale” μ_0 should be accessible to both .

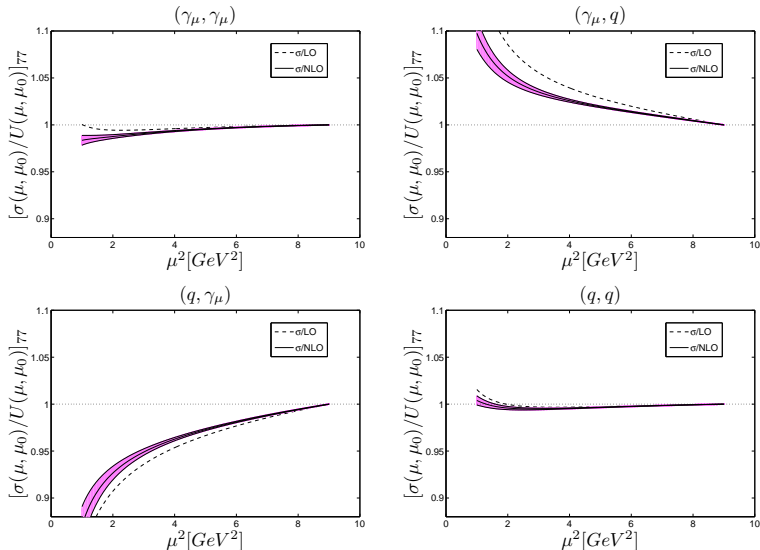


Results for the (27,1) operator in different schemes



Results for the (8, 8) matrix in the different schemes

Comparison of $\sigma_{7,7}(\mu, \mu_0)$ with perturbation theory



More on NPR

Tremendous improvements on NPR in the last years :

- Combine volume source [QCDSF] with twisted boundary conditions and non-exceptional kinematics: high statistical precision, suppress unwanted IR contribution and well defined continuum limit.
- Step scaling: universal non-perturbative scale evolution
⇒ opens the Rome-Southampton windows
- Generalized to the operator mixing case

⇒ Z factors can now be obtained non-perturbatively at the sub-% level

Limitation is the perturbative matching (only one-loop for the 4-quark operators)

⇒ Essential ingredient in the RBC-UKQCD light flavor physics program: allows us to renormalize the data from the IDSDR lattice.

Results for $A_{0,2}$

Experimentally

$$\text{from } K^+ \quad \text{Re } A_2 = 1.479(4) \times 10^{-8} \text{ GeV}$$

$$\text{from } K_S \quad \text{Re } A_2 = 1.416(35) \times 10^{-8} \text{ GeV}$$

we find [RBC-UKQCD (Blum et al '11)]

$$\text{Re } A_2 = (1.436 \pm 0.063_{\text{stat}} \pm 0.258_{\text{syst}}) \times 10^{-8} \text{ GeV},$$

$$\text{Im } A_2 = -(6.29 \pm 0.46_{\text{stat}} \pm 1.20_{\text{syst}}) \times 10^{-13} \text{ GeV}.$$

and

$$\text{Im}A_2/\text{Re}A_2 = (-4.76 \pm 0.37_{\text{stat}} \pm 0.81_{\text{syst}}) \times 10^{-5}$$

More phenomenology

Combine our result for $\text{Im } A_2$ with the experimental result of $\text{Re } A_2$ from K^+ , with $\text{Re } A_0 = 3.33 \times 10^{-7} \text{ GeV}$ and ϵ'/ϵ

$$\frac{\text{Im } A_0}{\text{Re } A_0} = -1.60(19)_{\text{stat}}(20)_{\text{syst}} \times 10^{-4}.$$

Absorptive long-distance contribution to κ_ϵ [Buras & Guadagnoli '08]

$$(\kappa_\epsilon)_{\text{abs}} = 0.923 \pm 0.006$$

Electroweak penguin contribution to ϵ'/ϵ

$$\text{Re}(\epsilon'/\epsilon)_{\text{EWP}} = -(6.52 \pm 0.49_{\text{stat}} \pm 1.24_{\text{syst}}) \times 10^{-4}$$

and experimentally

$$\text{Re} \left(\frac{\epsilon'}{\epsilon} \right) = (1.65 \pm 0.26) \times 10^{-3}$$

Toward a computation of the $\Delta I = 1/2$ amplitudes

Pilot computation [Blum et al 2011]

- Unphysical: Small volume and non-physical kinematics ($m_\pi \sim 400$ MeV, pions at rest)
- All the contractions are computed
- Renormalisation done non-perturbatively

We believe that the first physical computation will be possible with the new generation of supercomputer

Toward a computation of the $\Delta I = 1/2$ amplitudes



P.Boyle and the Blue Gene/Q in Edinburgh

Conclusion and outlook

- First realistic *ab initio* hadronic decay has been computed: $K \rightarrow (\pi\pi)_{I=2}$
- Technical improvement (realistic action, dynamical fermions, ...)
- New theoretical tools are being developed, e.g. for the renormalization
- Complete but unphysical computation of both $K \rightarrow (\pi\pi)$ amplitudes has been achieved [RBC-UKQCD '11]
- Aim for a physical computation of the $\Delta I = 1/2$ part (Blue Gene /Q)
- Plans for simulating a finer lattice spacing