# Probabilistic description of inmedium jet evolution 

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(In preparation)

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## Outline

$\sqrt{ }$ Motivation
$\checkmark$ Jets in vacuum
$\checkmark$ In medium single-gluon emission (BDMPS-Z)
Decoherence and resummation scheme
$\checkmark$ Generating functional and Master Equation
$\checkmark$ Application: Evolution Equations

## Motivation



## Motivation


(I) Significant dijet energy asymmetry (II) Soft particles at large angles (III) Vacuum-like fragmentation

$$
A_{J}=\frac{p_{\mathrm{T}, 1}-p_{\mathrm{T}, 2}}{p_{\mathrm{T}, 1}+p_{\mathrm{T}, 2}}
$$

## Jets in vacuum



- Originally a hard parton (quark/gluon) which fragments into many partons with virtuality down to a non-perturbative scale where it hadronizes
- LPHD: Hadronization does not affect inclusive observables (jet shape, energy distribution etc..)


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Large time domain for PQCD: $\quad \frac{1}{\sqrt{s}}<t<\frac{\sqrt{s}}{\Lambda_{\mathrm{QCD}}^{2}}$

## Jets in vacuum

[Bassetto, Mueller, Ciafaloni, Marchesini, Dokshitzer, Khoze, Troyan, Fadin, Lipatov, 80's]


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## Leading Logarithms

$$
d P \propto \frac{\alpha_{s} C_{R}}{\pi} \frac{d \omega}{\omega} \frac{d \theta}{\theta}
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$d^{2} P \propto \Theta\left(\theta_{1}-\theta_{2}\right) d P_{1} d P_{2}$

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- AO?
- Ordering variable?

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## I-gluon emission

$$
(2 \pi)^{2} \omega \frac{d N}{d \omega d^{2} \boldsymbol{k}}=\frac{4 C_{F} \alpha_{s}}{\omega} \int_{0}^{L} d t \int \frac{d^{2} \boldsymbol{q}}{(2 \pi)^{2}} \mathcal{P}(\boldsymbol{k}-\boldsymbol{q}, L-t) \sin \left(\frac{\boldsymbol{q}^{2}}{2 \boldsymbol{k}_{\mathrm{br}}^{2}}\right) \exp \left(-\frac{\boldsymbol{q}^{2}}{2 \boldsymbol{k}_{\mathrm{br}}^{2}}\right)
$$

- prob. of acquiring mom. k after $\xi \quad \mathcal{P}(\boldsymbol{k}, \xi)=\frac{4 \pi}{\hat{q} \xi} e^{-\frac{k^{2}}{\hat{\sigma}}}$
- branching time $t_{\mathrm{br}}$
- Static scat. centers

broadening!


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Baier, Dokshitzer, Mueller, Peigné, Schiff (I995-2000) Zakharov (I996)

$$
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$$

-1-gluon $\sim$ n-gluon when

$$
\alpha_{s} \frac{L}{t_{\mathrm{br}}} \sim 1
$$

- Emission time $t \sim L \gg t_{\mathrm{br}}$



## N -gluon emissions

- Factorized contribution $t_{1} \sim t_{2} \sim L \gg t_{\text {br }}$ or $\left(\omega_{1} \sim \omega_{2} \ll \omega_{c}=\hat{q} L^{2}\right)$



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- Interferences $t_{2}-t_{1} \sim t_{\mathrm{br}}$

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$$
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& \text { • Decoherence of } \\
& \text { successive splittings: } \\
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$$

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suppressed in a dense medium $\Rightarrow$ No Angular $\propto \alpha_{s}\left(\alpha_{s} \frac{L}{t_{\mathrm{br}}}\right)$

Ordering!
Y. M.-T. , K.Tywoniuk, C.A. Salgado (201I)
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## $\Rightarrow$ Probabilistic Scheme

- Resummation:

$$
\sigma=\sum_{n} a_{n}\left(\alpha_{s} \frac{L}{t_{\mathrm{br}}}\right)^{n}
$$

## Generating Functional Method

- n-gluon probability $P_{n}$
- Probability conservation

$$
\mathcal{Z}(u)=\sum_{n=1}^{\infty} P_{n} u^{n}
$$

$$
\mathcal{Z}(u=1)=1
$$

- Average gluon number

$$
\langle n\rangle \equiv \frac{d}{d u} \mathcal{Z}(u=1)
$$

- Higher moments

$$
\langle n(n-1) \ldots(n-m+1)\rangle=\left(\frac{d}{d u}\right)^{m} \mathcal{Z}(u=1)
$$

- To compute differential distributions in k

$$
u \longrightarrow u(k)
$$

$$
\frac{\delta u(k)}{\delta u(p)}=\delta^{(3)}(k-p)
$$

## Master Equation

$$
\begin{aligned}
& \text { L- } \mathcal{Z}(u))=\frac{p}{\left.\mathcal{P} p^{\prime} \mid u\right)=\Delta\left(p^{+}, L-t_{0}\right) \int \frac{d \boldsymbol{p}^{\prime}}{(2 \pi)^{2}} \mathcal{P}\left(\boldsymbol{p}^{\prime}-\boldsymbol{p}, L-t_{0}\right) u\left(p^{\prime}\right)} \\
& +\alpha_{s} \int_{t_{0}}^{L} d t \Delta\left(p^{+}, t-t_{0}\right) \int_{0}^{1} \frac{d z}{z} \int \frac{d \boldsymbol{p}^{\prime}}{(2 \pi)^{2}} \mathcal{P}\left(\boldsymbol{p}^{\prime}-\boldsymbol{p}, L-t_{0}\right) \int \frac{d \boldsymbol{q}}{(2 \pi)^{2}} \mathcal{K}\left(\boldsymbol{q}-z \boldsymbol{p}^{\prime} \mid z\right) \mathcal{Z}(\mathbf{q}, L-t \mid u) \mathcal{Z}\left(\mathbf{p}^{\prime}-\mathbf{q}, L-t \mid u\right)
\end{aligned}
$$

## Master Equation

$$
\begin{aligned}
& \mathcal{Z}\left(\mathbf{p}, L-t_{0} \mid u\right)=\Delta\left(p^{+}, L-t_{0}\right) \int \frac{d \boldsymbol{p}^{\prime}}{(2 \pi)^{2}} \underbrace{t_{0}}_{\mathcal{P}\left(\boldsymbol{p}^{\prime}-\boldsymbol{p}, L-t_{0}\right) u\left(\mathbf{p}^{\prime}\right)} \\
& \left.+\alpha_{s} \int_{t_{0}}^{L} d t \Delta\left(p^{+}, t-t_{0}\right) \int_{0}^{1} \frac{p^{\prime}}{t_{0}} \frac{p^{\prime}}{z} \frac{d \boldsymbol{p}^{\prime}}{(2 \pi)^{2}} \mathcal{P}\left(p^{\prime}\right)+\boldsymbol{p}, L-t_{0}\right) \int \frac{d \boldsymbol{q}}{(2 \pi)^{2}} \mathcal{K}\left(\boldsymbol{q}-z \boldsymbol{p}^{\prime} \mid z\right) \mathcal{Z}(\mathbf{q}, L-t \mid u) \mathcal{Z}\left(\mathbf{p}^{\prime}-\mathbf{q}, L-t \mid u\right)
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## Master Equation



## Master Equation



- In-medium splitting function
- Relative pT at branching time $\mathcal{K}_{B C}^{A}(\boldsymbol{q}-z \boldsymbol{p}, z)=\frac{2}{p^{+}} P_{A B}(z) \sin \left[\frac{(\boldsymbol{q}-z \boldsymbol{p})^{2}}{2 \boldsymbol{k}_{\mathrm{br}}^{2}}\right] \exp \left[-\frac{(\boldsymbol{q}-z \boldsymbol{p})^{2}}{2 \boldsymbol{k}_{\mathrm{br}}^{2}}\right] \quad \boldsymbol{k}_{\mathrm{br}}^{2}=\sqrt{z(1-z) p^{+} \hat{q}_{\mathrm{eff}}}$


## Master Equation



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$$

- Sudakov form factor:

Prob. not to emit (Unitarity)

$$
\Delta\left(p^{+}, L-t_{0}\right)=\exp \left[-\alpha_{s}\left(L-t_{0}\right) \int_{0}^{1} \frac{d z}{z} \mathcal{K}(z)\right]
$$

## Application I:gluon distribution

- Integrating over transverse momenta: $\quad \int \frac{d^{2} \boldsymbol{k}}{(2 \pi)^{2}} \mathcal{P}(\boldsymbol{k}-\boldsymbol{q}, L-t)=1$

$$
\frac{\partial}{\partial L} \mathcal{Z}(E, u)=\alpha_{s} \int_{0}^{1} \frac{d z}{z} \mathcal{K}(z)[\mathcal{Z}(z E, u) \mathcal{Z}((1-z) E, u)-\mathcal{Z}(E, u)]
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$$

- Gluon distribution

$$
D(x, E) \equiv x \frac{d N}{d x}=\left.\omega \frac{\delta \mathcal{Z}(E, u)}{\delta u(\omega)}\right|_{u=1}
$$

$$
x=\omega / E
$$

$$
\frac{\partial}{\partial L} D(x)=\alpha_{s} \int_{0}^{1} \frac{d z}{z} \mathcal{K}(z)\left[D\left(\frac{x}{z}\right)+D\left(\frac{x}{1-z}\right)-D(x)\right]
$$

## Application II: Correlations

- 2-particle correlations inside the jet

$$
\begin{gathered}
\left.D\left(x_{1}, x_{2}\right) \equiv \omega_{1} \omega_{2} \frac{\delta^{2} \mathcal{Z}(E, u)}{\delta u\left(\omega_{1}\right) \delta u\left(\omega_{2}\right)}\right|_{u=1} \\
\frac{\partial}{\partial L} D\left(x_{1}, x_{2}\right)=\alpha_{s} \int_{0}^{1} \frac{d z}{z} \mathcal{K}(z)\left[D\left(\frac{x_{1}}{z}, \frac{x_{2}}{z}\right)+D\left(\frac{x_{1}}{z}\right) D\left(\frac{x_{2}}{1-z}\right)+\operatorname{sym}-D\left(x_{1}, x_{2}\right)\right]
\end{gathered}
$$



$+$


## Summary

$\sqrt{ }$ In the limit of a dense medium, parton branchings are decoherent due to rapid color randomization.
$\checkmark$ A probabilistic description of in-medium jet evolution is formulated in terms of a Master Eq. for Generating Functional
$\checkmark \Rightarrow$ Fully exclusive description of the jet including momentum
broadening
$\checkmark$ Possible implementation in a Monte Carlo generator

