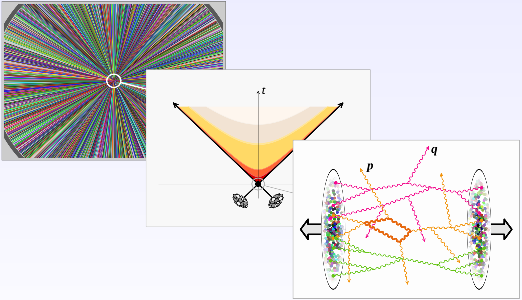
QCD in Heavy-ion collisions

RPP 2012, Montpellier



François Gelis
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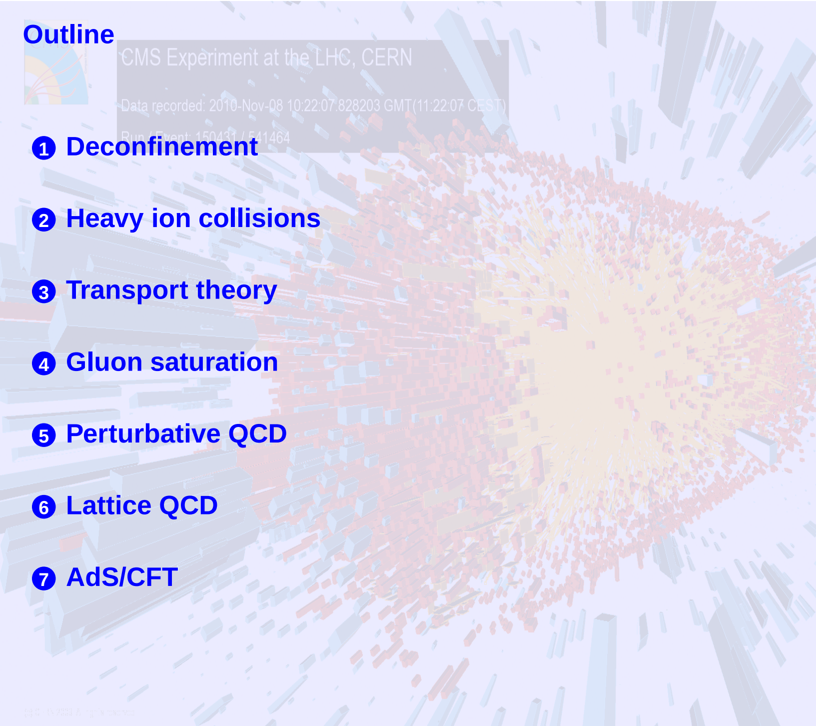
Outline



1. **Deconfinement**
2. **Heavy ion collisions**
3. **Transport theory**
4. **Gluon saturation**
5. **Perturbative QCD**
6. **Lattice QCD**
7. **AdS/CFT**

1. **Deconfinement**
   - QCD reminder
   - Deconfinement transition
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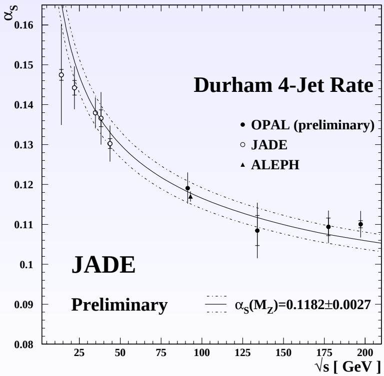
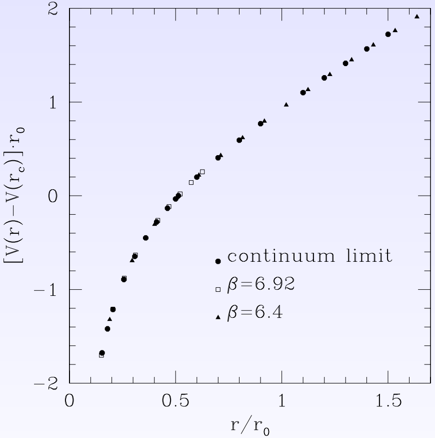
Asymptotic freedom

- Running coupling : $\alpha_s = g^2/4\pi$

$$\alpha_s(r) = \frac{2\pi N_c}{(11N_c - 2N_f)\log(1/r\Lambda_{QCD})}$$



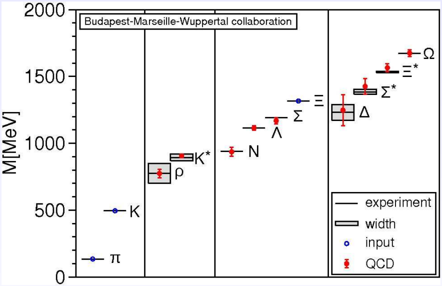
Color confinement



- The quark potential increases linearly with distance

Color confinement

- In nature, we do not see free quarks and gluons (the closest we have to actual quarks and gluons are jets)
- Instead, we see hadrons (quark-gluon bound states):



- The hadron spectrum is uniquely given by $\Lambda_{\rm QCD}, m_f$
- But this dependence is non-perturbative (it can now be obtained fairly accurately by lattice simulations)

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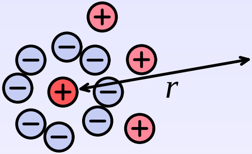
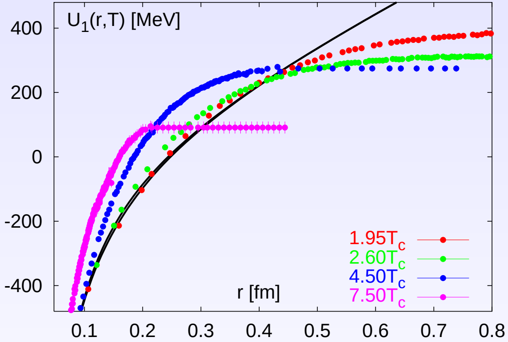
Debye screening



$$V(r) = \frac{\exp(-r/r_{\text{debye}})}{r}$$

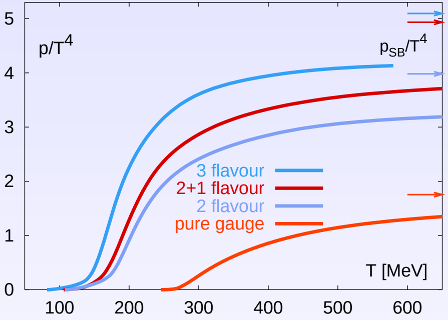
- In a dense medium, color charges are screened by their neighbors
- The interaction potential decreases exponentially beyond the Debye radius r_{debye}
- Hadrons whose radius is larger than r_{debye} cannot bind

Debye screening



- In lattice calculations, one sees the $q\bar{q}$ potential flatten at long distance as T increases

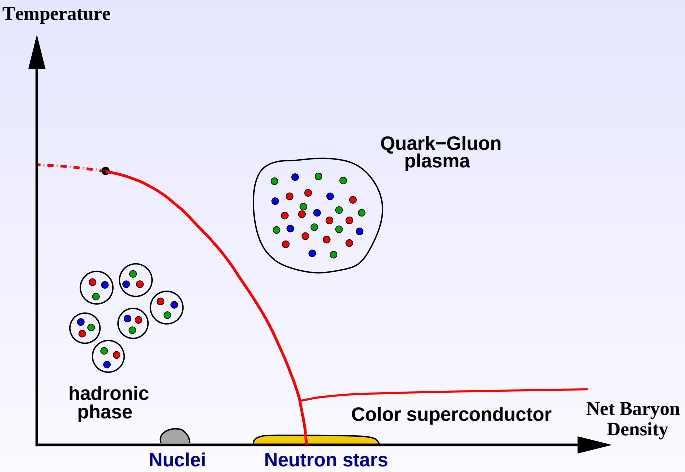
Deconfinement transition



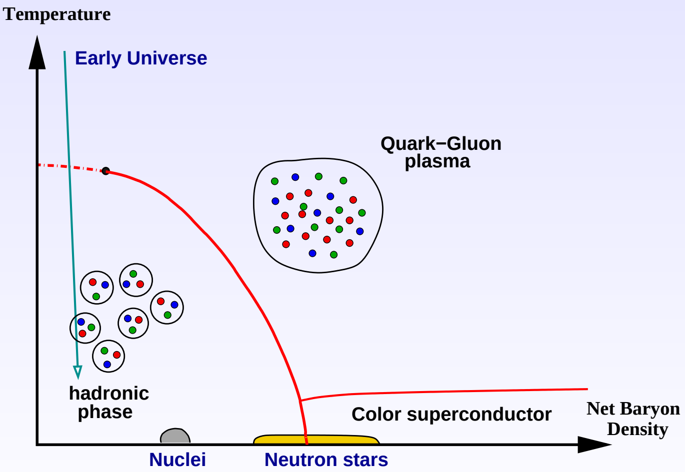
- Rapid increase of the pressure :
  - at $T \sim$ 270 MeV, with gluons only
  - at $T \sim$ 150 to 180 MeV, with light quarks

▷ interpreted as the increase in the number of degrees of freedom due to the liberation of quarks and gluons

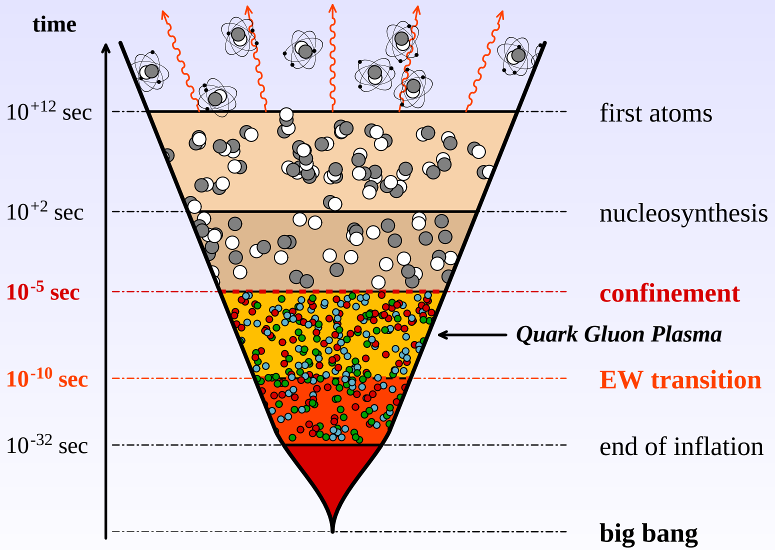
QCD phase diagram



QGP in the early universe



QGP in the early universe



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1. Deconfinement
2. **Heavy ion collisions**
   - Heavy Ion Collisions
   - Effective descriptions
3. Transport theory
4. Gluon saturation
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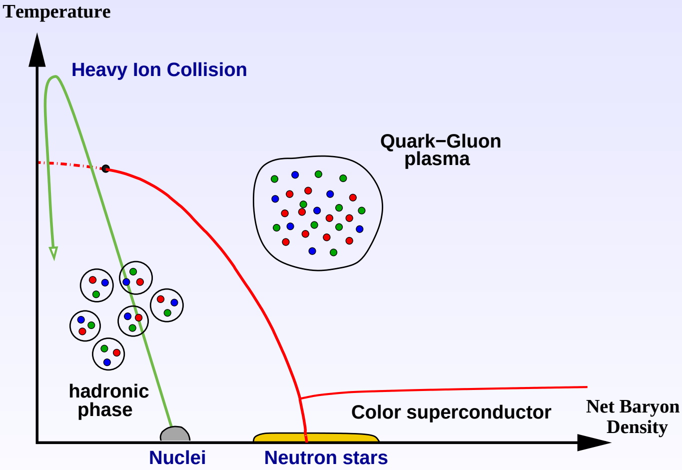
Heavy ion collisions



What would we like to learn?

i. Parameters of the transition: T_c, ϵ_c

ii. Equation of state of nuclear matter

iii. Transport properties of nuclear matter

iv. Do some hadrons survive in the QGP?

v. Formation of the QGP and thermalization

What we must get out of the way first...

- Unfortunately, heavy ion collisions also depend on a number of other trivial facts:

i. Lead nuclei are approximately spherical

ii. Their diameter is about 12 fermis

iii. They contain $A \approx 200$ nucleons

iv. The positions of these nucleons fluctuate

- These properties have all an incidence on observables
- None of them is interesting from the point of view of QCD
- We need ways to make observables independent of these trivial aspects of nuclear physics

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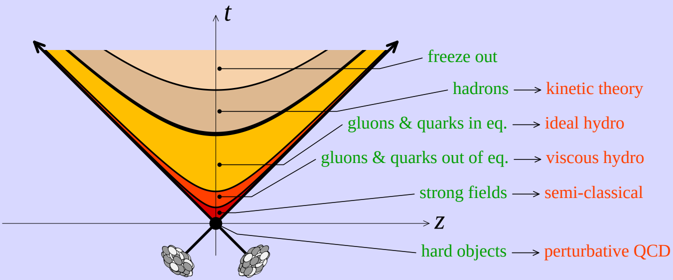
The multiple facets of QCD in Heavy Ion Collisions

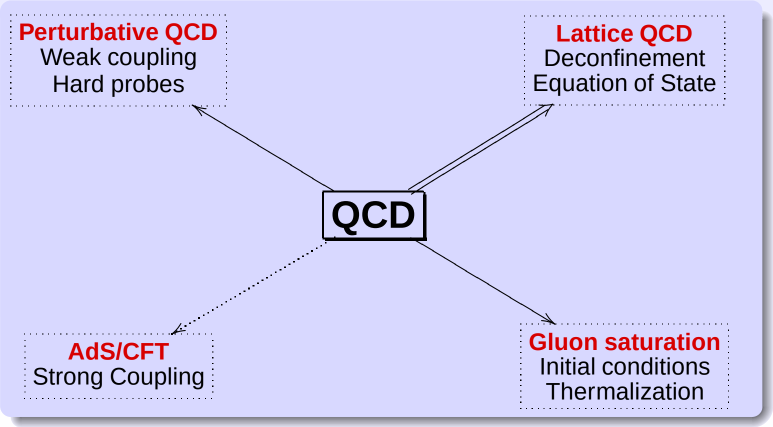


- Except for the production of hard objects (jets, heavy quarks, direct photons) at the impact of the two nuclei, we have to deal with strong interactions in a non-perturbative regime
NOTE: non-perturbative $\neq$ strongly coupled!!!

- Treated with a range of effective descriptions (semi-classical methods, hydrodynamics, kinetic theory) that are more or less closely related to QCD, but always require some QCD input

The multiple facets of QCD in Heavy Ion Collisions

- The simple formulation of QCD is deceptive: Ab initio calculations are very difficult, and feasible only for a handful of questions
- In many instances, it is more efficient to use an effective theory in which inessential degrees of freedom have been integrated out



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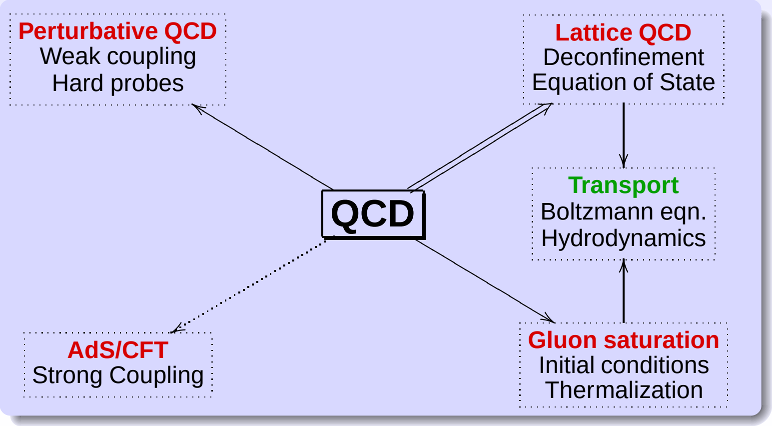


Transport models

- In many cases, the description of the system can be done at a scale large enough for the microscopic details to become irrelevant:
  - Kinetic theory
  - Hydrodynamics
- To a large extent, the evolution of the system is driven by conservation laws (energy, momentum, baryon number...)
- The microscopic dynamics is relegated into a handful of quantities that enter in these mesoscopic descriptions

1 Deconfinement

2 Heavy ion collisions

3 Transport theory

Kinetic theory

Hydrodynamics

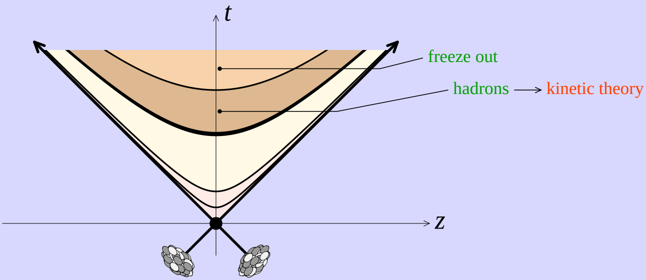
4 Gluon saturation

5 Perturbative QCD

6 Lattice QCD

7 AdS/CFT

Kinetic theory



Kinetic theory

- The system is described by a particle distribution

$$f(t, \vec{\boldsymbol{x}}, \vec{\boldsymbol{p}}) = \frac{dN}{d^3\vec{\boldsymbol{x}}d^3\vec{\boldsymbol{p}}}$$

(in most cases, this distribution is spin and color averaged)

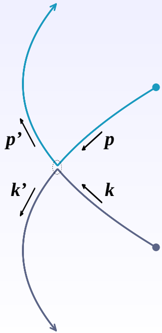
- The evolution of f is driven by the interactions between these particles
- The only QCD input is a set of cross-sections

Boltzmann equation

- The Boltzmann equation describes the evolution of a distribution of particles that undergo short range collisions

$$\left[\partial_t + \vec{v}_{\boldsymbol{p}} \cdot \vec{\nabla}_{\boldsymbol{x}}\right] f(t, \vec{x}, \vec{p}) = \underbrace{\mathcal{C}_{\boldsymbol{p}}[f]}_{\text{collisions}} \qquad \text{with } \vec{v}_{\boldsymbol{p}} \equiv \frac{\vec{p}}{E_{\boldsymbol{p}}}$$

- Elementary 2-body collision :



Boltzmann equation

- For $2 \to 2$ collisions, the collision term reads :

$$\mathcal{C}_{\boldsymbol{p}}[f] = \frac{1}{2E_{\boldsymbol{p}}} \int \frac{d^3\vec{\boldsymbol{p}}'}{(2\pi)^3 2E_{\boldsymbol{p}'}} \int \frac{d^3\vec{\boldsymbol{k}}}{(2\pi)^3 2E_{\boldsymbol{k}}} \int \frac{d^3\vec{\boldsymbol{k}}'}{(2\pi)^3 2E_{\boldsymbol{k}'}} \underbrace{(2\pi)^4\delta(p+k-p'-k')}_{E,\vec{\boldsymbol{p}}\text{ conservation}}$$

$$\times \underbrace{\Big[f(\vec{\boldsymbol{p}}')f(\vec{\boldsymbol{k}}')(1+f(\vec{\boldsymbol{p}}))(1+f(\vec{\boldsymbol{k}})) - f(\vec{\boldsymbol{p}})f(\vec{\boldsymbol{k}})(1+f(\vec{\boldsymbol{k}}'))(1+f(\vec{\boldsymbol{p}}'))\Big]}_{\text{micro-reversibility, detailed balance}} \underbrace{\left|\mathcal{M}\right|^2}_{\text{QCD}}$$

▷ Most of the equation relies on conservation laws and general principles of statistical physics. Only the cross-section depends on QCD

Inputs

i. Cross-sections

ii. Initial condition $f(t_0, \vec{\boldsymbol{x}}, \vec{\boldsymbol{p}})$

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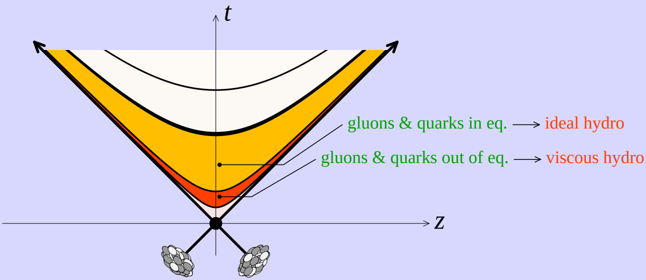
Hydrodynamics



Hydrodynamics : limit of kinetic theory when $\ell_{\rm mfp} \to 0$

Equations of hydrodynamics (conservation laws)

$$\partial_\mu T^{\mu\nu} = 0 \qquad , \qquad \partial_\mu J^\mu_B = 0$$

Assumptions and inputs

i. Near equilibrium form of $T^{\mu\nu}$:

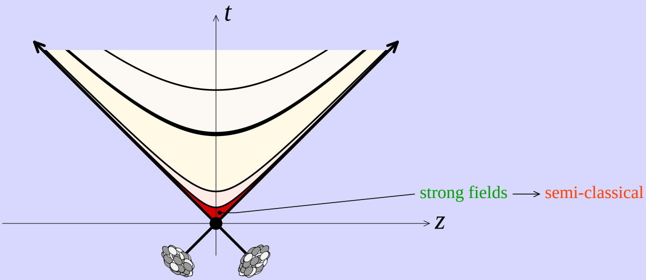
$$T^{\mu\nu} = \underbrace{(p+\epsilon)\,v^\mu v^\nu - p\,g^{\mu\nu}}_{\text{ideal hydro}} \oplus \underbrace{(\eta,\zeta)\partial v}_{\text{viscous terms}} \oplus \cdots$$

ii. Equation of State: $p = f(\epsilon)$

iii. Transport coefficients: $\eta, \zeta, \cdots$

iv. Initial condition for ϵ and $\vec{v}$ at some t_0

Gluon saturation



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Why small-x gluons matter

Gluon saturation

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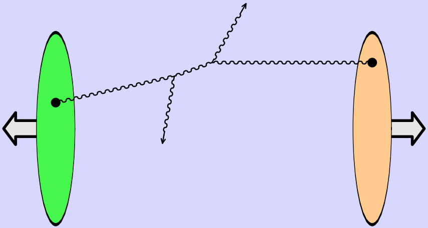
Longitudinal momentum fraction in AA collisions



- The partons that are relevant for the process under consideration carry the longitudinal momentum fractions:

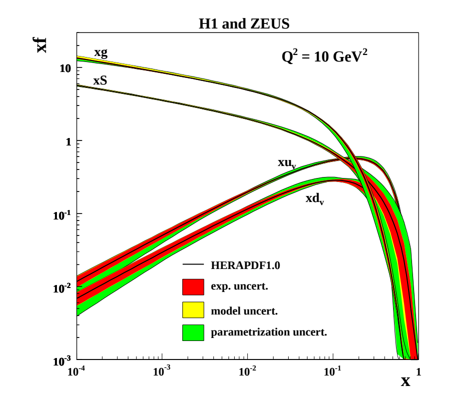
$$x_{1,2} = \frac{P_\perp}{\sqrt{s}}\, e^{\pm Y}$$

- $P_\perp \sim 1$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 10^{-3}$ at the LHC ($\sqrt{s} = 2.76$ TeV)

▷ partons at small x are the most important

Growth of the gluon distribution at small x

Parton distributions at small x

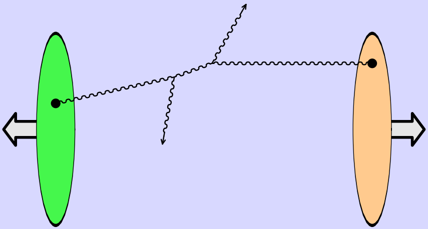


- Gluons dominate at any $x \lesssim 10^{-1}$

Multiple scatterings and gluon recombination

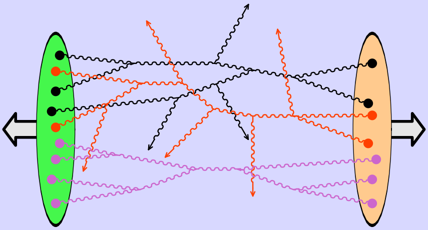
- Main difficulty: How to treat collisions involving a large number of partons?

Multiple scatterings and gluon recombination



- Dilute regime : one parton in each projectile interact
  - ▷ large Q^2, no small-x effects
  - ▷ usual PDFs + DGLAP evolution

Multiple scatterings and gluon recombination



- Dense regime : multiparton processes become crucial
  - ▷ gluon recombinations are important (saturation)
  - ▷ multi-parton distributions + JIMWLK evolution
  - ▷ new techniques are required (Color Glass Condensate):

$$\mathcal{L} = -\frac{1}{4}F^2 + J \cdot A$$

(gluons only, field A for $k^+ < \Lambda$, classical source J for $k^+ > \Lambda$)

Color Glass Condensate effective theory

- Power counting :
  - $J \sim g^{-1}$ in the saturated regime
  - Each g^2 order gets contributions from an infinite set of graphs
  - LO: all tree graphs, classical fields
  - NLO: one loop, small field fluctuations over a classical background
- Applications :
  - Initial conditions for hydrodynamics
  - Study of thermalization
- Main issue : the g^2 expansion is not uniform in time
  $\triangleright$ secular divergences

  [$\triangleright$ see T. Epelbaum's talk]

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6 Lattice QCD

7 AdS/CFT

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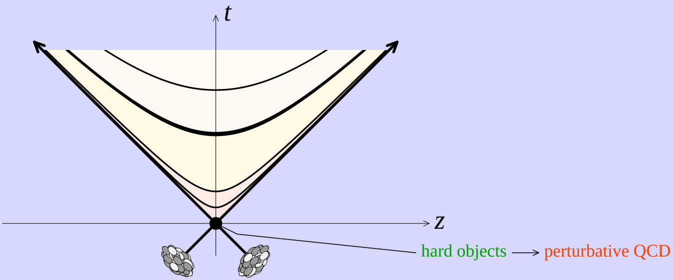
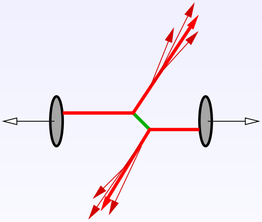
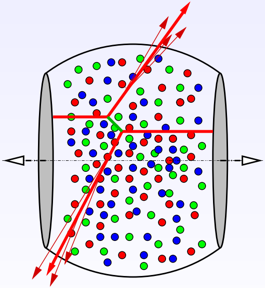
Perturbative QCD



Jet quenching

- The basis of perturbative QCD is asymptotic freedom
- pQCD is the tool of choice for computing the production of hard objects (high $p_\perp$ jets, direct photons, heavy quarks)
- In heavy ion collisions, a new challenge for QCD is the study of the propagation of a hard object in a dense quark-gluon medium





[▷ Y. Mehtar-Tani's talk]

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Partition function

Lattice QCD

7 AdS/CFT

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Partition function

- Partition function :

$$Z \equiv \mathrm{Tr}\left(e^{-\beta H}\right) = \int [\mathcal{D}A^\mu \mathcal{D}\overline{\psi}\mathcal{D}\psi]\; e^{-S_E[A^\mu,\overline{\psi},\psi]}$$

- S_E is the Euclidean action, with imaginary time in $[0, \beta = 1/T]$. The Matsubara formalism provides a way to do perturbative calculations at finite T

- Z knows everything about the QGP thermodynamics :

$$E = -\frac{\partial Z}{\partial \beta}$$

$$S = \beta E + \ln(Z)$$

$$F = E - TS = -\frac{1}{\beta}\ln(Z)$$

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   - Partition function
   - **Lattice QCD**
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Lattice QCD

- Lattice QCD : discretize space-time, and approximate the functional integration by a Monte-Carlo sampling
- "Sign problem" :
  - does not work for "real time" correlation functions
    ▷ limited to static properties of the QGP (thermodynamics)
  - does not work with a baryon chemical potential
- Light quarks with realistic masses are computationally expensive

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Gauge-gravity duality

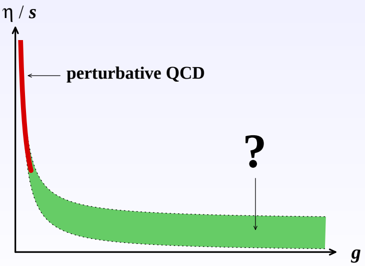
Viscosity in N=4 SYM

Limitations

Viscosity at weak coupling

- It all started with the observation that hydrodynamics reproduces well the data provided one uses a very small viscosity
- The shear viscosity has been calculated in QCD at weak coupling ($g \to 0$), and it is quite large :

$$\frac{\eta}{s} = \frac{5.12}{g^4 \ln\left(\frac{2.42}{g}\right)}$$



AdS/CFT duality at T=0

- Maximally super-symmetric $SU(N)$ Yang-Mills theories in the limit $g^2N \to +\infty$ are dual to classical super-gravity on an $\mathrm{AdS}_5 \times S_5$ manifold with metric

$$ds^2 = \frac{R^2}{z^2}(\underbrace{-dt^2 + d\vec{x}^2}_{\text{we live here... (at z=0)}} + dz^2) + R^2 d\Omega_5^2$$

- If an operator $\mathcal{O}$ of our world is coupled on the boundary to a field φ_0 that extends in the bulk, the duality states that :

$$e^{-S_{\mathrm{cl}}[\phi]} = \left\langle e^{\int_{\mathrm{boundary}} \mathcal{O}\varphi_0} \right\rangle$$

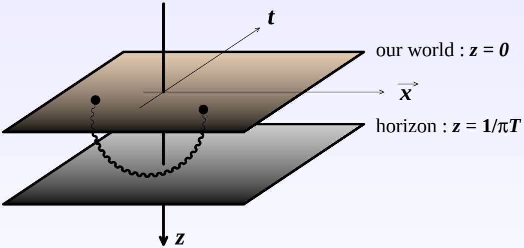
  - The right hand side is a generating functional for the correlators of operators $\mathcal{O}$ in the 4-dim gauge theory
  - The left hand side is calculable in the gravity dual (solve the classical EOM for ϕ with the boundary condition φ_0)

AdS/CFT duality at high T

- At finite temperature T :

$$-dt^2 + dz^2 \to -f(z)dt^2 + dz^2/f(z) \quad \text{with } f(z) = 1 - (\pi z T)^4$$

- $f(z) = 0$ at $z = 1/\pi T \quad \Rightarrow$ black hole horizon



- Ordinary particles in 4-dimensions are the end points of strings living in the bulk. Thermal effects occur when a string gets close to the BH horizon

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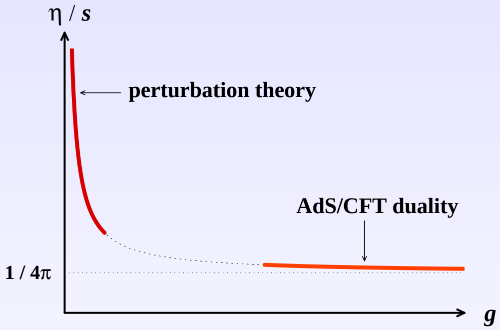
   - **Viscosity in N=4 SYM**
   - Limitations

Viscosity in SUSY Yang-Mills



- In SYM at $g^2N \to \infty$, one gets $\eta/s = 1/4\pi$
- Conjecture : $1/4\pi$ is the lowest possible value for η/s

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Viscosity in N=4 SYM

Limitations

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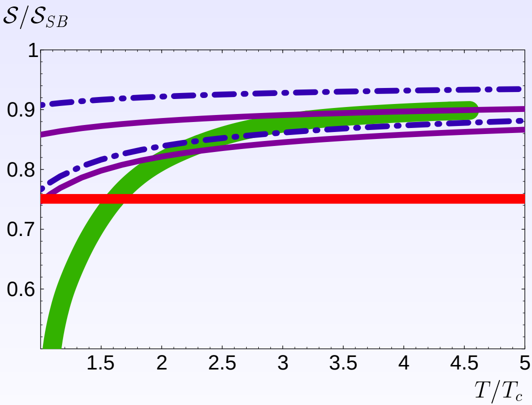
Importance of scale violations near T_c

- Is the QGP at $T/T_c \sim 2-3$ really strongly coupled? For quantities such as the entropy, perturbative techniques (+resummations) lead to sensible results in this region



- At $T < 3T_c$, the coupling may indeed be strong, but scale violations make AdS/CFT unreliable

Summary

- QCD in heavy ion collisions displays a very rich spectrum of phenomena
- Ab initio methods (lattice) are practical only for certain quantities
- The consequence of this is the diversity of tools and techniques that have been developed to study various aspects of strong interactions in heavy ion collisions
- QCD also plays a role in providing inputs into a number of effective descriptions such as kinetic theory and hydrodynamics