

Lattice QCD and the search for new physics from the perspective of the Budapest-Marseille-Wuppertal collaboration (BMWc)

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Excuses to ALPHA, CLS, ETMC, JLQCD, MILC, PACS-CS, QCDSF,
RBC/UKQCD, ..., for not having the time to cover some of their very nice
work

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The place of low-energy searches for New Physics

Standard model: $SU(3)_c \times SU(2)_L \times U(1)_Y$ is an effective theory

$$\mathcal{L}_{SM} = \mathcal{L}_{G\&M}(A_\mu, \psi, \phi) + \mathcal{L}_{H\&Y}(\phi, \psi, v) + \frac{1}{M} \mathcal{O}_{Maj}^{(5)} + \sum_{d \geq 6} \sum_i \frac{C_{d,i}}{\Lambda_i^{d-4}} \mathcal{O}_i^{(d)}$$

$\mathcal{L}_{G\&M}$: 3 couplings; well tested

$\mathcal{L}_{H\&Y}$: ≥ 15 couplings ; less well tested \supset 10 quark Yukawas \rightarrow flavor physics

- Naturalness: $\delta m_H^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 \Lambda^2 \simeq (0.3\Lambda)^2$
 $\rightarrow \Lambda \lesssim \text{few TeV}$
- Constraints on flavor conserving ops from precision EW data
 $\rightarrow \Lambda \gtrsim 5 \text{ TeV}$
- FCNC: e.g. $K^0 - \bar{K}^0$ mixing
 $\rightarrow \Lambda \sim 10^3 \text{ TeV}$
- If there are no RH ν 's
 $\rightarrow M \sim 10^{15} \text{ GeV}$

\Rightarrow low-energy processes can be sensitive to very high-energy scales

\Rightarrow if quarks are involved, have to deal w/ confinement

Motivation for lattice QCD

- Verify that QCD is theory of strong interaction at low energies
 - ↔ verify the validity of the computational framework
 - light hadron masses
 - hadron widths
 - look for exotics
 - ...
- Fix fundamental parameters and help search for new physics
 - m_u, m_d, m_s, \dots
 - $\langle N | m_q \bar{q} q | N \rangle, q = u, d, s$ for dark matter
 - $F_K/F_\pi \leftrightarrow \frac{G_q}{G_\mu} [|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2] = 1 ?$
 - $B_K \leftrightarrow$ consistency of CPV in K and B decays ?
 - ...
- Make predictions in nuclear physics?
- Full description of low energy particle physics → include QED

Why do we need lattice QCD?

- QCD fundamental d.o.f.: q and g
 - QCD observed d.o.f.: p , n , π , K , ...
 - q and g are permanently confined w/in hadrons
 - hadrons hugely different from d.o.f. present in the Lagrangian
- ⇒ perturbation in α_s has no chance
- ⇒ Need a tool to solve low energy QCD to address important questions above
- numerical lattice QCD

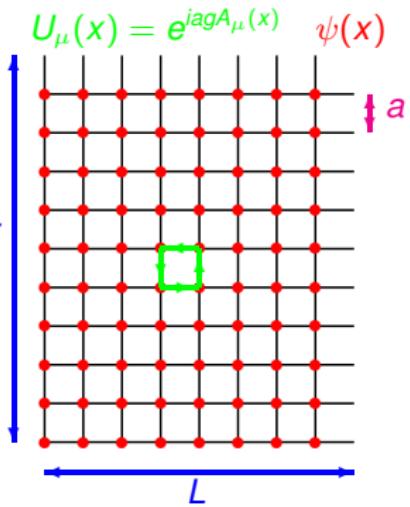
What is lattice QCD?

Lattice gauge theory → mathematically sound definition of NP QCD:

- UV (and IR) cutoffs and a well defined path integral in Euclidean spacetime:

$$\begin{aligned}\langle O \rangle &= \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S_G - \int \bar{\psi} D[M] \psi} O[U, \psi, \bar{\psi}] \\ &= \int \mathcal{D}U e^{-S_G} \det(D[M]) O[U]_{\text{Wick}}\end{aligned}$$

- $\mathcal{D}U e^{-S_G} \det(D[M]) \geq 0$ and finite # of dof's
→ evaluate numerically using stochastic methods



NOT A MODEL: LQCD is QCD when $a \rightarrow 0$, $V \rightarrow \infty$ and stats $\rightarrow \infty$

In practice, limitations ...

Ingredients of my dream calculation

- $N_f = 2 + 1$ simulations to include u , d and s sea quark effects
- Simulations all the way down to $M_\pi \lesssim 135 \text{ MeV}$ to allow small interpolation to physical mass point
- Large $L \gtrsim 5 \text{ fm}$ to have sub-percent finite V errors
- At least three $a \lesssim 0.1 \text{ fm}$ for controlled continuum limit
- Reliable determination of the scale w/ a well measured physical observable
- Unitary, local gauge and fermion actions
- Full nonperturbative renormalization and nonperturbative continuum running if necessary
- Complete analysis of systematic uncertainties

Huge challenge

of d.o.f. $\sim \mathcal{O}(10^9)$ and large overhead for computing $\det(D[M])$ ($\sim 10^9 \times 10^9$ matrix)

☞ Capri 1989: LQCD will require “both a 10^8 increase in computing power AND spectacular algorithmic advances before a useful interaction with experiments starts taking place”

$$\Rightarrow 1\text{--}10 \text{ Exaflop/s} = 10^6\text{--}10^7 \text{ Tflop/s}$$

→ nevertheless quenched LQCD provided useful interaction

☞ Berlin wall ca. 2001: beginning of unquenched calculations

$$\text{cost} \sim N_{\text{conf}}(L^3 \times T)^{c_L} m_{ud}^{-c_m} a^{-c_a}$$

with $c_m \sim 2.5\text{--}3$, $c_a \sim 7$, $c_L \sim 5/4$ and large prefactor (Gottlieb '02, Ukawa '02)

$\Rightarrow M_\pi \lesssim 300 \text{ MeV}$ and $a \lesssim 0.1 \text{ fm}$ completely out of reach

☞ $\gtrsim 2004$:

- algorithmic breakthroughs: DD-HMC (Lüscher '03-'04), multiple time scale integration and mass preconditioning (Sexton et al '92, Hasenbusch '01, Urbach et al '06, BMWc '07), ...

- more effective unitary discretizations of QCD which allow $m_{ud} \searrow m_{ud}^{\text{ph}}$ (BMWc '07 using Morningstar et al '04 & Hasenfratz et al '01)

$\Rightarrow c_m \searrow 1\text{--}2$, $c_a \searrow 4\text{--}6$, $c_L \searrow 1\text{--}5/4$ and much reduced prefactor (e.g. Jansen '08)

☞ $\gtrsim 2008$: supercomputers capable of $10^2\text{--}10^3 \text{ Tflop/s}$

\Rightarrow calculations w/ **full control over errors** are becoming possible

Not far from dream in 2008

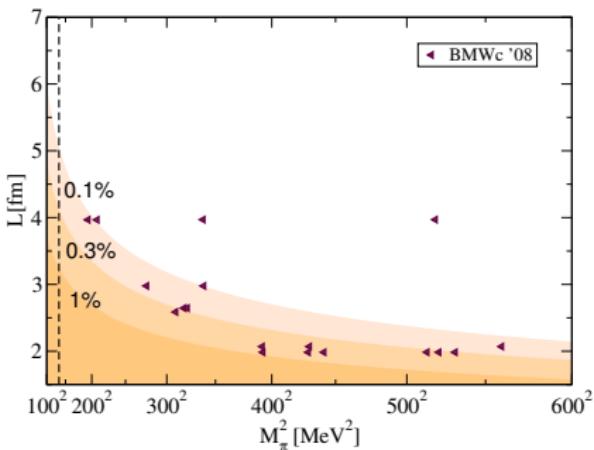
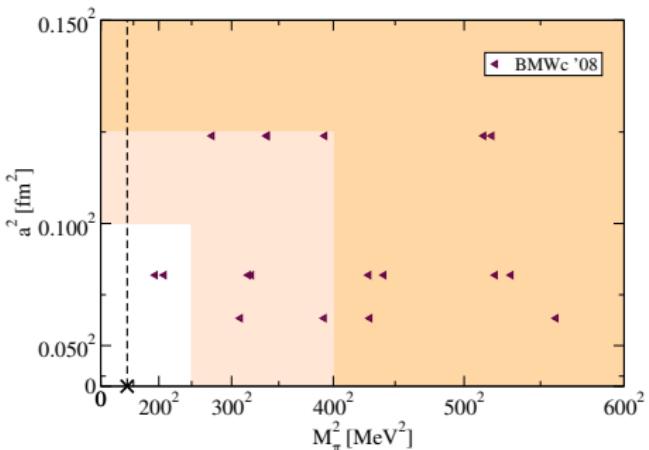
Dürr et al (BMWc) Science 322 '08, PRD79 '09

20 large scale $N_f = 2 + 1$ Wilson fermion simulations

$$M_\pi \gtrsim 190 \text{ MeV}$$

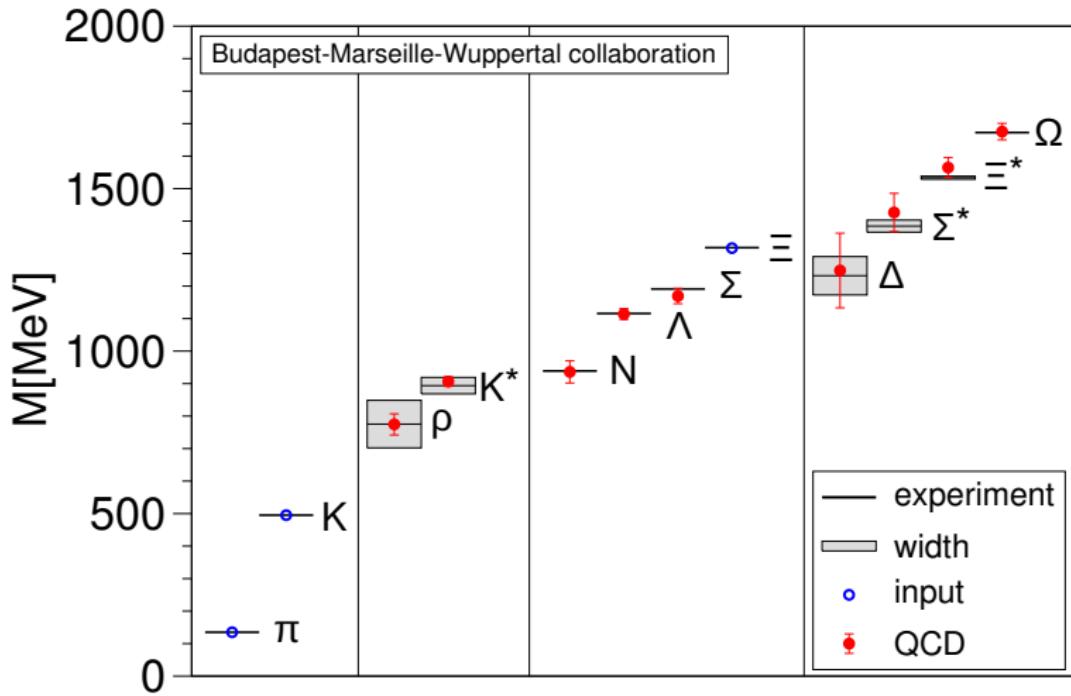
$$a \approx 0.065, 0.085, 0.125 \text{ fm}$$

$$L \rightarrow 4 \text{ fm}$$



Good enough for *ab initio* calculation of light hadron masses, . . .

Postdiction of the light hadron masses



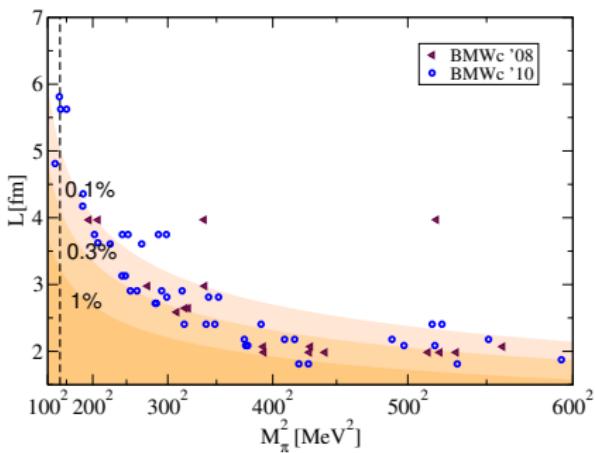
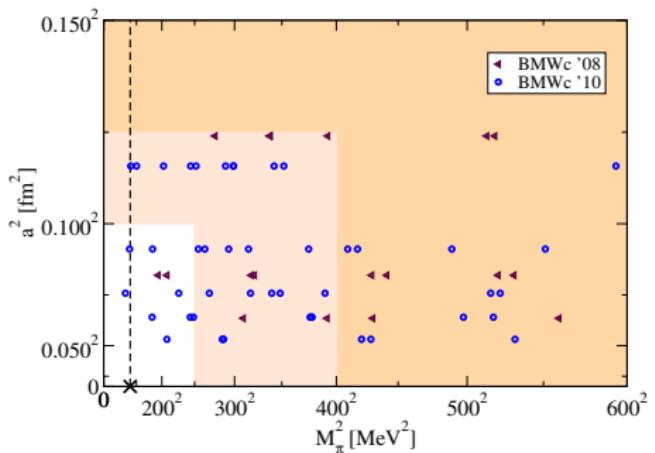
(Partial calculations by MILC '04-'09, RBC-UKQCD '07, Del Debbio et al '07, JLQCD '07, QCDSF '07-'09, Walker-Loud et al '08, PACS-CS '08-'10, ETM '09, Gattringer et al '09, . . .)

Dream comes true in 2010

Dürr et al (BMWc) PLB 701 (2011); JHEP 1108 (2011)

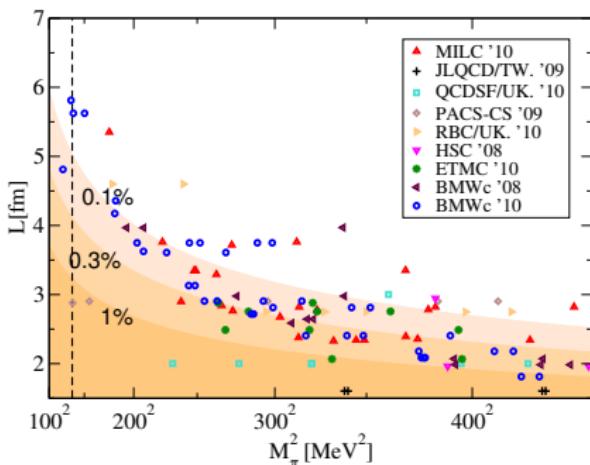
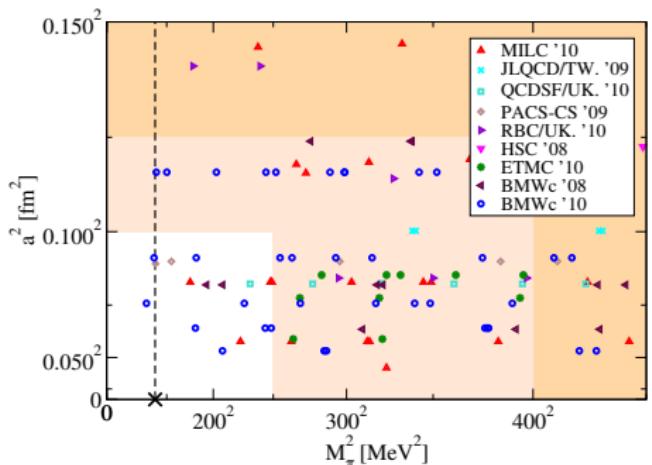
47 large scale $N_f = 2 + 1$ Wilson fermion simulations

$$M_\pi \gtrsim 120 \text{ MeV} \quad 5a's \approx 0.054 \div 0.116 \text{ fm} \quad L \rightarrow 6 \text{ fm}$$



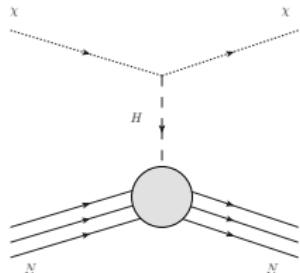
State of the art in 2011

Comparison of parameters reached by different LQCD collaborations



(only $N_f \geq 2 + 1$ simulations are shown)

Direct WIMP dark matter detection



$$\mathcal{L}_{q\chi} = \sum_q \lambda_q \bar{q} q \bar{\chi} \chi \rightarrow \mathcal{L}_{N\chi} = \lambda_N \bar{N} N \bar{\chi} \chi$$

→: requires nonperturbative tool

Spin-independent WIMP-N cross section (Ellis et al '08)

$$\sigma_{\text{SI}} = \frac{4M_r^2}{\pi} [Z f_p + (A - Z) f_n]^2 \quad M_r = M_\chi M_N / (M_\chi + M_N)$$

where

$$\frac{f_N}{M_N} = \sum_{q=[ud],s} f_{qN} \frac{\lambda_q}{m_q} + \frac{2}{27} f_{GN} \sum_{q=c,b,t} \frac{\lambda_q}{m_q} \quad f_{GN} = 1 - \sum_{q=[ud],s} f_{qN}$$

and, in terms of sigma terms,

$$f_{udN} M_N = \sigma_{\pi N} = m_{ud} \langle N | \bar{u} u + \bar{d} d | N \rangle$$

$$f_{sN} M_N = \sigma_{\bar{s}sN}/2 = m_s \langle N | \bar{s} s | N \rangle$$

Sigma terms from phenomenology

$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

$$\sigma_{\bar{s}sN} = 2m_s \langle N | \bar{s}s | N \rangle$$

$$y_N = 2 \langle N | \bar{s}s | N \rangle / \langle N | \bar{u}u + \bar{d}d | N \rangle$$

Besides DM, important for: hadron spectrum, m_s/m_{ud} , πN and KN scattering, counting rates in Higgs search ...

Not measured directly in experiment

⇒ history of phenomenological determinations from relation to Born-subtracted, $I=0$, πN amplitude at Cheng-Dashen point, Σ (Brown et al '71)

Using Gasser et al '91 and Bernard et al '96 to get $\sigma_{\pi N}$, then Borasoy et al '97 for y_N and $m_s/m_{ud} = 24.4(1.5)$ (Leutwyler '96) for f_{sN} , find

Canonical result

$$\Sigma = 59(2) \text{ MeV} \quad [3\%]$$

(Gasser et al '88)

$$\rightarrow \sigma_{\pi N} = 44(3) \text{ MeV} \quad [6\%]$$

$$\rightarrow f_{udN} = 0.047(3) \quad [7\%]$$

$$\rightarrow y_N = 0.18(17) \quad [95\%]$$

$$\rightarrow f_{sN} = 0.10(10) \quad [100\%]$$

← differ by $> 3\sigma$ →

$$\Sigma = 81(6) \text{ MeV} \quad [7\%]$$

(Hite et al '05)

$$\rightarrow \sigma_{\pi N} = 66(6) \text{ MeV} \quad [10\%]$$

$$\rightarrow f_{udN} = 0.070(7) \quad [10\%]$$

$$\rightarrow y_N = 0.45(12) \quad [26\%]$$

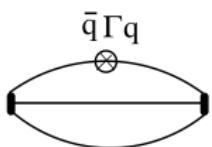
$$\rightarrow f_{sN} = 0.39(11) \quad [29\%]$$

Sigma terms from the lattice

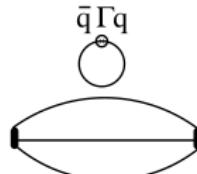
$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

$$\sigma_{\bar{s}sN} = 2m_s \langle N | \bar{s}s | N \rangle$$

Matrix element (ME) method



straightforward



challenging

Spectrum method: Feynman-Hellmann (FH) theorem, $X = N, \Lambda, \Sigma, \Xi$

$$\sigma_{\pi X} = m_{ud} \frac{\partial M_X}{\partial m_{ud}} \rightarrow M_\pi^2 \frac{\partial M_X}{\partial M_\pi^2}$$

$$\sigma_{\bar{s}sX} = 2m_s \frac{\partial M_X}{\partial m_s} \rightarrow 2M_{\bar{s}s}^2 \frac{\partial M_X}{\partial M_{\bar{s}s}^2}$$

Pros: - Can use hadron spectrum result (e.g. from BMWc '08)

- Can use whole octet and symmetry constraints

Con: - Model dependence when no lattice results at M_π^{ph} or symmetry constraints not checked w/ LQCD

→ fix w/ e.g. BMWc '10 simulations at M_π^{ph}

Combined analysis of baryon octet

Dürr et al PRD85 '12 w/ BMWc '08 simulations

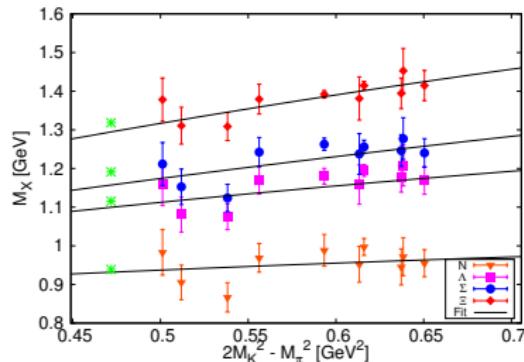
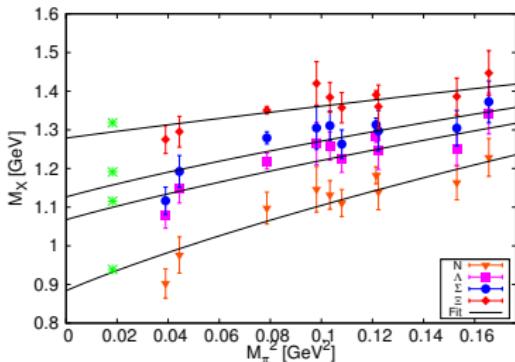
$$\sigma_{\pi X} \simeq M_\pi^2 \frac{\partial M_X}{\partial M_\pi^2}$$

$$\sigma_{\bar{s}sX} \simeq 2M_{\bar{s}s}^2 \frac{\partial M_X}{\partial M_{\bar{s}s}^2}$$

Example CB χ PT fits

$$M_X = M_0 - 4c_{\pi X}M_\pi^2 - 4c_{\bar{s}sX}M_{\bar{s}s}^2 + \sum_{\alpha=\pi, K, \eta} \frac{g_{\alpha X}}{F_\alpha^2} M_0^3 h\left(\frac{M_\alpha}{M_0}\right) + 4d_\pi M_\pi^4 + 4d_{\bar{s}s} M_{\bar{s}s}^4$$

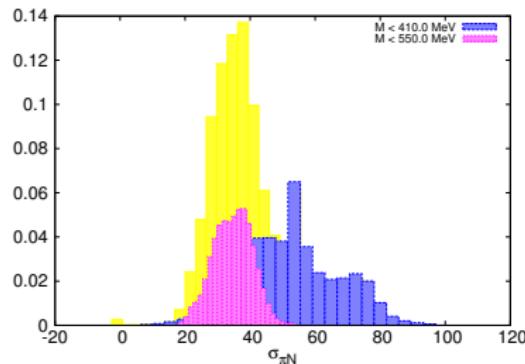
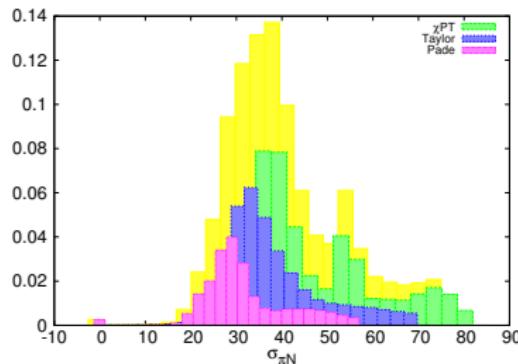
$$h(x) = -\frac{x^3}{4\pi^2} \left\{ \sqrt{1 - \left(\frac{x}{2}\right)^2} \arccos \frac{x}{2} + \frac{x}{2} \log x \right\}$$



$M_\pi < 410$ MeV, $\chi^2/\text{dof} = 39/34$

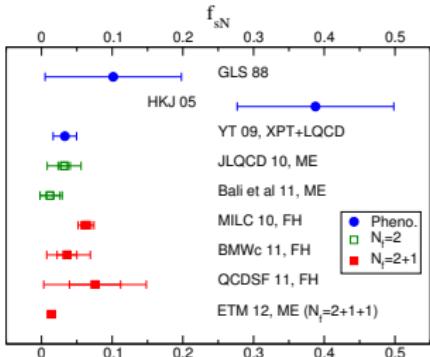
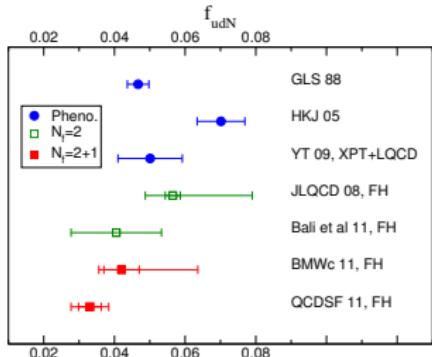
Error analysis and results

- 864 distinct analyses of octet spectrum corresponding to different choices for:
 $a \searrow 0$, $M_\pi \searrow 135 \text{ MeV}$, ...
- Weigh each one by fit quality → systematic error distribution
- repeat for 2000 bootstrap samples

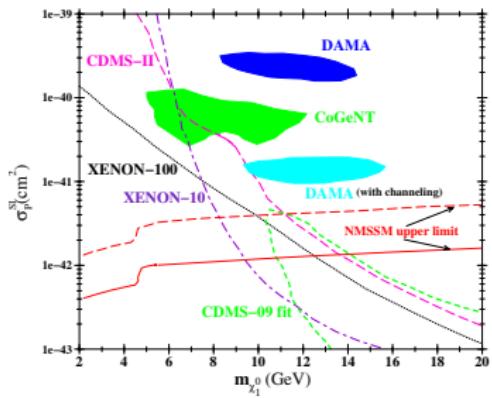


	$\sigma_{\pi X} [\text{MeV}]$	$\sigma_{\bar{s}sX} [\text{MeV}]$	y_X	f_{udX}	f_{sX}
N	$39(4)(^{+18})_{-7}$	$67(27)(^{+55})_{-47}$	$0.20(7)(^{+13})_{-17}$	$0.042(5)(^{+21})_{-4}$	$0.036(14)(^{+30})_{-25}$
Λ	$29(3)(^{+11})_{-5}$	$180(26)(^{+48})_{-77}$	$0.51(15)(^{+48})_{-27}$	$0.027(3)(^{+5})_{-10}$	$0.083(12)(^{+23})_{-31}$
Σ	$23(3)(^{+19})_{-3}$	$245(29)(^{+50})_{-72}$	$0.82(21)(^{+87})_{-39}$	$0.019(3)(^{+17})_{-3}$	$0.104(12)(^{+23})_{-31}$
Ξ	$16(2)(^{+8})_{-3}$	$312(32)(^{+72})_{-77}$	$1.7(5)(^{+1.9})_{-0.7}$	$0.0116(18)(^{+59})_{-22}$	$0.120(13)(^{+30})_{-30}$

Comparison and impact



[caveat: most calculations have incomplete systematic error analysis]



Impact of lattice results on DM search

	f_{udN}	f_{sN}
dashed-red:	0.078	0.63
red:	0.059	0.29
BMWc '11:	$0.042(5)(^{+21}_{-4})$	$0.036(14)(^{+30}_{-25})$

$$\Rightarrow \sigma_{SI}(\text{lattice}) < \sigma_{SI}(\text{red})$$

e.g. Das et al '10

Light quark masses: motivation

Determine m_u , m_d , m_s *ab initio*

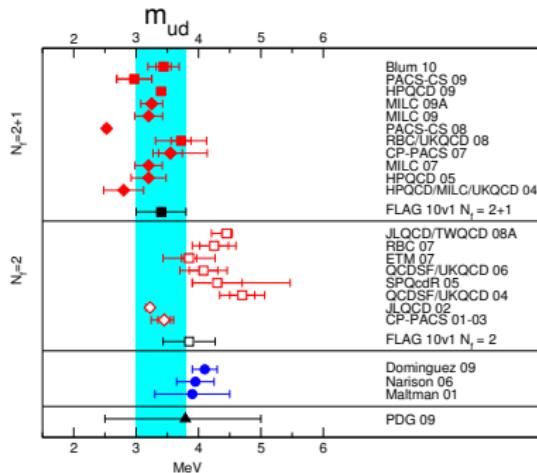
- Fundamental parameters of the standard model
- Precise values → stability of matter, N - N scattering lengths, presence or absence of strong CP violation, etc.
- Information about BSM: theory of fermion masses must reproduce these values
- Nonperturbative (NP) computation is required
- Would be needle in a haystack problem if not for χ SB

⇒ interesting first “measurement” w/ physical point LQCD

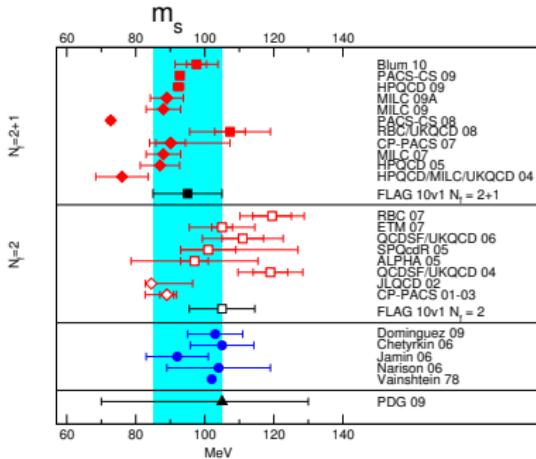
Light quark masses circa Aug. 2010

FLAG → analysis of unquenched lattice determinations of light quark masses

(arXiv:1011.4408v1)



$$m_{ud}^{\overline{MS}}(2 \text{ GeV}) = \begin{cases} 3.4(4) \text{ MeV [12%]} & \text{FLAG} \\ 3.0 \div 4.8 \text{ MeV [25%]} & \text{PDG} \end{cases}$$



$$m_s^{\overline{MS}}(2 \text{ GeV}) = \begin{cases} 95.(10) \text{ MeV [11%]} & \text{FLAG} \\ 80 \div 130 \text{ MeV [25%]} & \text{PDG} \end{cases}$$

Even extensive study by MILC still has:

- $M_\pi^{\text{RMS}} \geq 260 \text{ MeV} \Rightarrow m_{ud}^{\text{MILC,eff}} \geq 3.7 \times m_{ud}^{\text{phys}}$
- perturbative renormalization (albeit 2 loops)

Light quark masses: overall strategy

Dürr et al (BMWc) PLB 701 (2011); JHEP 1108 (2011)

- BMWc '10 set: 47 large scale $N_f = 2 + 1$ simulations w/ $M_\pi \gtrsim 120$ MeV,
 $5a's \approx 0.054 \div 0.116$ fm and $L \rightarrow 6$ fm
- improved definition of quark masses
- full nonperturbative renormalization in RI/MOM scheme w/ improvements
⇒ 21 additional $N_f = 3$ simulations at same $5a's$
- full nonperturbative running up to high μ and four-loop conversion to RGI
⇒ masses in other schemes w/ only errors proper to that scheme
- renormalized $m_{ud}(\mu)$ and $m_s(\mu)$ fitted as fn of $(M_\pi^2, 2M_K^2 - M_\pi^2, a, L)$
 - fully controlled $V \rightarrow \infty$ extrapolation
 - fully controlled **interpolation** to m_{ud}^{ph} and m_s^{ph}
 - fully controlled $a \rightarrow 0$ extrapolation
- use phenomenology to determine individual m_u and m_d
- extensive systematic and statistical error analysis: 288 full analyses on 2000 bootstrap samples

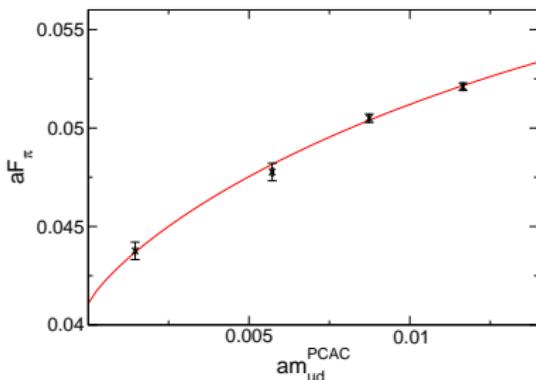
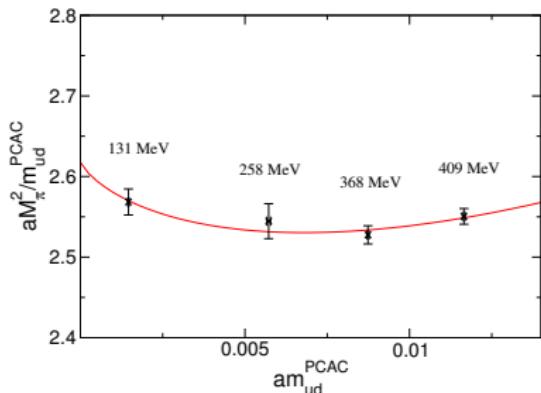
Combined mass interp. and continuum extrap. (1)

Project onto m_{ud} axis: chiral interpolation to M_π^{ph}

Illustration of chiral behavior (4/47 simulations)

- Fixed $a \approx 0.09 \text{ fm}$ and $M_\pi \sim 130 \div 410 \text{ MeV}$

- Fit to NLO $SU(2) \chi\text{PT}$ (Gasser et al '84)



✓ Consistent w/ NLO χPT for $M_\pi \lesssim 410 \text{ MeV}$

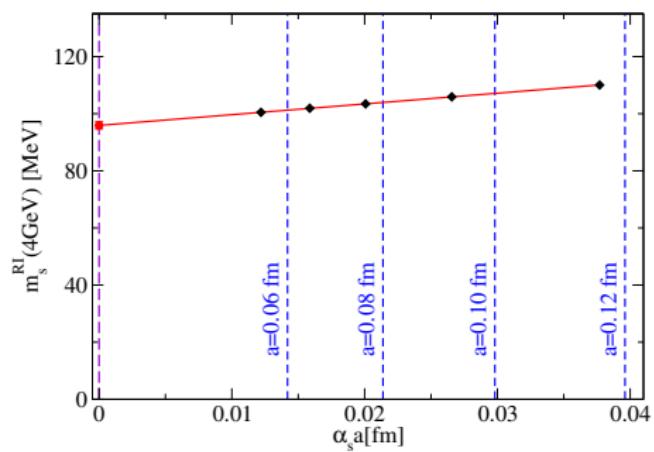
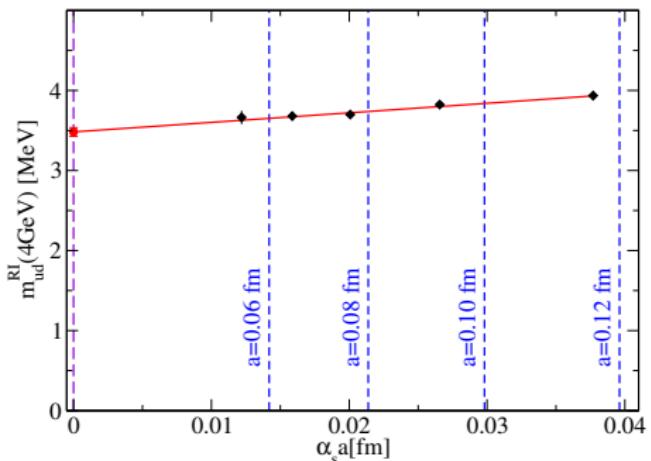
⇒ 2 safe interpolation ranges: $M_\pi < 340, 380 \text{ MeV}$

⇒ $SU(2)$ NLO χPT & Taylor interpolations to physical point

Combined mass interp. and continuum extrap. (2)

Project onto a axis: continuum extrapolation

- Leading order is $O(\alpha_s a)$
- Allow also domination of sub-leading $O(a^2)$



(continuum extrapolation examples – errors on points are statistical)

⇒ fully controlled continuum limit

Individual m_u and m_d

Calculation performed in isospin limit:

- $m_u = m_d$ • NO QED
⇒ leave *ab initio* realm
- Use dispersive Q from $\eta \rightarrow \pi\pi\pi$

$$Q^2 \equiv \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}$$

- Precise m_{ud} and $m_s/m_{ud} \Rightarrow$

$$m_{u/d} = m_{ud} \left\{ 1 \mp \frac{1}{4Q^2} \left[\left(\frac{m_s}{m_{ud}} \right)^2 - 1 \right] \right\}$$

- Use conservative $Q = 22.3(8)$ (Leutwyler '09)

Results

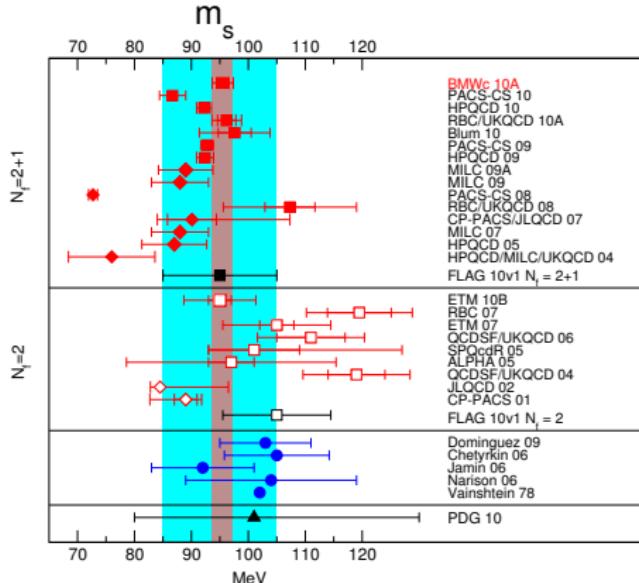
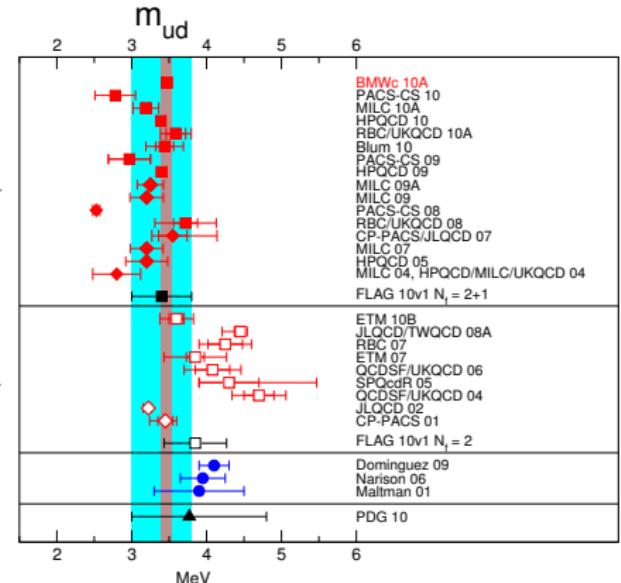
	RI 4 GeV	RGI	MS 2 GeV
m_s	96.4(1.1)(1.5)	127.3(1.5)(1.9)	95.5(1.1)(1.5)
m_{ud}	3.503(48)(49)	4.624(63)(64)	3.469(47)(48)
m_u	2.17(4)(3)(10)	2.86(5)(4)(13)	2.15(4)(3)(9)
m_d	4.84(7)(7)(10)	6.39(9)(9)(13)	4.79(7)(7)(9)

$$\frac{m_s}{m_{ud}} = 27.53(20)(8) \quad \frac{m_u}{m_d} = 0.449(6)(2)(29)$$

Additional consistency checks

- ✓ Additional continuum, chiral and FV terms
 - ☞ all compatible with 0
- ✓ Unweighted final result and systematic error
 - ☞ negligible impact
- ✓ Use m^{PCAC} only
 - ☞ compatible, slightly larger error
- ✓ Full quenched check of procedure ☞ cf. reference computation (Garden et al '00)

Comparison



- m_{ud} and m_s are now known to 2%, m_s/m_{ud} to 0.7%
- ... m_u to 5% and m_d to 3% w/ help of phenomenology

Quark flavor mixing constraints in the SM and beyond

Test SM paradigm of quark flavor mixing and CP violation and look for new physics

Unitary CKM matrix

$$\sim V_{ub} \rightarrow \nu = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \begin{pmatrix} d & s & b \\ 1 - \frac{\lambda}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Test CKM unitarity/quark-lepton universality and constrain NP using, e.g.

1st row unitarity:

$$\frac{G_q^2}{G_\mu^2} |V_{ud}|^2 \left[1 + |V_{us}/V_{ud}|^2 + |V_{ub}/V_{ud}|^2 \right] = 1 + \mathcal{O}\left(\frac{M_W^2}{\Lambda_{NP}^2}\right)$$

Unitarity triangle:

$$\frac{G_q^2}{G_\mu^2} (V_{cd} V_{cb}^*) \left[1 + \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right] = \mathcal{O}\left(\frac{M_W^2}{\Lambda_{NP}^2}\right)$$

CKM matrix and lattice QCD

CKM	process	LQCD	precision (%)
$ V_{us} $	$K \rightarrow \ell\nu$	f_K	≤ 1.5
	$K \rightarrow \pi\ell\nu$	$f_+^{K\pi}(0)$	1.0
$ V_{us} / V_{ud} $	$K \rightarrow \mu\nu/\pi \rightarrow \mu\nu$	f_K/f_π	≤ 1.5
	$D \rightarrow \ell\nu$	f_{D_s}/f_D	4.0
$ V_{cd} $	$D \rightarrow \pi\ell\nu$	$f_+^{D\pi}(0)$	~ 10
	$D_s \rightarrow \ell\nu$	f_{D_s}	2.5
$ V_{cs} $	$D \rightarrow K\ell\nu$	$f_+^{DK}(0)$	~ 7
	$B \rightarrow \ell\nu$	f_B	6.0
$ V_{ub} $		f_{B_s}/f_B	4.0
	$B \rightarrow \pi\ell\nu$	$f_+^{B\pi}(q^2)$	~ 15
$ V_{cb} $	$B \rightarrow D^{(*)}\ell\nu$	$\mathcal{F}_{B \rightarrow D^{(*)}}(1)$	4.0
$(\bar{\rho}, \bar{\eta})$	ϵ	\hat{B}_K	≤ 1.5
	ϵ'	$K \rightarrow \pi\pi$	
$ V_{tb}^* V_{tq} $	Δm_d	$B_{B_s}/B_{B_d}, \xi$	~ 3.0
	Δm_s	\hat{B}_{B_s}	~ 6.0

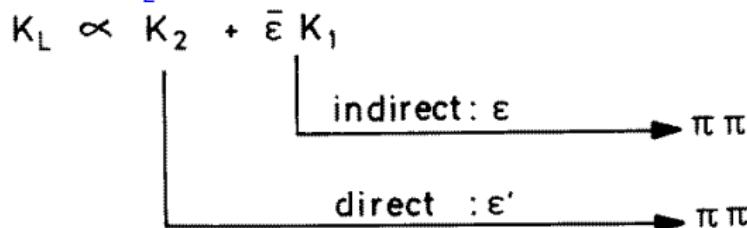
CPV and the K^0 - \bar{K}^0 system

Two neutral kaon flavor eigenstates: $K^0(d\bar{s})$ & $\bar{K}^0(s\bar{d})$

In experiment, have predominantly:

- $K_S^0 \rightarrow \pi\pi \Rightarrow K_S^0 \sim K_1$, the CP even combination
- $K_L^0 \rightarrow \pi\pi\pi \Rightarrow K_L^0 \sim K_2$, the CP odd combination

However, CP violation $\Rightarrow K_L^0 \rightarrow \pi\pi$



$$\Delta M_K \equiv M_{K_L} - M_{K_S} = 3.483(6) \times 10^{-12} \text{ MeV} \quad [0.2\%]$$

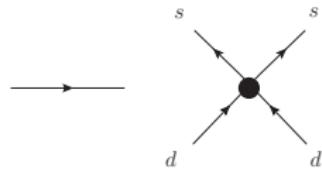
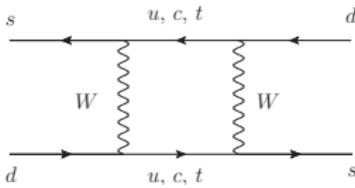
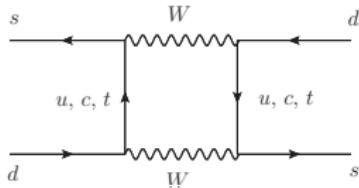
$$\Delta \Gamma_K \equiv \Gamma_{K_S} - \Gamma_{K_L} = 7.339(4) \times 10^{-12} \text{ MeV} \quad [0.05\%]$$

$$|\epsilon| = 2.228(11) \cdot 10^{-3} \quad [0.5\%]$$

$$\text{Re}(\epsilon'/\epsilon) = 1.65(26) \cdot 10^{-3} \quad [16\%]$$

Experimentally:
(PDG '11)

K^0 - \bar{K}^0 mixing in the SM



$$M_{12} - \frac{i}{2}\Gamma_{12} = \frac{\langle K^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | \bar{K}^0 \rangle}{2M_K} - \underbrace{\frac{i}{2M_K} \int d^4x \langle K^0 | \mathcal{T}\{\mathcal{H}_{\text{eff}}^{\Delta S=1}(x)\mathcal{H}_{\text{eff}}^{\Delta S=1}(0)\} | \bar{K}^0 \rangle + O(G_F^3)}$$

gives Γ_{12} and LD contributions to M_{12}

Neglecting long-distance (LD) corrections

$$2M_K M_{12}^* \stackrel{\text{SD}}{=} \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C_1^{\text{SM}}(\mu) \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle$$

$$O_1 = (\bar{s}d)_{V-A} (\bar{s}d)_{V-A} \quad \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \frac{16}{3} M_K^2 F_K^2 B_K(\mu)$$

Indirect CPV in $K \rightarrow \pi\pi$

Parametrized by

$$\text{Re } \epsilon = \text{Re} \frac{T[K_L \rightarrow (\pi\pi)_0]}{T[K_S \rightarrow (\pi\pi)_0]} = \cos \phi_\epsilon \sin \phi_\epsilon \left[\frac{\text{Im} M_{12}}{2 \text{Re} M_{12}} - \frac{\text{Im} \Gamma_{12}}{2 \text{Re} \Gamma_{12}} \right]$$

w/ $\phi_\epsilon = \tan^{-1}(2\Delta M_K / \Delta \Gamma_K) = 43.51(5)^\circ$, $\Delta M_K \simeq 2 \text{Re} M_{12}$, $\Delta \Gamma_K \simeq -2 \text{Re} \Gamma_{12}$

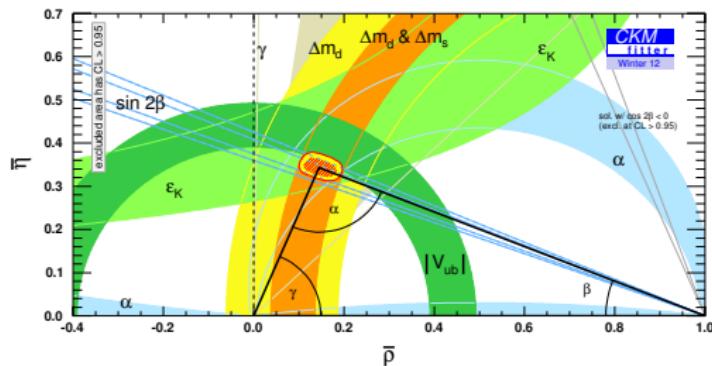
$$\rightarrow \epsilon = \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{\sqrt{2}} \frac{\text{Im} M_{12}}{\Delta M_K}$$

w/ $\kappa_\epsilon = 0.94(2)$ (Buras et al. (2010))

To NLO in α_s

$$\begin{aligned} |\epsilon| &= \kappa_\epsilon C_\epsilon \text{Im} \lambda_t \{ \text{Re} \lambda_c [\eta_1 S_{cc} \\ &\quad - \eta_3 S_{ct}] - \text{Re} \lambda_t \eta_2 S_{tt} \} \hat{B}_K \\ &\propto A^2 \lambda^6 \bar{\eta} [\text{cst} + \text{cst} \\ &\quad \times A^2 \lambda^4 (1 - \bar{\rho})] \hat{B}_K \end{aligned}$$

w/ $\lambda_q = V_{qd} V_{qs}^*$



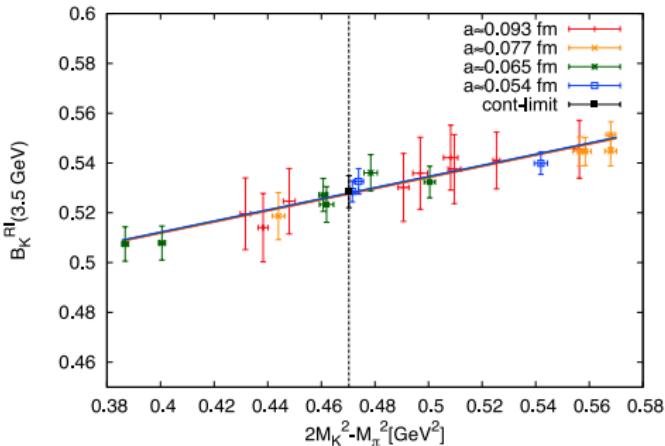
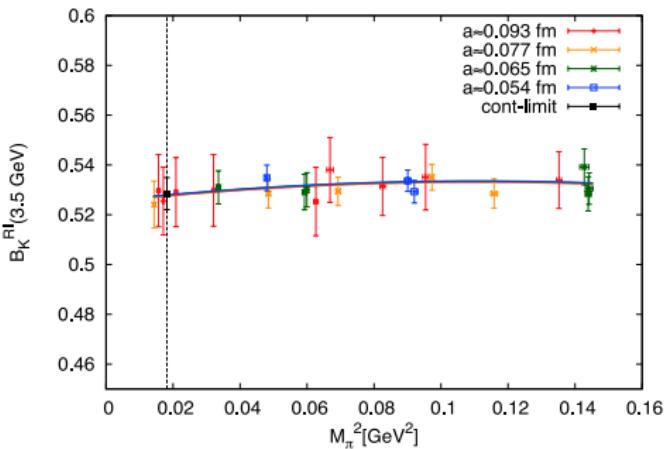
Overall strategy for B_K

Dürr et al [BMWc], PLB705 '11

- BMWc '10 set: 47 large scale $N_f = 2 + 1$ simulations w/ $M_\pi \gtrsim 120$ MeV, 5a's $\approx 0.054 \div 0.116$ fm and $L \rightarrow 6$ fm
- bare $Q_{1,\dots,5}$ computed on $N_f = 2 + 1$ ensembles
- full nonperturbative renormalization and mixing in RI/MOM scheme w/ improvements
⇒ 21 additional $N_f = 3$ simulations at same 5a's
- full nonperturbative running up to high μ and two-loop conversion to RGI
⇒ B_K in other schemes w/ small conversion errors
- renormalized $B_K(\mu)$ fitted as fn of $(M_\pi^2, 2M_K^2 - M_\pi^2, a, L)$
 - fully controlled $V \rightarrow \infty$ extrapolation (using Becirevic et al. '04)
 - fully controlled interpolation to m_{ud}^{ph} and m_s^{ph}
 - fully controlled $a \rightarrow 0$ extrapolation w/ 4 smallest a's
- extensive systematic and statistical error analysis: 5760 full analyses on 2000 bootstrap samples

Combined chiral interp. and continuum extrap. (1)

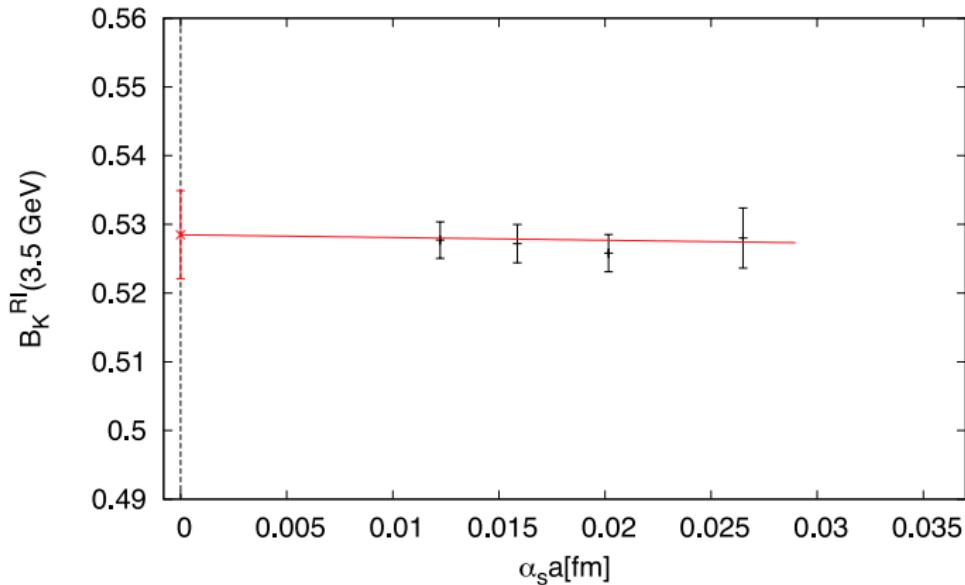
Project onto M_π^2 and M_{ss}^2 axes: interpolation to $M_\pi = 134.8(3)$ MeV and $M_K = 494.2(5)$ MeV



- nearly flat m_{ud} -dependence near m_{ud}^{ph}
- much steeper m_s -dependence near m_s^{ph}

Combined chiral interp. and continuum extrap. (2)

Project onto a axis: continuum extrapolation

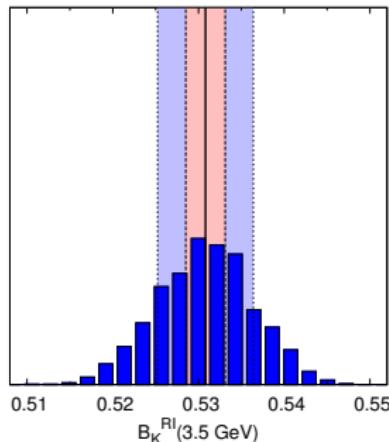
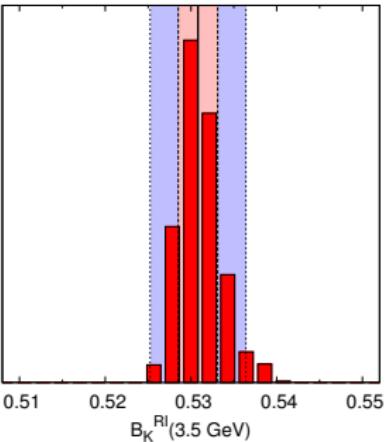


→ mild extrapolation for $\beta \geq 3.5$

→ $\beta \geq 3.31$ was found to be outside scaling regime

Systematic and statistical error

- 2 time-fit ranges for π and K masses
- 2 time-fit ranges for $Q_{1,\dots,5}$
- $O(\alpha_s a)$ or $O(a^2)$ for running
- 3 intermediate renormalization scales
- 2 fit fns and 4 ranges in p^2 for Δ_{1i}
- 5 fit fns for mass interpolation
- 2 pion mass cuts ($M_\pi < 340, 380$ MeV)



→ 5760 analyses, each of which is a reasonable choice, weighted by fit quality

→ median and central 68% give central value and systematic uncertainty

2000 bootstraps of the median give statistical error

Many cross checks

Results

Procedure gives B_K^{RI} (3.5 GeV) fully nonperturbatively

Can convert to other schemes w/ perturbation theory (PT)
→ perturbative uncertainty

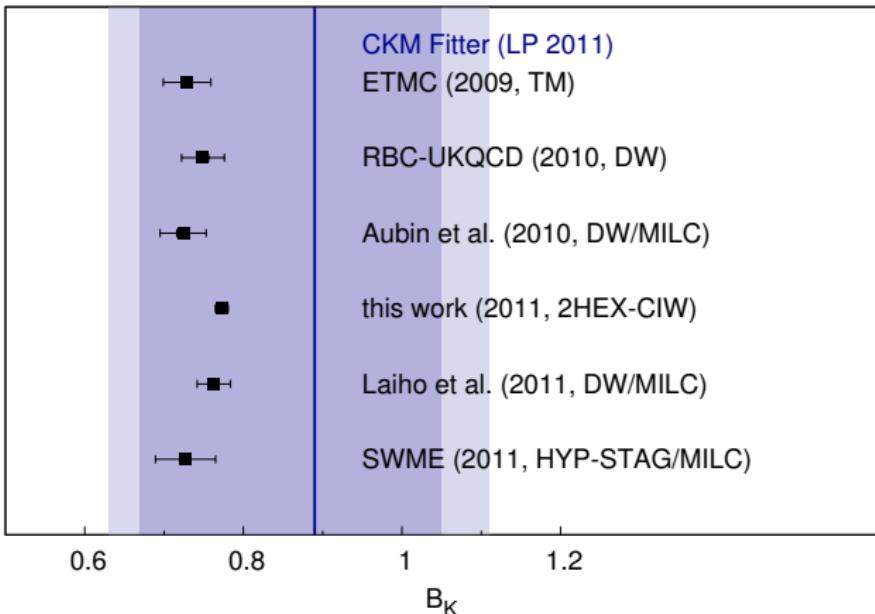
	conv.	RGI	MS-NDR 2 GeV
4-loop β , 1-loop γ	1.427	1.047	
4-loop β , 2-loop γ	1.457	1.062	
ratio	1.021	1.01376	

Take blanket 1% for 3 loop uncertainty

	RI @ 3.5 GeV	RGI	MS-NDR @ 2 GeV
B_K	0.5308(56)(23)	0.7727(81)(34)(77)	0.5644(59)(25)(56)

Total error 1.1-1.5%, statistical (and PT) dominated

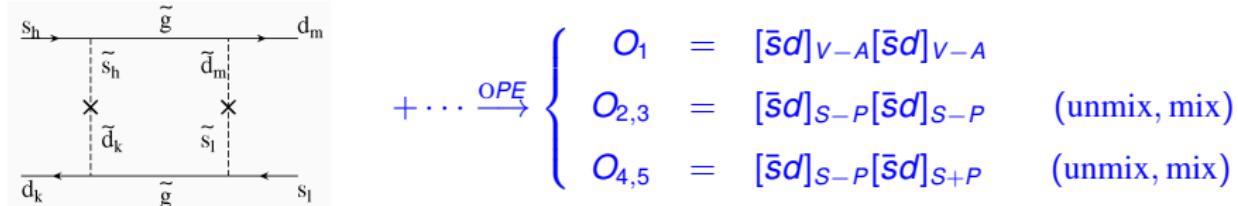
B_K comparison



Dominant error in CKMfitter global fit results is $|V_{cb}|^4 \sim (A\lambda^2)^4$
→ our result is an encouragement to reduce uncertainties in other parts of the calculation of ϵ

$\Delta S = 2$ processes beyond the SM

e.g. gluino mediated FCNC in mass insertion approximation (Ciuchini et al '99)



$$\langle K^0 | \mathcal{H}_{\text{eff,BSM}}^{\Delta S=2} | \bar{K}^0 \rangle = \sum_{i=1}^5 C_i^{\text{BSM}}(\mu) Q_i(\mu) = \sum_{i=1}^5 C_i^{\text{BSM}}(\mu) \langle K^0 | O_i(\mu) | \bar{K}^0 \rangle$$

- $C_i^{\text{BSM}}(\mu)$ short distance coefficients \supset flavor mixing parameters of BSM model
- $Q_i(\mu)$ are chirally enhanced

$$\frac{Q_i(\mu)}{Q_1(\mu)} \sim \frac{\langle \bar{q}q \rangle(\mu)}{F_\pi^2(m_s + m_{ud})(\mu)} \sim 10, \quad i = 2, \dots, 5$$

$\Rightarrow \epsilon$ and ΔM_K impose strong constraints on Im and Re of BSM parameters

- only 2 old quenched calculations of $Q_i(\mu)$, $i = 2, \dots, 5$ (Donini et al '99, Babich et al '06)
- \Rightarrow modern calculation required

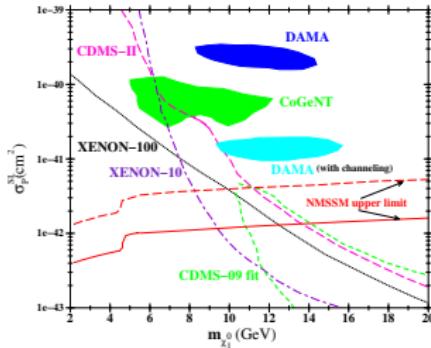
Conclusion

- After > 30 years we are finally able to perform **fully controlled LQCD** computations all the way down to $M_\pi \leq 135$ MeV
- Presented fully controlled results for sigma terms and for **light quark masses** and B_K w/ $\sigma_{\text{tot}} < 2\%$
- There $N_f = 2 + 1$ LQCD calculations of many observables relevant for particle phenomenology, some of which have fully controlled errors, see e.g.

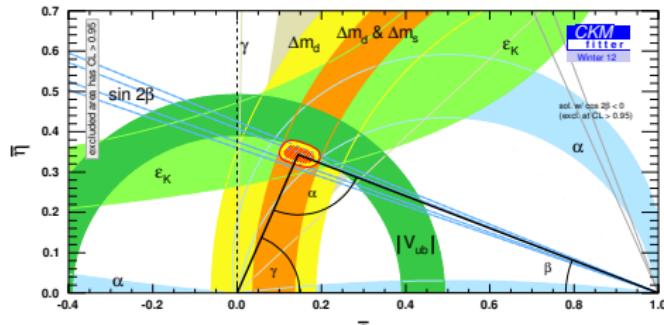
<http://itpwiki.unibe.ch/flag> (Colangelo et al EPJC71 '11)

<http://www.latticeaverages.org> (Laiho et al PRD81 '10)

⇒ LQCD has become an indispensable tool in the search for new physics



$$\sigma_{\text{SI}}(\text{lattice}) < \sigma_{\text{SI}}(\text{red})$$



Conclusion

- Lattice QCD is undergoing a major shift in paradigm
 - it is now possible to control and reliably quantify all systematic errors with “data” (for processes w/ at most 1 initial and/or final hadron state)
⇒ we are getting **QCD NOT LQCD predictions**
 - requires numerous simulations with $M_\pi < 200 \text{ MeV}$ and preferably
↘ 135 MeV , more than $3 a < 0.1 \text{ fm}$ and lattice sizes $L \rightarrow 4 \div 6 \text{ fm}$
 - requires trying all reasonable analyses of “data” and combining results in sensible way to obtain a **reliable systematic error**
- A LQCD result is only as good as its systematic error
- Calculations with anything less can yield very interesting results which are not, however, QCD only predictions
- Expect many more very interesting nonperturbative QCD predictions in coming years