Lepton flavour violation and neutrinos

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Outline

$\mathcal{I}$. Introduction: ν's

$\mathcal{II}$. Charged Lepton Flavour violation

$\mathcal{III}$. SUSY, neutrino masses and LFV

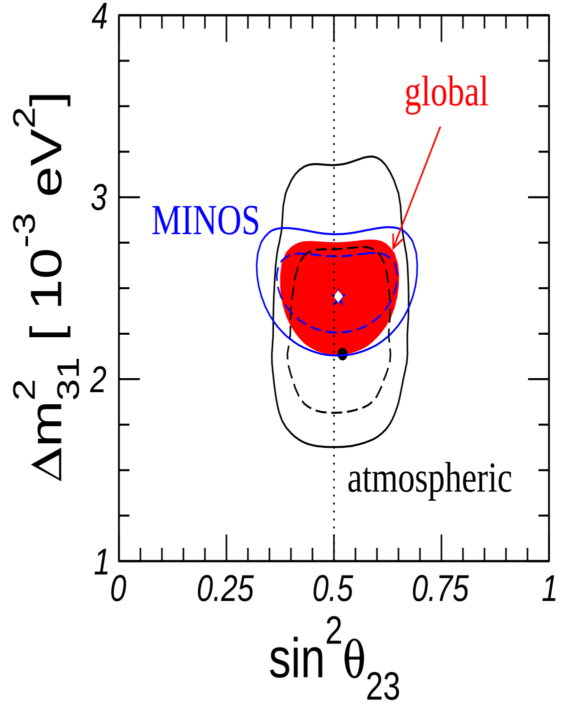
$\mathcal{IV}$. Discrete symmetries and LFV

$\mathcal{I}.$

Introduction

Lepton flavour is violated!

Status ν's

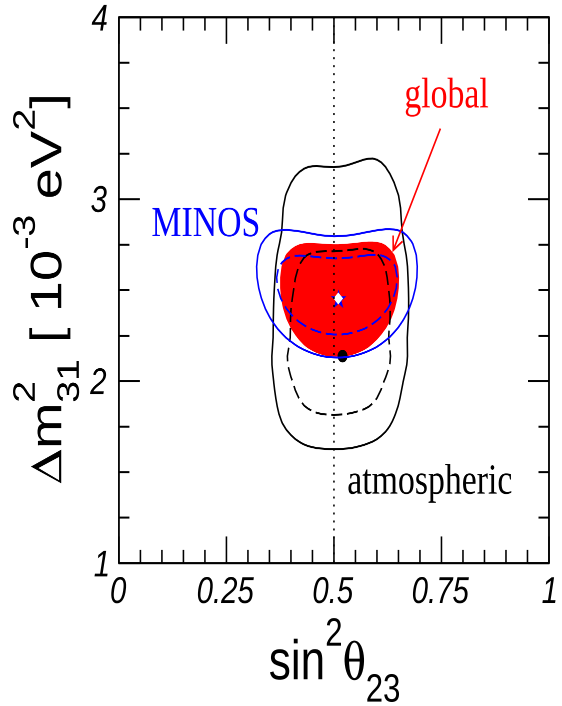


Super-K
fixes θ_{Atm}

MINOS
fixes $\Delta m^2_{\mathrm{Atm}}$

Schwetz, Tortola & Valle; arXiv:1103.0734 and arXiv:1108.1376

Status ν´s



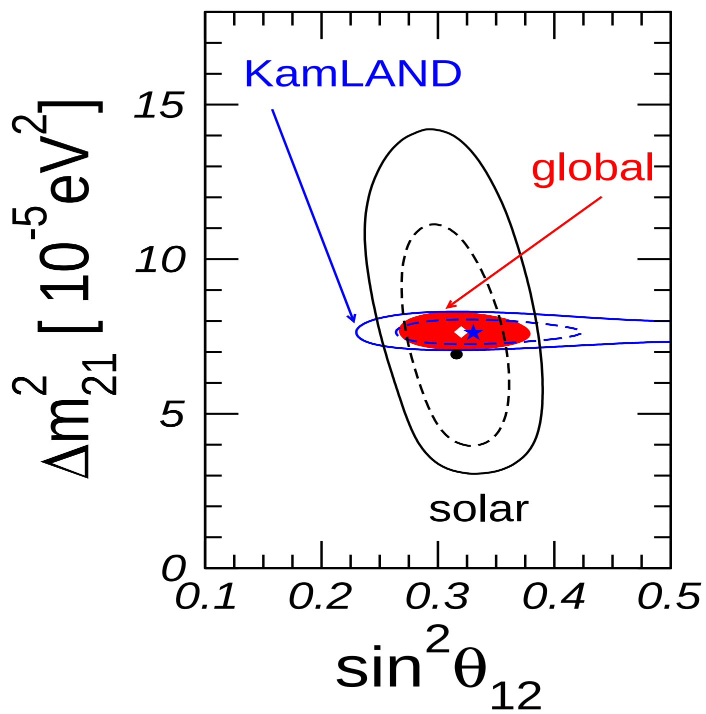
Super-K
fixes $\theta_{\rm Atm}$

MINOS
fixes $\Delta m^2_{\rm Atm}$

Schwetz, Tortola & Valle; arXiv:1103.0734 and arXiv:1108.1376

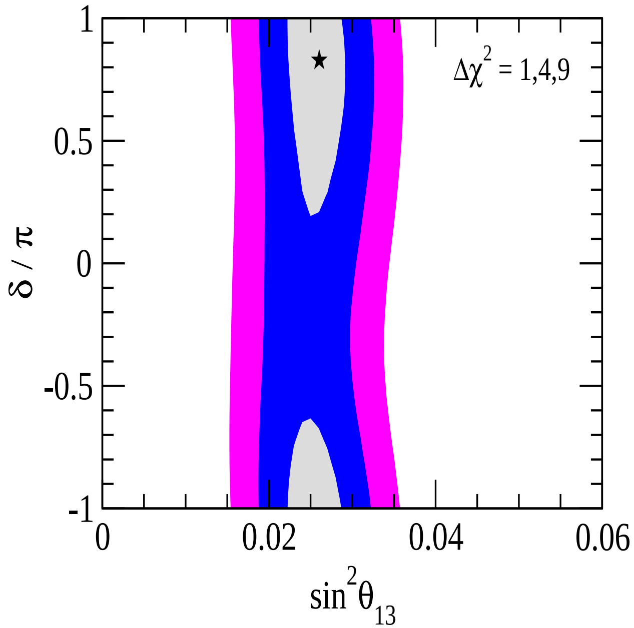
Solar data
fixes $\theta_\odot$

KamLAND
fixes $\Delta m^2_\odot$

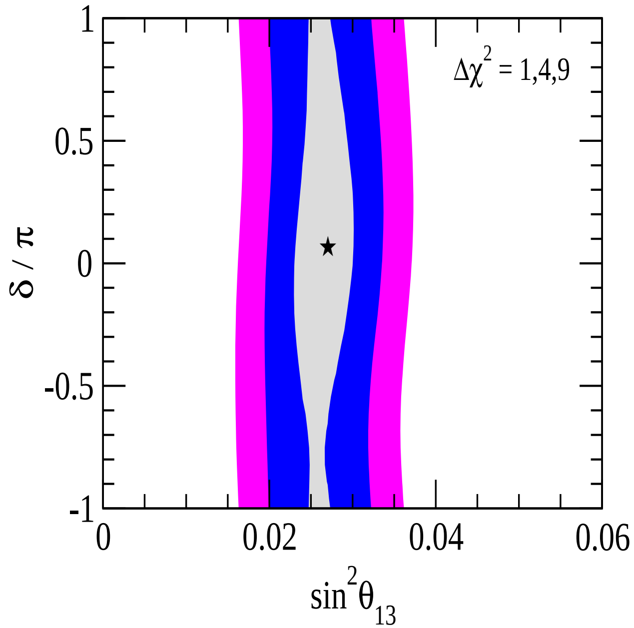


Latest ν's

Normal hierarchy:



Inverted hierarchy:

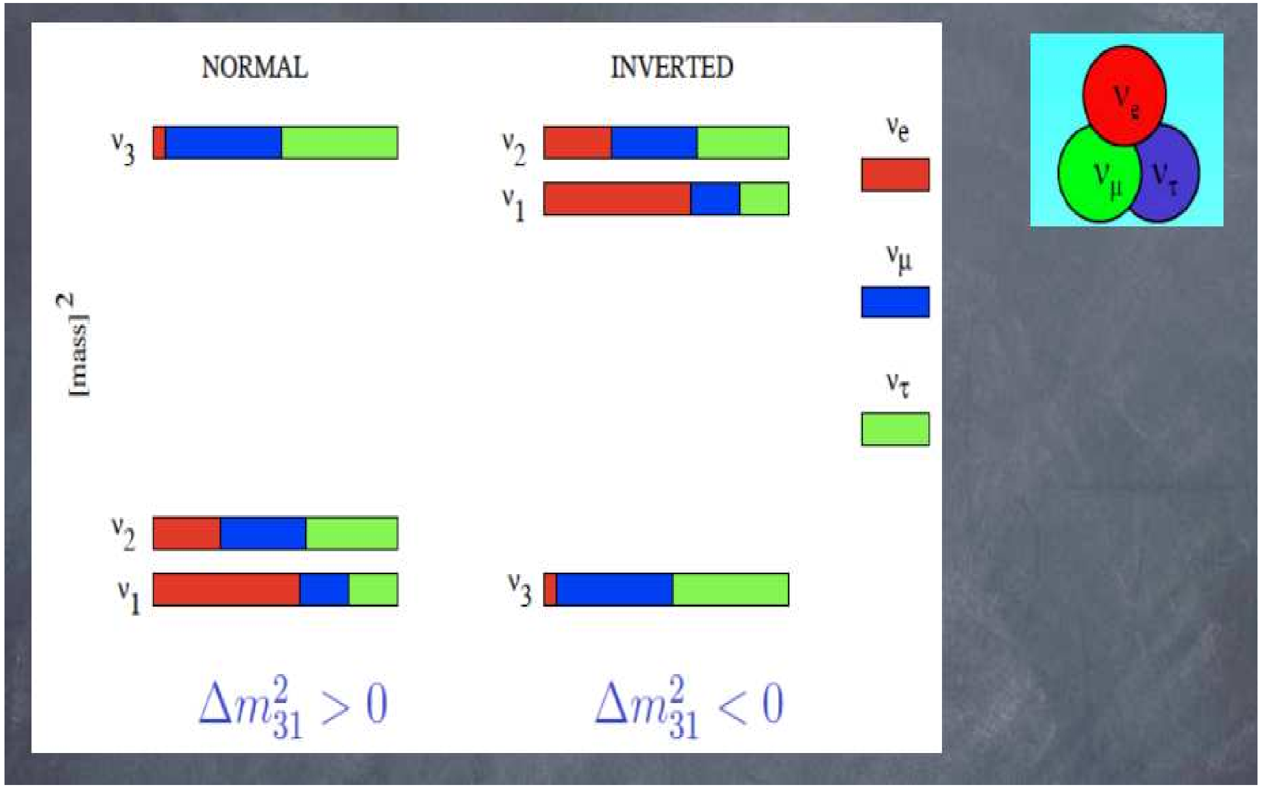


Thanks to:
M. Tortola

T2K (arXiv:1106.2822):
0.03(0.04)$< \sin^2(2\theta_{13}) <$ 0.28(0.34)
Double Chooz (arXiv:1112.6353):
$\sin^2(2\theta_{13}) = 0.086 \pm 0.041(stat) \pm 0.030(syst)$
Daya Bay (arXiv:1203.1669):
$\sin^2(2\theta_{13}) = 0.092 \pm 0.016(\mathrm{stat}) \pm 0.005(\mathrm{syst})$
RENO (arXiv:1204.0626):
$\sin^2(2\theta_{13}) = 0.113 \pm 0.013(\mathrm{stat.}) \pm 0.019(\mathrm{syst.})$

Last mixing angle has finally been measured: θ_{13}

Don't know ν's



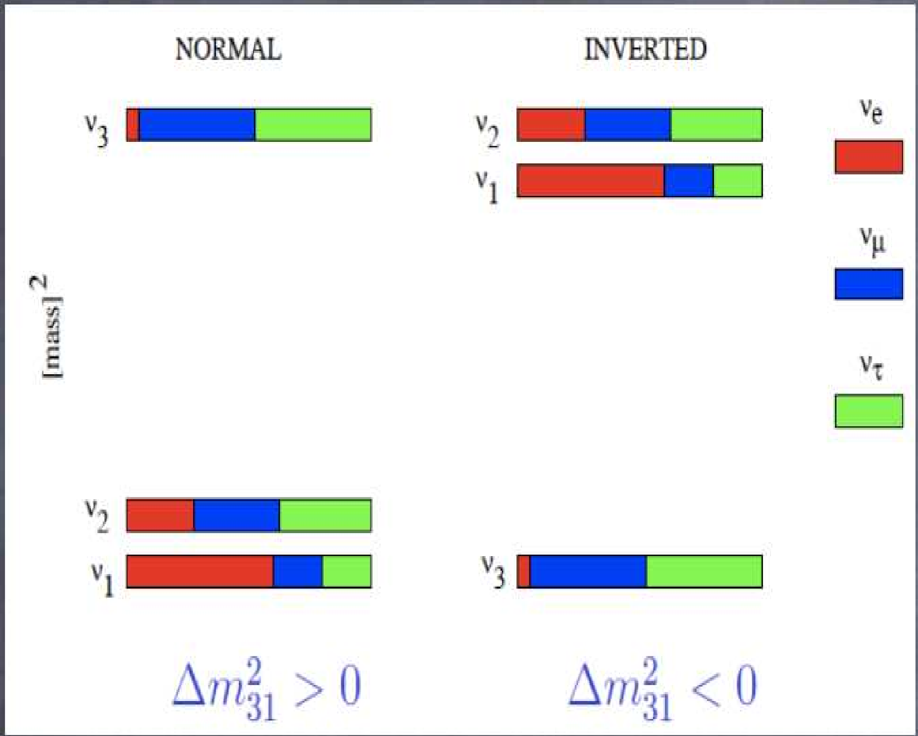
Open questions:

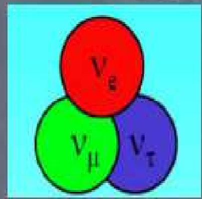
Which hierarchy: Normal or inverted?

What is the absolute neutrino mass scale?

Is there CP violation in the lepton sector?

Don't know ν's





Open questions:

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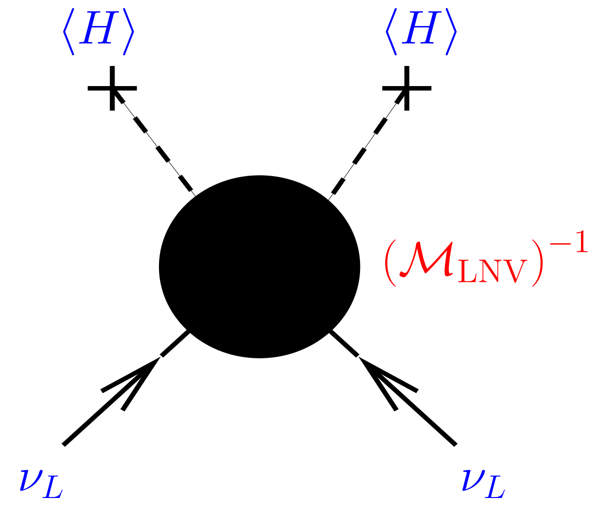
What is the absolute neutrino mass scale?

Is there CP violation in the lepton sector?

Is lepton number violated???

Majorana $\mathcal{M}_\nu$

If Lepton Number is Violated:



Weinberg, 1979

$$m_\nu = \frac{1}{\mathcal{M}_{LNV}}(LH)(LH)$$

Many possible models:

(i) Seesaw mechanism: Type-I, Type-II, Type-III, Inverse seesaw, etc ...

(ii) Radiative models: Zee, Babu, LQs ...

(iii) SUSY neutrino masses: $R\!\!\!/_p$

(iv) $\cdots$

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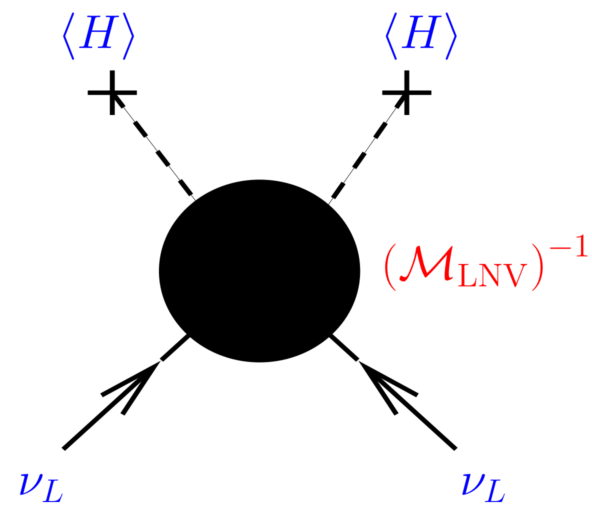

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Experimental tests?

$0\nu\beta\beta$ decay!

LHC (???)

$\mathcal{II}.$

Charged Lepton Flavour violation

Experimental status: CLFV

Decay	Current Limit
$\tau \to \mu\gamma$	$4.4 \cdot 10^{-8}$
$\tau \to e\gamma$	$3.3 \cdot 10^{-8}$
$\mu \to e\gamma$	$1.2 \cdot 10^{-11}$
$\tau \to 3\mu$	$2.1 \cdot 10^{-8}$
$\tau^- \to e^-\mu^+\mu^-$	$2.7 \cdot 10^{-8}$
$\tau^- \to e^+\mu^-\mu^-$	$1.7 \cdot 10^{-8}$
$\tau^- \to \mu^-e^+e^-$	$1.8 \cdot 10^{-8}$
$\tau^- \to \mu^+e^-e^-$	$1.5 \cdot 10^{-8}$
$\tau \to 3e$	$2.7 \cdot 10^{-8}$
$\mu \to 3e$	$1 \cdot 10^{-12}$

All values from:
Particle Data Group
http://pdg.lbl.gov/

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MEG 2011,
PRL107, 171801:
Br($\mu \to e\gamma$)$\leq 2.4 \times 10^{-12}$

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MEG 2011,

PRL107, 171801:

Br($\mu \to e\gamma$)$\leq 2.4 \times 10^{-12}$

Limit on $\mu \to 3e$ from:

SINDRUM 1988

New experiment could reach:

Br($\mu \to 3e$) $\simeq 10^{-16}$ (?)

A. Schöning et al.,
Physics Procedia 17 (2011) 181

Experimental status: CLFV

Capture	Current Limit
$\mu^- \, {}^{32}S \rightarrow e^- \, {}^{32}S$	$7 \cdot 10^{-11}$
$\mu^- \, {}^{32}S \rightarrow e^+ \, {}^{32}Si$	$9 \cdot 10^{-10}$
$\mu^- Ti \rightarrow e^- Ti$	$4.3 \cdot 10^{-12}$
$\mu^- Ti \rightarrow e^+ Ca$	$3.6 \cdot 10^{-11}$
$\mu^- Pb \rightarrow e^- Pb$	$4.6 \cdot 10^{-11}$
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http://pdg.lbl.gov/

Future experiments:

Sensitivities of $\mathcal{O}(10^{-16})$ (?)

COMET:
Letter of interest @:
http://j-parc.jp/

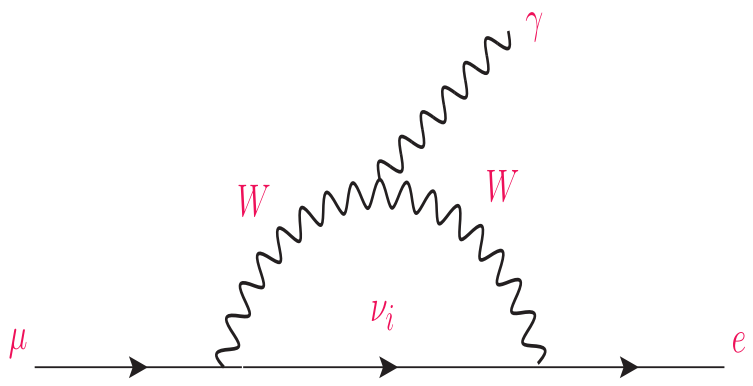
Mu2E:

Proposal @:
http://mu2e.fnal.gov/

Timeline: 2016 (?)

Guaranteed CLFV

Oscillations experiments have shown that $m_\nu \neq 0$:



$$\mathrm{Br}(\mu \to e\gamma) \sim \frac{3\alpha}{32\pi}\left(\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{i1}}{m^2_W}\right)^2 \leq 10^{-53}$$

$\Rightarrow$ GIM suppressed by small neutrino masses

Guaranteed CLFV

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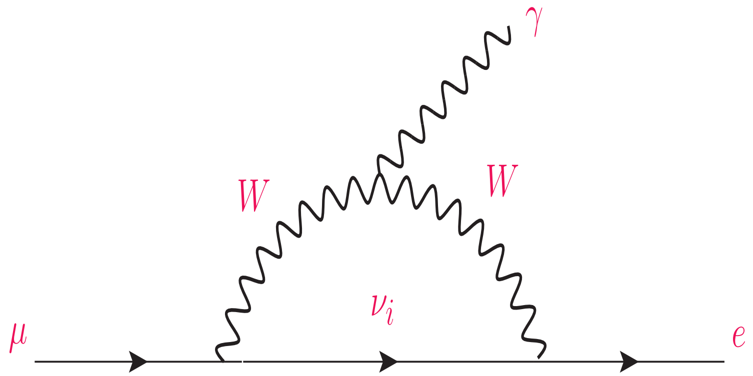

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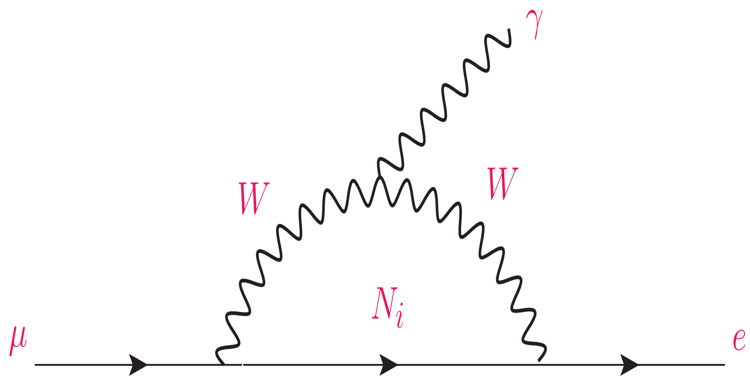
$\Rightarrow$ GIM suppressed by small neutrino masses

Any observation of charged LFV
points to physics beyond neutrino masses

CLFV beyond m_ν

Simple example: Heavy neutrinos (N) with m_N $\mathcal{O}(TeV)$:



$$\mathrm{Br}(\mu \to e\gamma) \sim$$

$$\frac{\alpha^3 s_W^2}{256\pi^2} \frac{m_\mu^5}{m_W^4 \Gamma_\mu} \Big(\sum_i K_{\mu i}^* K_{ei} G(\frac{m_{N_k}^2}{m_W^2}) \Big)^2$$

$$\leq 9 \times 10^{-6} \Big(\sum_i K_{\mu i}^* K_{ei} G(\frac{m_{N_k}^2}{m_W^2}) \Big)^2$$

– K_{ik} heavy neutrino - lepton mixing

– G(x) loop function, $G(1) = 1/8$

CLFV beyond m_ν

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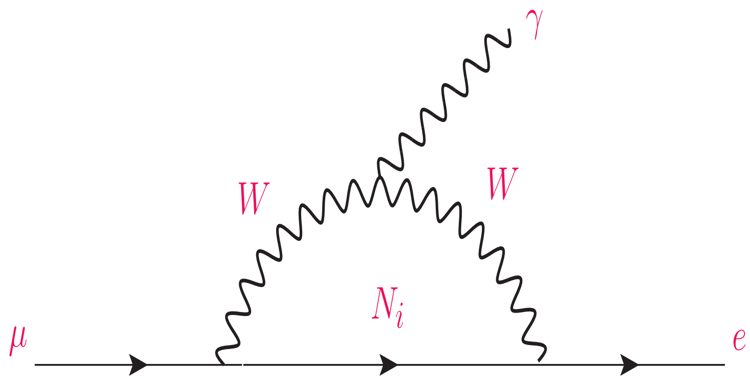


$$\mathrm{Br}(\mu \to e\gamma) \sim \frac{\alpha^3 s_W^2}{256\pi^2} \frac{m_\mu^5}{m_W^4 \Gamma_\mu} \Big(\sum_i K^*_{\mu i} K_{ei} G(\frac{m^2_{N_k}}{m^2_W}) \Big)^2$$

$$\leq 9 \times 10^{-6} \Big(\sum_i K^*_{\mu i} K_{ei} G(\frac{m^2_{N_k}}{m^2_W}) \Big)^2$$

– K_{ik} heavy neutrino - lepton mixing
– G(x) loop function, $G(1) = 1/8$

Practically any extension of SM
with new states at TeV scale
generates large charged LFV!

(C)LFV - Models

⇒ Example models that produce sizeable CLFV:

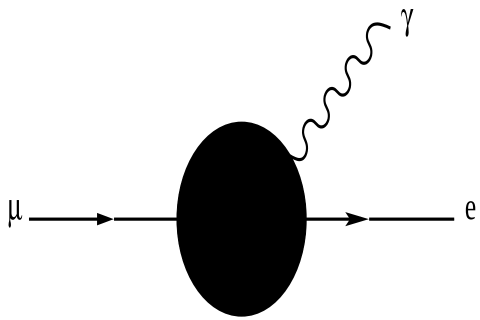
- TeV scale seesaw: Inverse seesaw, linear seesaw, etc.
- Radiative neutrino mass models: Zee-, Babu-Zee model, etc.

- RPC Supersymmetry
- RPV Supersymmetry
- Practically any extended Higgs sector:
  Little Higgs models, additional Higgs doublets, triplets, etc...
- Extra (large) dimensions
- etc ...

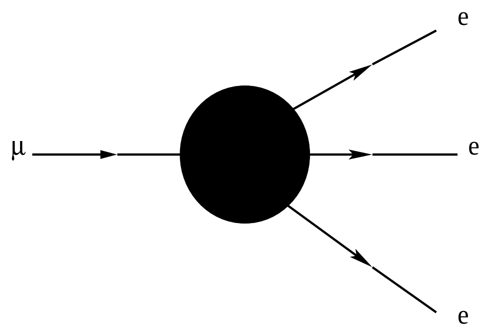
⇒ In fact, many models generate way to much CLFV:
"Flavour problem" of BSM

Schematically:

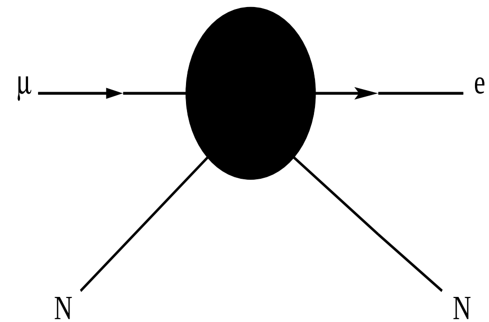
$\mu \to e\gamma$:



$\mu \to 3e$



μ-capture:

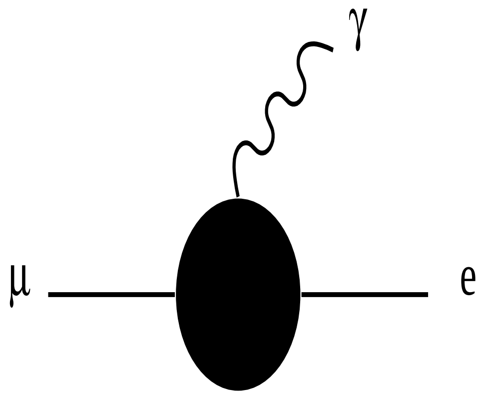


Can we learn about
different BSM models
from different LFV processes?

$\mu \to e\gamma$ versus $\mu \to 3e$

Consider $\mu \to e\gamma$:

Some physics
beyond SM
generates blob:



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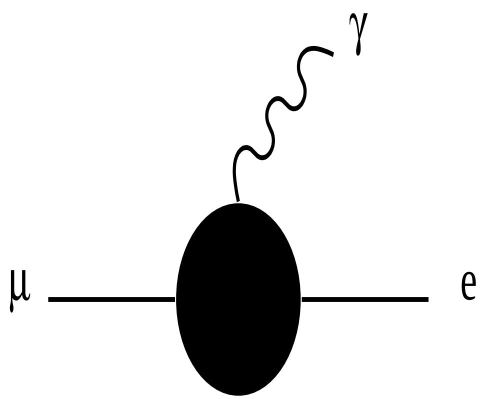
Some physics beyond SM generates blob:

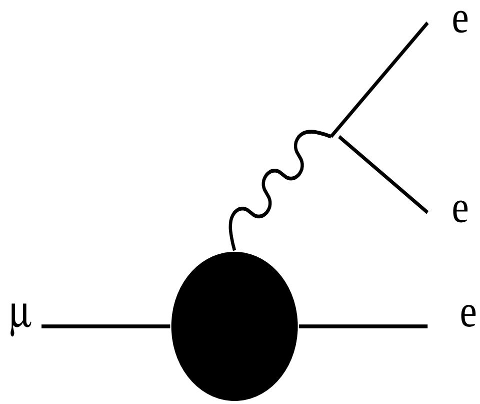


If photon diagram dominates:

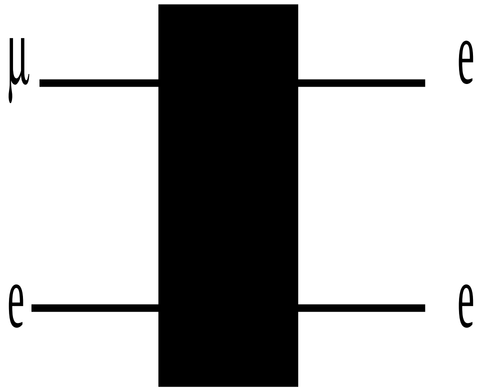
$$Br(l_i \to l_j l_k l_k) \sim$$
$$\alpha \times Br(e \to l_j + \gamma)$$

Compare $\mu \to 3e$:

Same blob appears in $\mu \to 3e$



+

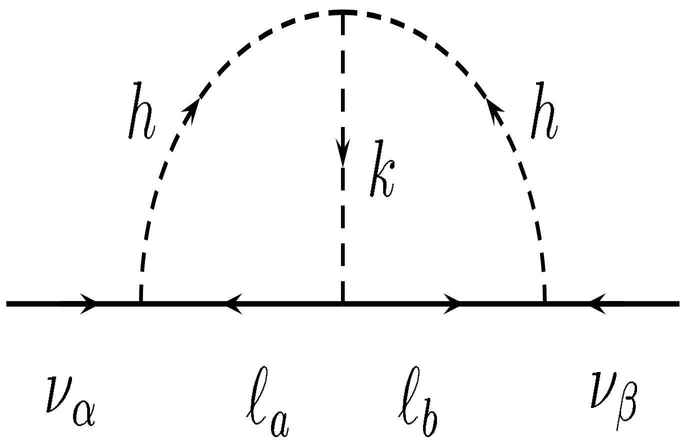


Simple example

Babu-Zee model for neutrino mass:

Cheng & Li, 1980
Zee, 1985
Babu, 1988

$$\mathcal{L} = f(L^T L)h^+ + g(e_R^T e_R)k^{++} - \mu h^+ h^+ k^{--}$$



Neutrino mass is
2-loop suppressed!

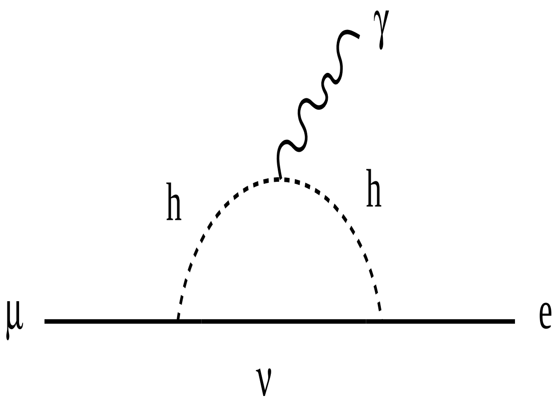
Babu & Macesanu, 2003
Aristizabal & Hirsch, 2006:

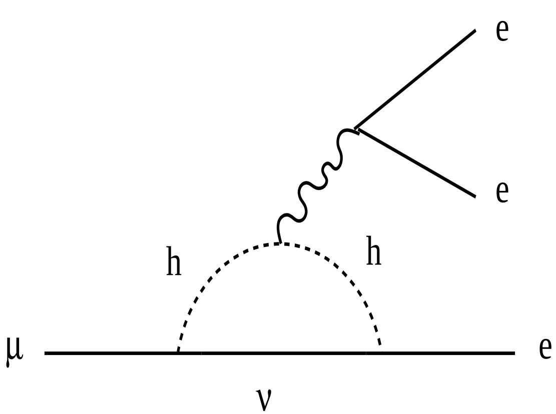
$$\mathcal{M}^{\nu}_{\alpha\beta} = \frac{8\mu}{(16\pi^2)^2 m_h^2} f_{\alpha a} m_a g_{xy} m_b f_{b\beta} \mathcal{I}(\frac{m_k^2}{m_h^2}),$$

Large neutrino mixing angles
require large CLFV

CLFV in Babu-Zee model

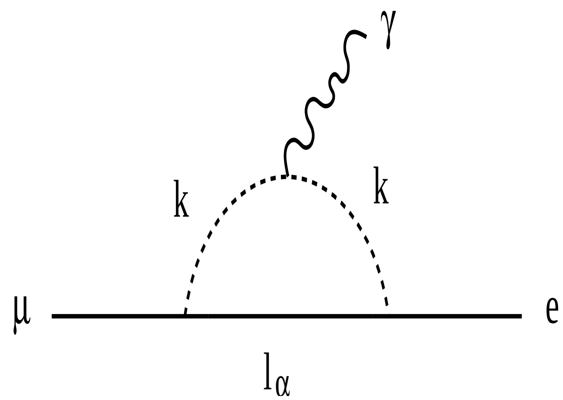
If $\frac{g^2}{m_k^2} \ll \frac{f^2}{m_h^2}$:

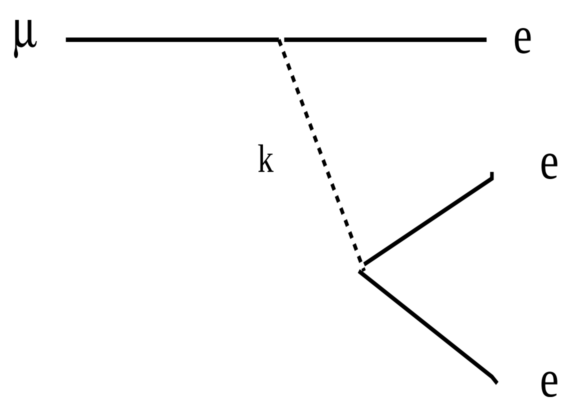




Photon dominance!

if $\frac{f^2}{m_h^2} \ll \frac{g^2}{m_k^2}$:

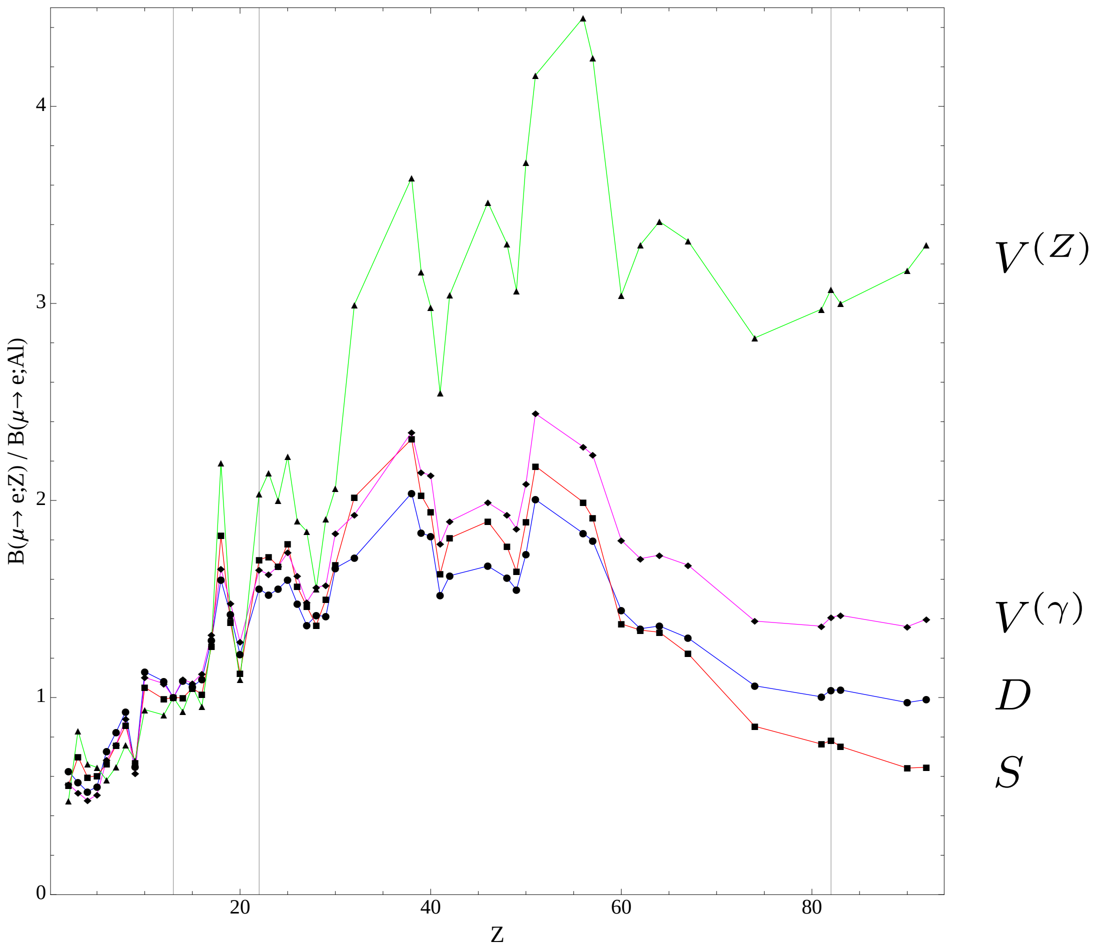




$\mu \to 3e$ tree-level!

Target dependence

Fig. from Cirigliano et al., 2009



Kitano et al., 2002

μ-capture on different nuclei

$\Rightarrow$ use different nuclear targets to distinguish different operators

$\mathcal{III}.$

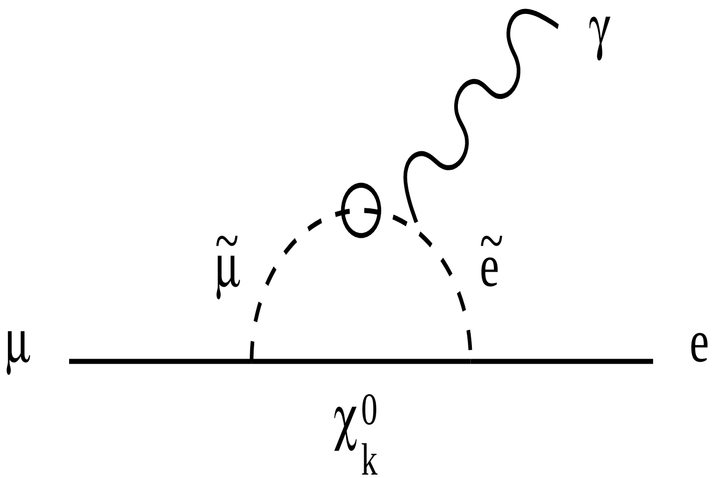
SUSY, neutrino masses and LFV

SUSY flavour problem

Soft SUSY breaking:

$$V = (m^2_{\tilde{L}})_{ij} \tilde{L}^*_i \tilde{L}_j + \cdots$$

Off-diagonal elements induce decays, such as:



Example only!

$\sim (m^2_{\tilde{L}})_{21}$

SUSY flavour problem

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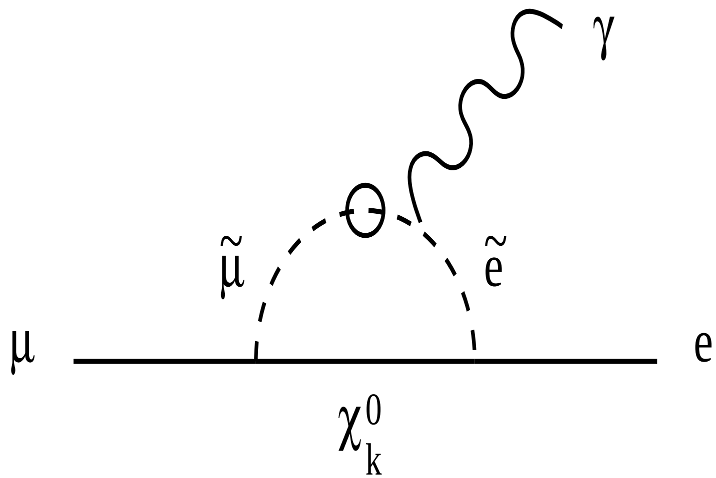


Example only!

$\sim (m^2_{\tilde{L}})_{21}$

$\delta_{12} = \frac{(m^2_{\tilde{L}})_{21}}{m^2_{SUSY}} \lesssim 10^{-4}$

A very old problem indeed!

Ellis and Nanopoulos, 1981
Donoghue et al., 1983
Gerard et al., 1984
Hall et al., 1985
Romao et al., 1985
Borzumati, Masiero, 1986
$\cdots$ many

mSugra

Boundary conditions: mSUGRA ("minimal Supergravity") :

$M_1 = M_2 = M_3 = M_{1/2}$,
$m^2_{H_u} = m^2_{H_d} = m^2_0$,
$M^2_{\tilde{Q}} = M^2_{\tilde{U}} = M^2_{\tilde{D}} = M^2_{\tilde{L}} = M^2_{\tilde{E}} = m^2_0 1_3$, $\Leftarrow$ Flavour blind SUSY breaking!
$A_d = A_0 Y_d$, $A_u = A_0 Y_u$, $A_e = A_0 Y_e$.

$\Rightarrow$ # of parameters: $4\frac{1}{2}$ ($m_0, M_{1/2}, A_0, \tan\beta, sgn(\mu)$)

$\Rightarrow$ Sometimes also called the CMSSM (C = constrained)

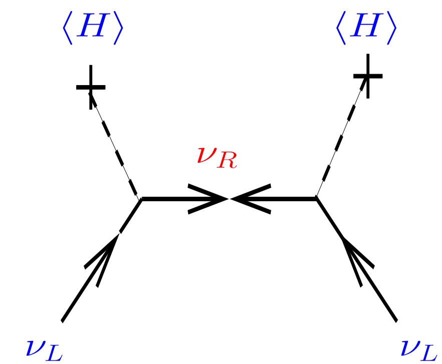
$\Rightarrow$ All low energy masses can then be calculated by RGE ("renormalization group equations")

$\Rightarrow$ No neutrino masses and no LFV

Seesaw mechanism

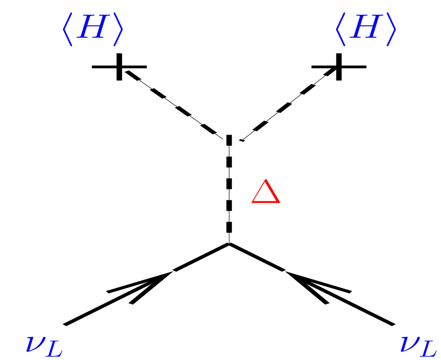
Seesaw type-I, right-handed neutrinos:

$$m_{1/2} \simeq (-\frac{Y_\nu^2 v^2}{M_M}, M_M)$$



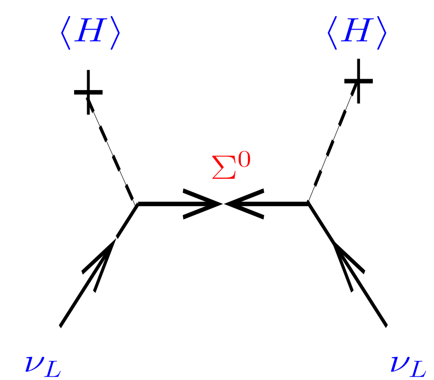
Seesaw type-II, scalar triplet:

$$m_\nu \simeq Y_T \langle \Delta_L^0 \rangle \simeq Y_T \frac{v^2}{m_\Delta}$$



Type-III: Replace ν_R by $\mathbf{\Sigma} = (\Sigma^+, \Sigma^0, \Sigma^-)$:

$$m_{1/2} \simeq (-\frac{Y_\Sigma^2 v^2}{M_\Sigma}, M_\Sigma)$$



mSugra and RGEs

Seesaw type-I: Borzumati & Masiero, 1986

$$(\Delta M^2_{\tilde{L}})_{ij} \sim -\frac{1}{8\pi^2} f(m_0, A_0, M_{1/2}, ...)(Y_\nu^\dagger L Y_\nu)_{ij}$$

Note: $L_i = \log[M_G/M_i]$.

$\Rightarrow$ 9 new independent parameters

Seesaw type-II: Rossi, 2002

$$(\Delta M^2_{\tilde{L}})_{ij} \sim -\frac{1}{8\pi^2} g(m_0, A_0, M_{1/2}, ...)(Y_T^\dagger Y_T)_{ij} \log(M_G/M_T)$$

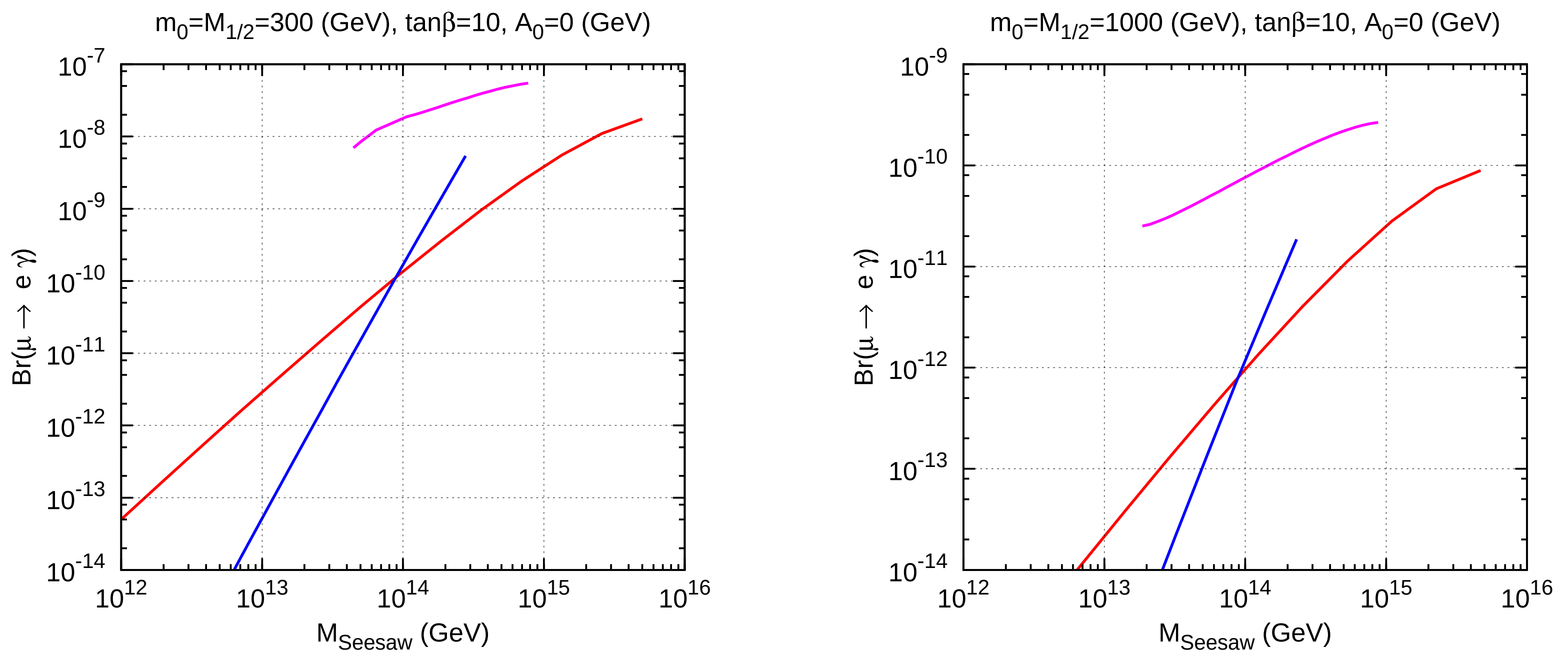
$\Rightarrow$ 9 entries, but proportional to Y_T^2

$\Rightarrow$ Measuring all entries in $(\Delta M^2_{\tilde{L}})_{ij}$ "over-constrains" type-II seesaw!

Note: type-III equation as type-I, but larger LFV ... see below

$\mu \to e\gamma$ in mSugra sesaw

Esteves et al., 2011



$\Rightarrow$ The three different seesaws are: type-III, type-II and type-I

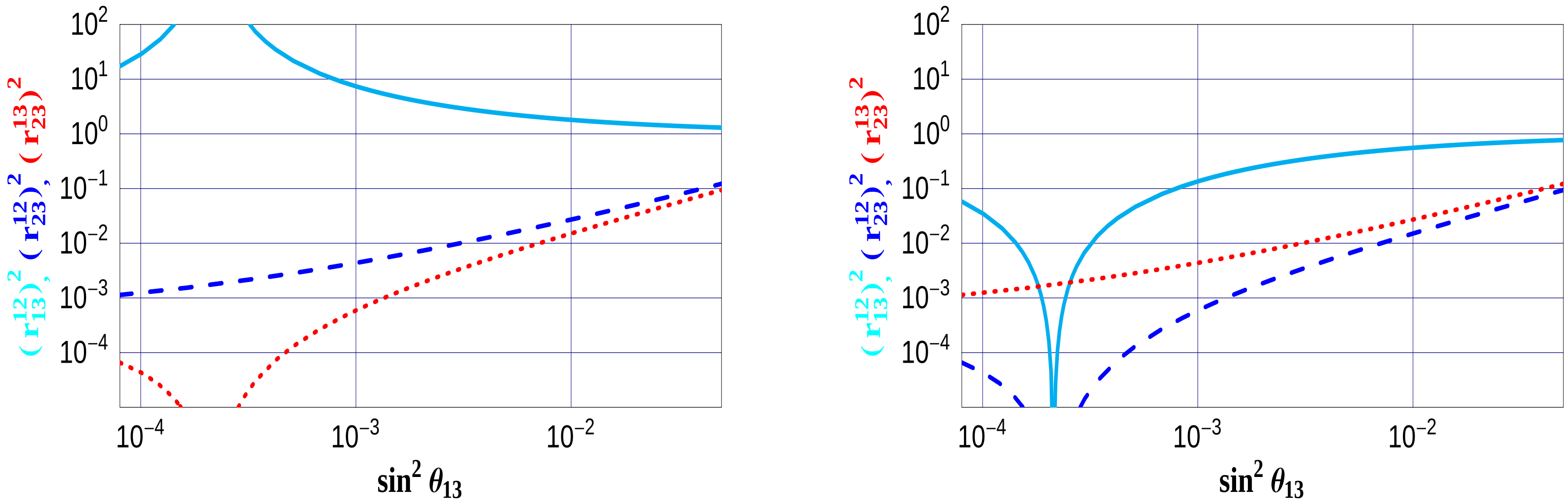
$\Rightarrow$ General expectation: "Large" LFV for "large" $M_{\rm Seesaw}$

$\Rightarrow$ General expectation LFV in type-III $\gg$ type-I

See talk by:
A. Villanova

Only for Type-II

Neutrino angles fix relative size of entries in Y_T: Hirsch et al., 2008

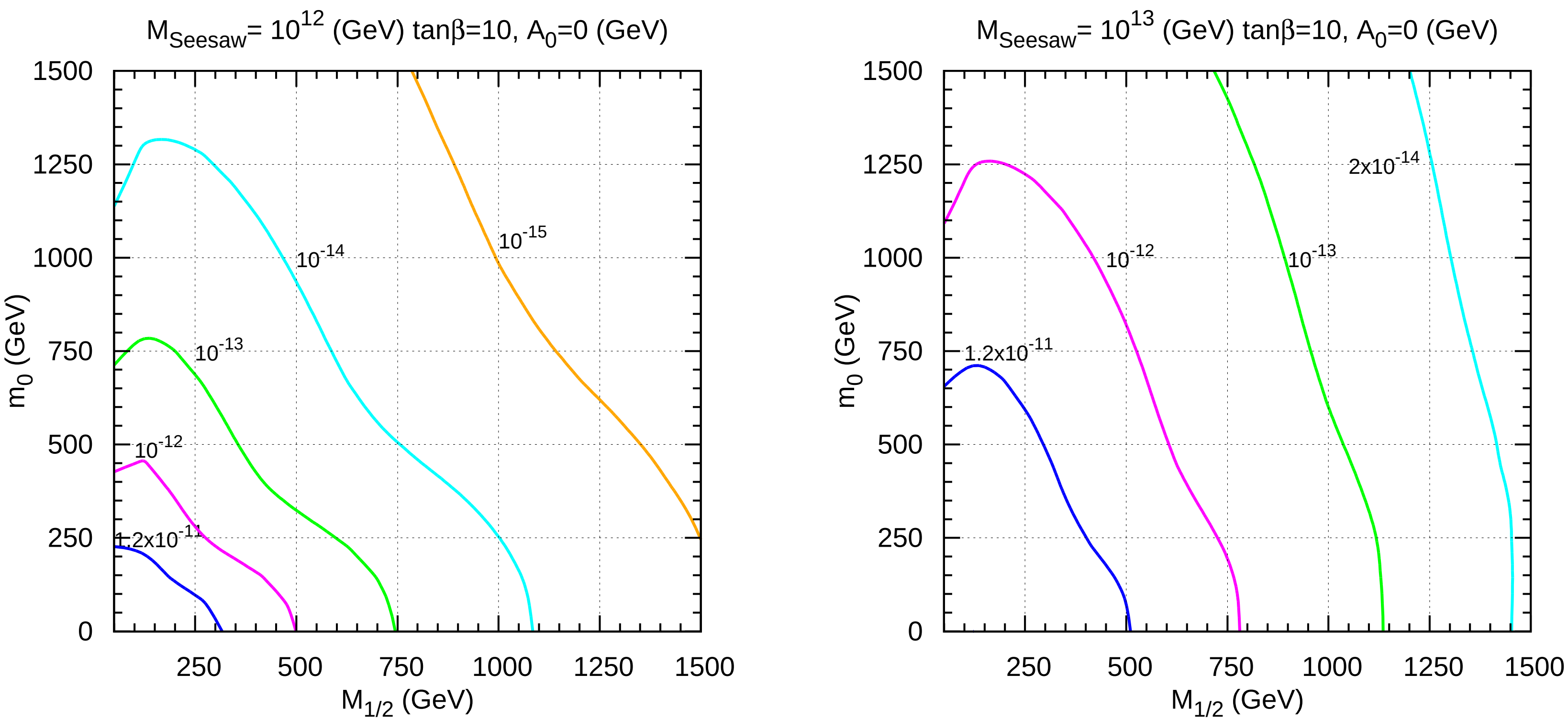


Here: $(r^{13}_{23})^2$ = Br($\tau \to e\gamma$)/Br($\tau \to \mu\gamma$) etc.

Ratios of BR's "predicted" as function of neutrino parameters

LFV in SUSY LR model

Esteves et al., 2010

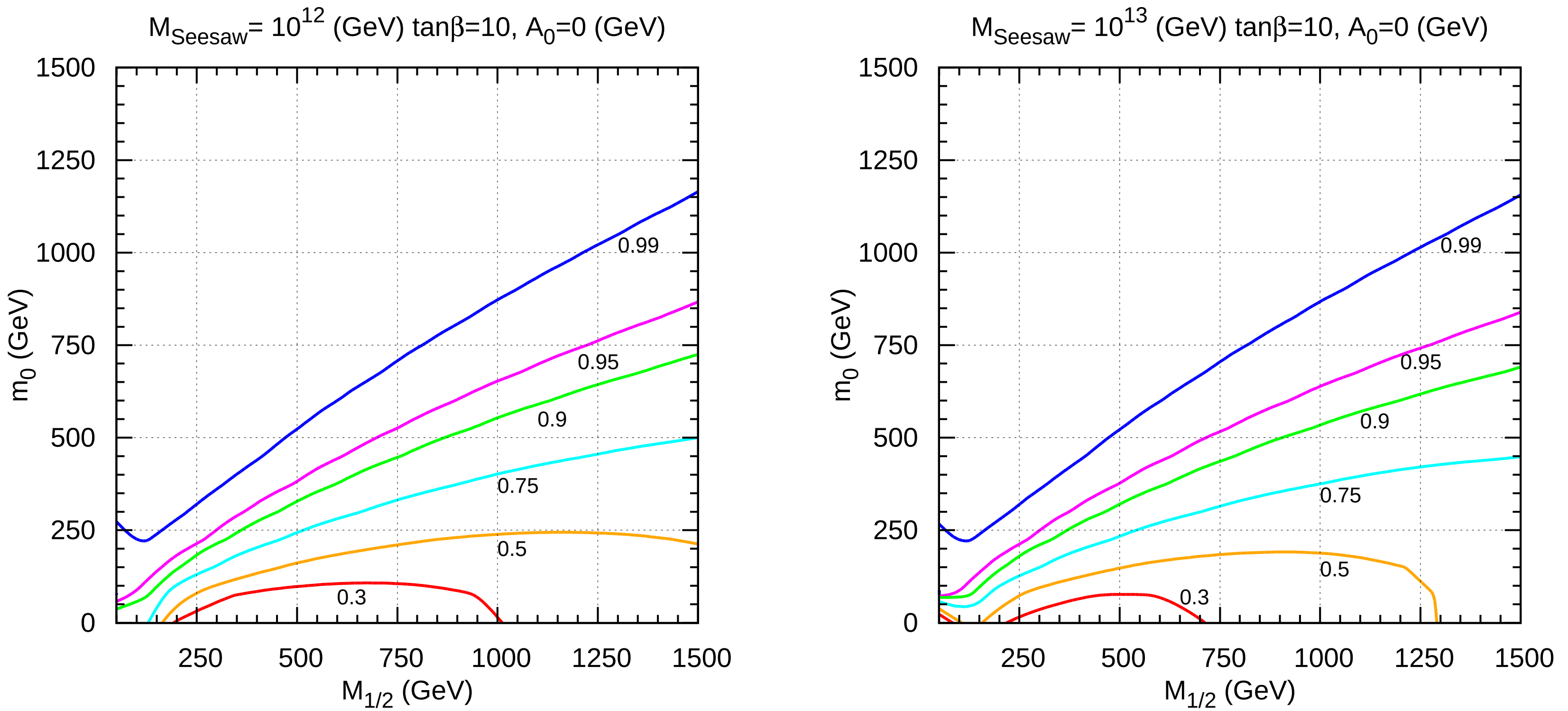


$\Rightarrow$ As in seesaw Br($\mu^+ \to e^+\gamma$) strong function of $M_{\rm Seesaw}$

... but ...

LFV in SUSY LR model

Esteves et al., 2010



Asymmetry:

$$\mathcal{A}(\mu^+ \to e^+\gamma) = \frac{|A_L|^2 - |A_R|^2}{|A_L|^2 + |A_R|^2},$$

$\Rightarrow$ Note: In mSugra seesaw $\mathcal{A} = 1$ always

Linear & inverse seesaw

Inverse seesaw, basis (ν, ν^c, S):

Mohapatra & Valle, 1986

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix},$$

After EWSB the effective light neutrino mass matrix is given by

$$M_\nu = m_D {M^T}^{-1} \mu M^{-1} m_D^T.$$

Linear seesaw:

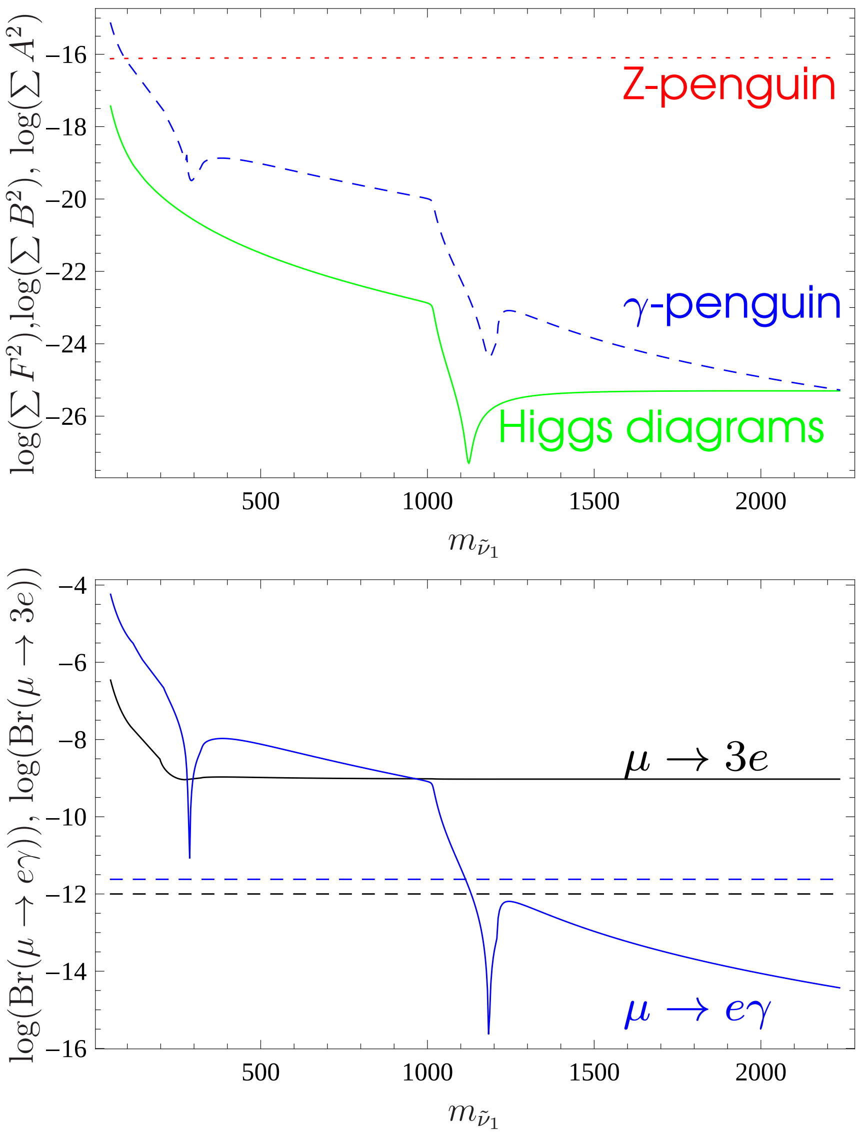
Akhmedov et al., 1995

$$M_\nu = \begin{pmatrix} 0 & m_D & M_L \\ m_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix}.$$

Light neutrino mass:

$$M_\nu = m_D (M_L M^{-1})^T + (M_L M^{-1}) {m_D}^T$$

SUSY Inverse seesaw



Hirsch, Staub
& Vicente, 2012

Z-penguin dominates
when MSSM extended:

(i) particle content ($\hat{\nu}^c$)
(ii) new Yukawa-like interactions
(example also: RPV)

See talk by:
C. Weiland

$\mathcal{IV}.$

Discrete symmetries and LFV

Discrete flavour symmetries

A very partial list:

S_3: Kubo et al., 2003; Chen et al., 2004; Grimus and Lavoura, 2005; Lavoura and Ma, 2005; Teshima, 2006; Koide, 2006; Mohapatra et al., 2006; $\cdots$, $\cdots$

Many Refs in Reviews by:

S_4: Ma, 2006; Hagedorn et al., 2006; Cai & Yu, 2006; Zhang, 2006; Koide, 2007; $\cdots$, $\cdots$

Altarelli & Feruglio, 2010 arXiv:1002.0211

A_4: Ma & Rajasekaran, 2001; Ma, 2002; Babu et al., 2003; Hirsch et al., 2003; Altarelli and Feruglio, 2005; Babu and He, 2005; Koide, 2007; $\cdots$, $\cdots$

Ishimori et al. arXiv:1003.3552

Q_4: Frigerio et al. 2005, $\cdots$

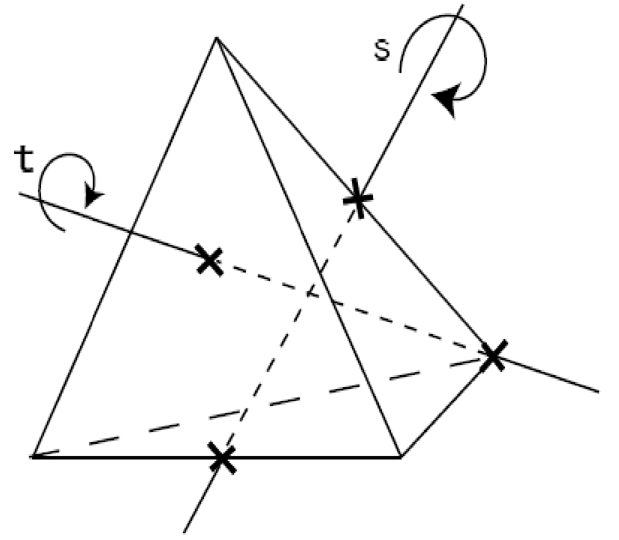
D_4: Grimus & Lavoura, 2003; Grimus et al., 2004; $\cdots$, $\cdots$

$\cdots$

Summary: A_4

- 12 elements: rotations
- 4 irreps: $\mathbf{1}$, $\mathbf{1'}$, $\mathbf{1''}$ and $\mathbf{3}$
- smallest group with $\mathbf{3}$

$\Rightarrow$ Symmetry of the tetrahedron:



Summary: A_4

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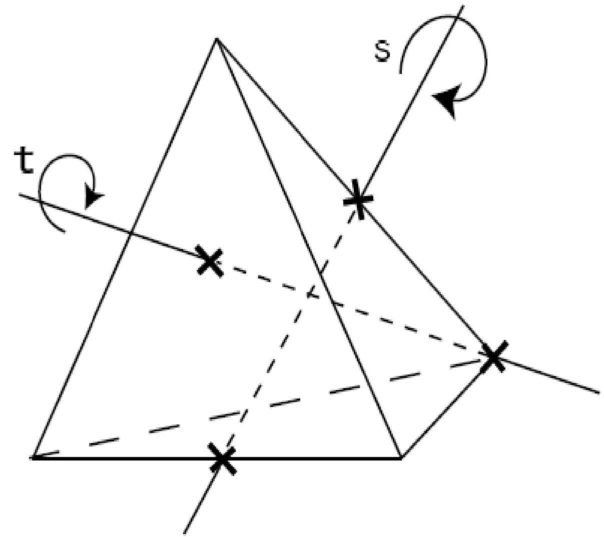


A4 is spontaneously broken in

Z3 in the charged sector

Z2 in the neutrino sector

TBM

Assign:
L_i, l^c_j, H_k $(\nu^c_m, \cdots)$
to different irreps of A_4

$\Rightarrow \cdots$

A_4: models

Type	L_i	ℓ_i^c	ν_i^c	Δ	References
A1				-	[1–14] [15]#
A2	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[16–18]
A3				$\underline{1}, \underline{3}$	[19]
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-	[4, 20–27]# [28–30]* [31–48]
B2				$\underline{1}, \underline{3}$	[49]#
C1				-	[2, 50, 51] [52]#
C2	$\underline{3}$	$\underline{3}$	-	$\underline{1}$	[53, 54] [55]#
C3				$\underline{1}, \underline{3}$	[56]
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[57]
D1				-	[58, 59]# [60, 61]* [62]
D2	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	[63] [64]*
D3				$\underline{1}'$	[65]*
D4				$\underline{1}', \underline{3}$	[66]*
E1	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	[67, 68]
E2				$\underline{1}$	[69]
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$	[70]
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	-	[71]
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	-	-	[72]
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}$	-	[73]*
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}$	-	[74]* [75]
K	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$	-	[76]*
L	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}', \underline{1}''$	-	[77]
M	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$	-	[12, 39, 78, 79]
N	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}$	$\underline{1}$	[80]*
O	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-	[81]
P	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}'', \underline{1}'$	$\underline{3}, \underline{1}$	-	[82, 83]
Q	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}'', \underline{1}'$	$\underline{3}, \underline{1}', \underline{1}''$	-	[84]

Barry & Rodejohann,
PRD81 093002 (2010)

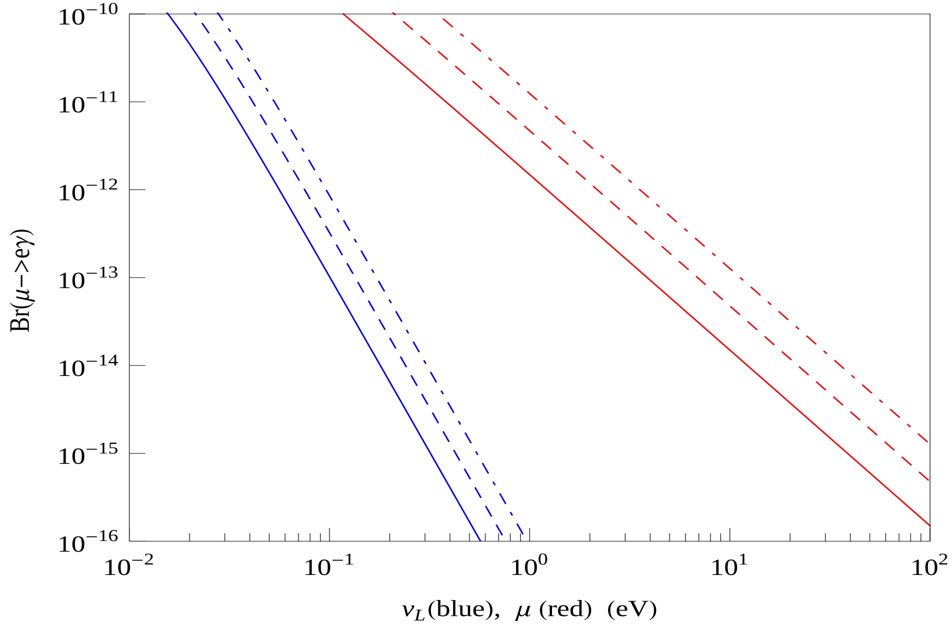
Many - but not all! - can give TBM

Classify:

(a) High-scale models

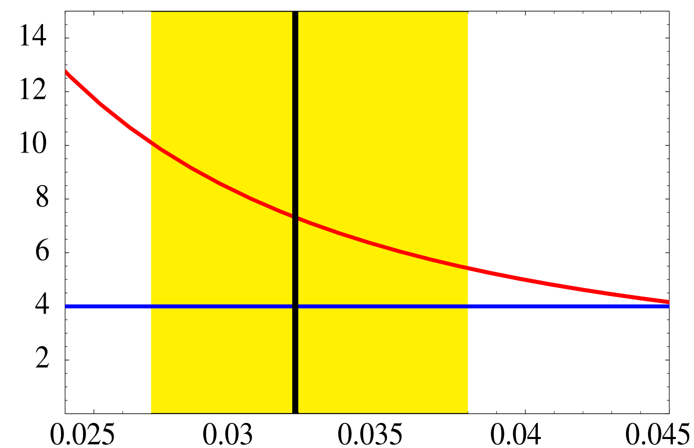
(b) EW-scale models

Linear and inverse SS in A_4

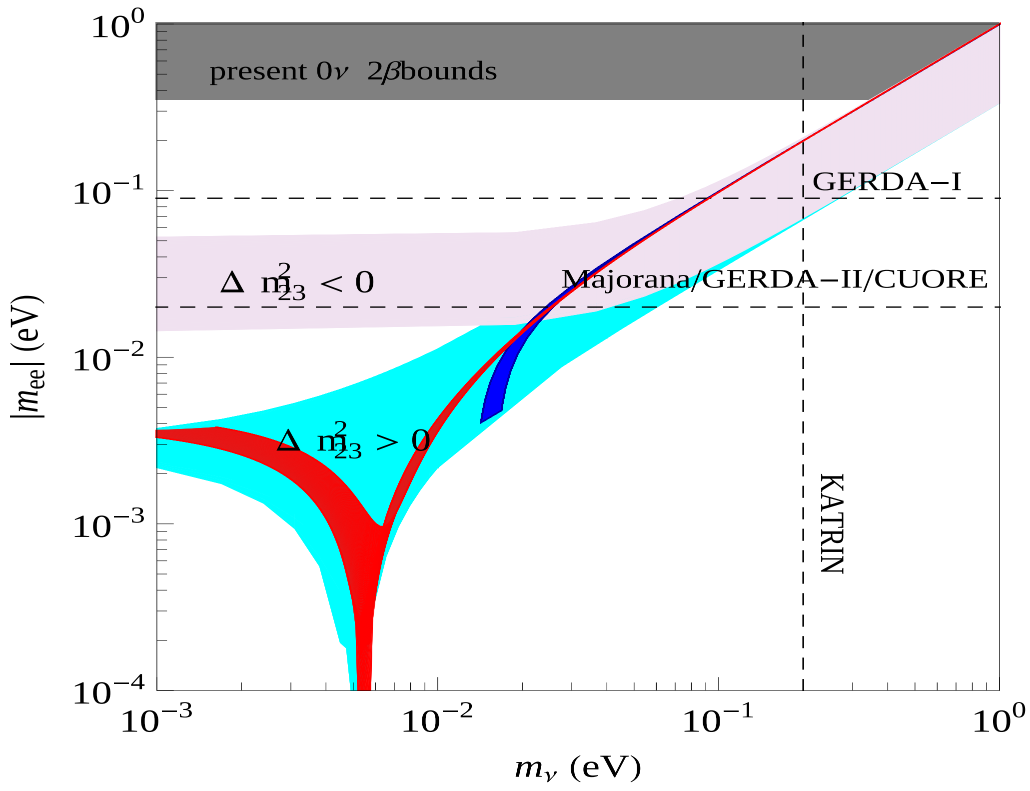


$\mathrm{Br}(\mu \to e\gamma)$ for 3 different values of m_N for inverse and linear seesaw

Ratio:
$\mathrm{Br}(\tau \to \mu\gamma)/\mathrm{Br}(\tau \to e\gamma)$ for inverse and linear seesaw assuming exact TBM mixing as function of $\alpha = \frac{\Delta m^2_\odot}{\Delta m^2_{\mathrm{Atm}}}$



Discrete sym's and $0\nu\beta\beta$



Hirsch, Morisi & Valle, 2009:

Inverse and linear seesaw

both A_4 based TBM

Discrete Dark Matter

A4 is spontaneously broken in

Z3 in the charged sector   Z2 in the neutrino sector

TBM

Discrete Dark Matter

A4 is spontaneously broken in

Z3 in the charged sector   Z2 in the neutrino sector

TBM

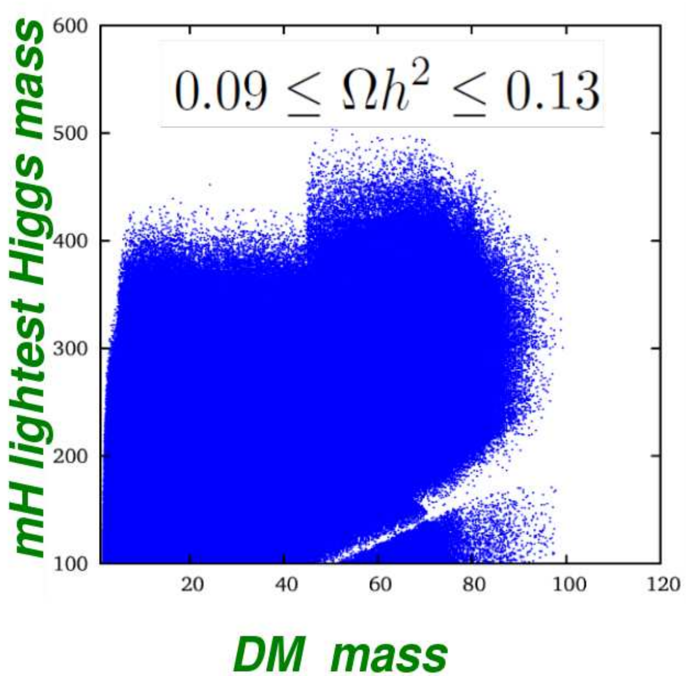
A4 is spontaneously broken in

Z3 in the charged sector X   Z2 in the neutrino sector

stabilize the DM

TBM X

Discrete Dark Matter



Hirsch, Morisi, Peinado & Valle
arXiv:1007.0871

Boucenna et al.
arXiv:1101.2874

Discrete Dark Matter

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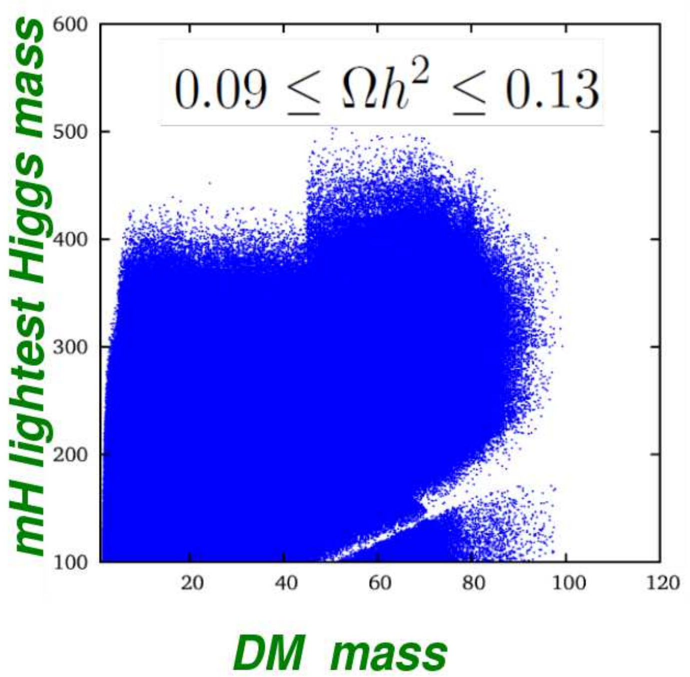
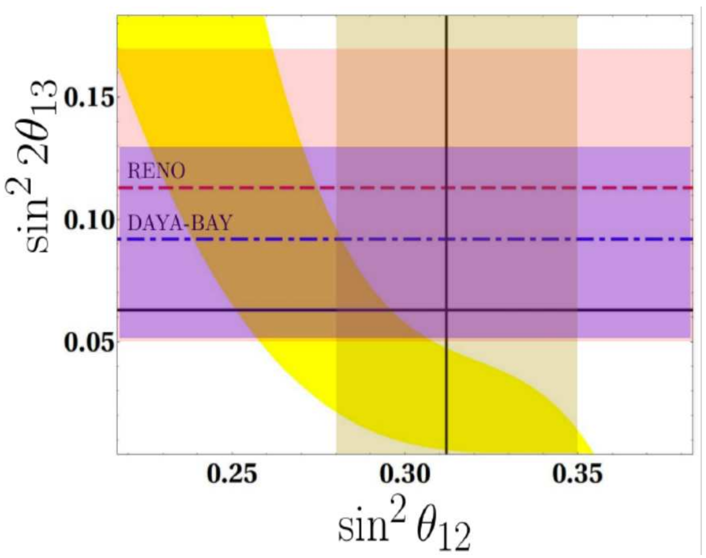


Group: $\Delta(54)$
DM as in original,
but $\theta_{13} <> 0$



Boucenna et al.
arXiv:1204.4733

Summary

⇒ Neutrino oscillations show LF is violated

⇒ last neutrino angle θ_{13} has been measured

⇒ Charged LVF not seen

⇒ CLVF interesting model discriminator

⇒ Discrete symmetries may help, but $\cdots$

⇒ Flavour problem not understood!

⇒ New ideas needed!