

Reconstruction of non-analytic functions

An Application to Heavy Quark Correlators

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16th May 2012

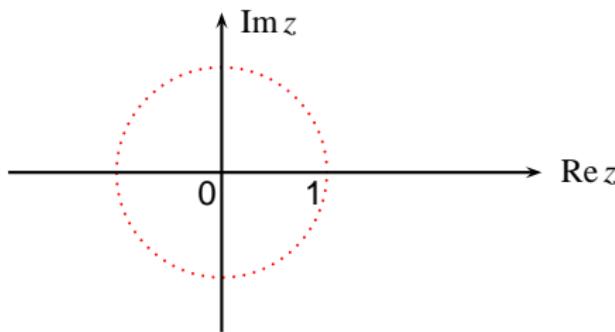
in collaboration with Pere Masjuan and Santiago Peris

Phys. Rev. D **82**, 034030 (2010)
Phys. Rev. D **85**, 054008 (2012)

Rencontres de Physique Particules — Montpellier

Hypothesis

Let consider of a 2 point Green's function, a form factor or more generally a complex function Π :



- Π is analytic on a disk $|z| < 1$: $\Pi(z) \underset{|z| < 1}{=} \sum_{n=0}^{\infty} C(n) z^n$
- Π admits a branching cut $[1, \infty[$ and has the threshold expansion:

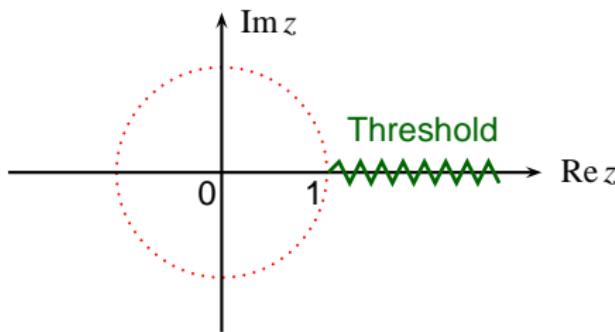
$$\Pi(z) \underset{z \rightarrow 1}{\sim} \sum_{p,k} A^{TH}(p,k) (1-z)^p \log^k(1-z)$$

- Π has the OPE expansion:

$$\Pi(z) \underset{z \rightarrow -\infty}{\sim} \sum_{p,k} B^{OPE}(p,k) \frac{1}{z^p} \log^k(-4z)$$

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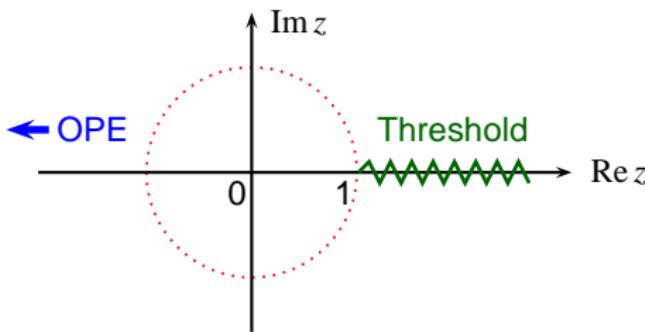
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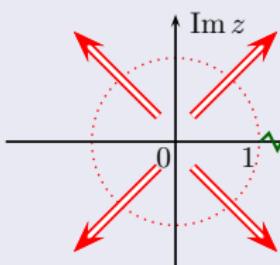
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Problem

How to do analytic continuation ?

From the infinite number of Taylor coefficients: this is mathematics



$$\Pi(z) = \sum_{n=0}^{\infty} C(n) z^n$$

By resummation or by construction order by order, with an infinite number of $C(n)$ the analytic continuation can be easily obtained.

From the finite number of Taylor coefficients

$$\Pi(z) = \sum_{n=0}^{N^*} C(n) z^n$$

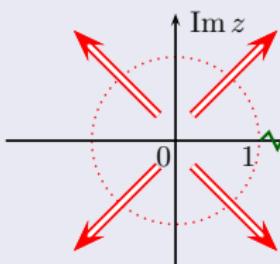
There is no way from a finite number N^* of $C(n)$, one needs the threshold and/or the OPE.

- Padé approximants under certain conditions.
- Mellin-Barnes reconstruction, the method that we propose.

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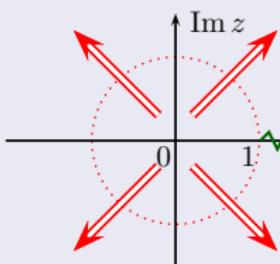
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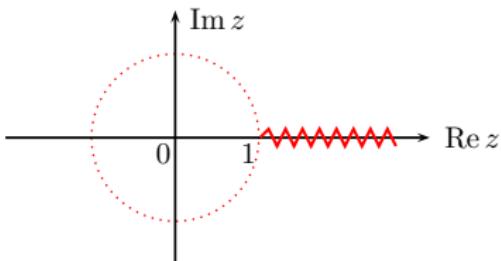
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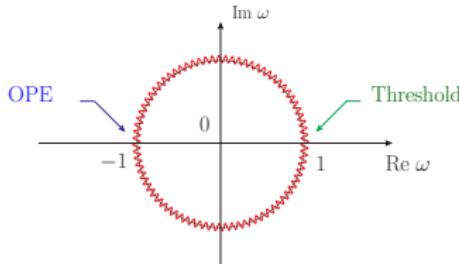
A new method based on Mellin transform

- It is an analytic reconstruction of the function.
- It is systematic and convergent order by order.
- It is a controlled approximation.
- All is based on a mathematical theorem.

Conformal Mapping



$$z = \frac{4\omega}{(1+\omega)^2}$$



$$\omega = \frac{1 - \sqrt{1-z}}{1 + \sqrt{1-z}}$$

$$\Pi(z) \underset{|z| < 1}{=} \sum_{n=0}^{\infty} C(n) z^n$$

$$\hat{\Pi}(\omega) \underset{|\omega| < 1}{=} \sum_{n=0}^{\infty} \Omega(n) \omega^n$$

$$\Pi(z) \underset{z \rightarrow 1}{\sim} \sum_{p,k} A^{TH}(p,k) (1-z)^p \log^k(1-z) \quad \Omega(n) \underset{n \rightarrow \infty}{\sim} \Omega^{AS}(n) = \alpha_{0,0} + \alpha_{0,1} \log n + \dots$$

$$\Pi(z) \underset{z \rightarrow -\infty}{\sim} \sum_{p,k} B^{OPE}(p,k) \frac{1}{z^p} \log^k(-4z)$$

$$\begin{aligned} &+ \frac{1}{n} (\alpha_{1,0} + \alpha_{1,1} \log n + \dots) + \dots \\ &+ (-1)^n [\alpha \rightarrow \beta] \end{aligned}$$

The radius of convergence is related to the behaviour of the Ω for large value of n . This relation is best expressed in terms of the Mellin transform.

$$\Omega(n) \underset{n \rightarrow \infty}{\sim} \Omega^{AS}(n) = \alpha_{0,0} + \alpha_{0,1} \log n + \cdots + \frac{1}{n} (\alpha_{1,0} + \alpha_{1,1} \log n + \cdots) + \cdots$$

$$+ (-1)^n [\alpha \rightarrow \beta]$$

Main result provided by the Converse Mapping Theorem

$\alpha_{k,\ell}$ is only a linear function of $A^{TH}(p, k)$ threshold coefficients

$\beta_{k,\ell}$ is only a linear function of $B^{OPE}(p, k)$ OPE coefficients

The approximation

$$\widehat{\Pi}(\omega) = \underbrace{\sum_{n=0}^{N^*} \Omega(n) \omega^n}_{\text{EXACT}} + \underbrace{\sum_{n=N^*+1}^{\infty} \Omega^{AS}(n) \omega^n}_{\text{APPROXIMATION}} + \underbrace{\sum_{n=N^*+1}^{\infty} [\Omega(n) - \Omega^{AS}(n)] \omega^n}_{\text{ERROR}}$$

- N^* is the number of known Taylor coefficients.
- In practice $\Omega^{AS}(n)$ are really efficient.

$$\Omega(n) \underset{n \rightarrow \infty}{\sim} \Omega^{AS}(n) = \alpha_{0,0} + \alpha_{0,1} \log n + \cdots + \frac{1}{n} (\alpha_{1,0} + \alpha_{1,1} \log n + \cdots) + \cdots$$

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- N^* is the

$$= \alpha_{0,0} \frac{\omega}{1-\omega} + \alpha_{0,1} \text{Li}^{(1)}(0, \omega) + \cdots + \left[\begin{array}{l} \alpha \rightarrow \beta \\ \omega \rightarrow -\omega \end{array} \right]$$

- In practice

A perfect application example: Heavy-Quark Correlators

The correlators ($s = \bar{\psi}\psi$, $p = i\bar{\psi}\gamma_5\psi$, $v_\mu = \bar{\psi}\gamma_\mu\psi$ et $a_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$)

$$\begin{aligned} q^2 \Pi^{s,p}(q^2) &\doteq i \int d^4x e^{iqx} \langle 0 | T \begin{bmatrix} s(x) & s(0) \\ p(x) & p(0) \end{bmatrix} | 0 \rangle \\ (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi^{v,a}(q^2) + q_\mu q_\nu \Pi_L^{v,a}(q^2) &\doteq i \int d^4x e^{iqx} \langle 0 | T \begin{bmatrix} v_\mu(x) & v_\nu(0) \\ a_\mu(x) & a_\nu(0) \end{bmatrix} | 0 \rangle \end{aligned}$$

may be decomposed as

$$\Pi(q^2) = \Pi^{(0)}(q^2) + \left(\frac{\alpha_s}{\pi}\right) \Pi^{(1)}(q^2) + \left(\frac{\alpha_s}{\pi}\right)^2 \Pi^{(2)}(q^2) + \left(\frac{\alpha_s}{\pi}\right)^3 \Pi^{(3)}(q^2) + \mathcal{O}(\alpha_s^4)$$

Π obeys an once-subtracted dispersion relation

$$\Pi(q^2) = q^2 \int_0^\infty \frac{d\xi}{\xi} \frac{1}{\xi - q^2 - i\varepsilon} \frac{1}{\pi} \text{Im } \Pi(\xi + i\varepsilon) .$$

$\text{Im } \Pi(\xi \leq 4m^2) = 0$, where m is the heavy quark pole mass.

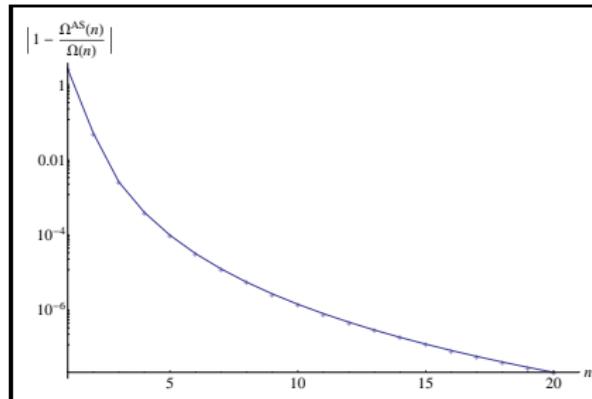
$\Pi^{\text{v}(0)}$: a warming-up example

The function $\Pi^{\text{v}(0)}$ is known explicitly, as

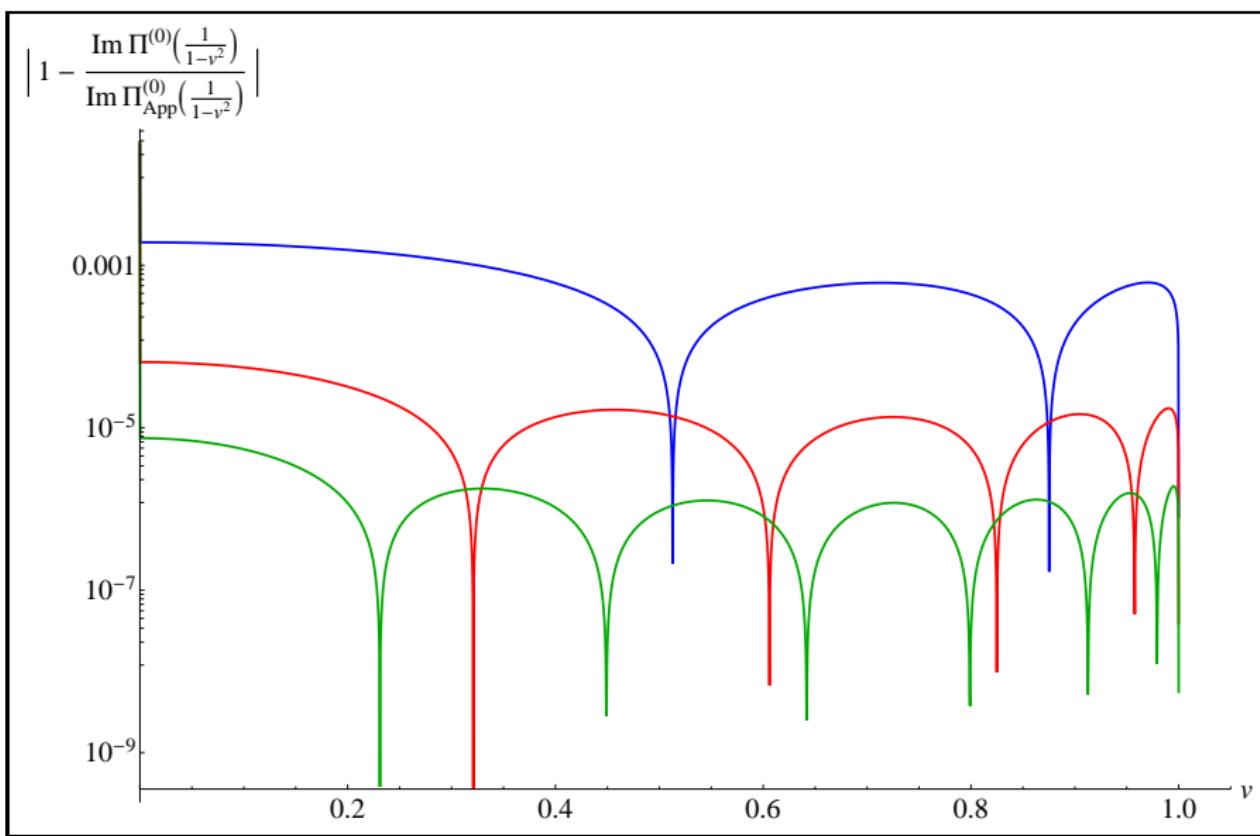
$$\Pi^{\text{v}(0)}(z) = \frac{3}{16\pi^2} \left[\frac{20}{9} + \frac{4}{3z} - \frac{4(1-z)(1+2z)}{3z} \frac{2 \frac{\sqrt{1-1/z}-1}{\sqrt{1-1/z+1}} \log \left(\frac{\sqrt{1-1/z}-1}{\sqrt{1-1/z+1}} \right)}{\left(\frac{\sqrt{1-1/z}-1}{\sqrt{1-1/z+1}} \right)^2 - 1} \right]$$

It is the easy to obtain some terms at $z = 0$, $z = 1$ and $z = -\infty$, and then generate

$$\Omega^{\text{AS}}(n) = (-1)^n \left[-\frac{1}{2\pi^2} \frac{1}{n} + \frac{9}{32\pi^2} \frac{1}{n^5} + \dots \right]$$



$\Omega^{\text{AS}}(n)$ is efficient



$\Pi^{\nu(2)}$

Taylor Expansion

$$\Pi^{\nu(2)}(z) \underset{|z| < 1}{=} \sum_{n=1}^{30} C(n) z^n \quad \text{and } N^* = 30 \text{ and } z \doteq \frac{q^2}{4m^2}$$

Threshold expansion

$$\begin{aligned} \Pi^{\nu(2)}(z) \underset{z \rightarrow 1}{\sim} & \frac{A(-\frac{1}{2}, 0)}{\sqrt{1-z}} + \left\{ A(0, 2) \log^2(1-z) + A(0, 1) \log(1-z) + K^{(2)} \right\} \\ & + \left\{ A(\frac{1}{2}, 1) \log(1-z) + A(\frac{1}{2}, 0) \right\} \sqrt{1-z} + \dots \end{aligned}$$

$K^{(2)}$ is unknown.

OPE expansion

$$\begin{aligned}\Pi^{\text{v}(2)}(z) \underset{z \rightarrow -\infty}{\sim} & \left\{ B(0, 2) \log^2(-4z) + B(0, 1) \log(-4z) + B(0, 0) \right\} \\ & + \left\{ B(-1, 2) \log^2(-4z) + B(-1, 1) \log(-4z) + B(-1, 0) \right\} \frac{1}{z} \\ & + \left\{ B(-2, 3) \log^3(-4z) + B(-2, 2) \log^2(-4z) \right. \\ & \quad \left. + B(-2, 1) \log(-4z) + B(-2, 0) \right\} \frac{1}{z^2} + \dots\end{aligned}$$

From the **Threshold** and the **OPE** expansions, one can obtain easily

$$\begin{aligned}\Omega^{AS}(n) = & \alpha_{0,0} + \left\{ \alpha_{1,0} + \alpha_{1,1} \log n \right\} \frac{1}{n} + \alpha_{2,0} \frac{1}{n^2} + \mathcal{O} \left(\frac{1}{n^3} \log^{\ell_1} n \right) \\ & + (-1)^n \left[\left\{ \beta_{1,0} + \beta_{1,1} \log n \right\} \frac{1}{n} + \left\{ \beta_{3,0} + \beta_{3,1} \log n \right\} \frac{1}{n^3} \right. \\ & \quad \left. + \left\{ \beta_{5,0} + \beta_{5,1} \log n + \beta_{5,2} \log^2 n \right\} \frac{1}{n^5} + \mathcal{O} \left(\frac{1}{n^7} \log^{\ell_2} n \right) \right]\end{aligned}$$

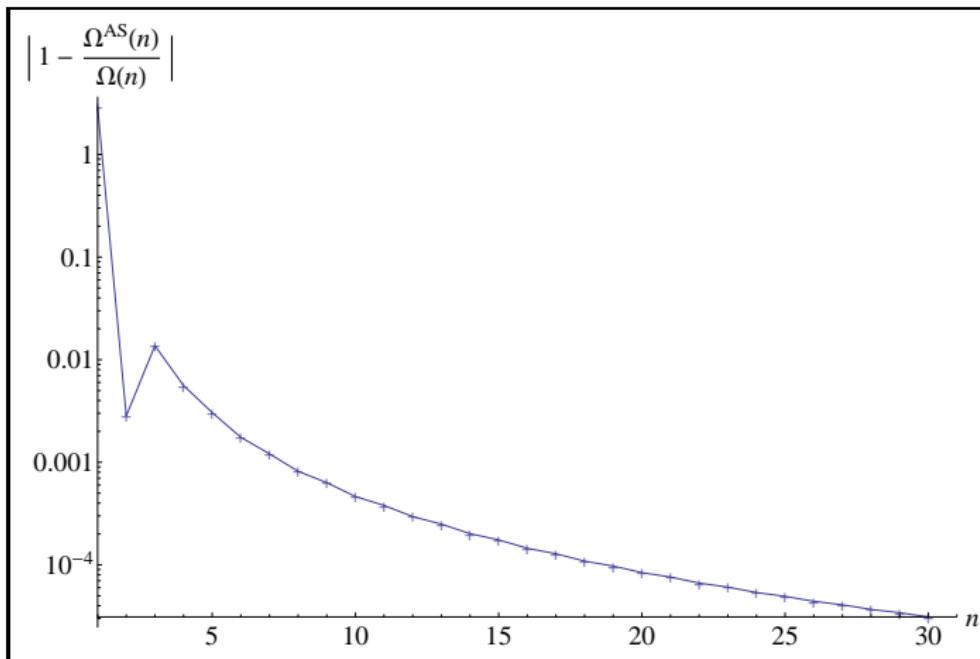
where the α 's and the β 's are known analytically by identification

$$\left\{ \begin{array}{l} \alpha_{0,0} = 2 A(-\frac{1}{2}, 0) \simeq 3.44514 \\ \alpha_{1,0} \simeq -0.492936 \\ \alpha_{1,1} = 2.25 \\ \alpha_{2,0} \simeq 3.05433 \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \beta_{1,0} \simeq 0.33723 \\ \beta_{1,1} \simeq 0.211083 \\ \beta_{3,0} \simeq 0.183422 \\ \beta_{3,1} \simeq -0.620598 \\ \beta_{5,0} \simeq -1.89016 \\ \beta_{5,2} \simeq 1.38684 \end{array} \right.$$

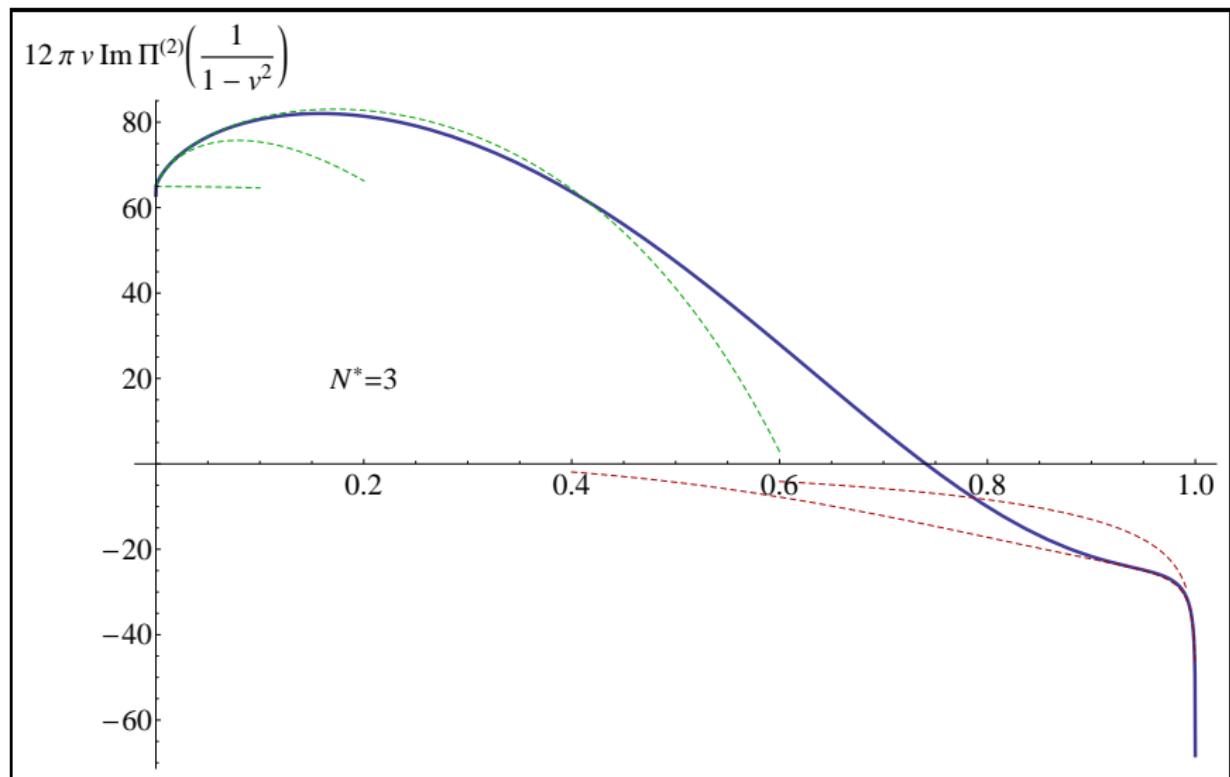
and we estimate the error,

$$\left[\Omega(n) - \Omega^{AS}(n) \right]_{n > N^*(=30)} \cong \left\{ \begin{array}{c} +1 \\ -0 \end{array} \right\} \frac{\log^{1.5} n}{n^3} \pm (-1)^n \mathcal{O} \left(\frac{\log^{\ell_2} n}{n^7} \right)$$

In the case of Π^2 we know $N^* = 30$ terms in the Taylor expansion.



Remembering that $\Pi^{v(2)}(z) = \widehat{\Pi}^{v(2)}\left(\frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right)$, one can reconstruct the imaginary part, (into velocity $\sqrt{1-1/z}$)



From this analytic expression of $\Pi^{V(2)}$, one can extract the value of the constant $K^{(2)}$

$$\begin{aligned}
 K^{(2)} = & -\frac{\alpha_{0,0}}{2} + \left(\frac{\pi^2}{12} + \frac{\gamma_E^2}{2} + \gamma_1 \right) \alpha_{1,1} + \frac{\pi^2}{6} \alpha_{2,0} - \zeta'(2) \alpha_{2,1} - \beta_{1,0} \log 2 \\
 & + \left(-\frac{\log^2 2}{2} + \gamma_E \log 2 \right) \beta_{1,1} - \frac{3\zeta(3)}{4} \beta_{3,0} + \left(\frac{\zeta(3) \log 2}{4} + \frac{3\zeta'(3)}{4} \right) \beta_{3,1} \\
 & - \frac{15}{16} \zeta(5) \beta_{5,0} + \left(\frac{\zeta(5) \log 2}{16} + \frac{15\zeta'(5)}{16} \right) \beta_{5,1} \\
 & + \left(\frac{\zeta(5) \log^2 2}{16} - \frac{\zeta'(5) \log 2}{8} - \frac{15\zeta''(5)}{16} \right) \beta_{5,2} + \sum_{n=1}^{N^*} \left[\Omega(n) - \Omega^{AS}(n) \right] + \mathcal{E}(N^*, 1)
 \end{aligned}$$

$$K^{(2)} = 3.783 {}^{+0.004}_{-0.000}$$

A. H. Hoang et al., Nucl. Phys. B 813, 349 (2009).

3.81 ± 0.02

P. Masjuan and S. Peris, Phys. Lett. B 686, 307 (2010)

3.71 ± 0.03

$\Pi^{\text{v}(3)}$

One can apply the same method to reconstruct $\Pi^{(3)}$, but this time $N^* = 3$,

$$\begin{aligned} \Omega^{\text{AS}}(n) = & \alpha_{-1,0} n + \left\{ \alpha_{0,0} + \alpha_{0,1} \ln n \right\} + \left\{ \alpha_{1,0} + \alpha_{1,1} \ln n + \alpha_{1,2} \ln^2 n \right\} \frac{1}{n} + \mathcal{O}\left(\frac{1}{n^2} \log^{\ell_1} n\right) \\ & + (-1)^n \left[\left\{ \beta_{1,0} + \beta_{1,1} \ln n + \beta_{1,2} \ln^2 n \right\} \frac{1}{n} + \left\{ \beta_{3,0} + \beta_{3,2} \ln^2 n \right\} \frac{1}{n^3} \right. \\ & \left. + \left\{ \beta_{5,0} + \beta_{5,1} \ln n + \beta_{5,2} \ln^2 n + \beta_{5,3} \ln^3 n \right\} \frac{1}{n^5} + \mathcal{O}\left(\frac{1}{n^7} \log^{\ell_2} n\right) \right], \end{aligned}$$

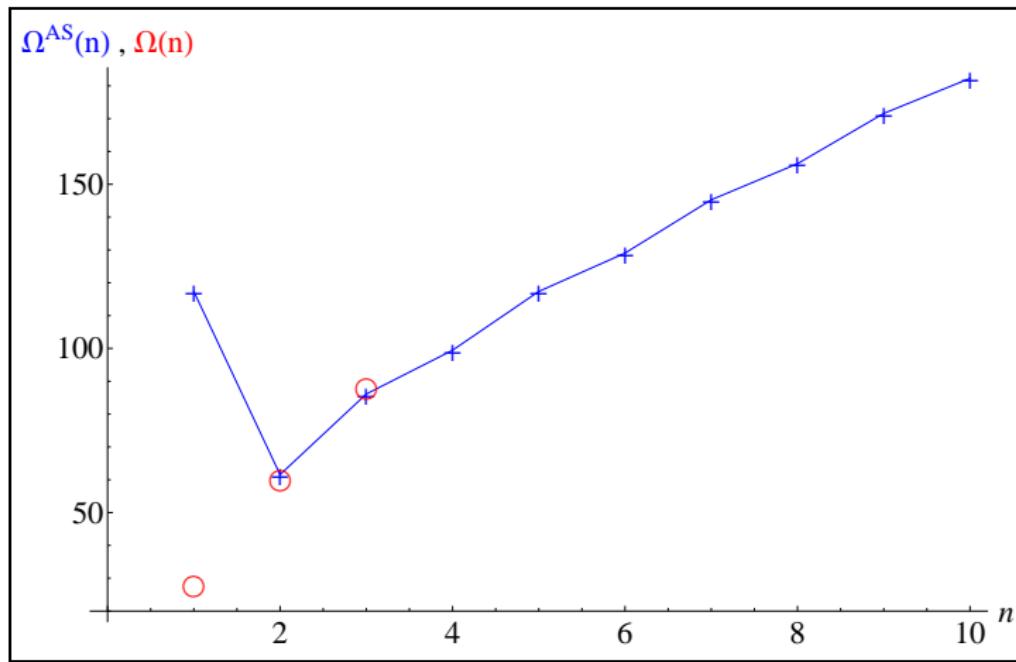
with

$$\left\{ \begin{array}{l} \alpha_{-1,0} \simeq 10.5456 \\ \alpha_{0,1} \simeq 31.0063 \\ \alpha_{0,0} \simeq -11.0769 \\ \alpha_{1,0} \simeq 36.3318 \\ \alpha_{1,1} \simeq 37.1514 \\ \alpha_{1,2} \simeq 10.125 \end{array} \right. , \quad \left\{ \begin{array}{l} \beta_{1,0} \simeq -0.181866 \\ \beta_{1,1} \simeq -2.4852 \\ \beta_{1,2} \simeq -0.879515 \\ \beta_{3,0} \simeq -10.4385 \\ \beta_{3,2} \simeq 3.82702 \end{array} \right. , \quad \left\{ \begin{array}{l} \beta_{5,0} \simeq -70.9277 \\ \beta_{5,1} \simeq 56.3093 \\ \beta_{5,2} \simeq 20.9951 \\ \beta_{5,3} \simeq -7.55063 \end{array} \right. .$$

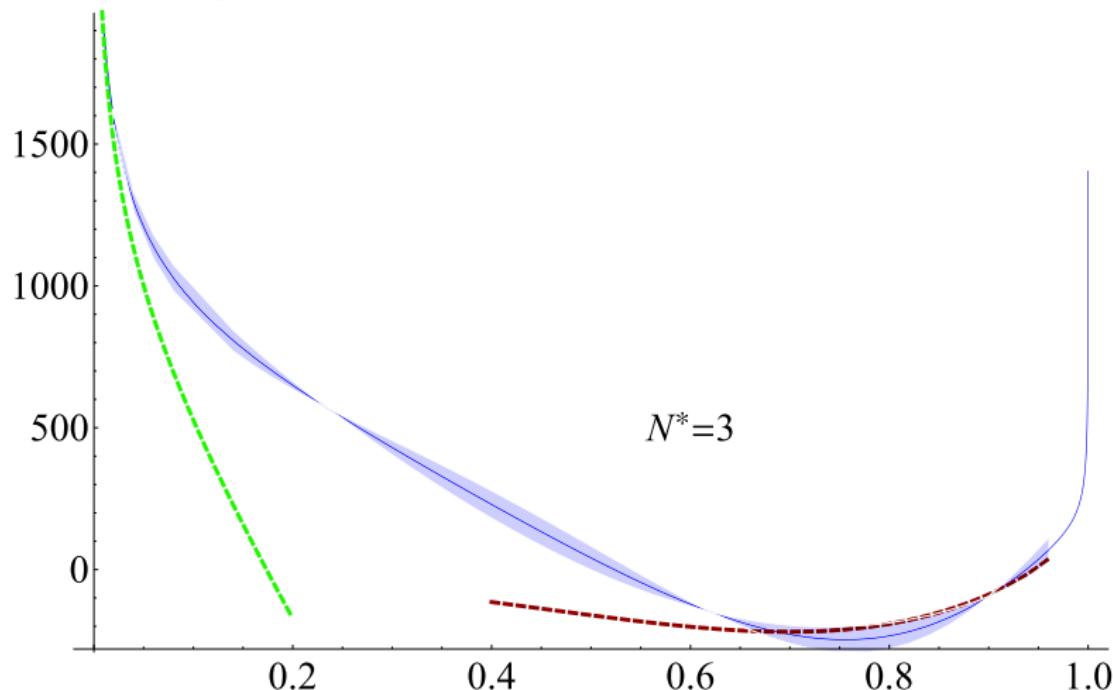
and we estimate the error,

$$\left[\Omega(n) - \Omega^{\text{AS}}(n) \right]_{n > N^*(=30)} \cong \pm 15 \frac{\log^3 n}{n^2} \pm (-1)^n \mathcal{O}\left(\frac{\log^{\ell_2} n}{n^7}\right)$$

In the case of Π^3 we know $N^* = 3$ terms in the Taylor expansion.



$$12 \pi \nu \operatorname{Im} \Pi^{(3)} \left(\frac{1}{1 - \nu^2} \right)$$

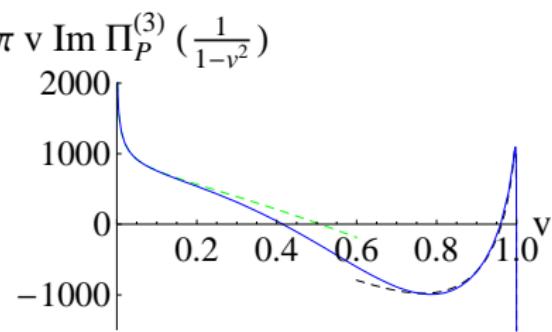
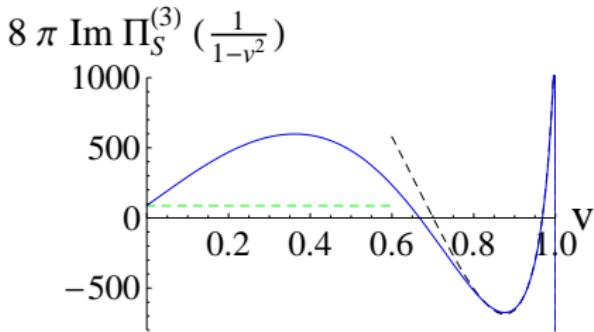
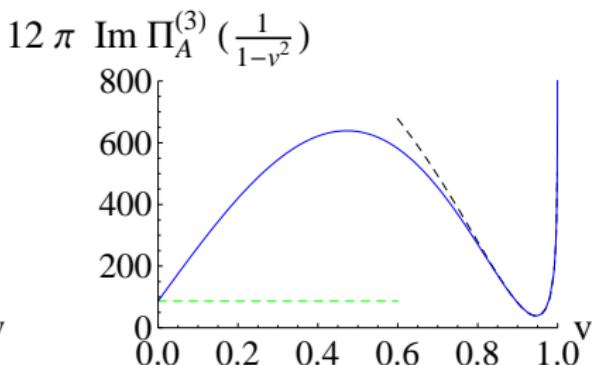
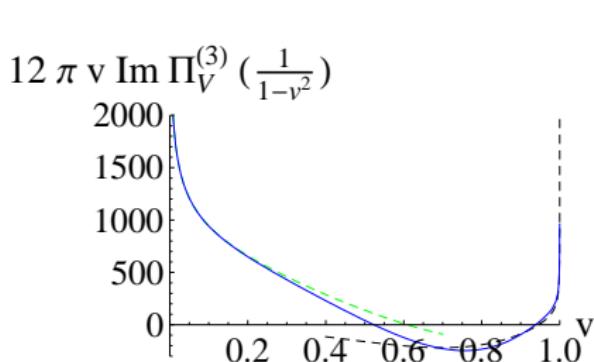


All channels

D.G., P. Masjuan and S. Peris, hep-ph 1104.3425, Phys. Rev. D 85, 054008 (2012)

It is easy to apply the method to all the channels, and obtaining the following α 's and β 's for each:

Ω_{AS}^X	v		a		p		s	
	$n_l = 3$	$n_l = 4$						
$\alpha_{-1,0}^X$	10.5456	10.5456	0	0	10.5456	10.5456	0	0
$\alpha_{0,0}^X$	-11.0769	-12.6382	0	0	-6.4835	-8.0448	0	0
$\alpha_{0,1}^X$	31.0063	28.7095	0	0	31.0063	28.7095	0	0
$\alpha_{1,0}^X$	36.3318	33.0585	1.4622	1.4622	40.6575	36.7189	2.1932	2.1932
$\alpha_{1,1}^X$	37.1514	33.8404	0	0	51.8488	48.3155	0	0
$\alpha_{1,2}^X$	10.1250	8.6805	0	0	10.1250	8.6805	0	0
$\beta_{1,0}^X$	-0.1819	-0.0555	-0.1819	-0.0555	9.9493	6.9861	9.9493	6.9861
$\beta_{1,1}^X$	-2.4852	-2.1312	-2.4852	-2.1312	-43.1187	-39.6735	-43.1186	-39.6735
$\beta_{1,2}^X$	-0.8795	-0.7444	-0.8795	-0.7444	1.6381	1.6688	1.6381	1.6688
$\beta_{1,3}^X$	0	0	0	0	5.1027	4.6298	5.1027	4.6298
$\beta_{3,0}^X$	-10.4385	-9.7282	26.2458	22.9826	3.1298	1.1687	93.7790	83.0590
$\beta_{3,1}^X$	-4.7750	-4.2501	-19.8617	-18.4878	-53.4944	-50.0465	-137.2835	-129.2810
$\beta_{3,2}^X$	3.8270	3.4724	-6.8349	-6.1103	0.8960	1.1335	-24.9630	-22.4605
$\beta_{3,3}^X$	0	0	2.5513	2.3149	5.0337	4.6960	15.1011	14.0879
$\beta_{5,0}^X$	-70.9277	-63.8573	100.2171	89.1103	-115.8498	-108.3750	440.1394	399.7520
$\beta_{5,1}^X$	56.3093	53.6862	-72.4918	-68.9185	62.1988	60.2675	-512.9781	-487.4843
$\beta_{5,2}^X$	20.9951	19.0619	-29.3263	-26.2676	38.4395	35.8466	-129.9058	-118.3556
$\beta_{5,3}^X$	-7.5506	-7.0439	10.1019	9.3589	-9.9903	-9.4668	60.1732	56.5763



Taylor Coefficients

Y. Kyio et al., Nucl. Phys. B 823, 269 (2009).

n	$(16\pi^2/3)C^{\text{pred}}(n)$ for $N^* = 3$	Error	$C(n)$ from Kyio et al.	Error
1	366.174	0	366.175	0
2	381.510	0	381.509	0
3	385.235	0	385.233	0
4	382.7	0.5	383.073	0.011
5	378.0	1.2	378.688	0.032
6	372.5	1.8	373.536	0.061
7	367.0	2.3	368.23	0.09
8	361.5	2.7	363.03	0.13
9	356.4	3.1	358.06	0.17
10	351.6	3.4	353.35	0.2

Taylor coefficients are crucial to determine the quark masses through the spectral moments

$$m_Q^{2n} \equiv \frac{1}{C(n)} \times \frac{1}{2\pi} \int \frac{ds}{s^{n+1}} \frac{1}{\pi} \text{Im } \Pi(s)$$

Taylor Coefficients

Y. Kyio et al., Nucl. Phys. B 823, 269 (2009).

n	$(16\pi^2/3)C^{\text{pred}}(n)$ for $N^* = 3$	Error	$C(n)$ from Kyio et al.	Error
1	366.174	0	366.175	0
2	381.510	0	381.509	0
3	385.235	0	385.233	0
4	382.7	0.5	383.073	0.011
5	378.0	1.2	378.688	0.032
6	372.5	1.8	373.536	0.061
7	367.0	2.3	368.23	0.09
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A remark on Padé inspired reconstruction

The alternative technique used to perform such reconstructions are "inspired" by Padé approximations, so far the results guessed with this method seem okay but

- Heavily numerical and not completely systematic (it exists some arbitrariness).
- How can control the convergence ?
- How understand the errors ?

An interesting example

Chetyrkin and Steinhauser Eur. Phys. J. C 21 (2001) 319

Czarnecki and Melnikov, PRD66 (2002) 011502(R)

In the heavy - light correlator, one has the coefficient

$$12\pi \text{Im } \Pi^{(2)}(z) = \dots + (c_1 + c_2 + c_3) \log(1 - 1/z) + \dots$$

where c_1, c_2 and c_3 are coming from different topologies.

		c_1	c_2	c_3
Chetyrkin and Steinhauser	Padé	21(6)	-2.3(7)	1.2(4)
Czarnecki and Melnikov	Exact	21.46	-2.585	4.894

About 9 sigma errors...

Conclusion

- We proved that it is possible to reconstruct in a systematic way a full function with a located cut from its Taylor expansion around 0, its threshold and OPE expansions.
- We show that it is possible to control the systematic error.
- Regarding the application to the heavy-quarks correlators, one can imagine now how to control in a systematic way the evaluation of the quark mass and α_s from Lattice calculations.
- This method is general enough to be applied to other situations (with similar analytic structure):
 - *Symbols* \mathcal{S} and integrals involved in $\mathcal{N} = 4$ SYM theories.
 - Applicable since one has partial information and need for local constraints.