Unlocking the Standard Model

the 1-generation case

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May 16 th, 2012

Preliminary

- Pseudoscalar $J = 0$ mesons have nice ordering / symmetry pattern
  scalar mesons = anarchy
- Higgs boson likely to be composite
- Standard Higgs boson = scalar ; can it be $\simeq J = 0$ scalar meson ?
- Could it be that [Higgs mystery] $\in$ [scalar maze] ?
- If Higgs $\propto \bar{u}u + \bar{d}d$, what are the 3 Goldstones in the Higgs multiplet? scalars? pseudoscalars? combinations of the two?
- If Goldstones $= \vec{\pi} \Rightarrow m_W = gf_\pi$ ☹
- Technicolor intricate and unaesthetic ☹
- So what to do ? Start by something unknown (to me) but likely to be simple enough :

find the sets of bilinear fermion operators stable by $SU(2)_L$ ☺

pause

Results

1

- The Standard Model with 1 generation of quarks is naturally endowed with 2-Higgs (complex) doublets H_1 and H_2 as soon as the scalar fields are considered (to transform) as $\bar{q}_i q_j$ / $\bar{q}_i \gamma_5 q_j$ bilinears
- I construct H_1 and H_2, which are parity transformed of each other

2

- Quark condensation $< \bar{u}u > \neq 0, < \bar{d}d > \neq 0$ is the *catalyst* of both chiral symmetry breaking and electroweak symmetry breaking
- There are 2 Higgs bosons h and ξ, with VEV's $v = f_\pi$ and $\sigma \approx \frac{2\sqrt{2}m_W}{g}$
- The pions keep being the (massive pseudo-)Goldstone bosons of the broken chiral $SU(2)_L \times SU(2)_R$ symmetry down to its diagonal $SU(2)_V$ subgroup
- The couplings of h, ξ to $\vec{W}$, quarks, pions, leptons are determined
- $m_h^2 = \lambda_{H_1} f_\pi^2 - m_\pi^2 < (2m_\pi)^2$; 4-pions coupling $\left|\frac{\lambda_{H_1}}{4}\right| \leq .162 \times 4\pi \approx 2$
- $h \stackrel{?}{=}$ candidate for dark matter
- $\xi \simeq$ "standard", $m_\xi = ?$ ☹

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pause

GSW. The $SU(2)_L$ group and its generators

- $SU(2)_L$ transformation

$$\mathcal{U}_L = e^{-i\alpha_i T_L^i}$$

- Lie Algebra

rep1

$$T_L^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, T_L^+ = T_L^1 + iT_L^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, T_L^- = T_L^1 - iT_L^2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\boxed{[T_L^i, T_L^j] = +i\epsilon_{ijk} T_L^k, \quad i,j = 1,2,3}$$

commut1

- $u_L, d_L \in$ *doublet* in the fundamental representation of $SU(2)_L$

$$\psi = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad T_L^i.\psi = T_L^i \psi$$

commut2 pause

GSW. The Higgs doublet H (and its alter ego $\tilde{H}$)

- Let $\chi^0, \chi^1, \chi^2, k^3$ be real and $\chi^3 = ik^3$

H1H2 pause

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^1 + i\chi^2 \\ \chi^0 - \chi^3 \end{pmatrix}, \quad \tilde{H} = \frac{i}{2} T^2 H^* = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^0 + \chi^3 \\ -(\chi^1 - i\chi^2) \end{pmatrix}$$

$$< \chi^0 >= v \Rightarrow < H >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad < \tilde{H} >= \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$$

- Defining $T^i_L.\chi^\alpha$ and $(T^i_L T^j_L).\chi^\alpha$ by

$$T^i_L.H = \frac{1}{\sqrt{2}} \begin{pmatrix} T^i_L.\chi^1 + iT^i_L.\chi^2 \\ T^i_L.\chi^0 - T^i_L.\chi^3 \end{pmatrix}, \quad (T^i_L T^j_L).H = \frac{1}{\sqrt{2}} \begin{pmatrix} (T^i_L T^j_L).\chi^1 + i(T^i_L T^j_L).\chi^2 \\ (T^i_L T^j_L).\chi^0 - (T^i_L T^j_L).\chi^3 \end{pmatrix}$$

the laws of transformations of the χ's satisfy commut commut2 rep1 rep2

$$[T^i_L, T^j_L].\chi^\alpha = +i\epsilon_{ijk} T^k_L.\chi^\alpha$$

$$\Leftrightarrow \quad \boxed{T^i_L.(T^j_L.\chi^\alpha) - T^j_L.(T^i_L.\chi^\alpha) = -i\epsilon_{ijk} T^k_L.\chi^\alpha = -[T^i_L, T^j_L].\chi^\alpha}$$

GSW. Transforming H by $SU(2)_L$

They write

$$T^1_L \cdot \chi^0 = +\tfrac{i}{2}\,\chi^2, \quad T^2_L \cdot \chi^0 = +\tfrac{i}{2}\,\chi^1, \quad T^3_L \cdot \chi^0 = +\tfrac{1}{2}\,\chi^3,$$
$$T^1_L \cdot \chi^1 = -\tfrac{1}{2}\,\chi^3, \quad T^2_L \cdot \chi^1 = -\tfrac{i}{2}\,\chi^0, \quad T^3_L \cdot \chi^1 = +\tfrac{i}{2}\,h^2,$$
$$T^1_L \cdot \chi^2 = -\tfrac{i}{2}\,\chi^0, \quad T^2_L \cdot \chi^2 = +\tfrac{1}{2}\,\chi^3, \quad T^3_L \cdot \chi^2 = -\tfrac{i}{2}\,\chi^1,$$
$$T^1_L \cdot \chi^3 = -\tfrac{1}{2}\,\chi^1, \quad T^2_L \cdot \chi^3 = +\tfrac{1}{2}\,\chi^2, \quad T^3_L \cdot \chi^3 = +\tfrac{1}{2}\,\chi^0.$$

Calling

$$\chi^0 = -\zeta^3, \quad \chi^1 = \zeta^1, \quad \chi^2 = -\zeta^2, \quad \chi^3 = \zeta^0$$

they rewrite

$$\begin{aligned} T^i_L.\zeta^j &= -\tfrac{1}{2}\left(i\epsilon_{ijk}\,\zeta^k + \delta_{ij}\,\zeta^0\right) \\ T^i_L.\zeta^0 &= -\tfrac{1}{2}\,\zeta^i \end{aligned}$$

$\equiv$ laws of transformations of composite multiplets (see transf2) pause

$\bar{q}_i(\gamma_5)q_j$. Action of the chiral group $U(2)_L \times U(2)_R$ (I)

1. Left- and right- transformations :

$$\mathcal{V}_L = e^{-i\beta_j \mathbb{T}^j_L}, \quad \mathcal{V}_R = e^{-i\kappa_j \mathbb{T}^j_R}$$

with generators $\mathbb{T}^j_L = \{\mathbb{I}_L, \vec{T}^j_L\}$ (the same for $\mathbb{T}^j_R$)

2. Any quark bilinear can be represented as $\bar{\psi}\mathbb{M}\psi$ or $\bar{\psi}\,\mathbb{M}\gamma_5\,\psi$, or $\bar{\psi}\,\frac{1+\gamma_5}{2}\mathbb{M}\,\psi$ or $\bar{\psi}\,\frac{1-\gamma_5}{2}\mathbb{M}\,\psi$, where $\mathbb{M}$ is a 2×2 matrix

3. The action of the chiral group on such bilinears is

$$\begin{aligned}
(\mathcal{V}_L \times \mathcal{V}_R)\,.\,\bar{\psi}\frac{1+\gamma_5}{2}\mathbb{M}\psi &= \bar{\psi}\,\mathcal{V}_L^{-1}\,\mathbb{M}\,\mathcal{V}_R\,\frac{1+\gamma_5}{2}\psi\\
(\mathcal{V}_L \times \mathcal{V}_R)\,.\,\bar{\psi}\frac{1-\gamma_5}{2}\mathbb{M}\psi &= \bar{\psi}\,\mathcal{V}_R^{-1}\,\mathbb{M}\,\mathcal{V}_L\,\frac{1-\gamma_5}{2}\psi
\end{aligned}$$

pause

$\bar{q}_i(\gamma_5)q_j$. Action of the chiral group $U(2)_L \times U(2)_R$ (I)

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pause

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$$(\mathcal{V}_L \times \mathcal{V}_R)\,.\,\bar\psi \frac{1+\gamma_5}{2} \mathbb{M}\psi = \bar\psi\, \mathcal{V}_L^{-1}\, \mathbb{M}\, \mathcal{V}_R\, \frac{1+\gamma_5}{2} \psi$$

$$(\mathcal{V}_L \times \mathcal{V}_R)\,.\,\bar\psi \frac{1-\gamma_5}{2} \mathbb{M}\psi = \bar\psi\, \mathcal{V}_R^{-1}\, \mathbb{M}\, \mathcal{V}_L\, \frac{1-\gamma_5}{2} \psi$$

pause

$\bar{q}_i(\gamma_5)q_j$. Action of the chiral group $U(2)_L \times U(2)_R$ (I)

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$$(\mathcal{V}_L \times \mathcal{V}_R)\,.\,\bar\psi\frac{1+\gamma_5}{2}\mathbb{M}\psi \;=\; \bar\psi\,\mathcal{V}_L^{-1}\,\mathbb{M}\,\mathcal{V}_R\,\frac{1+\gamma_5}{2}\psi$$

$$(\mathcal{V}_L \times \mathcal{V}_R)\,.\,\bar\psi\frac{1-\gamma_5}{2}\mathbb{M}\psi \;=\; \bar\psi\,\mathcal{V}_R^{-1}\,\mathbb{M}\,\mathcal{V}_L\,\frac{1-\gamma_5}{2}\psi$$

pause

$\bar{q}_i(\gamma_5)q_j$. Action of the chiral group $U(2)_L \times U(2)_R$ (II)

- Expanding the exponentials and using $\mathbb{T}^i_L.\psi = \mathbb{T}^i_L\psi$: ▸ SU(2) ▸ pause

$$\begin{aligned}
\mathbb{T}^j_L \,.\, \bar\psi \frac{1+\gamma_5}{2}\mathbb{M}\psi &= -\,\bar\psi\,\mathbb{T}^j\mathbb{M}\,\frac{1+\gamma_5}{2}\psi\\
\mathbb{T}^j_L \,.\, \bar\psi \frac{1-\gamma_5}{2}\mathbb{M}\psi &= +\,\bar\psi\,\mathbb{M}\mathbb{T}^j\,\frac{1-\gamma_5}{2}\psi\\
\mathbb{T}^j_R \,.\, \bar\psi \frac{1+\gamma_5}{2}\mathbb{M}\psi &= +\,\bar\psi\,\mathbb{M}\mathbb{T}^j\,\frac{1+\gamma_5}{2}\psi\\
\mathbb{T}^j_R \,.\, \bar\psi \frac{1-\gamma_5}{2}\mathbb{M}\psi &= -\,\bar\psi\,\mathbb{T}^j\mathbb{M}\,\frac{1-\gamma_5}{2}\psi
\end{aligned}$$

- These relations satisfy in particular ▸ commut1

$$[T^i_{L,R}, T^j_{L,R}].\,\bar\psi\frac{1\pm\gamma_5}{2}\mathbb{M}\psi = +\epsilon_{ijk}\,T^k_{L,R}.\bar\psi\frac{1\pm\gamma_5}{2}\mathbb{M}\psi$$

and

$$\boxed{\left(T^i_{L,R}.(T^j_{L,R} - T^j_{L,R}.(T^i_{L,R}\right).\,\bar\psi\frac{1\pm\gamma_5}{2}\mathbb{M}\psi) = -i\epsilon_{ijk}\,T^k_{L,R}\,.\,\bar\psi\frac{1\pm\gamma_5}{2}\mathbb{M}\psi}$$

$\bar{q}_i(\gamma_5)q_j$. Action of the chiral group $U(2)_L \times U(2)_R$ (III)

- $[L, R] = 0$:

$$\Big(T^i_{L,R}.(T^j_{R,L} - T^j_{R,L}.(T^i_{L,R}\Big) . \bar\psi\frac{1\pm\gamma_5}{2}\mathbb{M}\psi) = 0 = [T^i_{L,R}, T^j_{R,L}] . \bar\psi\frac{1\pm\gamma_5}{2}\mathbb{M}\psi$$

- The diagonal subgroup acts as expected by commutation

$\bar{q}_i(\gamma_5)q_j$. Action of the chiral group $U(2)_L \times U(2)_R$ (IV). $U(1)_L \times U(1)_R \to U(1)_V$ and parity

$$\mathbb{I}_R\,.\,\bar\psi\frac{1\pm\gamma_5}{2}\mathbb{M}\psi = \pm\bar\psi\frac{1\pm\gamma_5}{2}\mathbb{M}\psi, \quad \mathbb{I}_L\,.\,\bar\psi\frac{1\mp\gamma_5}{2}\mathbb{M}\psi = \pm\bar\psi\frac{1\mp\gamma_5}{2}\mathbb{M}\psi$$

$$\Leftrightarrow \boxed{\quad \mathbb{I}_L\,.\,(S,P) = -(P,S)\ , \quad \mathbb{I}_R\,.\,(S,P) = +(P,S)}$$

$$\Leftrightarrow \qquad (\mathbb{I}_R - \mathbb{I}_L)\,.\,(S,P) \equiv \gamma_5\mathbb{I}\,.\,(S,P) = (P,S)$$

$$(\mathbb{I}_R + \mathbb{I}_L)\,.\,(S,P) \equiv \mathbb{I}\,.\,(S,P) = 0$$

$U(1)_L \times U(1)_R \xrightarrow{breaking} U(1) \equiv$ breaking of (*scalar* $\leftrightarrow$ *pseudo*) symmetry

Which Goldstone boson ?

▸ Goldst ▸ pause

The two composite quadruplets stable by $SU(2)_{L,R}$ (I)

One has

pause

$$T^i_L \,.\, \bar\psi \mathbb{I} \psi \;=\; -\frac{1}{2}\,\bar\psi\,\gamma_5\,2T^i\,\psi$$

$$T^i_L \,.\, \bar\psi\,\gamma_5\,2T^j\,\psi \;=\; -\frac{1}{2}\left(i\epsilon_{ijk}\;\bar\psi\,\gamma_5\,2T^k\,\psi + \delta_{ij}\;\bar\psi \mathbb{I}\psi\right)$$

and

$$T^i_L \,.\, \bar\psi\,\gamma_5 \mathbb{I}\,\psi \;=\; -\frac{1}{2}\,\bar\psi\,2T^i\psi$$

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The two composite quadruplets stable by $SU(2)_{L,R}$ (II). Transformation laws

$\Rightarrow$ the two quadruplets (parity-transformed of each other) ▸ pause

$$\begin{aligned} \Phi &= (\phi^0, \vec{\phi}) = \bar{\psi}\left(\mathbb{I}, 2\gamma_5\,\vec{T}\right)\psi \\ \Xi &= (\xi^0, \vec{\xi}) = \bar{\psi}\left(\gamma_5\mathbb{I}, 2\,\vec{T}\right)\psi \end{aligned}$$

are stable by $SU(2)_L$ and have the same laws of transformation as $\equiv H$ ▸ transf1

$$\begin{aligned} T_L^i\,.\,\phi^j &= -\tfrac{1}{2}\left(i\epsilon_{ijk}\,\phi^k + \delta_{ij}\,\phi^0\right) \\ T_L^i\,.\,\phi^0 &= -\tfrac{1}{2}\,\phi^i \end{aligned}$$

Likewise, by $SU(2)_R$

$$\begin{aligned} T_R^i\,.\,\phi^j &= -\tfrac{1}{2}\left(i\epsilon_{ijk}\,\phi^k - \delta_{ij}\,\phi^0\right) \\ T_R^i\,.\,\phi^0 &= +\tfrac{1}{2}\,\phi^i \end{aligned}$$

Hermitian conjugation $(\phi^0)^\dagger = \phi^0, \quad (\xi^0)^\dagger = -\xi^0, \quad (\vec{\phi})^\dagger = -\vec{\phi}, \quad (\vec{\xi})^\dagger = \vec{\xi}$

The two composite Higgs doublets (I)

The two composite doublets isomorphic to H are ▸ H ▸ mhmxi ▸ pause

$$H_1 = \frac{1}{\sqrt{2}}\frac{v}{\mu^3}\begin{pmatrix} \phi^1 - i\phi^2 \\ -(\phi^0 + \phi^3) \end{pmatrix} = \frac{v\sqrt{2}}{\mu^3}\begin{pmatrix} \bar{d}\gamma_5 u \\ -\frac{1}{2}(\bar{u}u + \bar{d}d) - \frac{1}{2}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) \end{pmatrix}$$

$$\tilde{H}_1 = \frac{1}{\sqrt{2}}\frac{v}{\mu^3}\begin{pmatrix} \phi^0 - \phi^3 \\ -(\phi^1 + i\phi^2) \end{pmatrix} = \frac{v\sqrt{2}}{\mu^3}\begin{pmatrix} \frac{1}{2}(\bar{u}u + \bar{d}d) - \frac{1}{2}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) \\ -\bar{u}\gamma_5 d \end{pmatrix}$$

$$H_2 = \frac{1}{\sqrt{2}}\frac{\sigma}{\nu^3}\begin{pmatrix} \xi^1 - i\xi^2 \\ -(\xi^0 + \xi^3) \end{pmatrix} = \frac{\sigma\sqrt{2}}{\nu^3}\begin{pmatrix} \bar{d}u \\ -\frac{1}{2}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) - \frac{1}{2}(\bar{u}u - \bar{d}d) \end{pmatrix}$$

$$\tilde{H}_2 = \frac{1}{\sqrt{2}}\frac{\sigma}{\nu^3}\begin{pmatrix} \xi^0 - \xi^3 \\ -(\xi^1 + i\xi^2) \end{pmatrix} = \frac{\sigma\sqrt{2}}{\nu^3}\begin{pmatrix} \frac{1}{2}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) - \frac{1}{2}(\bar{u}u - \bar{d}d) \\ -\bar{u}d \end{pmatrix}$$

The two composite Higgs doublets (II)

- $$< \phi^0 > = < \bar{u}u + \bar{d}d > = \mu^3, \quad < \xi^3 > = < \bar{u}u - \bar{d}d > = \nu^3 \quad \Rightarrow$$
$$< H_1 > = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad , \quad < H_2 > = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \sigma \end{pmatrix}$$
$$< \tilde{H}_1 > = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix} \quad , \quad < \tilde{H}_2 > = -\frac{1}{\sqrt{2}}\begin{pmatrix} \sigma \\ 0 \end{pmatrix}$$

- Usual shift(s) for $h_1^0 = \frac{1}{\sqrt{2}}\frac{v}{\mu^3}\,\phi^0$ and $h_2^3 = \frac{1}{\sqrt{2}}\frac{\sigma}{\nu^3}\,\xi^3$

$$h_1^0 = -\frac{v}{\sqrt{2}} + h, \quad h_2^3 = -\frac{\sigma}{\sqrt{2}} + \xi$$

In the 4-bases $(h_1^0, h_1^3, h_1^+, h_1^-)$ and $(h_2^0, h_2^3, h_2^+, h_2^-)$

- The three generators of $SU(2)_L$ and those of $SU(2)_R$ are 4×4 "Pauli-like" matrices which satisfy

$$\{T_L^i, T_L^j\} = \frac{1}{2}\delta_{ij}\,\mathbb{I}_L, \quad \{T_R^i, T_R^j\} = \frac{1}{2}\delta_{ij}\,\mathbb{I}_R$$

- The generators of the diagonal $SU(2)_V$ are $\vec{T} = \vec{T}_L + \vec{T}_R$

$$T^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = Q, \quad T^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

- $$T^3 = Q = T_L^3 + T_R^3 \quad \text{and} \quad Y = Q - T_L^3 \quad \Rightarrow Y = T_R^3$$

- $(h_1^0, h_1^3, h_1^+, h_1^-)$ and $(h_2^0, h_2^3, h_2^+, h_2^-)$ are *not* eigenvectors of $\vec{T}_{L,R}$, only invariant subsets. They are (singlet + triplet) of $SU(2)_V$

pause end

▸ prelim ▸ results

▸ SU(2) ▸ H ▸ transf1 ▸ chiral ▸ chiral

▸ quadrup ▸ parity ▸ transf2 ▸ H1H2 ▸ 4-base

▸ breaking ▸ mw ▸ mfpi ▸ crossed ▸ pot ▸ hw

▸ hq ▸ mhmxi ▸ WWscat ▸ leptons ▸ hl ▸ dark ▸ 3gen

▸ concl ▸ invest

▸ end

▸ rep1 ▸ rep2 ▸ Goldst

Kinetic terms ; the masses of $SU(2)_L$ gauge bosons

pause

They appear in the kinetic terms

$$\frac{1}{2}\left((D_\mu H_1)^\dagger D^\mu H_1 + (D_\mu H_2)^\dagger D^\mu H_2\right)$$

hw

$$m_W^2 = g^2\,\frac{v^2+\sigma^2}{8}$$

We shall see

sigma v

$$v = f_\pi \quad \Rightarrow \quad \sigma \approx 2\sqrt{2}\frac{m_W}{g}$$

such that

$$\frac{< h_1^0 >}{< h_2^3 >} = \frac{g f_\pi}{2\sqrt{2} m_W} = f_\pi \sqrt{G_f/\sqrt{2}} \approx \frac{1}{4200}$$

The unitary gauge

invest pause

Crossed terms occur in the kinetic Lagrangian, for example leptons WWscat

$$\frac{igf_\pi}{2\sqrt{2}} W_\mu^3\, \partial^\mu \pi^0 \quad \text{and} \quad \frac{ig\sigma}{2\sqrt{2}} W_\mu^3\, \partial^\mu h_2^0$$

- The first term accounts for *leptonic pion decays* in agreement with PCAC and should not be "gauged away"
- So, the pseudoscalar singlet h_2^0 and the two scalars h_2^+, h_2^- are the 3 Goldstones doomed to become the longitudinal W's. lambda Goldst

The breaking pattern

- The scalar potential lives in an 8-dimensional space. There are 6 "flat" directions and 2 massive ones which are the 2 Higgs bosons
- $< \bar{u}u + \bar{d}d > \neq 0$ fully breaks $SU(2)_L$, $SU(2)_R$, but *not* $SU(2)_V$. The 3 Goldstones of this $SU(2)_L$ breaking can be the same as those of $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$: the 3 pions
- $< \bar{u}u - \bar{d}d > \neq 0$ fully breaks $SU(2)_L$, $SU(2)_R$, but $SU(2)_V$ is only broken down to $U(1)_{em}$ with generator $Q = T^3$.
  The 3 Goldstones of $SU(2)_L$ breaking yield the 3 longitudinal $W_{||}$
- $S \leftrightarrow P$ symmetry $U(1)_L \times U(1)_R$ broken down to $U(1)_V$.
  Goldstone $= h_1^0 \equiv \frac{f_\pi}{\sqrt{2}} + h$ or more traditionally $h_2^0 \propto \eta$? ▸ Goldst
- Yukawa couplings + PCAC + Gell-Mann-Oakes-Renner $\rightarrow$ pion masses by soft $SU(2)_L$ invariant breaking of $SU(2)_L \times SU(2)_R$
- Only $SU(2)_L$ or $SU(2)_R$ can get gauged if one wants to keep physical pions : not enough Goldstones to break $SU(2)_L \times SU(2)_R$ ▸ invest ▸ pause

Yukawa couplings

pause

Yukawa couplings between u, d and H_1, H_2 H1H2 trigger

- fermion masses

lambda

$$m_u = \frac{v\rho_u + \sigma\lambda_u}{\sqrt{2}}, \quad m_d = \frac{v\rho_d + \sigma\lambda_d}{\sqrt{2}}$$

- other couplings

$$\frac{1}{2\sqrt{2}}\frac{v}{\mu^3}\Big((\rho_u+\rho_d)(\bar{u}\gamma_5 d\,\phi^- + \bar{d}\gamma_5 u\,\phi^+) + (\rho_u-\rho_d)(\bar{d}u\,\phi^+ - \bar{u}d\,\phi^-)$$
$$+(\rho_u+\rho_d)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)\,\phi^3 + (\rho_u-\rho_d)(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)\,\phi^3\Big)$$

and

$$-\frac{1}{2\sqrt{2}}\frac{\sigma}{\nu^3}\Big((\lambda_u+\lambda_d)(\bar{d}\gamma_5 u\,\xi^+ - \bar{u}\gamma_5 d\,\xi^-) + (\lambda_u-\lambda_d)(\bar{d}u\,\xi^+ + \bar{u}d\,\xi^-)$$
$$-(\lambda_u+\lambda_d)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)\,\xi^0 - (\lambda_u-\lambda_d)(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)\,\xi^0\Big)$$

Yukawa couplings ; the masses of fermions and pions (I)

- PCAC

$$\partial^\mu A_\mu^{1+i2} = i(m_u + m_u)\bar{u}\gamma_5 d = \sqrt{2} f_\pi m_\pi^2\, \pi^+$$

+ Gell-Mann-Oakes-Renner (GMOR) ▸ pi+pi0

$$(m_u + m_d) < \bar{u}u + \bar{d}d >= 2 f_\pi^2 m_\pi^2$$

- $\frac{i}{\sqrt{2}}\frac{v}{\mu^3}\phi^+ \equiv i\sqrt{2}\frac{v}{\mu^3}\bar{u}\gamma_5 d \equiv \pi^+$ with $\bar{u}\gamma_5 d \stackrel{PCAC}{=} -i\frac{\sqrt{2}m_\pi f_\pi^2}{m_u+m_d}\pi^+$ compatible with GMOR ($< \bar{u}u + \bar{d}d >= \mu^3$) $\Leftrightarrow$ ▸ mw ▸ minV

$$v = f_\pi$$

- Can we set $\frac{i}{\sqrt{2}}\frac{v}{\mu^3}\phi^+ = \pi^+$? Yukawa couplings $\ni$ pions mass terms $\Leftrightarrow$

$$\sqrt{2} f_\pi (\rho_u + \rho_d) = m_u + m_d$$

- Cancel classical (scalar $\leftrightarrow$ pseudoscalar) transitions ▸ hq ▸ invest

$$\rho_u = \rho_d = \frac{m_u + m_d}{2\sqrt{2} f_\pi}$$

Yukawa couplings ; the masses of fermions and pions (II)

- v small $\Rightarrow$ the mass of W comes mostly from σ mw minV

$$\sigma \approx 2\sqrt{2}\frac{m_W}{g}$$

- One gets from the expressions of the fermion masses mfpi hq

$$\lambda_u = g\frac{3m_u - m_d}{8m_W}, \quad \lambda_d = g\frac{3m_d - m_u}{8m_W}$$

- The Yukawa couplings $\propto \lambda_{u,d}$ are "quasi-standard" and describe transitions between fermion pairs and ξ Goldstones that can be gauged away
- So, we can indeed identify

$$\vec{\pi} = \vec{h}_1 = \frac{i}{\sqrt{2}}\frac{v}{\mu^3}\,\vec{\phi}$$

Kinetic terms : coupling of Higgs bosons to gauge bosons

pause

The couplings of Higgs bosons to gauge bosons come from the kinetic terms kin and write WWscat

$$\frac{gm_W}{2} W_\mu^2\,\xi, \quad \frac{g^2 f_\pi}{4\sqrt{2}} W_\mu^2\,h$$

- ξ couples in a "standard" way $\simeq gm_W$ to two W's
- h's coupling $\mathcal{O}(g^2 f_\pi)$ are much smaller by a factor $\mathcal{O}(10^{-3})$ dark

Yukawa couplings : coupling of Higgs bosons to quarks

▸ pause

Yukawa couplings include

$$-h\,(\rho_u \overline{u}u + \rho_d \overline{d}d) - \xi\,(\lambda_u \overline{u}u + \lambda_d \overline{d}d),$$

that is using the values of $\rho_u = \rho_d$ and $\lambda_{u,d}$ ▸ lambda ▸ rho ▸ Veff

$$-\frac{m_u + m_d}{2\sqrt{2}f_\pi}\,h\,(\overline{u}u + \overline{d}d) - g\,\xi\,\left(\left(\frac{3m_u - m_d}{8m_W}\right)\overline{u}u + \left(\frac{3m_d - m_u}{8m_W}\right)\overline{d}d\right)$$

ξ has "quasi-standard" couplings ▸ invest

h has potentially large couplings ▸ dark

What follows is evolving. Take it with care

The scalar potential $V(H_1, H_2)$

▸ pause

The most general V includes

- $-\frac{m^2_{H_1}}{2} H_1^\dagger H_1$, $\frac{\lambda_{H_1}}{4}(H_1^\dagger H_1)^2$, $-\frac{m^2_{H_2}}{2} H_2^\dagger H_2$, $\frac{\lambda_{H_2}}{4}(H_2^\dagger H_2)^2$, with real $m_{H_1}, m_{H_2}, \lambda_{H_1}, \lambda_{H_2}$
- $(m^2 H_1^\dagger H_2 + h.c)$, with m complex: induces classical scalar $\leftrightarrow$ pseudoscalar transitions. Set it to 0
- $\lambda_3(H_1^\dagger H_2)^2 + h.c.$, $\lambda_4(H_1^\dagger H_1)(H_1^\dagger H_2) + h.c.$, $\lambda_5(H_2^\dagger H_2)(H_1^\dagger H_2) + h.c.$. with complex $\lambda_3, \lambda_4, \lambda_5$: set it to 0 for the same reasons
- $\lambda_1(H_1^\dagger H_1)(H_2^\dagger H_2)$, $\lambda_2(H_1^\dagger H_2)(H_2^\dagger H_1)$, with real λ_1, λ_2 : the first induces $m^2_{\pi} \propto \lambda_1 \sigma^2$ and the second $m^2_{\pi^0} \propto \lambda_2 \sigma^2$; wrong $\pi^+ \pi^0$ mass difference and wrong origin of π mass $\Rightarrow$ set $\lambda_1 = 0 = \lambda_2$ ▸ GMOR
- $\Rightarrow V(H_1, H_2) = V_1(H_1^\dagger H_1) + V_2(H_2^\dagger H_2)$
  $= -\frac{m^2_{H_1}}{2} H_1^\dagger H_1 + \frac{\lambda_{H_1}}{4}(H_1^\dagger H_1)^2 - \frac{m^2_{H_2}}{2} H_2^\dagger H_2 + \frac{\lambda_{H_2}}{4}(H_2^\dagger H_2)^2$

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The scalar potential $V(H_1, H_2)$

▸ pause

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V_{eff} and the masses of the Higgs bosons (I)

pause

- Since in H_1, H_2 H1H2

$$\bar{u}u + \bar{d}d = \frac{\sqrt{2}\mu^3}{v} h_1^0, \quad \bar{u}u - \bar{d}d = \frac{\sqrt{2}\nu^3}{\sigma} h_2^3$$

Yukawa couplings hq rewrite as quadratic terms in the Higgs bosons which add to the potential $V \Rightarrow$

$$V_{eff}(H_1, H_2) = -\frac{m_{H_1}^2}{2} H_1^\dagger H_1 + \frac{\lambda_{H_1}}{4} (H_1^\dagger H_1)^2 - \frac{m_{H_2}^2}{2} H_2^\dagger H_2 + \frac{\lambda_{H_2}}{4} (H_2^\dagger H_2)^2$$
$$+ \delta_1 (h_1^0)^2 + \delta_{12} h_2^3 h_1^0 + \delta_2 (h_2^3)^2$$

with $\delta_1 = m_\pi^2$, $\quad \delta_{12} = g \frac{f_\pi m_\pi^2}{2\sqrt{2} m_W}$, $\quad \delta_2 = g^2 (m_u - m_d) \frac{\nu^3}{8 m_W^2}$

- V_{eff} must have its minimum at v Goldst

$$< h_1^0 >= \frac{f_\pi}{\sqrt{2}}, \quad < h_2^3 >= \frac{2 m_W}{g}$$

- Diagonalize 2×2 *matrix* to get Higgs bosons and their masses

V_{eff} and the masses of the Higgs bosons (II)

The masses of the two Higgs bosons h and ξ are ▸ Goldst

$$m_h^2 \approx 2\lambda_{H_1} < h_1^0 >^2 - \delta_{12}\frac{< h_2^3 >}{< h_1^0 >} = \lambda_{H_1} f_\pi^2 - m_\pi^2$$

$$m_\xi^2 \approx 2\lambda_{H_2} < h_2^3 >^2 - \delta_{12}\frac{< h_1^0 >}{< h_2^3 >} = \lambda_{H_2}\frac{8m_W^2}{g^2} - m_\pi^2\,\frac{f_\pi^2}{8m_W^2/g^2}$$

- $\vec{h}_1 \equiv \vec{\pi} \Rightarrow \frac{\lambda_{H_1}}{4} =$ 4-pions effective coupling
  - Mahanthappa-Riazuddin (1966)
    $$\rightarrow \frac{\lambda_{H_1}}{4} \approx 4\pi(.17) \approx 2.13 \Rightarrow m_h \approx 234\ MeV < 2m_\pi$$
  - Auberson-Mahoux-Simão (1976)
    $$\rightarrow -2 < \frac{\lambda_{H_1}}{4} < +2 \Rightarrow 0 \leq m_h < 226\ MeV < 2m_\pi$$
- $$m_\xi = ???$$

$W_{\parallel} W_{\parallel}$ scattering

- Internal line with ξ or h

hw

- Even when $m_\xi \to \infty$, h should tame the growth in s of the amplitude and restore perturbative unitarity $\mathcal{A} \simeq G_F \left(s - \frac{s^2}{s - m_h^2} \right)$
- What about renormalizability ?
- Do we get a spontaneously broken gauged linear σ-model for $(h, \vec{\pi})$?

Note: the equivalence theorem now includes Goldstones (h_2^0, h_2^+, h_2^-) *and pions*

crossed

pause

Leptons : Yukawa couplings

pause

Do the same jobs as for quarks $\Rightarrow$

$$m_e = \rho_e \frac{f_\pi}{\sqrt{2}} + \lambda_e \frac{2m_W}{g}, \quad m_\nu = \rho_\nu \frac{f_\pi}{\sqrt{2}} + \lambda_\nu \frac{2m_W}{g}$$

- One generates couplings between $\vec{\pi}$ and leptons

crossed

$$\frac{1}{2\sqrt{2}} \frac{v}{\mu^3} \Big((\rho_\nu + \rho_e)(\bar{e}\gamma_5 \nu \phi^- + \bar{\nu}\gamma_5 e \phi^+) + (\rho_\nu - \rho_e)(\bar{e}\nu\phi^- - \bar{\nu} e \phi^+)$$
$$+ (\rho_\nu + \rho_e)(\bar{\nu}\gamma_5 \nu - \bar{e}\gamma_5 e)\phi^3 + (\rho_\nu - \rho_e)(\bar{\nu}\gamma_5 \nu + \bar{e}\gamma_5 e)\phi^3 \Big)$$

and between ξ^0, ξ^+, ξ^- and leptons

$$\frac{1}{2\sqrt{2}} \frac{\sigma}{\nu^3} \Big((\lambda_\nu + \lambda_e)(\bar{e}\gamma_5 \nu \xi^- - \bar{\nu}\gamma_5 e \xi^+) + (\lambda_\nu - \lambda_e)(\bar{e}\nu\xi^- + \bar{\nu} e \xi^+)$$
$$+ (\lambda_\nu + \lambda_e)(\bar{\nu}\gamma_5 \nu - \bar{e}\gamma_5 e)\xi^0 + (\lambda_\nu - \lambda_e)(\bar{\nu}\gamma_5 \nu + \bar{e}\gamma_5 e)\xi^0 \Big)$$

- No extra leptonic pion decays $\Leftrightarrow$

invest dark

$$\rho_e = 0 = \rho_\nu, \quad \lambda_e = g\frac{m_e}{2m_W}, \quad \lambda_\nu = g\frac{m_\nu}{2m_W}$$

$\Rightarrow$ the Yukawa couplings λ_e and λ_ν of H_2 are "standard"

Leptons : couplings to Higgs bosons

pause

Yukawa couplings also generate couplings between h, ξ and leptons

$$h(\rho_\nu \overline{\nu_L} \nu_R + \rho_e \overline{e_L} e_R) + \xi(\lambda_\nu \overline{\nu_L} \nu_R + \lambda_e \overline{e_L} e_R)$$

- $\rho_{e,\nu} = 0 \Rightarrow h$ is not coupled to leptons (or very weakly invest)
- ξ is coupled in the standard way

The h Higgs boson. A candidate for light dark matter ?

- $m_h^2 = \lambda_{H_1} f_\pi^2 - m_\pi^2$; $m_h < 2m_\pi$

$$\left|\frac{\lambda_{H_1}}{4}\right| \simeq |4\text{−pions coupling}| \leq 2$$

- hWW coupling: $\frac{g^2}{4\sqrt{2}} f_\pi$ very small ▸ hw
- $h(\overline{e_L} e_R + \overline{\nu_L} \nu_R)$ coupling: vanishing or very small ▸ leptons
- $h(\overline{u_L} u_R + \overline{d_L} d_R)$ coupling: $\frac{m_u + m_d}{2\sqrt{2} f_\pi}$ small (potentially large) ▸ hm

  $h\pi\pi$ coupling: $\lambda_{H_1} \frac{f_\pi}{\sqrt{2}}$

  $hh\pi\pi$ coupling: $\frac{\lambda_{H_1}}{2}$

  $hhhh$ coupling: $\frac{\lambda_{H_1}}{4}$

- h is light, very stable, difficult to detect, present wherever there is hadronic matter ⇒ scalar MeV dark matter ?

▸ pause

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- $h(\overline{u_L} u_R + \overline{d_L} d_R)$ coupling: $\frac{m_u + m_d}{2\sqrt{2} f_\pi}$ small (potentially large)

$h\pi\pi$ coupling: $\lambda_{H_1} \frac{f_\pi}{\sqrt{2}}$

$hh\pi\pi$ coupling: $\frac{\lambda_{H_1}}{2}$

$hhhh$ coupling: $\frac{\lambda_{H_1}}{4}$

- h is light, very stable, difficult to detect, present wherever there is hadronic matter ⇒ scalar MeV dark matter ?

hw leptons hm pause

The h Higgs boson. A candidate for light dark matter ?

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More generations, a few remarks

pause

- $2N^2$ Higgs doublets $\rightarrow 8N^2$ bilinears, N scalar $< \bar{q}_i q_i >$ condensates $\Rightarrow N$ Higgs bosons
- $N = 3$: 18 Higgs doublets, 72 (scalars + pseudoscalars). 6 Higgs bosons, 3 Goldstones eaten by the 3 W's.
- Diagonalize the 6×6 Higgs mass matrix
- Among the 6 Higgs bosons, presumably scalars, 1 will presumably have mass $\mathcal{O}(m_W)$; among the 5 others some may be "physical" and some maybe "dark", depending on their masses, couplings ...
- Conjecture : among the 72 $J = 0$ mesons, three $W_{\parallel}$'s and some "dark Higgses" cannot be observed. If W^3 has eaten a neutral pseudoscalar (singlet ?) this one cannot be observed $\Rightarrow$ 35 obervable pseudoscalars. Observable scalars : 36 - two $W_{\parallel}^{\pm}$'s - d dark Higgses = (34 - d)

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Is the Standard Model self-contained ?

This avatar of the Standard Model is (very close to being) self-contained. No need of extra fermions, extra interactions. Composite Higgs multiplets provide enough degrees of freedom to account for physical masses and energy scales *except* m_ξ

Naturalness, simplicity, economy, no fancy "BSM physics"

It also has its "dark sides"... ☺☹

Still only a sketch
Much work lies ahead to make accurate predictions and tests

pause

To investigate

- make precise calculations of well chosen processes
- write the hadronic Lagrangian in terms of bosonic fields only
- what to do with large ρ Yukawa couplings if any ? ▸ rho
- deviations from GSW: coupling $\xi q\bar{q}$ ▸ hm, leptonic π decays ... ▸ leptons
- light Higgs h contribution to $g-2$?
- what is m_ξ ??? Unitarity ? Renormalizability ? Decoupling limit
- $\xi^0 \simeq W^3_{||}$ = pseudoscalar / $\xi^\pm \simeq W^\pm_{||}$ = scalars
- $W_{||}$ composite $\stackrel{?}{\Rightarrow}$ W composite
- which Goldstone for which breaking ? ▸ Goldst
- production, abundance and detection of h dark matter ▸ dark
- realistic case of $N=3$ generations, FCNC etc ...
- why does only the $SU(2)_L$ subgroup of $U(2N)\times U(2N)$ get local ? ▸ gauged
- ... ▸ pause

Thanks for your attention !

▸ 4-base ▸ pause

Appendix : the fundamental representation of $SU(2)$ (I)

$V = \begin{pmatrix} a \\ b \end{pmatrix}$ in the fundamental representation of $SU(2)$: ▸ pause

$$T^i.V = T^i\, V$$

Defining $T^i.a$ and $T^i.b$ by ▸ SU(2)

$$T^i.V = \begin{pmatrix} T^i.a \\ T^i.b \end{pmatrix}$$

gives

$$T^1.a = \frac{b}{2}, \quad T^1.b = \frac{a}{2}, \quad T^2.a = -\frac{ib}{2}, \quad T^2.b = \frac{ia}{2}, \quad T^3.a = \frac{a}{2}, \quad T^3.b = -\frac{b}{2}$$

such that ▸ commut ▸ commut1

$$T^1.(T^2.a) - T^2.(T^1.a) = -\frac{ia}{2} = -iT^3.a = -[T^1, T^2].a$$

$$T^1.(T^2.b) - T^2.(T^1.b) = -\frac{ib}{2} = +iT^3.b = -[T^2, T^1].b$$

Appendix : the fundamental representation of $SU(2)$ (II)

▸ pause

$$\begin{aligned}[T^i, T^j].V &= (T^i T^j - T^j T^i).V \\ &= i\epsilon_{ijk} T^k.V = i\epsilon_{ijk} T^k V\end{aligned}$$

$$\Leftrightarrow (T^i T^j).V = (T^i T^j)V$$

Defining $(T^i T^j).a$ and $(T^i T^j).b$ by

$$(T^i T^j).V = \begin{pmatrix} (T^i T^j).a \\ (T^i T^j).b \end{pmatrix}$$

one gets

▸ commut1

$$\boxed{(T^i T^j).a = -T^i.(T^j.a), \quad (T^i T^j).b = -T^i.(T^j.b)}$$

such that

$$T^i.(T^j.a) - T^j.(T^i.a) = -[T^i, T^j].a\,, \quad T^i.(T^j.b) - T^j.(T^i.b) = -[T^i, T^j].b$$

Appendix : a Goldstone paraphernalia (I)

- because $< h_1^3 >= 0$, $\nexists\ W_\mu^3 \partial^\mu h_1^0$ coupling $\Rightarrow W_\parallel^3 \neq h_1^0$
  $\Rightarrow W_\parallel^3 = h_2^0 \simeq \eta$ which can be gauged away

- *traditional:* the Goldstone of $U(1)_L \times U(1)_R \to U(1)$ (broken parity) is also $h_2^0 \simeq \eta$ which disappears from the spectrum;
  h_1^0 has no special status but could be dark matter

- *daring:* Goldstone of broken parity $= h_1^0 \propto \bar u u + \bar d d = \frac{f_\pi}{\sqrt{2}} + h =$ light (MeV) dark matter $<< m_W$; $h_2^0 \simeq \eta$ is *not* the "$U(1)$ Goldstone"

- *crazy:* Goldstone of broken parity $= h_2^3 \propto \bar u u - \bar d d = \frac{2m_W}{g} + \xi$ gets mass $\mathcal{O}$(scale of parity violation $\approx m_W$); parity restoration at $\Lambda \gg m_W$; $h_2^0 \simeq \eta$ is *not* the "$U(1)$ Goldstone"; h_1^0 has no special status but could be dark matter

breaking mhmxi parity crossed pause

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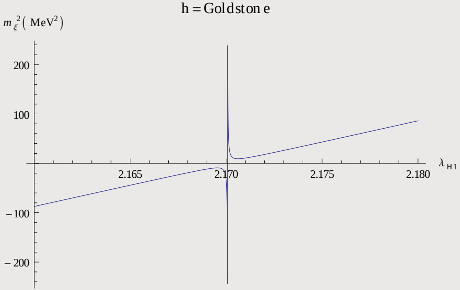
breaking mhmxi parity crossed pause

Appendix : a Goldstone paraphernalia (II)

For h = Goldstone of $U(1)_L \times U(1)_R \rightarrow U(1)$

$H_1 = (h_1^0, \vec{\pi}) \ni 4$ (pseudo)Goldstones of $U(2)_L \times U(2)_R \rightarrow U(2)_V$

$$\lambda_{H_1} \approx \frac{m_\pi^2}{f_\pi^2} + \frac{g^2 f_\pi^2 m_\pi^2}{4 m_W^2 m_\xi^2} \approx 2.17 \overset{m_\xi = 115\ GeV}{+} 1.63\,10^{-13}$$



To escape from $m_\xi = \infty$, λ_{H_1} must be tuned at 10^{-13}

if $m_\xi \rightarrow \infty$, what is the role of H_2 ?

pause