Probing the $N_f = 2$ chiral dynamics with topological zero-modes in the mixed chiral regime

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Ingredients : Mixed-actions setup

- valence : Neuberger fermions

Giusti *et al.* (2003-2006)

- sea : $N_f = 2$ $\mathcal{O}(a)$-improved Wilson fermions
  $\rightarrow$ use of CLS gauge configurations

Del Debbio *et al.* (2006, 2007)

What about observables when sea and valence masses don't belong to the same chiral regime ?

Matching of ChPT and Lattice QCD

Benefit from testing different chiral regimes

$\rightarrow$ reduce systematic uncertainties.

Advantages

- + Chiral symmetry preserved in the valence sector
- + Overlap $\rightarrow$ well-defined topological charge
- + Wilson $\rightarrow$ all topological sectors are sampled
- + Renormalization easier (no operator mixing)

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- + Overlap → well-defined topological charge
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Disadvantages

- – Overlap → numerical cost much more higher than Wilson
- – Possible large unitarity violations

Dürr *et al.* PoSLAT 2007

p-regime

Applicable if : $M_\pi L \gg 1,\ 4\pi F \gg L^{-1}$
Standard ChPT in finite volume : $m \sim p^2, \quad L^{-1} \sim T^{-1} \sim p$

Gasser and Leutwyler (1983, 1987)

Mass and volume effects are relevant.

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Mass and volume effects are relevant.

ϵ-regime

Applicable if : $m\Sigma V \sim \mathcal{O}(1),\ M_\pi L \sim 1,\ 4\pi F \gg L^{-1}$
Reordering of the chiral expansion : $m \sim p^4 \sim \epsilon^4, \quad L^{-1} \sim T^{-1} \sim \epsilon$

Hansen (1990) – Hansen and Leutwyler (1991)

Volume effects are relevant, **mass effects are suppressed**.

Pion zero-momentum modes become non-perturbative. Factorization :

$$U = U_0 \, \mathrm{e}^{2i\xi/F} \text{ with } \int \mathrm{d}^4x \, \xi(x) = 0.$$

$\Rightarrow$ "Exact" factorization of the lagrangian :

$$S_{\mathrm{LO}}(\xi, U_0) = \underbrace{\int \mathrm{d}^4x \; \mathrm{Tr}\,[\partial_\mu \xi \partial^\mu \xi]}_{\text{usual perturbative treatment}} - \underbrace{\frac{\Sigma V}{2} \mathrm{Tr}\left[M U_0 + U_0^\dagger M\right]}_{\text{non-perturbative integration}}.$$

The integation over the zero-modes manifold can be done exactly via master integrals.

Topology plays here an important role : it becomes here an extra scaling variable.

$\rightarrow$ compute observables at fixed topology.

Pseudoscalar correlator in ChPT

N_v valence quarks with mass m_v and N_s sea quarks with mass m_s

Pseudoscalar correlator

$$\mathrm{Tr}[T^a T^b] C(t) = \int \mathrm{d}^3\vec{x}\, \langle P^a(x) P^b(0) \rangle, \qquad P^a \equiv \bar{\psi}_v T^a i\gamma_5 \psi_v$$

T^a generator of the valence group $SU(N_v)$

$\longrightarrow$ ChPT predictions at NLO in finite volume $V = L^3 T$.

Propagators

$$\begin{aligned}
G\left(x, M^2\right) &= \frac{1}{V} \sum_p \frac{\mathrm{e}^{ip\cdot x}}{p^2 + M^2}, \\
E\left(x, M_a^2, M_b^2\right) &= \frac{1}{V} \sum_p \frac{\mathrm{e}^{ip\cdot x}}{(p^2 + M_a^2)(p^2 + M_b^2)\left(\frac{N_v}{p^2 + M_{\mathrm{vv}}^2} + \frac{N_s}{p^2 + M_{\mathrm{ss}}^2}\right)}, \\
\bar{G}\left(x\right) &= \frac{1}{V} \sum_{p^*} \frac{\mathrm{e}^{ip\cdot x}}{p^2}.
\end{aligned}$$

Take all masses in the p-regime

$m_{s,v}\Sigma V \gg 1$

$$C(t) = \frac{2\Sigma_{\text{eff}}}{F^2}\frac{\cosh\left[M_{vv,\text{eff}}\left(t - T/2\right)\right]}{2M_{vv,\text{eff}}\ \sinh\left[M_{vv,\text{eff}}\,T/2\right]} \quad \text{with} \left\{ \begin{array}{lcl} M^2_{vv,\text{eff}} & = & M^2_{vv}\left(1+\Delta_M\right) \\ \Sigma_{\text{eff}} & = & \Sigma\left(1+\Delta_\Sigma\right) \end{array} \right.$$

NLO chiral corrections on the mass and chiral condensate

$$\begin{aligned} \Delta_M &= \frac{E\left(0, M^2_{vv}, M^2_{vv}\right)}{F^2} - \frac{8}{F^2}\left[\left(N_v M^2_{vv} + N_s M^2_{ss}\right)\left(L_4 - 2L_6\right) \right. \\ &\quad \left. +\ M^2_{vv}\left(L_5 - 2L_8\right)\right] \\ \Delta_\Sigma &= -\frac{N_v}{F^2}G\left(0, M^2_{vv}\right) - \frac{N_s}{F^2}G\left(0, M^2_{vs}\right) + \frac{E\left(0, M^2_{vv}, M^2_{vv}\right)}{F^2} \\ &\quad +\ \frac{16}{F^2}\left[\left(N_v M^2_{vv} + N_s M^2_{ss}\right)L_6 + M^2_{vv}L_8\right] \end{aligned}$$

Spectral decomposition of the propagator

For correlators computed in fixed topological sectors, exact poles in $1/m^n$ may appear when some propagators are saturated by zero-modes :

$$D^{-1}_{xy} = \sum_{i,\mathrm{zero-modes}} \frac{v_i(x)v_i^\dagger(y)}{mV} + \sum_{i,\mathrm{non\,zero-modes}} \frac{v_i(x)v_i^\dagger(y)}{(\lambda_i + m)\,V}$$

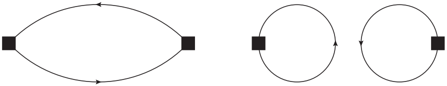
$\rightarrow$ One can match the residues instead of the correlators themselves in the ϵ-regime.

$$\begin{aligned} C_\nu\,(x-z, y-z) &= \frac{\mathrm{Res_n}}{(mV)^n}, \\ \mathrm{Res_n} &= \lim_{m\to 0}\,(mV)^n\,C_\nu\,(x-z, y-z) \end{aligned}$$

Giusti, Hernández, Laine, Weisz, Wittig (2004).

Take all masses in the ϵ-regime. With the spectral representation of the quark propagator :

$$\lim_{m\to 0} (mV)^2\, C_\nu^{ab}(x) = \mathrm{Tr}\,\left[T^a T^b\right] A_\nu(x) + \mathrm{Tr}\,[T^a]\mathrm{Tr}\,[T^b]\tilde{A}_\nu(x)$$



"Connected" (left) and "disconnected" (right) external flavours contractions.

Damgaard, Hernández, Jansen, Laine and Lellouch (2003).

Amplitudes

$$\begin{aligned} A_\nu(x-y) &= \left\langle \sum_{i,j} v_j^\dagger(x) v_i(x) v_i^\dagger(y) v_j(y) \right\rangle_\nu , \\ \tilde{A}_\nu(x-y) &= -\left\langle \sum_i v_i^\dagger(x) v_i(x) \sum_j v_j^\dagger(y) v_j(y) \right\rangle_\nu \end{aligned}$$

Giusti, Hernández, Laine, Weisz and Wittig (2004).

$m_{s,v}\Sigma V \sim 1$, $N = N_v + N_s$

$$C_\nu^{aa}(x) = C_\nu + \alpha_a \bar{G}(x) + \beta_a [\bar{G}(x)]^2 + \gamma_a \int \mathrm{d}^4 y\, \bar{G}(x-y)\bar{G}(y) + \varepsilon_a \delta(x)$$

Time derivative of the residues in $1/m^2$

$$\begin{aligned} \lim_{m\to 0} (mV)^2 \frac{\mathrm{d}}{\mathrm{d}t} \int \mathrm{d}^3\vec{x}\, C_\nu^{aa}(x) &= \frac{1}{2} A'(t), \\ \lim_{m\to 0} (mV)^2 \frac{\mathrm{d}}{\mathrm{d}t} \int \mathrm{d}^3\vec{x}\, C_\nu^{00}(x) &= N A'(t) + N^2 \tilde{A}'(t). \end{aligned}$$

Let $\tau = t/T$:

$$\begin{aligned} F^2 A'(t) &= \frac{2|\nu|}{N} \left[(1 + N|\nu|)\, h_1'(\tau) + \frac{T^2}{NF^2V} H_2(\tau) \right], \\ F^2 \tilde{A}'(t) &= -\frac{2|\nu|}{N} \left[(N + |\nu|)\, h_1'(\tau) + \frac{T^2}{NF^2V} \tilde{H}_2(\tau) \right]. \end{aligned}$$

$$
\begin{aligned}
H_2(\tau) &= -(1+N|\nu|)\,N^2\frac{\beta_1\sqrt{V}}{T^2}h_1'(\tau)+\left[N(6-N^2)|\nu|+4\right.\\
&\quad \left.+N^2(2\nu^2-1)\right]h_2'(\tau)+\left[N\left(2-\frac{N^2}{2}\right)|\nu|+1+\frac{N^2}{2}\right]g_1'(\tau),\\
\tilde{H}_2(\tau) &= -(N+|\nu|)\,N^2\frac{\beta_1\sqrt{V}}{T^2}h_1'(\tau)+\left[(4+N^2)|\nu|+2N(2+\nu^2)\right.\\
&\quad \left.-N^3\right]h_2'(\tau)+\left[\left(1+\frac{N^2}{2}\right)|\nu|+2N-\frac{N^3}{2}\right]g_1'(\tau).
\end{aligned}
$$

$$
\begin{aligned}
h_1(\tau) &= \frac{1}{2}\left[\left(\tau-\frac{1}{2}\right)^2-\frac{1}{12}\right],\ h_2(\tau)=\frac{1}{24}\left[\tau^2(\tau-1)^2-\frac{1}{30}\right]\\
g_1(\tau) &= [h_1(\tau)]^2+\sum_{\vec{n}\neq\vec{0}}\left\{\frac{\cosh\left[|\vec{p}|\,(\tau-1/2)\right]}{2|\vec{p}|\sinh\left(|\vec{p}|/2\right)}\right\}^2,\ |\vec{p}|=2\pi\frac{T}{L}\left[\sum_{i=1}^{3}n_i^2\right]^{1/2}
\end{aligned}
$$

$\Rightarrow$ Only depends on F at NLO ! !

Expansion around $z = \tau - 1/2 = 0$

$$\begin{aligned} \frac{1}{L^2} A'(t) &= D_\nu z + C_\nu z^3 + \mathcal{O}(z^5), \\ \frac{1}{L^2} \tilde{A}'(t) &= \tilde{D}_\nu z + \tilde{C}_\nu z^3 + \mathcal{O}(z^5). \end{aligned}$$

$$\begin{aligned} D_\nu &= \frac{2|\nu|}{N(FL)^2} \Bigg\{ (1+N|\nu|) \left(1 - N\frac{\beta_1}{F^2\sqrt{V}}\right) + \frac{T^2}{F^2 V} \Bigg[\gamma_1 \Bigg(\frac{2+N^2}{2N} \\ &\quad + \frac{4-N^2}{2}|\nu| \Bigg) + \zeta_2 \left(\left(6-N^2\right)|\nu| + \frac{4}{N} + N\left(2\nu^2 - 1\right) \right) \Bigg] \Bigg\}, \\ \tilde{D}_\nu &= -\frac{2|\nu|}{N(FL)^2} \Bigg\{ (N+|\nu|) \left(1 - N\frac{\beta_1}{F^2\sqrt{V}}\right) + \frac{T^2}{F^2 V} \Bigg[\gamma_1 \Bigg(\frac{4-N^2}{2} \\ &\quad + \frac{2+N^2}{2N}|\nu| \Bigg) + \zeta_2 \left(4 + 2\nu^2 - N^2 + \frac{4}{N}|\nu| + N|\nu| \right) \Bigg] \Bigg\} \end{aligned}$$

with

$$\zeta_2 = -\frac{1}{24} \text{ and } \gamma_1 = -\frac{1}{12} + \frac{1}{2} \sum_{\vec{n} \neq \vec{0}} \frac{1}{\sinh^2\left(|\vec{p}|/2\right)}.$$

Let's consider now N_v valence quark in the ϵ-regime and N_s sea quarks in the p-regime $\Rightarrow$ partial-quenching.

Valence pion zero-momentum modes still non-perturbative but factorization becomes :

$$U = \begin{pmatrix} \bar{U}_0 & 0 \\ 0 & e^{-i\frac{\bar{\eta}}{N_s}} \mathbb{1}_s \end{pmatrix} e^{i\frac{2\xi}{F}} \text{ with } \int d^4x \,\mathrm{Tr}\left[T^a \xi(x)\right] = 0, \begin{cases} T^a \text{ generator of } SU(N_v) \\ T^\eta = \begin{pmatrix} \frac{\mathbb{1}_v}{N_v} & 0 \\ 0 & -\frac{\mathbb{1}_s}{N_s} \end{pmatrix}. \end{cases}$$

Power counting

$$\begin{aligned} p &\sim \partial_\mu \sim \xi \sim \epsilon \\ L^{-1} &\sim T^{-1} \sim \epsilon \\ m_v &\sim \epsilon^4 \\ m_s &\sim \epsilon^2 \end{aligned}$$

Bernardoni and Hernández (2007),
Bernardoni, Damgaard, Fukaya, Hernández (2008).

$m_v\Sigma V \sim 1$, $m_s\Sigma V \gg 1$, $M_\eta^2 = \frac{N_v}{N} M_{\mathrm{ss}}^2$

$$
\begin{aligned}
C_\nu^{aa}(x) \;=\; & C_\nu + \alpha_a^{(1)}\bar{G}(x) + \alpha_a^{(2)}\bar{G}\left(x, M_\eta^2\right) + \alpha_a^{(3)}\frac{\mathrm{d}}{\mathrm{d}M_{\mathrm{ss}}^2}\left[M_{\mathrm{ss}}^2\bar{G}\left(x, M_\eta^2\right)\right] \\
& + \beta_a^{(1)}\left[\bar{G}(x)\right]^2 + \beta_a^{(2)}\left[\bar{G}\left(x, M_\eta^2\right)\right]^2 + \beta_a^{(3)}\left[\bar{G}\left(x, M_{\mathrm{ss}}^2\right)\right]^2 \\
& + \beta_a^{(4)}\bar{G}(x)\bar{G}\left(x, M_\eta^2\right) + \int \mathrm{d}^4 y\,\Big[\gamma_a^{(1)}\bar{G}(x-y)\bar{G}(y) \\
& + \;\gamma_a^{(2)}\bar{G}\left(x-y, M_\eta^2\right)\bar{G}\left(y, M_\eta^2\right) + \gamma_a^{(3)}\bar{G}\left(x-y\right)\bar{G}\left(y, M_\eta^2\right)\Big] \\
& + \varepsilon_a\delta(x)
\end{aligned}
$$

Time derivative of the residues in $1/m^2$

$$
\begin{aligned}
\lim_{m\to 0}(mV)^2\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3\vec{x}\,C_\nu^{aa}(x) \;&=\; \frac{1}{2}A'(t), \\
\lim_{m\to 0}(mV)^2\frac{\mathrm{d}}{\mathrm{d}t}\int \mathrm{d}^3\vec{x}\,C_\nu^{00}(x) \;&=\; N_v A'(t) + N_v^2\tilde{A}'(t).
\end{aligned}
$$

$m_v \Sigma V \sim 1,\ m_s \Sigma V \gg 1,\ M_\eta^2 = \frac{N_v}{N} M_{\rm ss}^2$

$$\begin{aligned} C_\nu^{aa}(x) &= C_\nu + \alpha_a^{(1)} \bar{G}(x) + \alpha_a^{(2)} \bar{G}\left(x, M_\eta^2\right) + \alpha_a^{(3)} \frac{\mathrm{d}}{\mathrm{d}M_{\rm ss}^2}\left[M_{\rm ss}^2 \bar{G}\left(x, M_\eta^2\right)\right] \\ &+ \beta_a^{(1)}\left[\bar{G}(x)\right]^2 + \beta_a^{(2)}\left[\bar{G}\left(x, M_\eta^2\right)\right]^2 + \beta_a^{(3)}\left[\bar{G}\left(x, M_{\rm ss}^2\right)\right]^2 \\ &+ \beta_a^{(4)} \bar{G}(x)\bar{G}\left(x, M_\eta^2\right) + \int \mathrm{d}^4 y \left[\gamma_a^{(1)} \bar{G}(x-y)\bar{G}(y)\right. \\ &+ \left. \gamma_a^{(2)} \bar{G}\left(x-y, M_\eta^2\right)\bar{G}\left(y, M_\eta^2\right) + \gamma_a^{(3)} \bar{G}(x-y)\,\bar{G}\left(y, M_\eta^2\right)\right] \\ &+ \varepsilon_a \delta(x) \end{aligned}$$

Effective couplings

$$\begin{aligned} \tilde{\Sigma} &= \Sigma\left\{1 - \frac{1}{F^2}\left[N_v \bar{G}(0) + N_s G\left(0, M_{\rm ss}^2/2\right) - \bar{E}(0,0,0) - 16 L_6 N_s M_{\rm ss}^2\right]\right\} \\ \tilde{F} &= F\left\{1 - \frac{1}{2F}\left[N_v \bar{G}(0) + N_s G\left(0, M_{\rm ss}^2/2\right) - 8 L_4 N_s M_{\rm ss}^2\right]\right\} \end{aligned}$$

Decoupling theorem

$\rightarrow$ Matching with the N_v ϵ-regime result but with a modified $\bar{F} = F\left(N_v, N_s, M_{\rm ss}^2, L_i\right)$ up to exponentially suppressed $M_{\rm ss}$ corrections :
Sea quarks in the mixed regime $\sim$ decoupling particles because the mixed regime probes much lower energy scale than $M_{\rm ss}$ since the valence quarks are much lighter and the size of the box is much larger than the Compton length of the sea pions :

$$M_{\rm vv}^2 \leq \frac{1}{L^2} \leq M_{\rm ss}^2 \ll (4\pi F)^2$$

Effective couplings

$$\begin{aligned}
\tilde{\Sigma} &= \Sigma\left\{1 - \frac{1}{F^2}\left[N_v \bar{G}(0) + N_s G\left(0, M_{\rm ss}^2/2\right) - \bar{E}(0,0,0) - 16 L_6 N_s M_{\rm ss}^2\right]\right\} \\
\tilde{F} &= F\left\{1 - \frac{1}{2F}\left[N_v \bar{G}(0) + N_s G\left(0, M_{\rm ss}^2/2\right) - 8 L_4 N_s M_{\rm ss}^2\right]\right\}
\end{aligned}$$

How to get quenched results from the full ones ?

- Supersymmetric method : add N_v spin 1/2 bosons with mass m_v to cancel the fermionic loops. Zero-modes integrals performed now with the supergroup $U(N|N)$.
- Replica limit : keep N_v dependence explicitly and take the limit $N_v \to 0$ add the very end of the computation. Zero-mode integrals as for the SUSY method.

$\rightarrow$ both methods equivalent at any order. We checked the consistency of the results in our computation.

Quenched propagator

$$\begin{aligned} \bar{E}\,(x, 0, 0) &= \frac{\alpha}{2N_c}\bar{G}(x) + \frac{m_0^2}{2N_c}\bar{F}(x), \\ \bar{F}(x) &= \frac{1}{V}\sum_{p^*}\frac{\mathrm{e}^{ip\cdot x}}{p^4}. \end{aligned}$$

$\rightarrow$ double poles as expected due to unitary violations.

Overall form of the correlator

$$\begin{aligned} C_\nu^{aa}(x) &= \tilde{C} + \tilde{\alpha}_a^{(1)}\bar{G}(x) + \tilde{\alpha}_a^{(2)}\bar{E}(x) + \tilde{\beta}_a^{(1)}\left[\bar{G}(x)\right]^2 + \tilde{\beta}_a^{(2)}\left[\bar{E}(x)\right]^2 \\ &+ \tilde{\beta}_a^{(3)}\left[\bar{G}\left(x, M_{\rm ss}^2/2\right)\right]^2 + \tilde{\beta}_a^{(4)}\bar{G}(x)\bar{E}(x) \\ &+ \int \mathrm{d}^4y\left[\tilde{\gamma}_a^{(1)}\bar{G}(x-y)\bar{G}(y) + \tilde{\gamma}_a^{(2)}\bar{G}(x-y)\bar{E}(y)\right. \\ &+ \left.\tilde{\gamma}_a^{(3)}\bar{E}(x-y)\bar{E}(y)\right] + \tilde{\varepsilon}_a\delta(x) \end{aligned}$$

Effective couplings

$$\begin{aligned} F_{\rm eff}^2 &= F^2\left\{1 - \frac{N_s}{F^2}\left[G\left(0, M_{\rm ss}^2/2\right) - 8L_4M_{\rm ss}^2\right]\right\} \\ \frac{\alpha_{\rm eff}}{2N_c} &= \frac{1}{N_s}\left\{1 - \frac{1}{F^2}\left[N_sG\left(0, M_{\rm ss}^2/2\right) - 8L_5M_{\rm ss}^2\right]\right\} \\ \frac{m_{0,\,\rm eff}^2}{2N_c} &= \frac{M_{\rm ss}^2}{N_s}\left\{1 - \frac{1}{F^2}\left[\frac{N_s^2-1}{N_s}G\left(0, M_{\rm ss}^2\right) - 16M_{\rm ss}^2\left(N_sL_6\right.\right.\right. \\ &+ \left.\left.\left. N_sL_7 + L_8\right)\right]\right\} \end{aligned}$$

Result

$$D_\nu = \frac{2|\nu|}{(F_{\rm eff} L)^2}\left\{|\nu| + \frac{\alpha_{\rm eff}}{2N_c} + \frac{\rho}{(F_{\rm eff}L)^2}\left[-\beta_1\rho^{-3/2} + \left(\frac{5}{N_s^2} + \frac{8|\nu|}{N_s} + 3 \right.\right.\right.$$
$$\left. + 2\nu^2 - 2\langle\nu^2\rangle_{\rm eff}\right)\zeta_2 + \left(\frac{1}{N_s^2} + \frac{2|\nu|}{N_s} + \frac{1}{2}\right)\gamma_1$$
$$+ 2\frac{M_{\rm ss}^2}{N_s}T^2\left(\frac{1}{N_s} + |\nu|\right)(\gamma_2 - 4\zeta_3) + \left(\frac{M_{\rm ss}^2}{N_s}\right)^2 T^4(\gamma_3 + 4\zeta_4)$$
$$\left.\left. - \frac{N_s}{2}|\nu|\gamma_4\left(M_{\rm ss}^2/2\right)\right]\right\}$$

Using the Witten-Veneziano formula at LO : $m_0^2 F^2 = 4N_c\frac{\langle\nu^2\rangle}{V}$.

To compare with Giusti, Hernández, Laine, Weisz, Wittig (2004) : $\frac{\alpha}{2N_c} \sim \mathcal{O}\left(\epsilon^2\right)$, $\frac{m_0^2}{2N_c} \sim \mathcal{O}\left(\epsilon^4\right)$

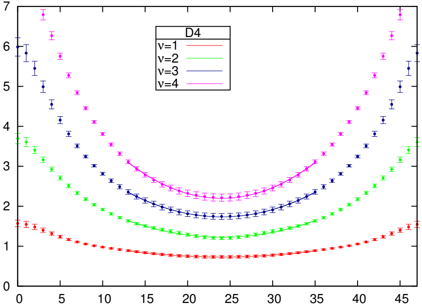
$$D_\nu = \frac{2|\nu|}{(FL)^2}\left\{|\nu| + \frac{\alpha}{2N_c} - \frac{2KF}{\Sigma} + \frac{\rho}{(FL)^2}\left[-\beta_1\rho^{-3/2}\right.\right.$$
$$\left.\left. + \left(3 + 2\nu^2 - 2\langle\nu^2\rangle\right)\zeta_2 + \frac{\gamma_1}{2}\right]\right\}$$

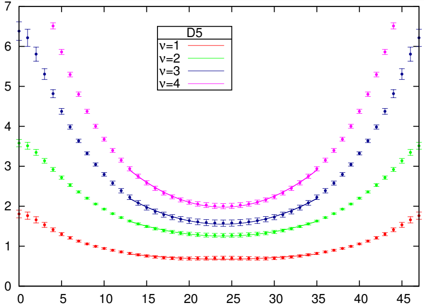
Statistics

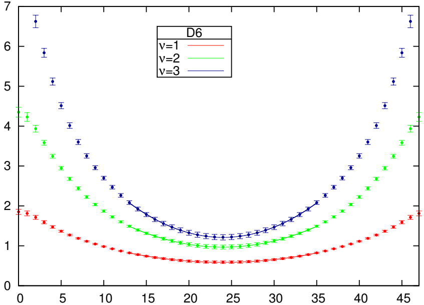
Lattice	Total # conf.	# of conf. by topological sector			
		$\nu = 1$	$\nu = 2$	$\nu = 3$	$\nu = 4$
D4	117	43	18	31	25
D5	129	40	43	23	23
D6	153	85	43	25	–

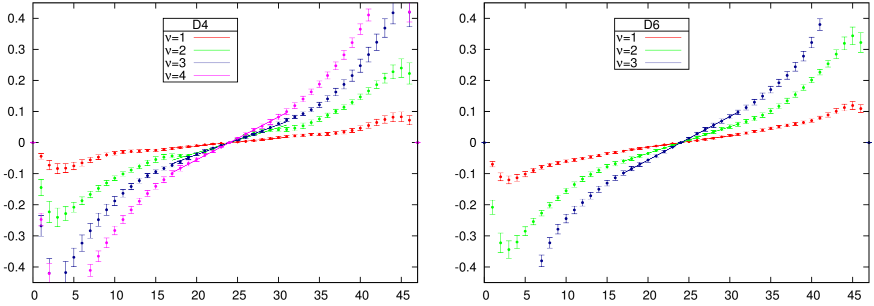
$$\begin{aligned} a &\sim 0.065 \text{ fm}, \\ V &= 24^3 \times 48. \end{aligned}$$

Pseudoscalar correlator in ChPT
Mixed regime - Quenched case





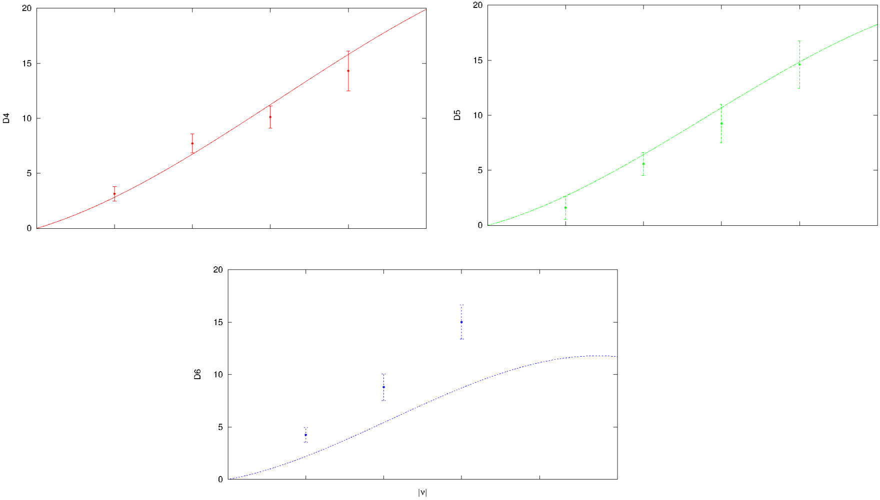



D4
ν=1
ν=2
ν=3
ν=4
D6
ν=1
ν=2
ν=3
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
5
10
15
20
25
30
35
40
45

Temporal dependance of the residue of $\langle PP \rangle$

$$\lim_{m\to 0} m^2 C_\nu(t) = \alpha_\nu + 2\beta_1 h_1(t) = \alpha_\nu - \frac{\beta_\nu}{12} + \beta_\nu \left(\frac{t}{T} - \frac{1}{2}\right)^2 + \mathcal{O}\left(\frac{t}{T} - \frac{1}{2}\right)^3$$

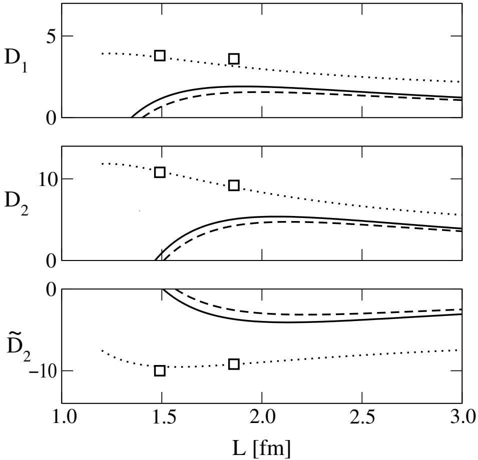
$\rightarrow$ fit of the time derivative of $\lim_{m\to 0} m^2 C_\nu(t)$ with functional form from mixed regime computation.



→ Bad convergence domain in the mixed regime.

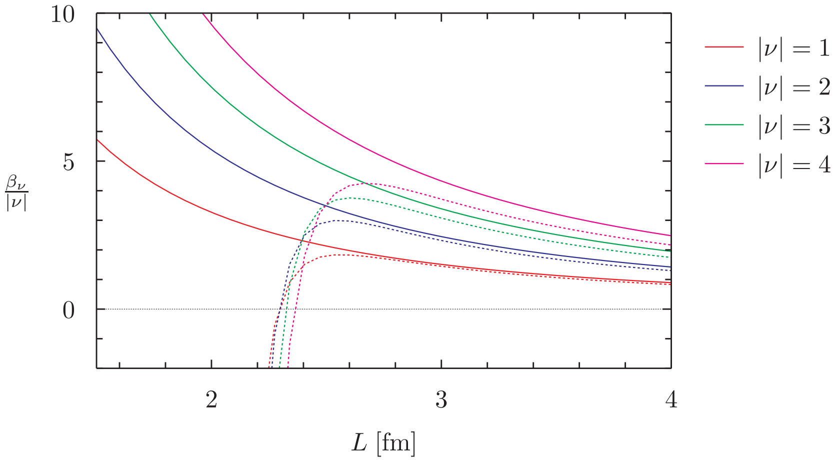
To compare with results from the ϵ-regime :

Giusti, Hernández, Laine, Weisz, Wittig (2004)



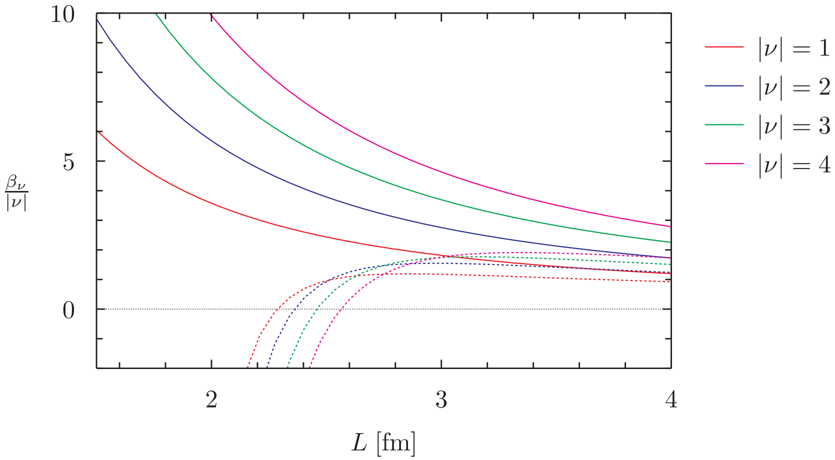
Solid : full $N_f = 3$ - Dashed : full $N_f = 2$ - Dotted : quenched $N_f = 0$.

β_ν vs. L for different values of $|\nu|$ – $M_{\rm ss} = 150$ MeV, $\rho = 1$



Solid : LO - Dashed : NLO.

β_ν vs. L for different values of $|\nu|$ – $M_{ss} = 150$ MeV, $\rho = 2$



Solid : LO - Dashed : NLO.

Conclusion

- Mixed action simulation is a tool to investigate QCD at low energy
  $\rightarrow$ Exact chiral symmetry with less numerical cost
- Explore "unusual" chiral regime : mixed regime
- Lattice artifacts can be very easily checked : use of ChPT at finite lattice spacing
- Allow to extract more precisely Σ, F and L_i's
- Role of the charm in $\Delta I = 1/2$ rule
- Weak LEC's : $K \rightarrow \pi\pi$ decay