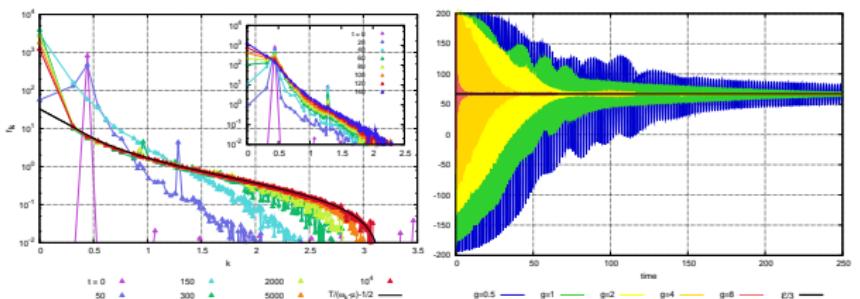


Thermalization in Scalar Field Theories



Montpellier, May 2012

Thomas EPELBAUM
IPhT

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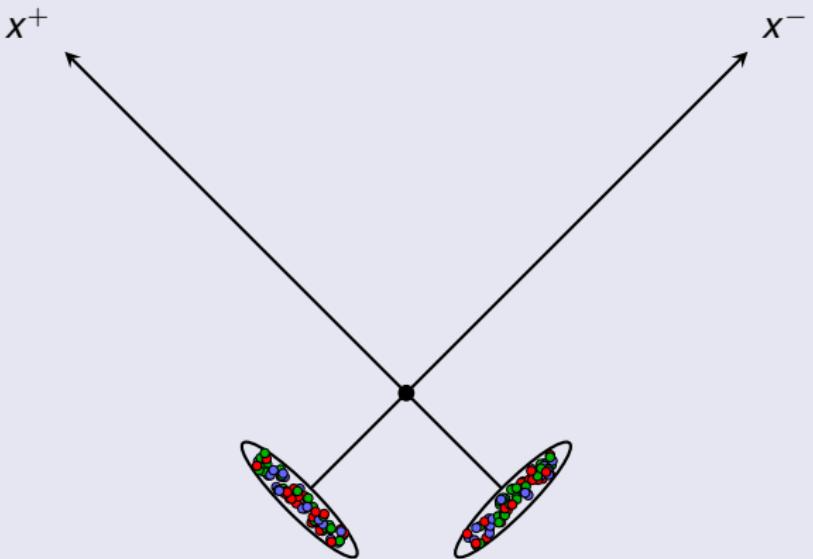
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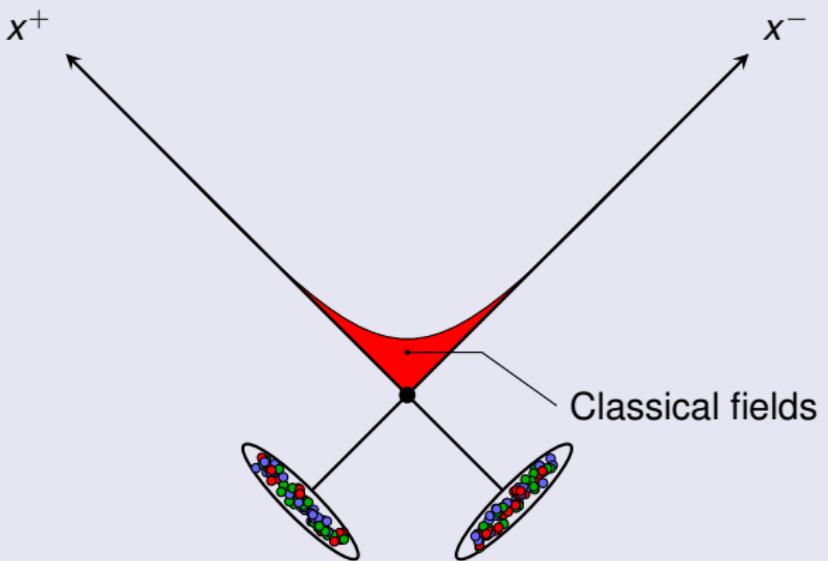
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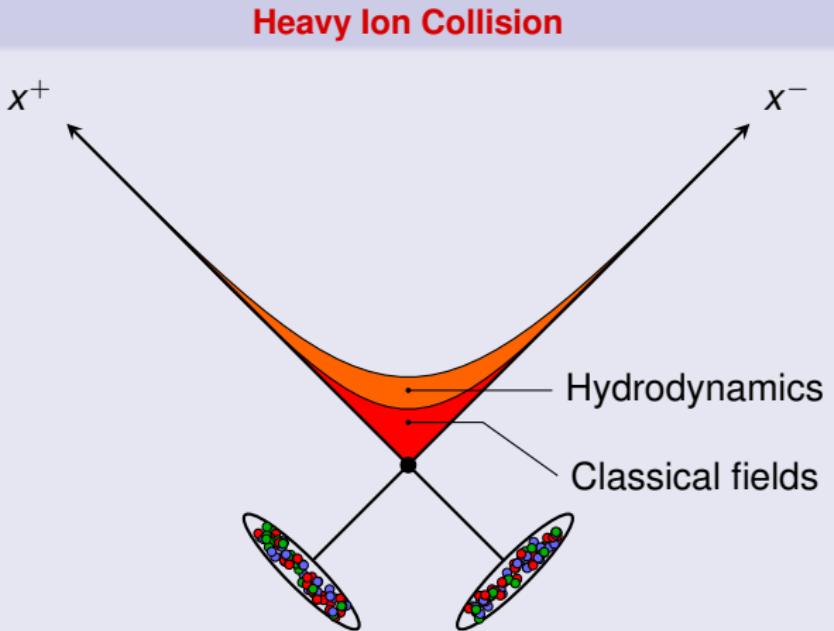
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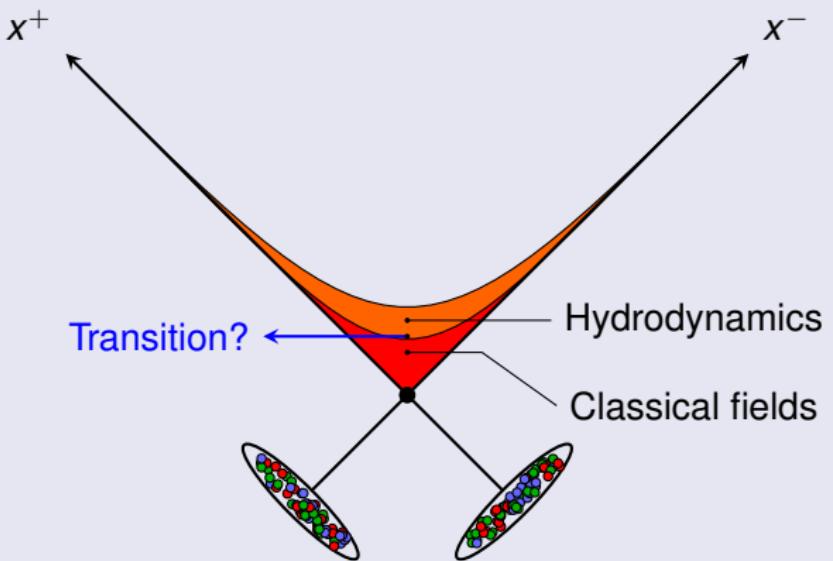
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The general picture

Hydro prerequisites

- Thermal equilibrium (Equation of state, BOSE-EINSTEIN distribution fonction...)
- Initial conditions: energy density, pressure
- Transport coefficients: viscosity...

Theoretical framework

- Color Glass Condensate (CGC) semi classical effective theory (unphysical momentum cutoff Λ between color fields and classical sources)
- JIMWLK equation (renormalization group equation for the evolution with Λ)

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Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Secular divergences

- LO \mapsto Finite results, but no equilibration
- NLO \mapsto Secular divergences \mapsto NLO > LO

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Secular divergences

- LO \mapsto Finite results, but no equilibration
- NLO \mapsto Secular divergences \mapsto NLO > LO

Energy-momentum tensor at LO

$$T_{\text{LO}}^{\mu\nu} = (\partial^\mu \varphi)(\partial^\nu \varphi) - g^{\mu\nu} \mathcal{L}(\varphi),$$

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \varphi)} = \frac{\delta \mathcal{L}}{\delta \varphi}$$

$$\varphi = \varphi_0 \text{ at } t_0$$

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Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Energy-momentum tensor at NLO

$$T_{\text{NLO}}^{\mu\nu} = \hat{O} T_{\text{LO}}^{\mu\nu}[\varphi_0]$$

$$\hat{O} = \frac{1}{2} \int_{u,v \in \Sigma} G(u,v) \frac{\delta}{\delta \varphi_0(u)} \frac{\delta}{\delta \varphi_0(v)}$$

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Resummed Energy-momentum tensor

$$T_{\text{resum}}^{\mu\nu} = e^{\hat{O}} T_{\text{LO}}^{\mu\nu}[\varphi_0] = T_{\text{LO}}^{\mu\nu} + T_{\text{NLO}}^{\mu\nu} + \dots$$

$$\hat{O} = \frac{1}{2} \int_{u,v \in \Sigma} G(u,v) \frac{\delta}{\delta \varphi_0(u)} \frac{\delta}{\delta \varphi_0(v)}$$

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$$\hat{O} = \frac{1}{2} \int_{u,v \in \Sigma} G(u,v) \frac{\delta}{\delta \varphi_0(u)} \frac{\delta}{\delta \varphi_0(v)}$$

Equivalent formulation

$$e^{\frac{\gamma}{2} \partial_x^2} f(x) = \int_{-\infty}^{+\infty} \frac{dz}{\sqrt{2\pi\gamma}} e^{-\frac{z^2}{2\gamma}} f(x+z)$$

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Resummed Energy-momentum tensor

$$T_{\text{resum}}^{\mu\nu} = e^{\hat{O}} T_{\text{LO}}^{\mu\nu}[\varphi_0] = T_{\text{LO}}^{\mu\nu} + T_{\text{NLO}}^{\mu\nu} + \dots$$

$$\hat{O} = \frac{1}{2} \int_{u,v \in \Sigma} G(u,v) \frac{\delta}{\delta \varphi_0(u)} \frac{\delta}{\delta \varphi_0(v)}$$

Equivalent formulation

$$T_{\text{resum}}^{\mu\nu} = \int [D\mathbf{a}(u)] e^{-\frac{1}{2} \int_{u,v \in \Sigma} \mathbf{a}(u) G^{-1}(u,v) \mathbf{a}(v)} T_{\text{LO}}^{\mu\nu}[\varphi_0 + \mathbf{a}]$$

- Solve the EOM for the initial condition φ_0 with random gaussian fluctuations \mathbf{a} on top of it.
- Semi-classical calculation that takes into account some quantum corrections.
- Functional integral done by Monte Carlo sampling.

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Scalar field theory

Lagrangian of the theory

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)} + J\phi$$

where

$$J \propto \theta(-x^0)$$

Why do we use this model?

- Scale invariance in 3 + 1 dimensions
- Parametric resonance
- A lot simpler!

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Form of the solution

Initial condition of the EOM

$$\phi_i(t, \mathbf{x}) = \varphi_0(\mathbf{x}) + \sum_{\mathbf{k}} \operatorname{Re} [c_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t} v_{\mathbf{k}}(\mathbf{x})]$$

with

$$[-\Delta + V''(\varphi_0)] v_{\mathbf{k}}(\mathbf{x}) = \omega_{\mathbf{k}}^2 v_{\mathbf{k}}(\mathbf{x})$$

$$\langle c_{\mathbf{k}} c_{\mathbf{l}}^* \rangle = \delta(\mathbf{k} - \mathbf{l})$$

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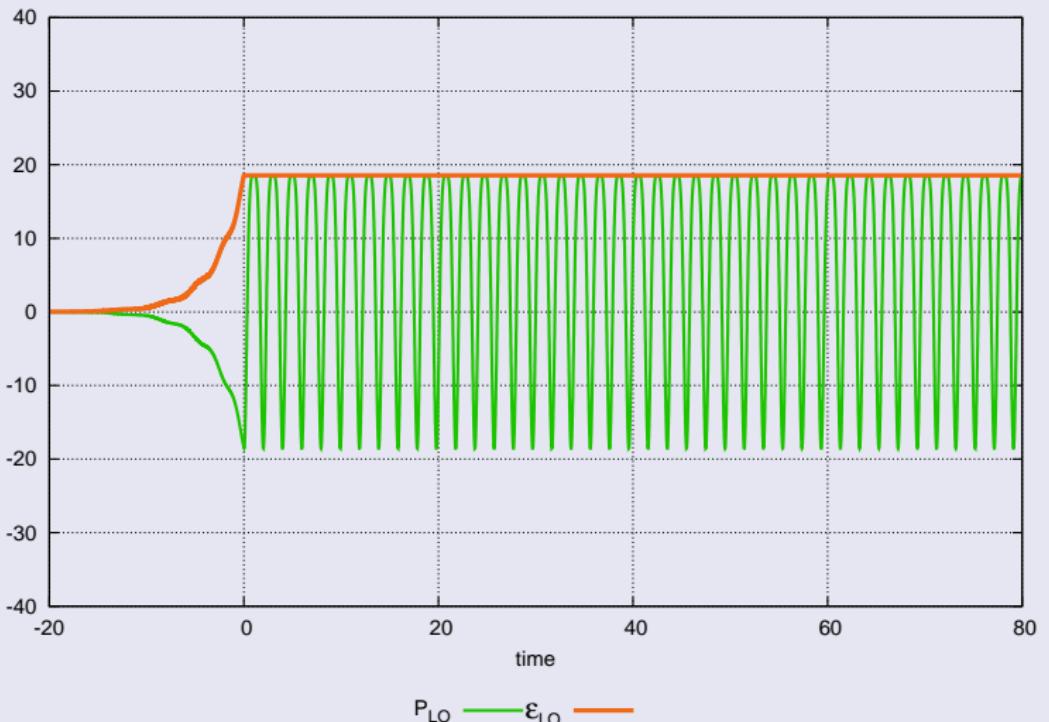
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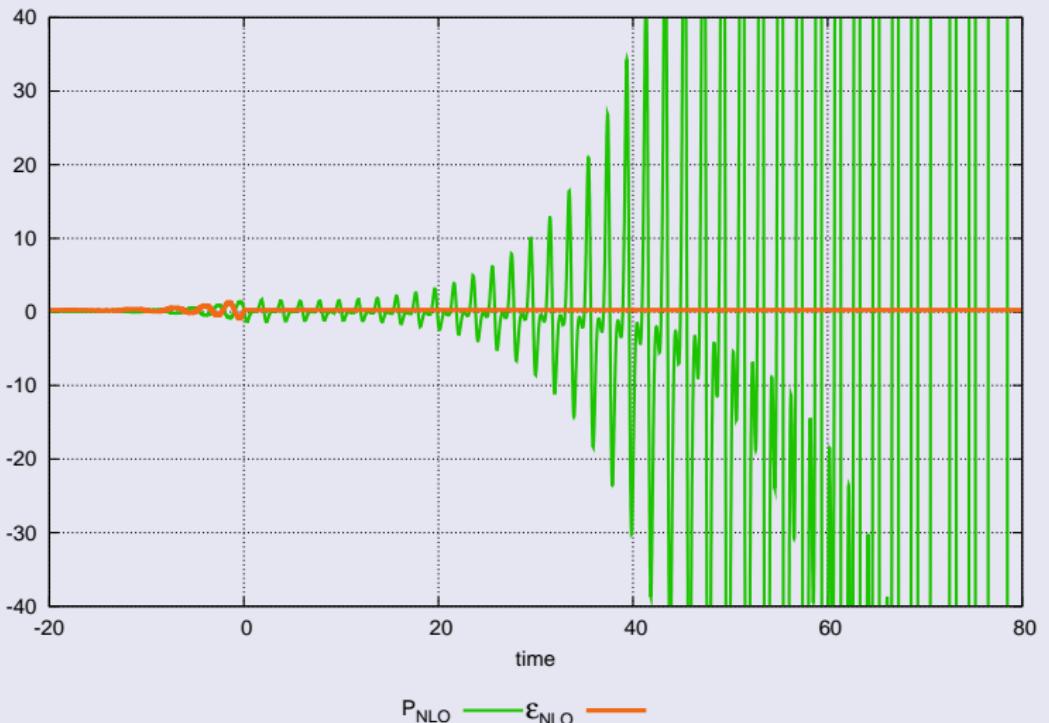
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$T^{\mu\nu}_{\text{NLO}}$: secular divergences

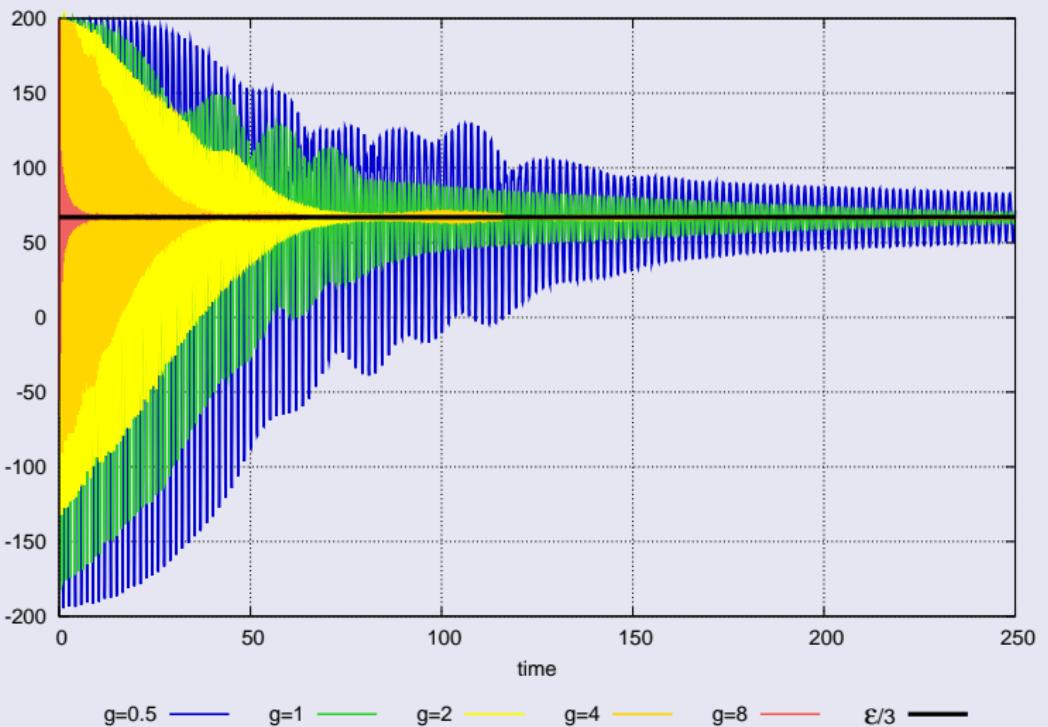


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$T^{\mu\nu}_{\text{resum}}$: Pressure equilibration

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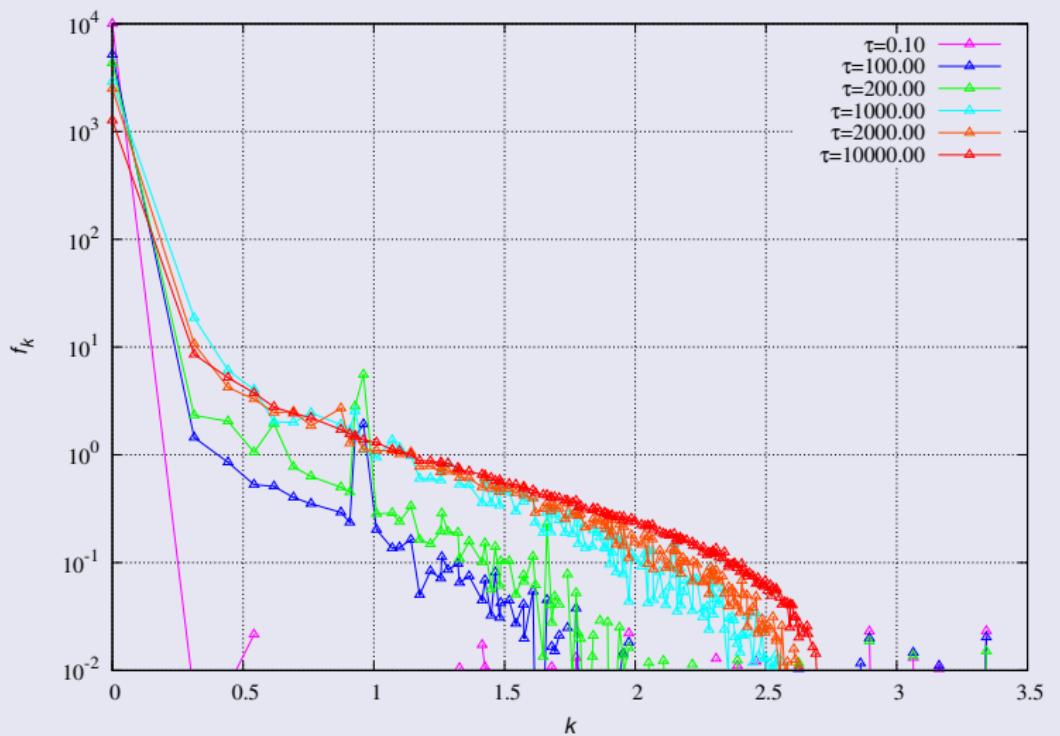
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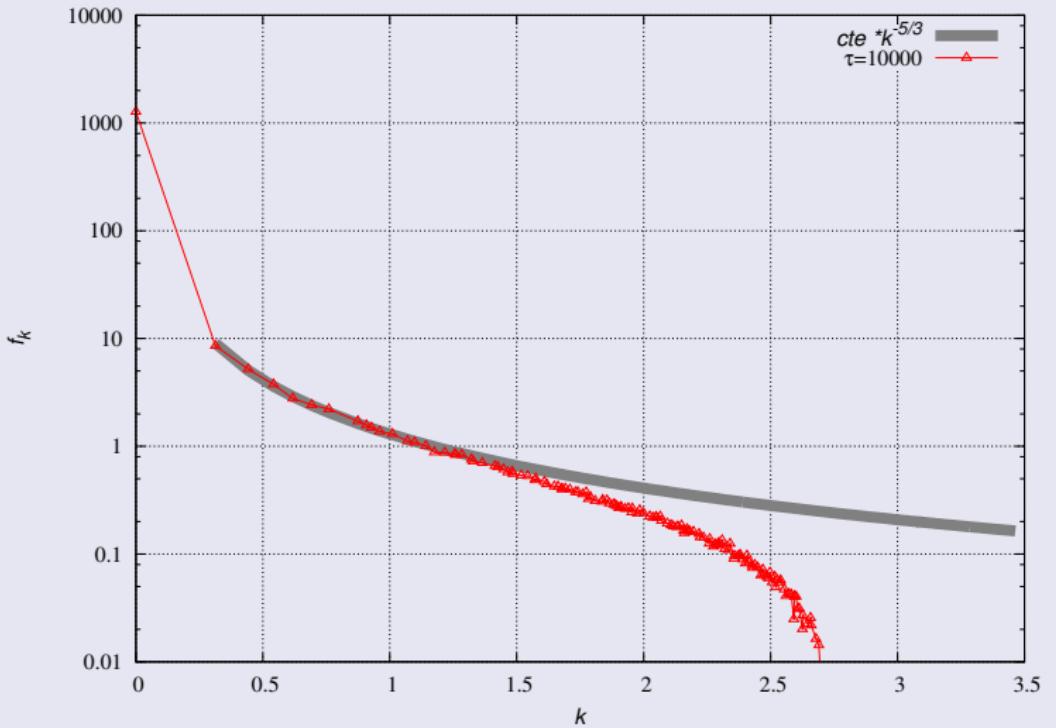
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Time evolution of the occupation number [TE, GELIS (2011)]

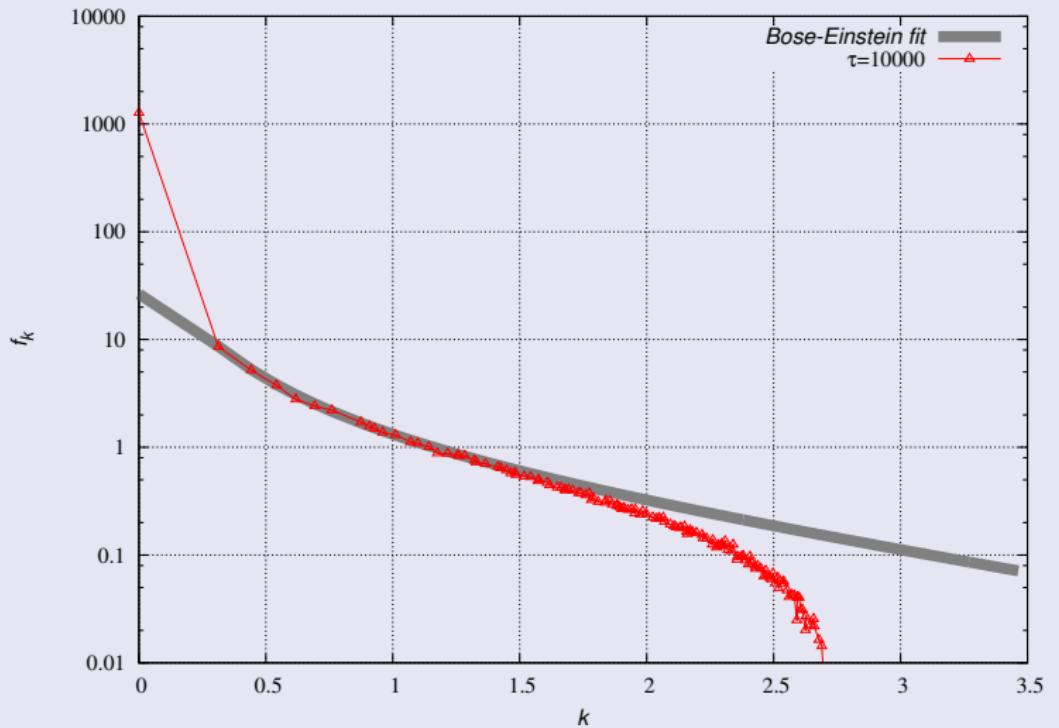


Kolmogorov scaling at late times?



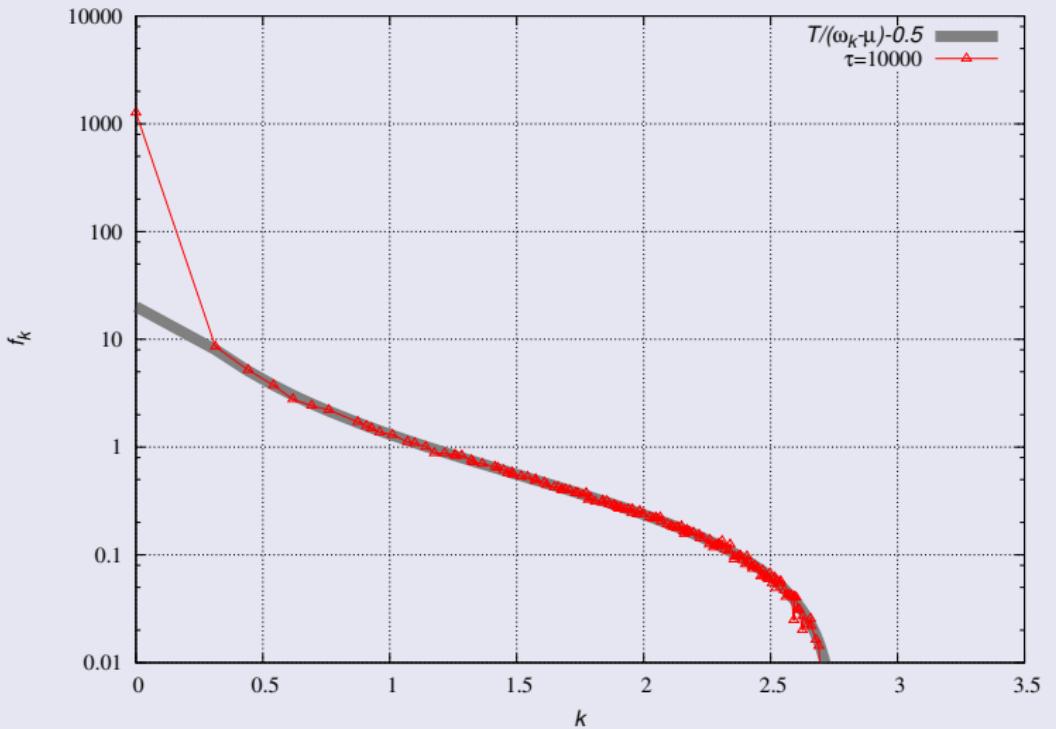
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BOSE-EINSTEIN equilibrium distribution?



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"Classical" equilibrium distribution



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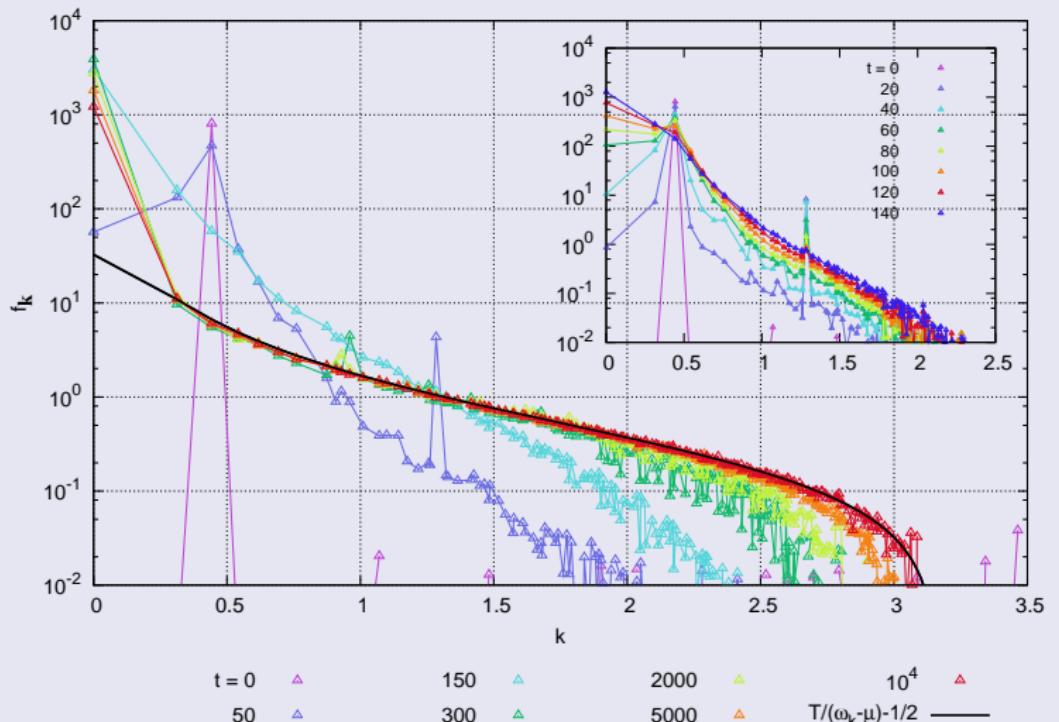
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Non-zero initial mode: $\varphi_0 \sim \cos k.x$



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Evolution of the condensate

$$f_{\mathbf{k}} = \frac{T}{\omega_{\mathbf{k}} - \mu} - \frac{1}{2} + n_0 \delta(\mathbf{k})$$

implies

$$\frac{f_0}{V} = \text{cte}$$

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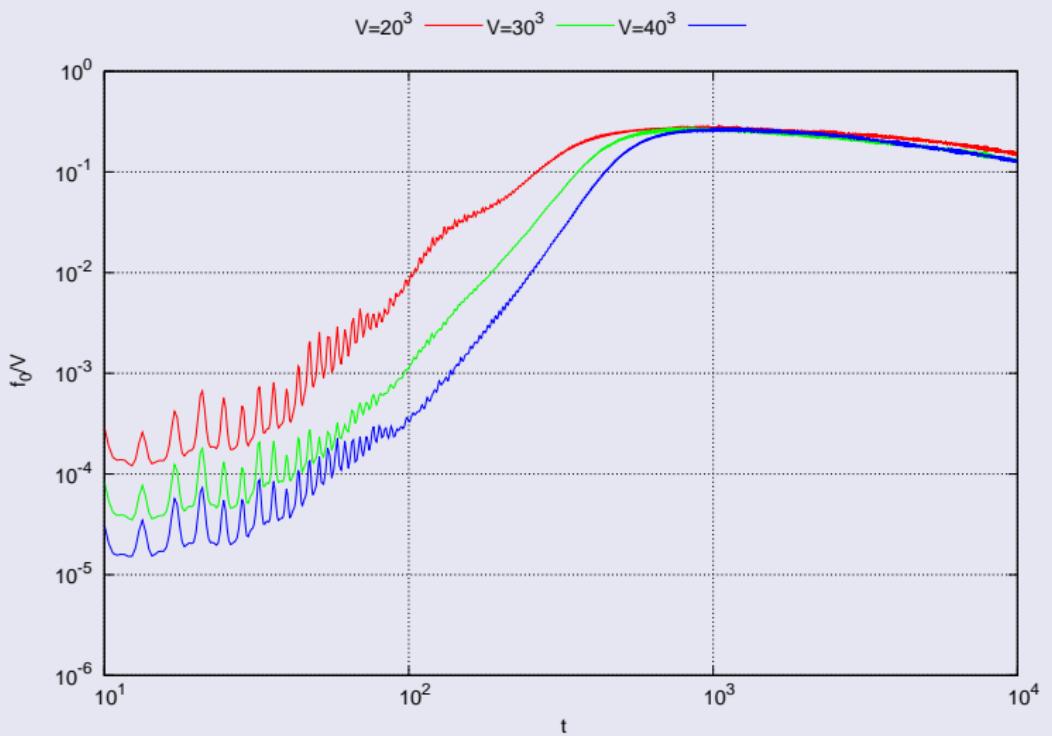
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Evolution of the condensate



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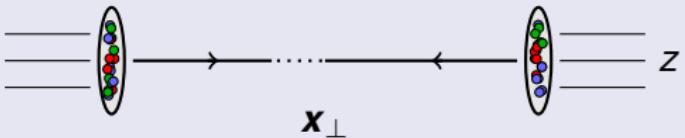
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Scalar field theory

Adapted coordinate system to describe a Heavy Ion Collision?



System boost invariant in z direction

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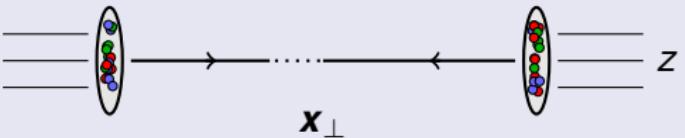
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System boost invariant in z direction

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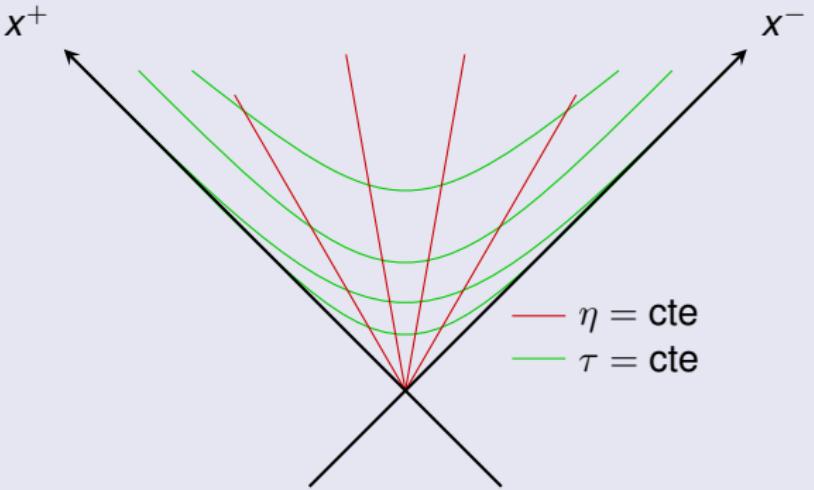
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Proper time/rapidity coordinate system

$$\eta = \frac{1}{2} \ln \frac{t+z}{t-z} \quad \tau = \sqrt{t^2 - z^2}$$

Scalar field theory

Proper time/rapidity coordinate system



Proper time/rapidity coordinate system

$$\eta = \frac{1}{2} \ln \frac{t+z}{t-z}$$

$$\tau = \sqrt{t^2 - z^2}$$

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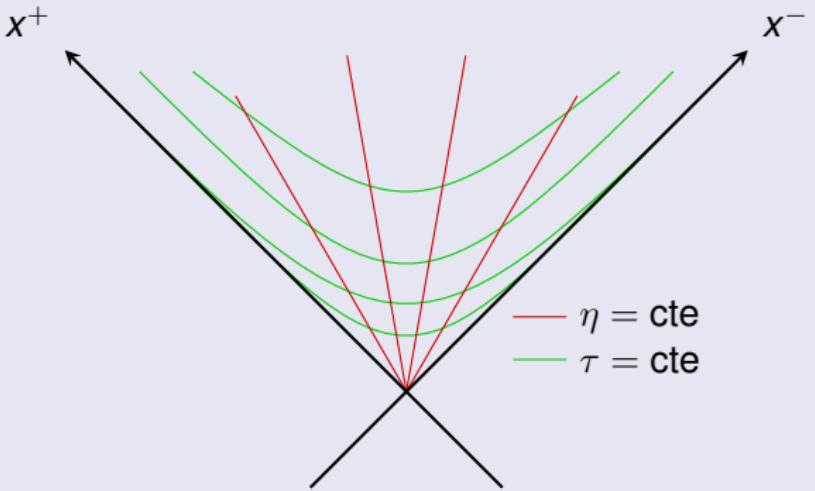
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Proper time/rapidity coordinate system

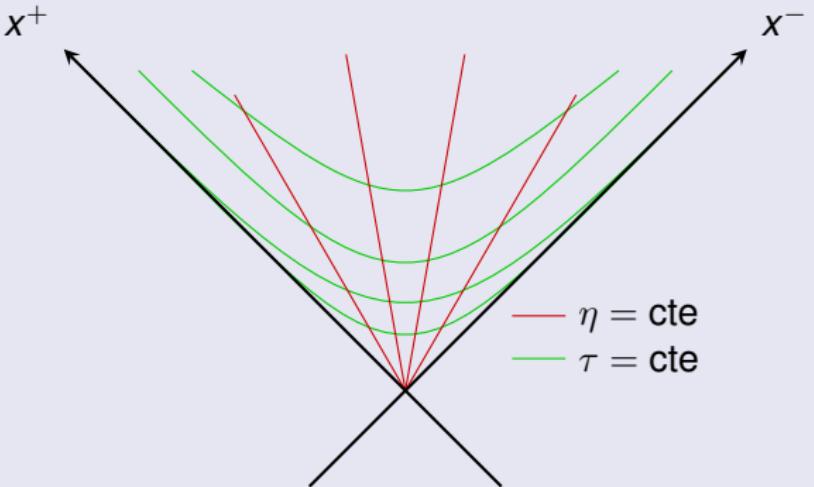


EOM for a boost-invariant field

$$\left[\frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial}{\partial \tau} - \nabla_{\perp}^2 \right] \varphi + V'(\varphi(\tau, \mathbf{x}_{\perp})) = 0$$

Scalar field theory

Proper time/rapidity coordinate system



EOM for a small fluctuation a

$$\left[\frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial}{\partial \tau} - \nabla_{\perp}^2 - \frac{1}{\tau^2} \frac{\partial^2}{\partial \eta^2} + V''(\varphi) \right] a(\tau, \eta, \mathbf{x}_{\perp}) = 0$$

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Form of the solution

Initial condition of the EOM

$$\phi_i(\tau, \mathbf{x}_\perp, \eta) = \varphi_0(\mathbf{x}_\perp) + \sum_{\mathbf{k}_\perp, \nu} \operatorname{Re} \left[c_{\mathbf{k}_\perp, \nu} H_{i\nu}^{(2)}(\omega_{\mathbf{k}_\perp} \tau) e^{i\nu\eta} v_{\mathbf{k}_\perp}(\mathbf{x}_\perp) \right]$$

with the Hankel functions $H_{i\nu}^{(2)}$ being the equivalent of the plane waves in the fixed volume case , and

$$[-\Delta_\perp + V''(\varphi_0)] v_{\mathbf{k}_\perp}(\mathbf{x}_\perp) = \omega_{\mathbf{k}_\perp}^2 v_{\mathbf{k}_\perp}(\mathbf{x}_\perp)$$

$$\langle c_{\mathbf{k}_\perp, \nu} c_{\mathbf{l}_\perp, \mu}^* \rangle = \delta(\mathbf{k}_\perp - \mathbf{l}_\perp) \delta(\nu - \mu)$$

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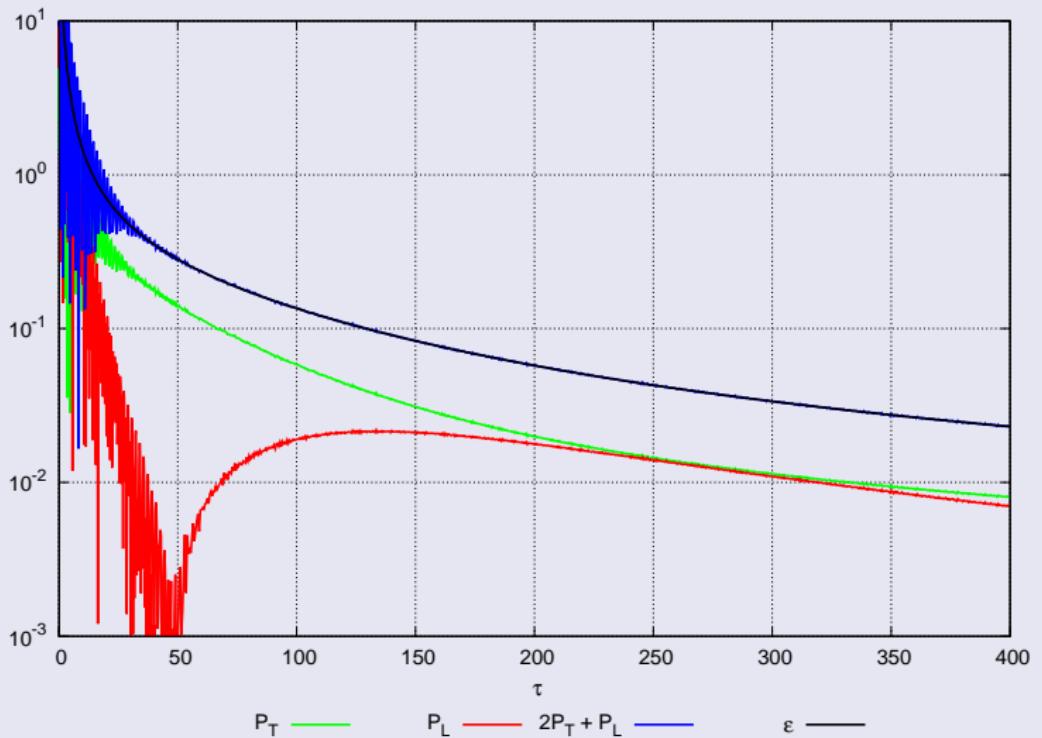
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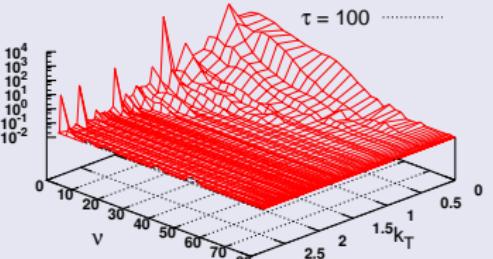
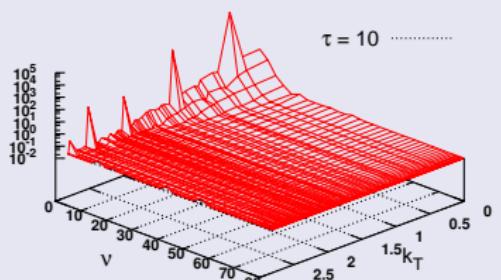
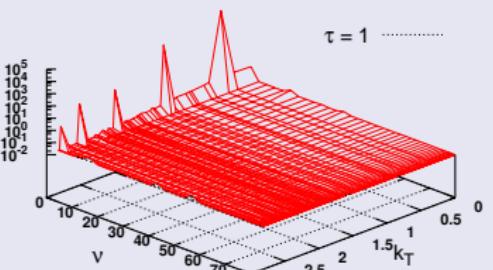
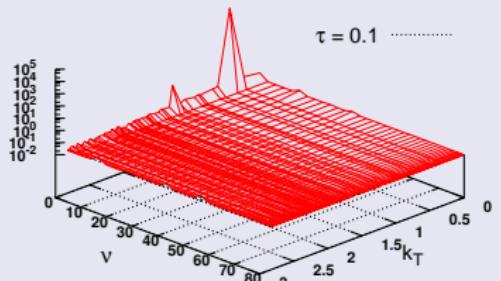
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Evolution of the distribution function [Work in progress]



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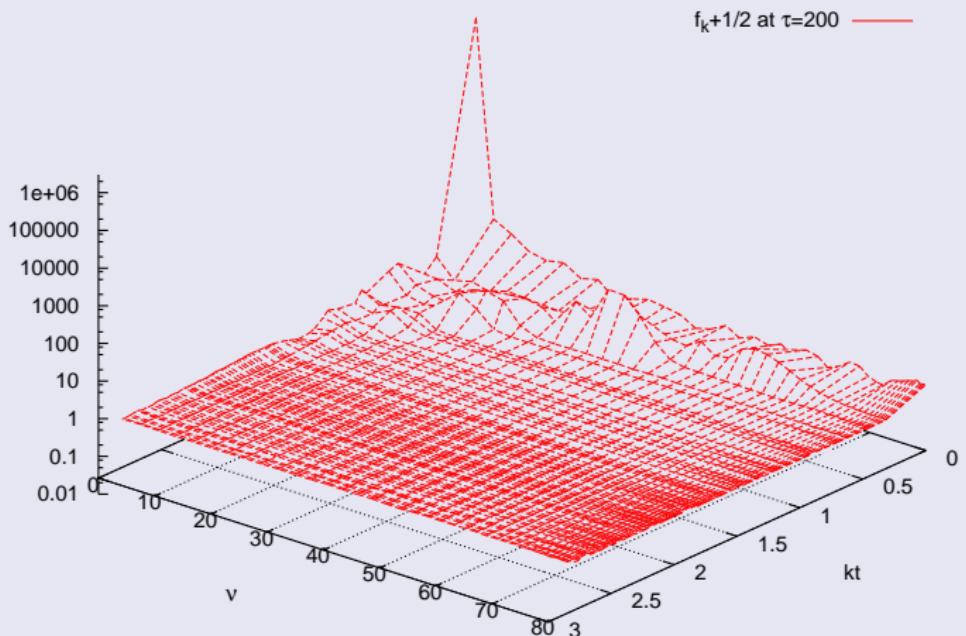
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Lattice artifacts [Work in progress]

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In thermal equilibrium $\nu \propto \tau^{\frac{2}{3}}$

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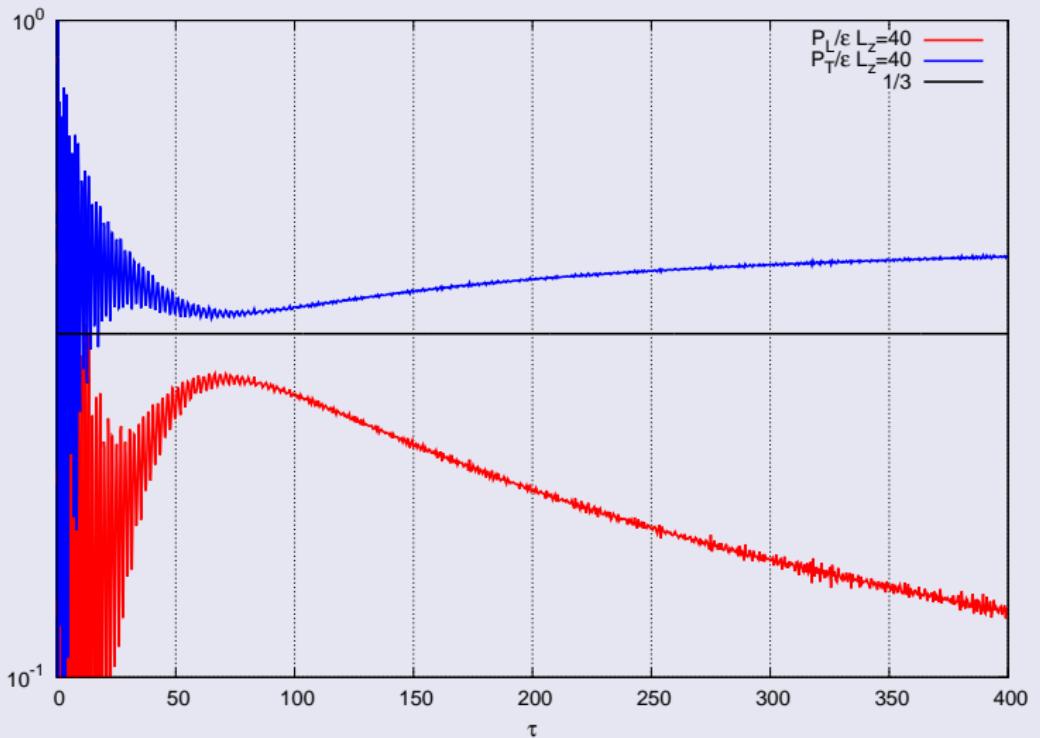
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$T_{\text{resum}}^{\mu\nu}$: Time evolution of $\frac{P_{L,T}}{\epsilon}$ [Work in progress]

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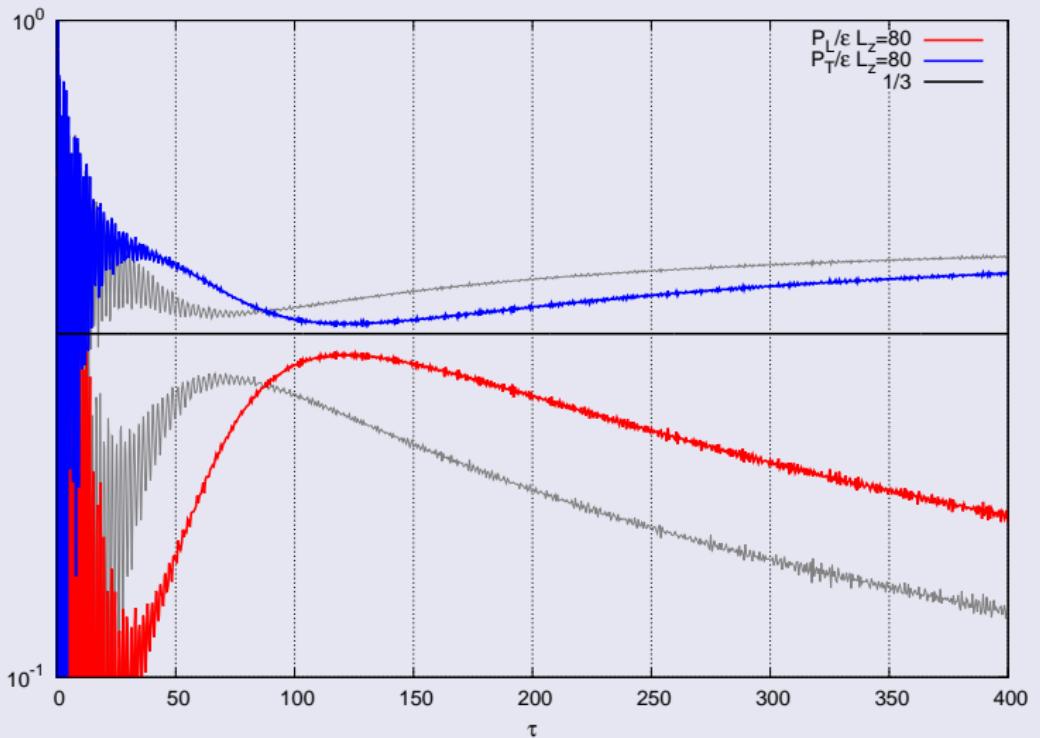
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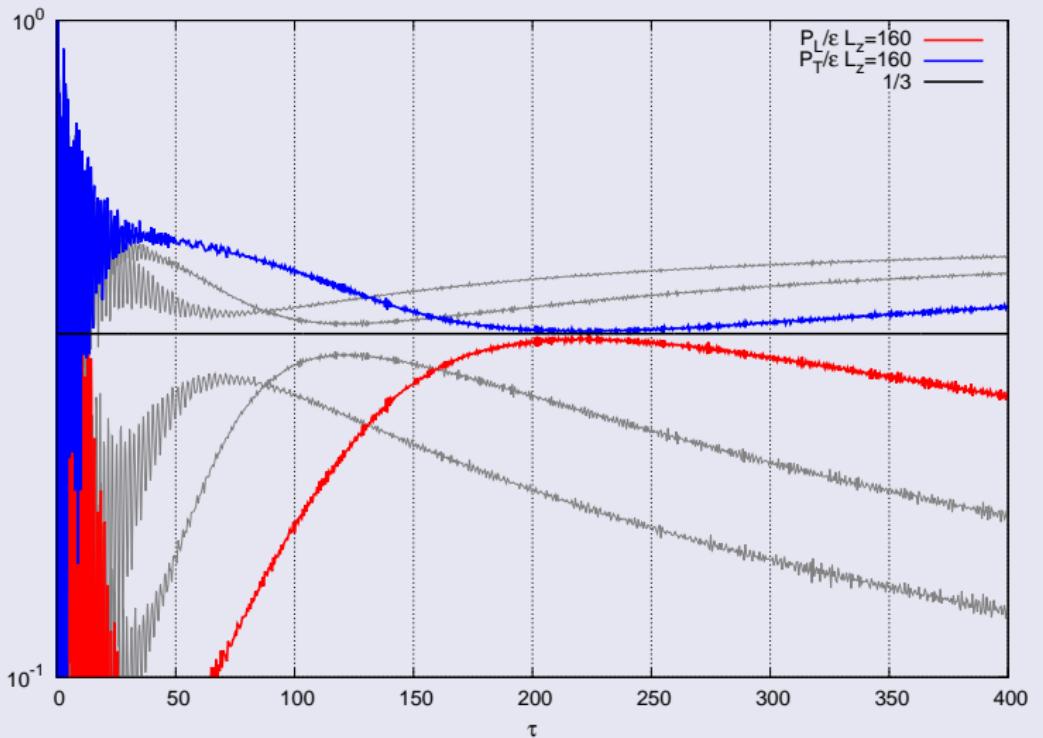
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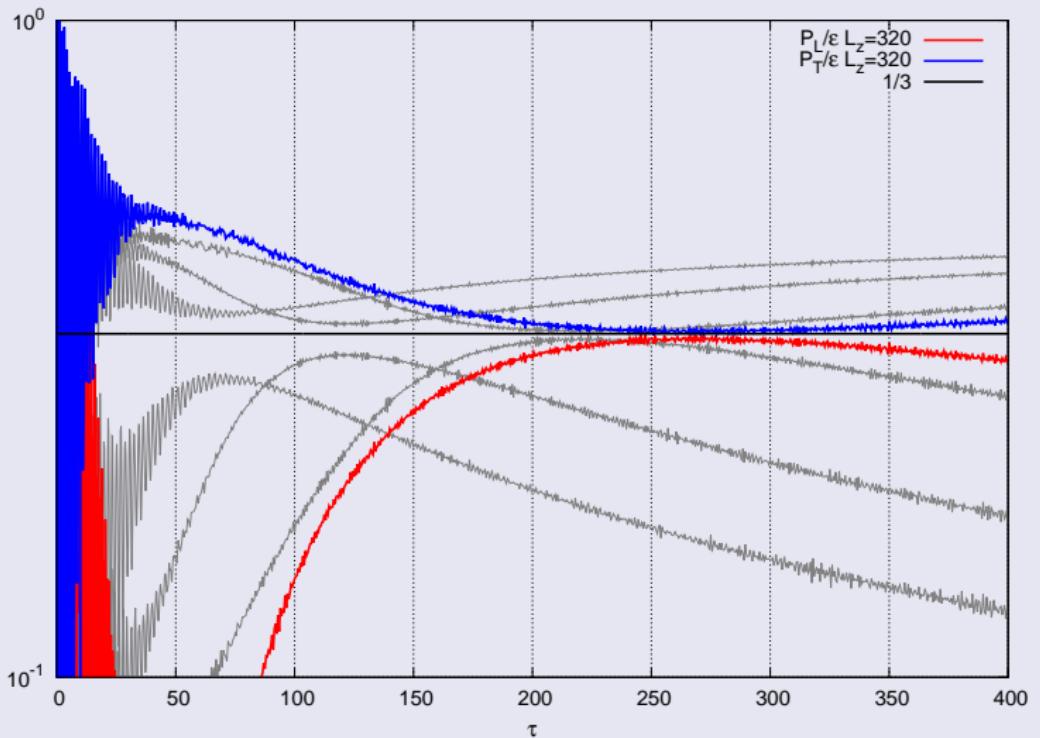
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Principal results for the fixed volume

- Equation of state $\epsilon = 3P$
- $P_x = P_y = P_z$
- $f_k \propto \frac{T}{\omega_k - \mu} - \frac{1}{2}$ at late times
- BOSE-EINSTEIN condensate

Principal results for the expanding volume

- Equation of state
- BOSE-EINSTEIN condensate?
- $P_L = P_T$?

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