Model independent view of $b \to s\ell^-\ell^+$

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A very brief history of $b \to s$ transitions

- $B \to K^* \gamma$, $B \to X_s \gamma$ – first penguins were seen [CLEO '93, '94]
- $B \to K^* \ell^+ \ell^-$, $B \to X_s \ell^+ \ell^-$
- B_s mixing observed [CDF '06]
- CP violation in B_s mixing (compatible with small SM prediction) [D0, CDF '09, LHCb '11]
- LHC is now closing in on the SM $\mathcal{B}(B_s \to \mu^+ \mu^-)$

$$\mathcal{B}^{\mathrm{th-SM}} = (3.1 \pm 0.2) \times 10^{-9}$$

$\in [2.8, 44] \times 10^{-9}$ [CDF]

$< 7.7 \times 10^{-9}$ [CMS]

$< 4.5 \times 10^{-9}$ [LHCb]

Motivation

- What information $B_s \to \mu^+\mu^-$ still holds about NP?
- Scalar operators first to show up
- Turn the anticipated enhancement in supersymmetry into a strong constraint. $B_s \to \mu^+\mu^-$ can be suppressed as well.
- Few structures enter the prediction of $\mathcal{B}(B_s \to \mu^+\mu^-)$

$$C_S - C_S', \quad C_P - C_P', \quad C_{10} - C_{10}'$$

  Decay constant f_{B_s} known with good precision.
- Orthogonal to this, $B \to K\mu^+\mu^-$, probes,

$$C_S + C_S', \quad C_P + C_P', \quad C_{10} + C_{10}'$$

  $+ \; C_{9,T,T5}^{(\prime)}$. Form factor uncertainties are moderate.
- $C_{10}^{(\prime)}$ could be further searched for in $B \to K^*\ell^+\ell^-$

Outline

$(\bar{s}\Gamma b)(\bar{\ell}\Gamma'\ell)$ effective Hamiltonian at μ_b

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[\sum_{i=1}^{6} C_i(\mu)\mathcal{O}_i(\mu) + \sum_{i=7,8,9,10,P,S} \left(C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu) \right) \right]$$

$$\mathcal{O}_7 = \frac{em_b}{g^2}(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}$$

$$\mathcal{O}_8 = \frac{m_b}{g}(\bar{s}\sigma_{\mu\nu}T^a P_R b)G^{\mu\nu,a}$$

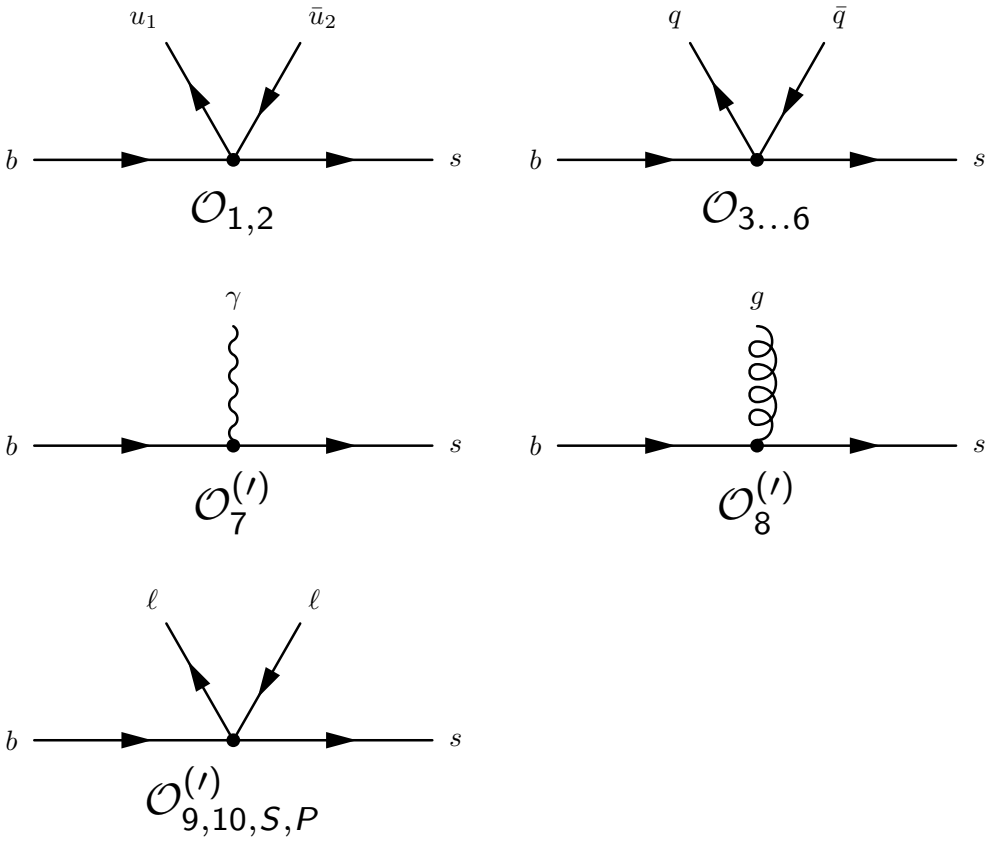
$$\mathcal{O}_9 = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_{10} = \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_S = \frac{e^2}{16\pi^2}(\bar{s}P_R b)(\bar{\ell}\ell)$$

$$\mathcal{O}_P = \frac{e^2}{16\pi^2}(\bar{s}P_R b)(\bar{\ell}\gamma_5 \ell)$$

$$+\mathcal{O}'_{7\ldots P}/\cdot\{P_L \leftrightarrow P_R\}$$



$(\bar{s}\Gamma b)(\bar{\ell}\Gamma'\ell)$ effective Hamiltonian at μ_b

- Amplitudes are compactly represented in terms of effective Wilson coefficients

$$C_7^{\rm eff} = \frac{4\pi}{\alpha_s} C_7 - \frac{1}{3} C_3 - \frac{4}{9} C_4 - \frac{20}{3} C_5 - \frac{80}{9} C_6$$

$$C_8^{\rm eff} = \frac{4\pi}{\alpha_s} C_8 + C_3 - \frac{1}{6} C_4 + 20 C_5 - \frac{10}{3} C_6 \qquad \text{likewise for } C_i'^{\rm eff}$$

$$C_9^{\rm eff} = \frac{4\pi}{\alpha_s} C_9 \quad (+ Y(q^2))$$

$$C_{10}^{\rm eff} = \frac{4\pi}{\alpha_s} C_{10}$$

- SM values, NNLL [Altmanshofer '08]

$C_1 = -0.257 \qquad C_2 = 1.009 \qquad C_3 = -0.005 \qquad C_4 = -0.078 \qquad C_5 = 0.0 \qquad C_6 = 0.001$

$$C_7^{\rm eff} = -0.304 \qquad C_8^{\rm eff} = -0.167$$

$$C_9^{\rm eff} = 4.211 \qquad C_{10}^{\rm eff} = -4.103$$

$B_s \to \mu^+\mu^-$

- Only $C_{10}^{(\prime)}$ helicity suppressed

$$\Gamma(B_s \to \ell^+\ell^-) = \frac{G_F^2\alpha^2}{64\pi^3}\sqrt{1-\frac{4m_\ell^2}{m_{B_s}^2}}\; m_{B_s}^3 f_{B_s}^2 |V_{tb}V_{ts}|^2 \Big[|C_S - C_S'|^2 \frac{m_{B_s}^2}{m_b^2}(1-4m_\ell^2/m_{B_s}^2) + \left|(C_P - C_P')\frac{m_{B_s}}{m_b} + \frac{2m_\ell}{m_{B_s}}(\mathbf{C_{10}} - C_{10}')\right|^2 \Big]$$

- Small hadronic uncertainties: $f_{B_s} = 234 \pm 6\ \mathrm{MeV}$, [Naive avg. of ETMC, Fermilab-MILC, HPQCD results]

- Prediction with C_{10}^{SM}, while $C_{10}' = C_S^{(\prime)} = C_P^{(\prime)} = 0$

$$\mathcal{B}(B_s \to \mu^+\mu^-) = (3.1 \pm 0.2) \times 10^{-9}$$

- LHCb bound

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 4.5 \times 10^{-9} \qquad (95\,\%\mathrm{CL})$$

- Decay with $\ell = e$ is further suppressed in the SM. Clean test of C_S, C_P.

$B \to K\ell^+\ell^-$ spectrum

$$\frac{d\Gamma}{dq^2}(B \to K\ell^+\ell^-) \sim \sqrt{\lambda(q^2)}\beta_\ell(q^2)\Big[q^2(\beta_\ell^2|F_S|^2 + |F_P|^2) + \frac{\lambda(q^2)}{6}(|F_A|^2 + |F_V|^2) + 2m_\ell(m_B^2 - m_K^2 + q^2)\mathrm{Re}\,(F_P F_A^*) + 4m_\ell^2 m_B^2 |F_A|^2\Big]$$

$$F_A(q^2) = f_+(q^2)(C_{10} + C_{10}') \qquad F_S(q^2) = \frac{1}{2}\frac{m_B^2 - m_K^2}{m_b} f_0(q^2)(C_S + C_S')$$

$$F_V(q^2) = f_+(q^2)(C_9 + C_9') + \frac{2m_b}{m_B + m_K} f_T(C_7 + C_7')$$

$$F_P(q^2) = \frac{1}{2}\frac{m_B^2 - m_K^2}{m_b} f_0(q^2)(C_P + C_P') - m_\ell(C_{10} + C_{10}')\left[f_+(q^2) - \frac{m_B^2 - m_K^2}{q^2}(f_0(q^2) - f_+(q^2))\right]$$

- $B \to K$ form factors

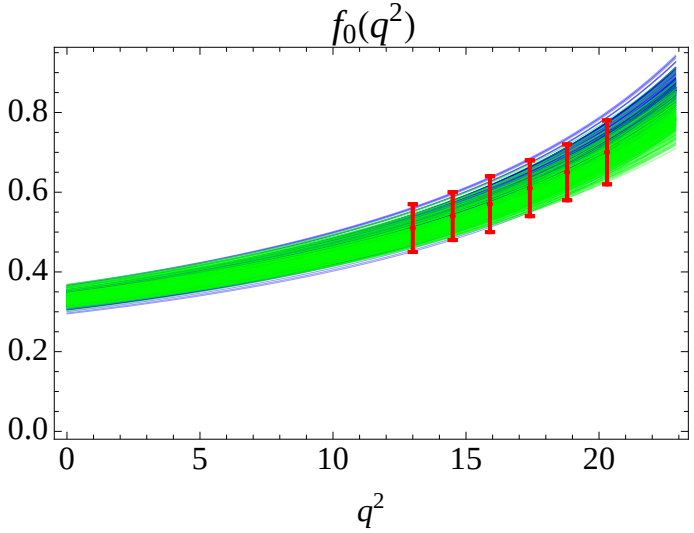
$$C_7^{(\prime)} \to f_T$$
$$C_{9,10}^{(\prime)} \to f_+(q^2), f_0(q^2)$$
$$C_{S,P}^{(\prime)} \to f_0/m_b$$

$$\langle K(k)\,|\,\bar{s}\gamma_\mu b\,|\,B(p)\rangle = \left[(p+k)_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu\right] f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q_\mu f_0(q^2)$$

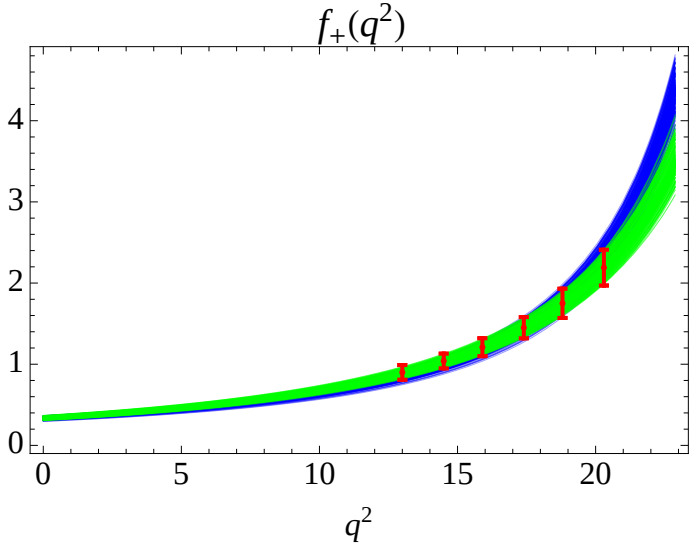
$$\langle K(k)\,|\,\bar{s}\sigma_{\mu\nu} b\,|\,B(p)\rangle = i(p_\mu k_\nu - p_\nu k_\mu)\frac{2f_T(q^2)}{m_B + m_K}$$

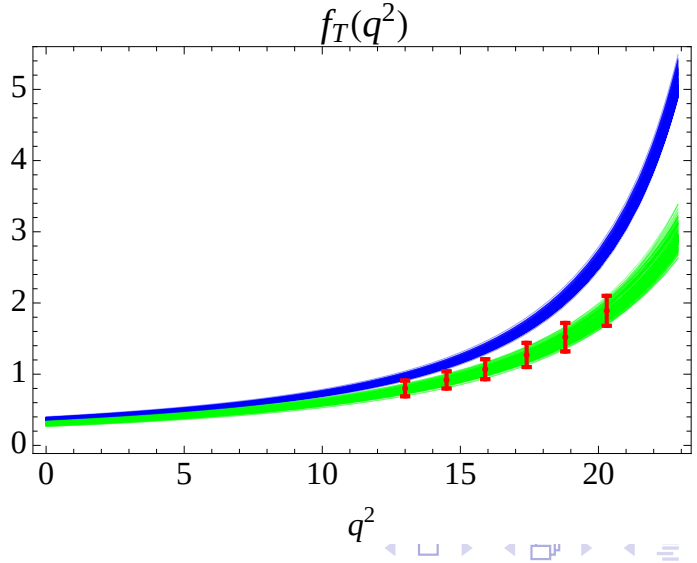
- Dominant uncertainties come from the form factors.
- Wide range of $q^2 \in [0, (m_B - m_K)^2]$. Opportunities for different nonperturbative techniques.

$B \to K\ell^+\ell^-$ form factors

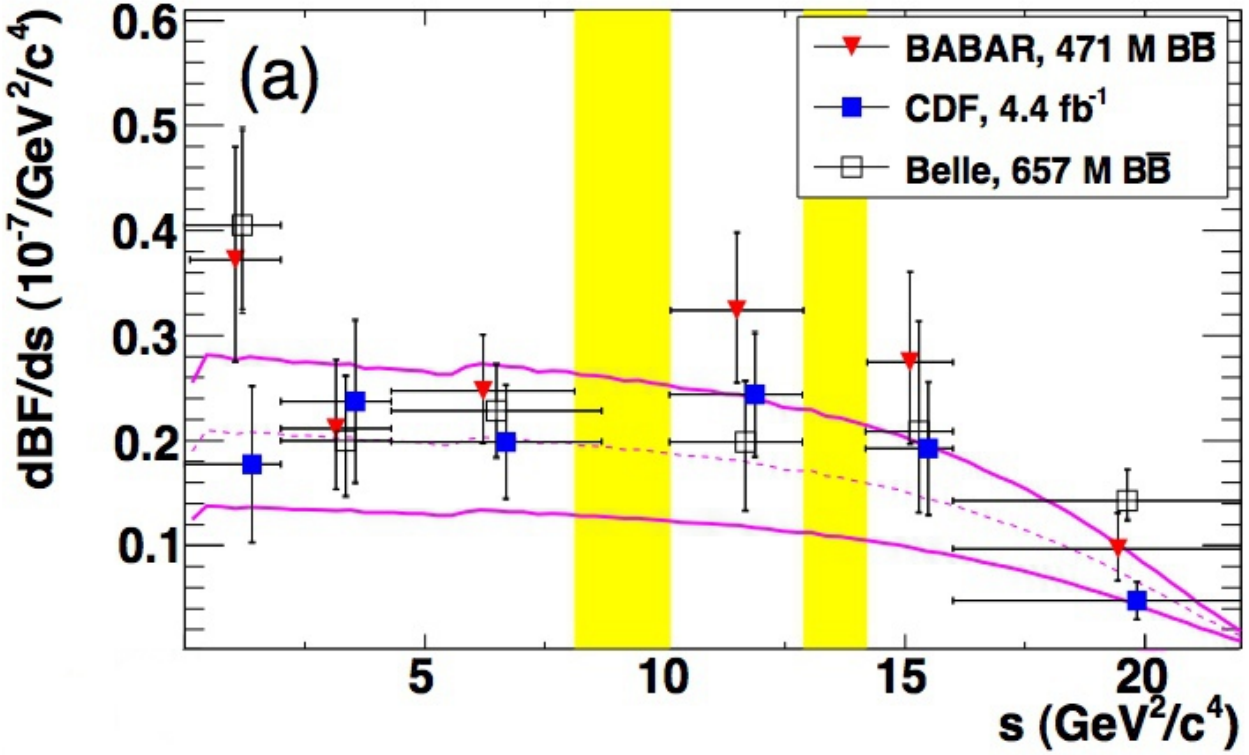


Light cone QCD sum rules [Ball '05; Khodjamirian '07],
Lattice QCD
Lattice points (quenched) [Abada '01]





$B \to K\ell^+\ell^-$ spectrum

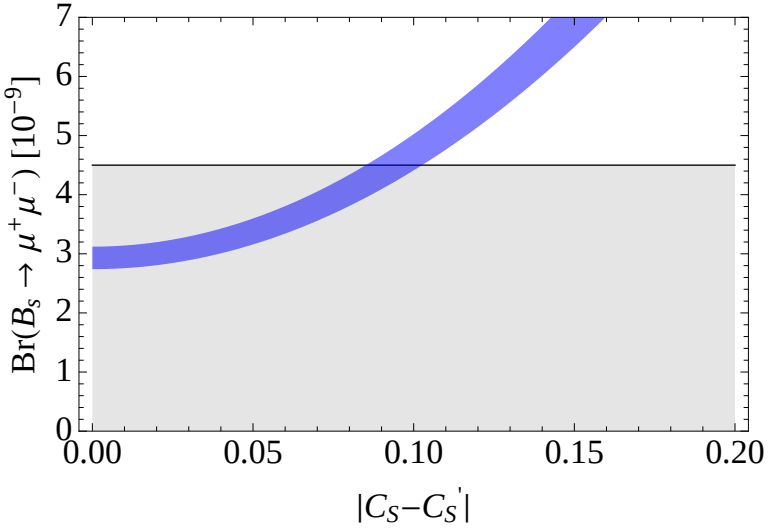


$$\mathcal{B}(B \to K\ell^+\ell^-) = (4.7 \pm 0.6 \pm 0.2) \times 10^{-7}$$ [BaBar '12]

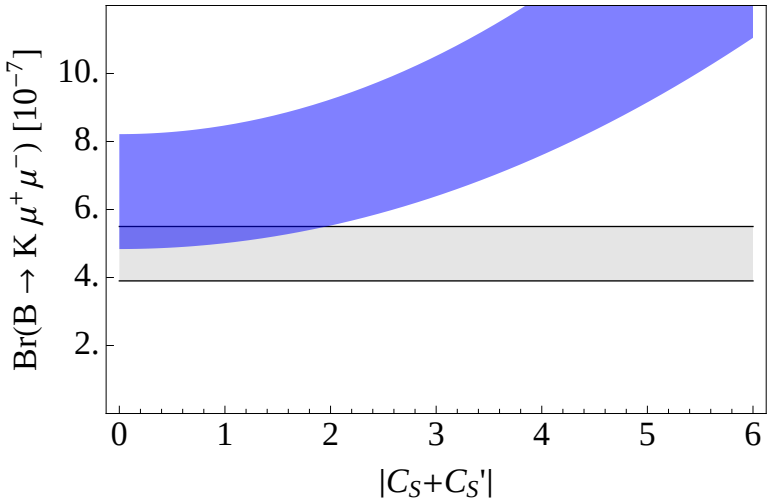
- Caveat: a sum of μ and e final states (assume lepton universality and work with μ)
- Narrow resonances (yellow region) cut out.

Scalar operators

- SM $+C_S,\ C_S'$
- $B_s \to \mu^+\mu^- \longrightarrow |C_S - C_S'|$ NO helicity suppression
- $B \to K\mu^+\mu^- \longrightarrow |C_S + C_S'|$



LHCb 2012, $\mathcal{B} < 4.5 \times 10^{-9}$

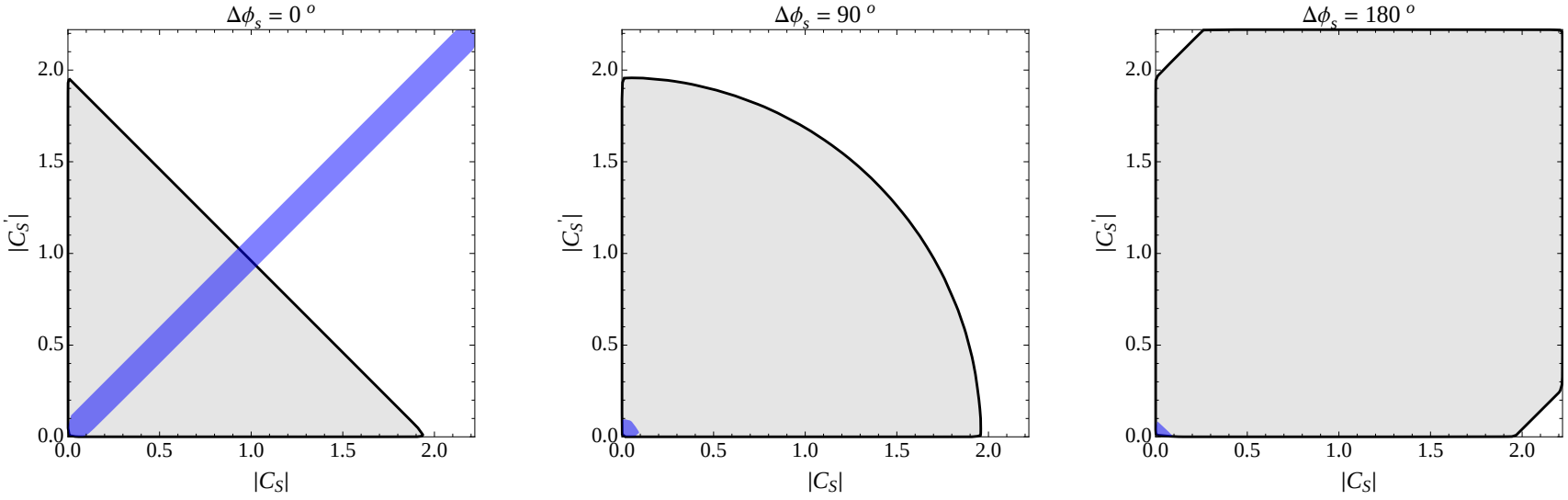


BaBar (Moriond 2012)
$\mathcal{B}_{B\to K\ell^+\ell^-} = (4.7 \pm 0.6 \pm 0.2)\,10^{-7}$

More precise form factors could exclude this scenario altogether

Scalar operators

- Both regions are ellipes in $|C_S|$, $|C_S'|$ plane
- Constraints strongly depend on relative phase $\Delta\phi_S$
  (**GREY** = $B \to K\mu^+\mu^-$, **BLUE** = $B_s \to \mu^+\mu^-$)

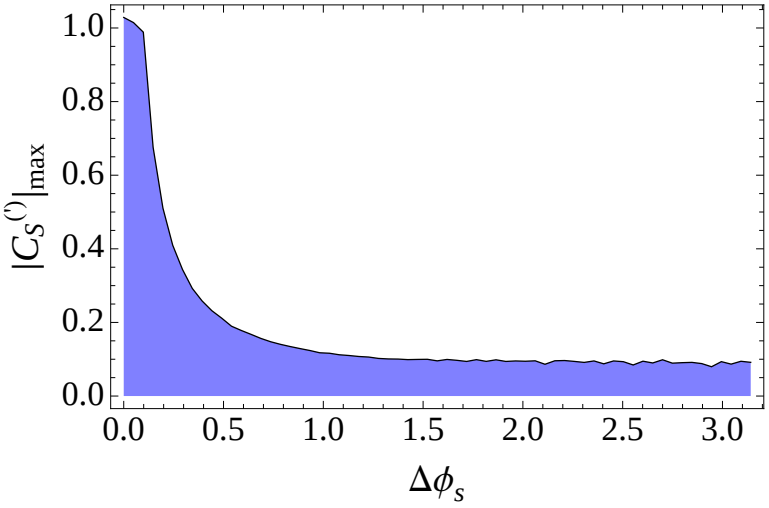


↑
the most conservative region

- Inclusive $B \to X_s\mu^+\mu^-$ is less sensitive
- No transverse asymmetry in $B \to K^*\mu^+\mu^-$

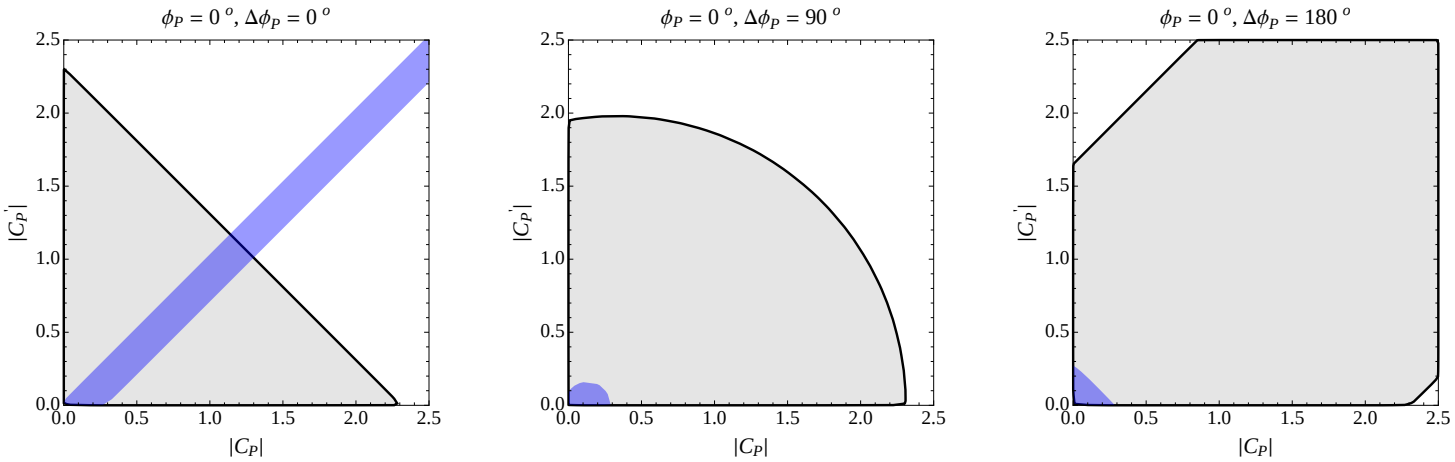
Scalar operators

Strong constraint $|C_S^{(\prime)}| \lesssim 0.1$, unless relative phase is small

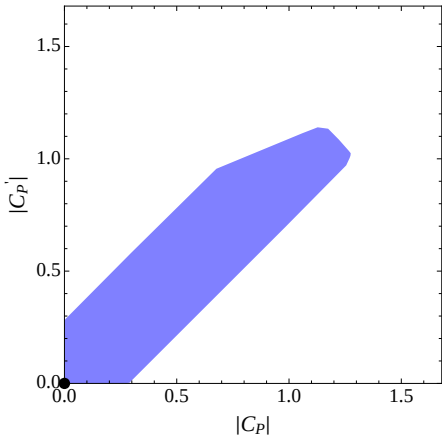


Pseudoscalar operators

- $B_s \to \mu^+\mu^- \longrightarrow |C_P - C_P' + 2m_\ell/m_{B_s} C_{10}^{\rm SM}|$ No helicity suppression
- $B \to K\mu^+\mu^- \longrightarrow |C_P + C_P' - \#m_\ell/m_B C_{10}^{\rm SM}|$
- Relative $\Delta\phi_P$ and absolute ϕ_P phase dependence
  (**GREY** = $B \to K\mu^+\mu^-$, **BLUE** = $B_s \to \mu^+\mu^-$)

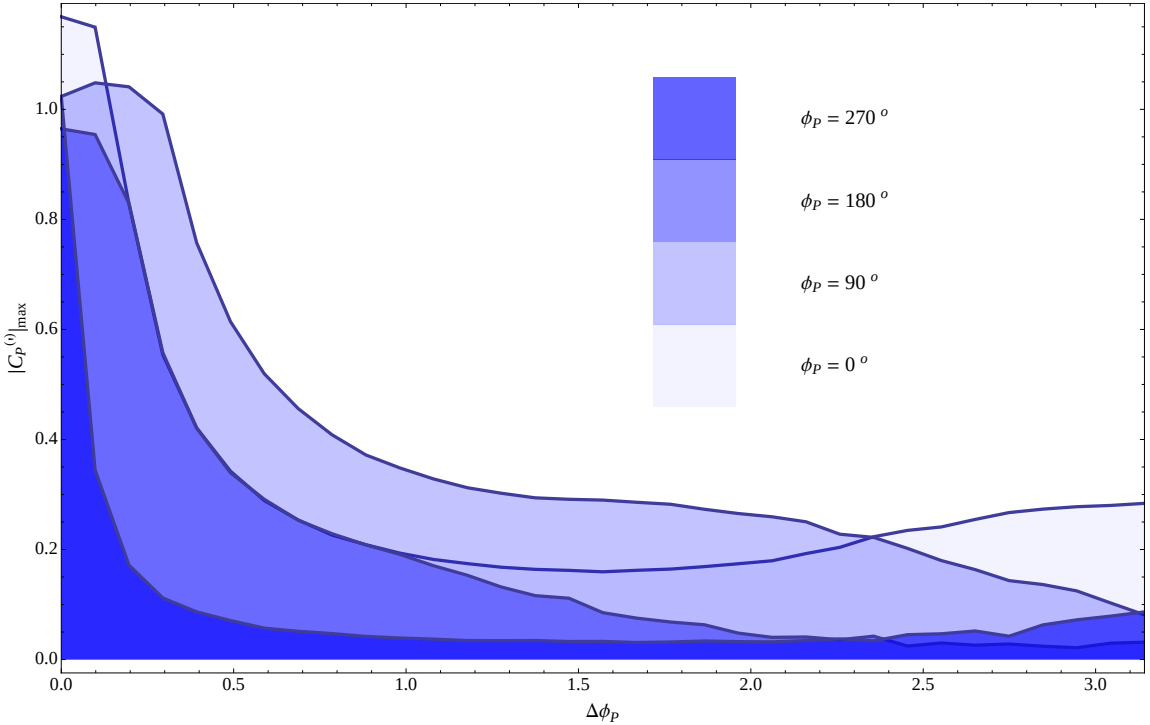


Phase-independent:



Pseudoscalar operators

$|C_P^{(\prime)}| \lesssim 0.4$ for $\mathcal{O}(1)$ relative phase.



Pseudoscalar and scalar

- $B_s \to \mu^+\mu^- \longrightarrow |C_S|, \quad |C_P + 2m_\ell/m_B\, C_{10}^{\rm SM}|$
- $B \to K\mu^+\mu^- \longrightarrow |C_S|, \quad |C_P + \#m_\ell/m_B\, C_{10}^{\rm SM}|$
- Phase of C_P enters

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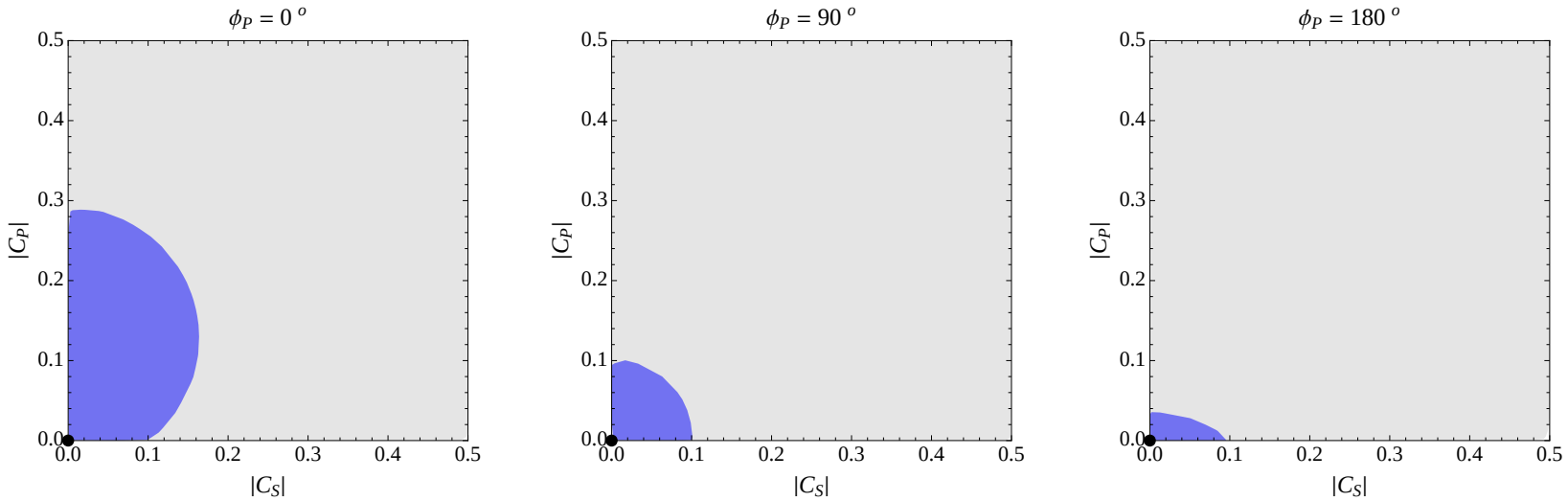
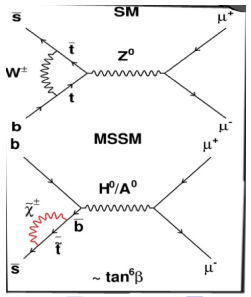


↑

the most conservative case

- MSSM prediction at large $\tan\beta$ (dominance of H^0 penguins diagram with $\tilde{\chi}$ and $\tilde{u}$)

$$C_S, C_P \sim \tan^3\beta$$



$B \to K\mu^+\mu^-$? Large theoretical uncertainty.

Pseudoscalar and scalar

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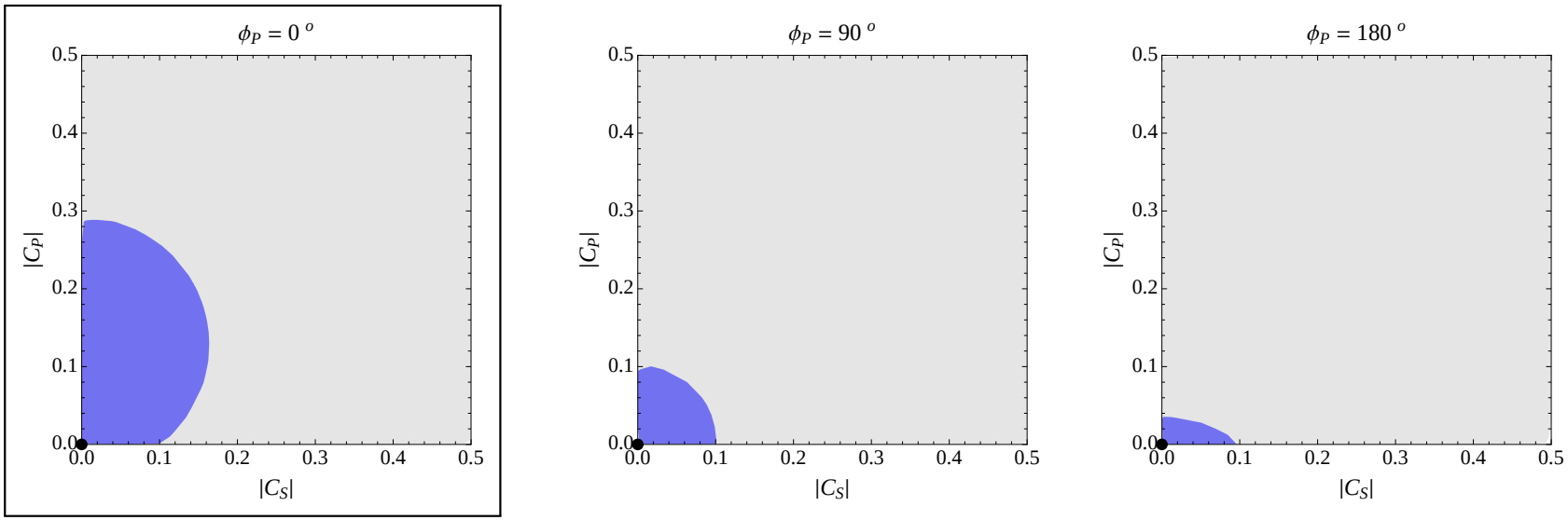

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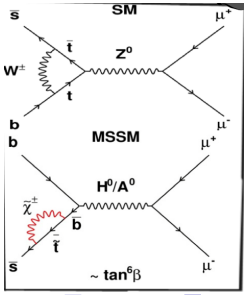

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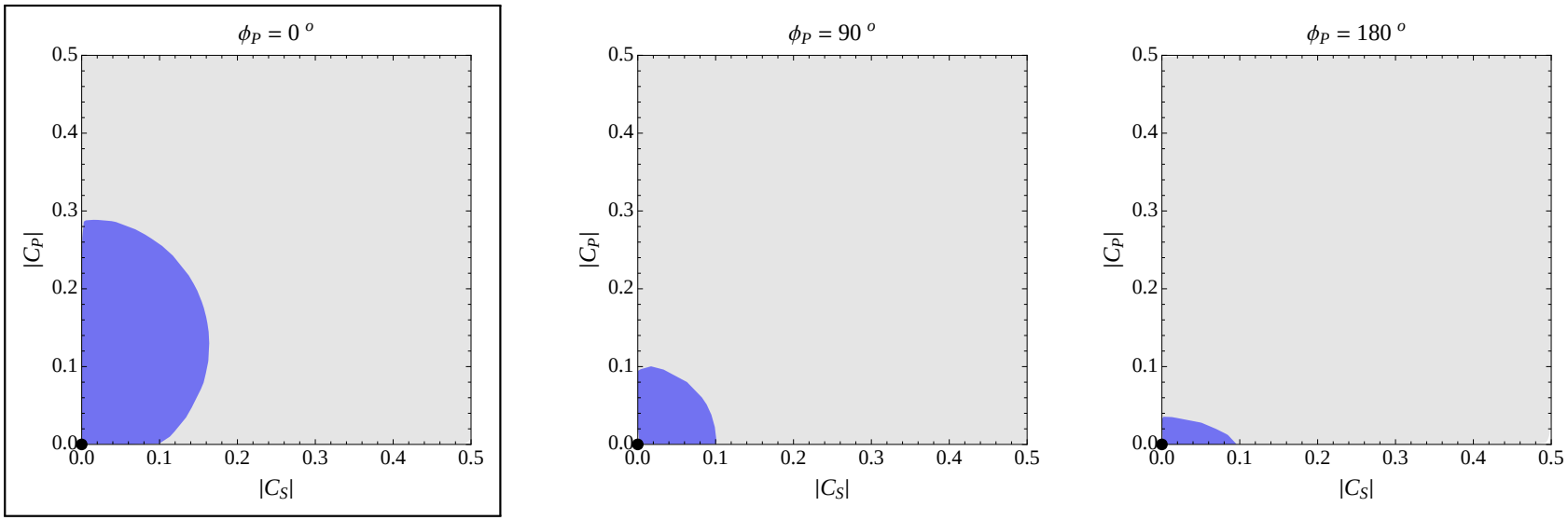

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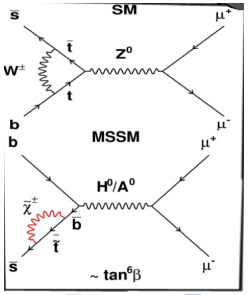

$B \to K\mu^+\mu^-$? Large theoretical uncertainty.

Pseudoscalar and scalar — smaller error bars?

- Scale form factor errors by 0.6, keep central values,
- May be achieved by unquenching the lattice QCD calculation (?)

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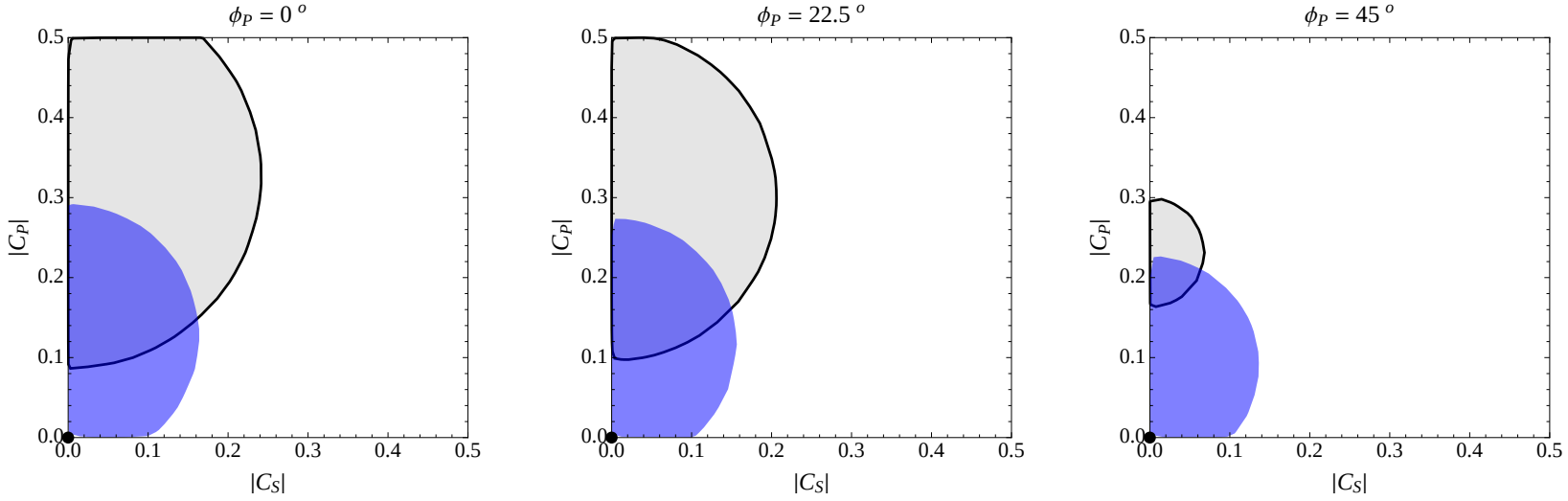

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This "toy-scenario" would prefer nonzero C_P.

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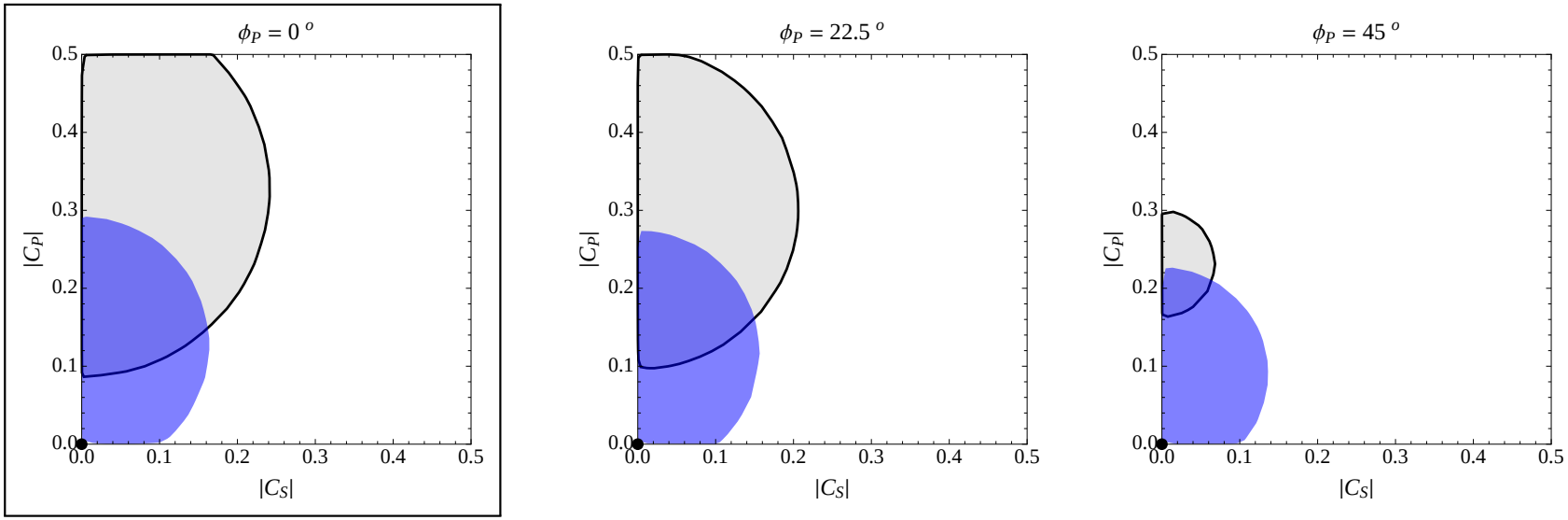

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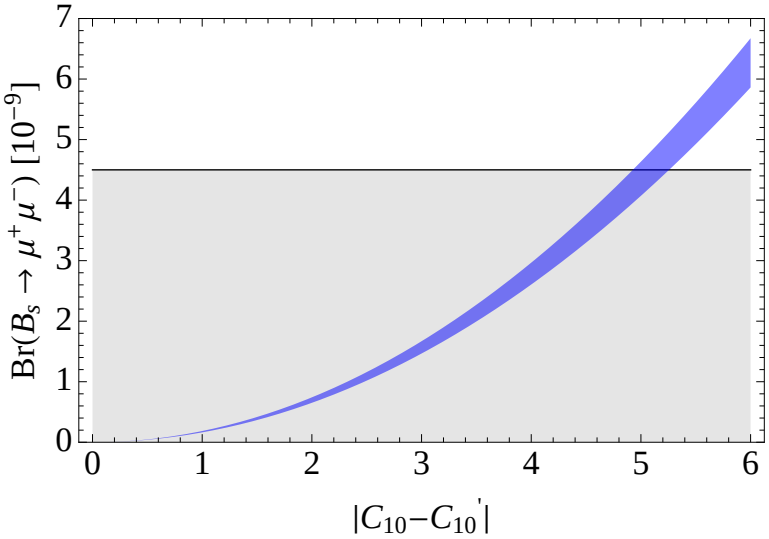
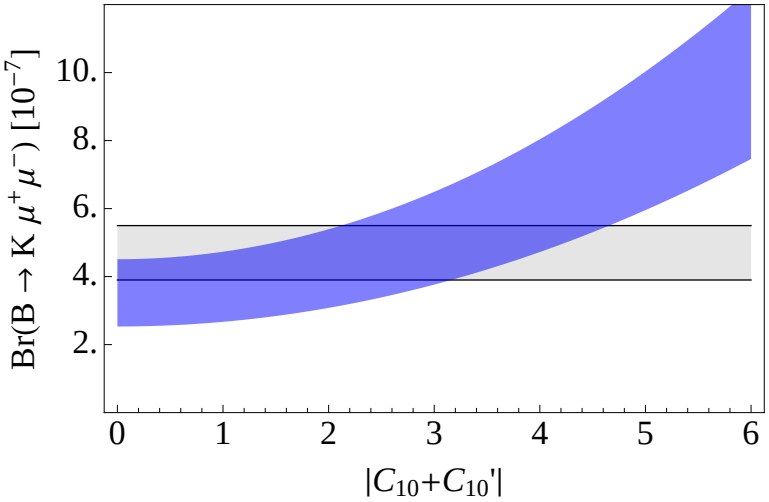
Axial vector operators

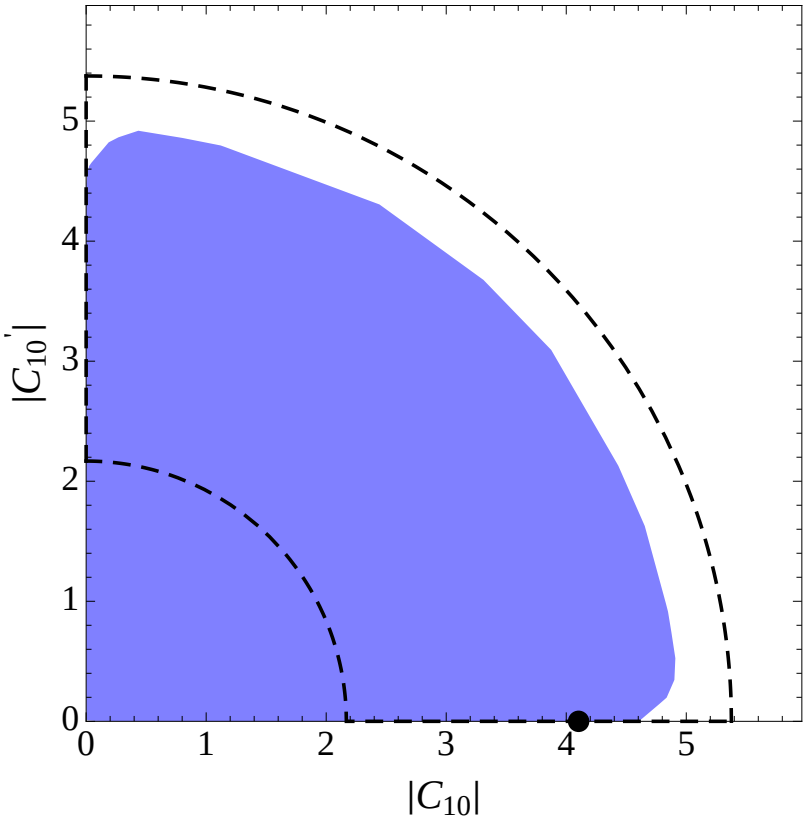
- Right-handed currents, colored scalars





Axial vector operators

- Both regions are ellipes in $|C_{10}|$, $|C'_{10}|$ plane



- Blue = $B_s \to \mu^+\mu^-$ & $B \to K\mu^+\mu^-$
- Dashed = $B \to X_s\ell^+\ell^-$

Presence of C'_{10} can be cross-checked in $B \to K^*\mu^+\mu^-$ transverse asymmetries

A word on tensors and FB asymmetry

- We assumed NO tensor contributions to $B \to K\mu^+\mu^-$

$$\mathcal{O}_T = \frac{e^2}{16\pi^2}(\bar{s}\sigma^{\mu\nu}b)(\bar{\ell}\sigma_{\mu\nu}\ell) \qquad \mathcal{O}_{T5} = \frac{e^2}{16\pi^2}(\bar{s}\sigma^{\mu\nu}b)(\bar{\ell}\sigma_{\mu\nu}\gamma_5\ell)$$

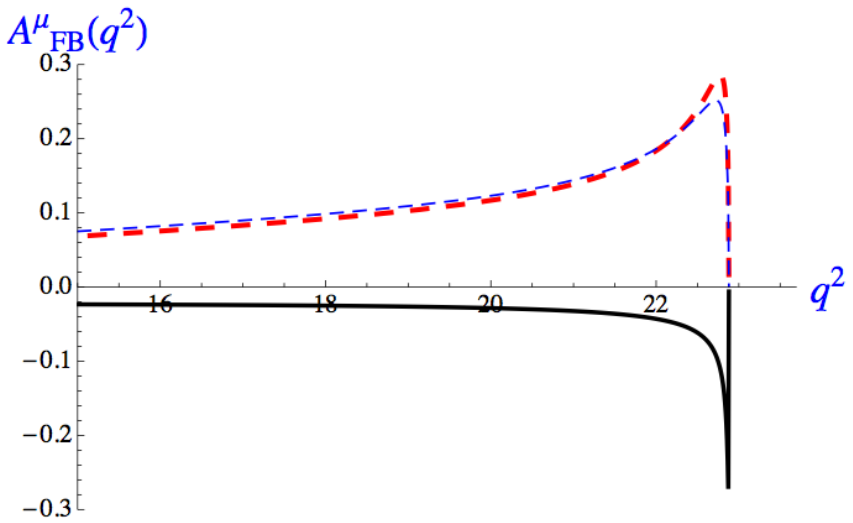
Assumption testable in

- $\mathcal{B}(B \to X_s\mu^+\mu^-)$

$$\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} \frac{d\mathcal{B}(B \to X_s\mu^+\mu^-)}{dq^2} dq^2 = 1.59(17) \times 10^{-6}[1 + 0.59(2)(|C_T|^2 + |C_{T5}|^2)]$$

- Forward-backward asymmetry of $B \to K\mu^+\mu^-$

$$A_{FB}^{\ell}(q^2) = \frac{2\,\mathcal{C}(q^2)}{\Gamma_\ell}\,\frac{m_B - m_K}{m_b}\,\sqrt{\lambda(q^2)}\,f_0(q^2)\,\Big\{\,(C_S C_T + C_P C_{T5})\;q^2 f_T(q^2)$$
$$+\,m_\ell\Big[C_S\,C_9(m_B + m_K)f_P(q^2) + 2m_b\,(C_S C_7 + 2C_{T5}C_{10})\,f_T(q^2)\Big] + \mathcal{O}(m_\ell^2)\Big\}$$



- $C_T = C_{T5} = 1.6$
- Thick $\to C_S = C_P = 0$
- Dashed $\to \{C_S, C_P\} = (1, 0)$

Conclusions

- $\mathcal{B}(B_s \to \mu^+\mu^-)$ is sensitive to (pseudo)scalar operators

$$(\bar{s}P_{L,R}b)(\bar{\ell}(\gamma_5)\ell)$$

- Only one hadronic parameter enters, f_{B_s}

- $\mathcal{B}(B \to K\ell^+\ell^-)$ is sensitive to (pseudo)scalars + vector operators (+tensors)

- With respect to $B_s \to \mu^+\mu^-$ it probes the effective Hamiltonian in the "orthogonal" direction
- Improvement in form factors calculation would make the two observables a high resolution probe of scalar operators

- For vector operators cross check is possible with spectrum of $B \to X_s\ell^+\ell^-$ and transverse asymmetries in $B \to K^*\mu^+\mu^-$

Backup

q^2-dependent contributions to $C_9^{\rm eff}$

$$Y(q^2) = \frac{4}{3}C_3 + \frac{64}{9}C_5 + \frac{64}{27}C_6 - \frac{1}{2}h(q^2,0)\left(C_3 + \frac{4}{3}C_4 + 16C_5 + \frac{64}{3}C_6\right)$$
$$+ h(q^2,m_c)\left(\frac{4}{3}C_1 + C_2 + 6C_3 + 60C_5\right) - \frac{1}{2}h(q^2,m_b)\left(7C_3 + \frac{4}{3}C_4 + 76C_5 + \frac{64}{3}C_6\right)$$

$$a_\ell(q^2) = \mathcal{C}(q^2)\beta_\ell(q^2)\left[q^2(\beta_\ell^2|F_S|^2 + |F_P|^2) + \frac{\lambda(q^2)}{4}(|F_A|^2 + |F_V|^2) + 2m_\ell(m_B^2 - m_K^2 + q^2)\mathrm{Re}\,(F_P F_A^*)4m_\ell^2 m_B^2|F_A|^2\right]$$

$$c_\ell(q^2) = \mathcal{C}(q^2)\beta_\ell(q^2)\left[q^2(\beta_\ell^2|F_T|^2 + |F_{T5}|^2) - \frac{\lambda(q^2)}{4}(|F_A|^2 + |F_V|^2) + 2m_\ell\sqrt{\lambda(q^2)}\beta_\ell\mathrm{Re}\,(F_T F_V^*)4m_\ell^2 m_B^2|F_A|^2\right]$$