

A Minimal Model of Neutrino Flavor

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Work in progress w/Christoph Luhn (IPPP, Durham) &
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Neutrino Mixing Matrix

- Neutrinos have mass and the different flavors can mix

Super-Kamiokande Collaboration, Y. Fukuda *et al.*, "Evidence for oscillation of atmospheric neutrinos," *Phys. Rev. Lett.* **81** (1998) 1562–1567, [hep-ex/9807003](#)

SNO Collaboration, Q. R. Ahmad *et al.*, "Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory," *Phys. Rev. Lett.* **89** (2002) 011301, [nucl-ex/0204008](#)

- Charged lepton and neutrino mass matrices cannot be simultaneously diagonalized

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

- Pontecorvo-Maki-Nakagawa-Sakata matrix

B. Pontecorvo, "Mesonium and antimesonium," *Sov. Phys. JETP* **6** (1957) 429

Z. Maki, M. Nakagawa, and S. Sakata, "Remarks on the unified model of elementary particles," *Prog. Theor. Phys.* **28** (1962) 870–880

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{D_L U_L^\dagger}_{U_{\text{PMNS}}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrino Mixing Matrix

What we know about the mixing angles ...

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}-s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12}-s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12}-s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

T. Schwetz, M. Tortola, and J. Valle, "Where we are on θ_{13} : addendum to 'Global neutrino data and recent reactor fluxes: status of three-flavour oscillation parameters', " *New J.Phys.* **13** (2011) 109401, [1108.1376](https://arxiv.org/abs/1108.1376)

Parameter	Best Fit	1σ range	3σ range
θ_{12}	33.96°	$33.02^\circ - 35.0^\circ$	$31.31^\circ - 36.87^\circ$
θ_{23}	46.15°	$42.13^\circ - 49.6^\circ$	$38.65^\circ - 53.13^\circ$
θ_{13}	6.55°	$5.13^\circ - 8.13^\circ$	$1.81^\circ - 10.78^\circ$

Tribimaximal Mixing

- Until recently, our best guess was tribimaximal mixing (TBM)

P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Phys. Lett.* **B530** (2002) 167, [hep-ph/0202074](#)

$$U_{\text{PMNS}} \stackrel{?}{=} U_{\text{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\hookrightarrow \theta_{12} = 35.26^\circ, \quad \theta_{23} = 45^\circ, \quad \theta_{13} = 0^\circ$$

- Agreement still quite good for θ_{12} , θ_{23} , but $\theta_{13} = 0^\circ$ excluded @5 σ

Schwetz et al, [1108.1376](#), DAYA-BAY Collaboration, [1203.1669](#), RENO Collaboration, [1204.0626](#)

Parameter	Tribimaximal	Global fit 1 σ	Daya Bay	Reno	
θ_{12}	35.26°	$33.02^\circ - 35.0^\circ$	-	-	✓
θ_{23}	45.00°	$42.13^\circ - 49.6^\circ$	-	-	✓
θ_{13}	0.00°	$5.13^\circ - 8.13^\circ$	8.8°	9.8°	✗

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Post-Daya Bay Confusion

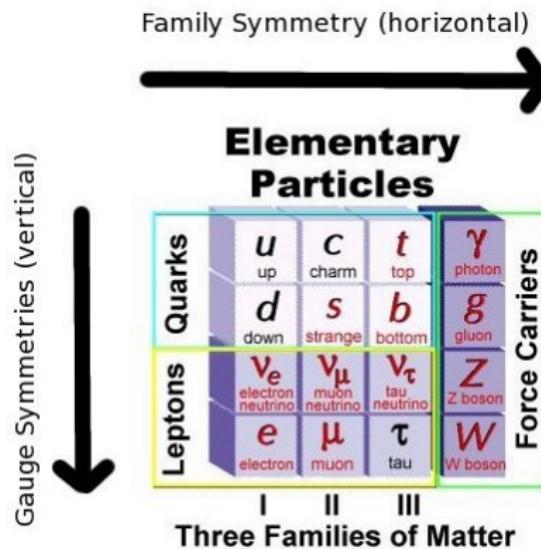
- Regular pattern U_{PMNS} is suggestive of a family symmetry

$$U_{\text{PMNS}} \stackrel{?}{=} U_{\text{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

- Daya bay and Reno rule out tribimaximal Mixing
- What are our options?
 - Give up family symmetries!
 - Look for groups that give $\theta_{13} \neq 0^\circ$
R. d. A. Toorop, F. Feruglio, and C. Hagedorn, "Discrete Flavour Symmetries in Light of T2K," *Phys.Lett.* **B703** (2011) 447–451, [1107.3486](#)
 - Keep TBM and calculate higher corrections → [This talk!](#)

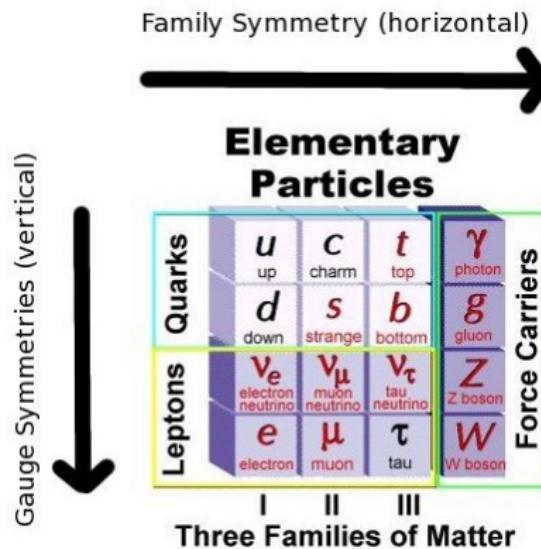
Discrete Flavor Symmetries

- Introduce relations between families of quarks and leptons
- But which discrete group do we take for the family symmetry?



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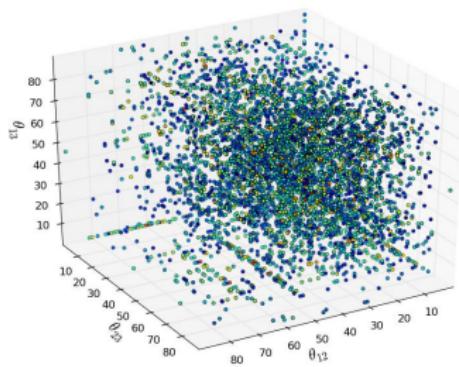
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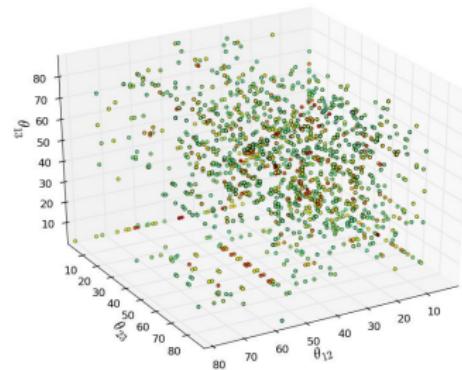
Discrete Flavor Symmetries

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys. Rev.* **D84** (2011) 013011,
[1012.2842](#)

- Neutrino mixing angles in a general class of $A_4 \times \mathbb{Z}_3$ models



(a) The 5528 bins that are ≥ 1 .

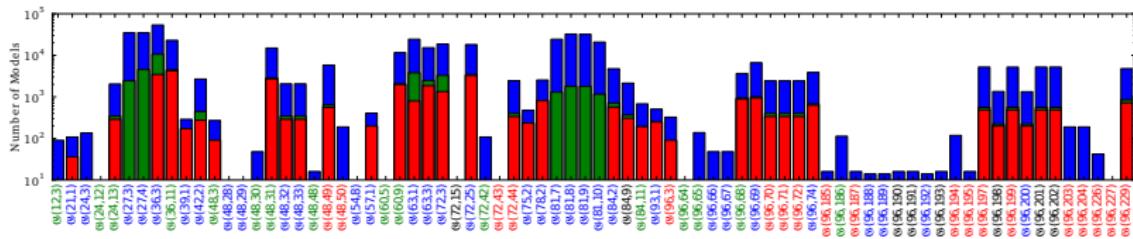


(b) The 1287 bins that are ≥ 1000 .

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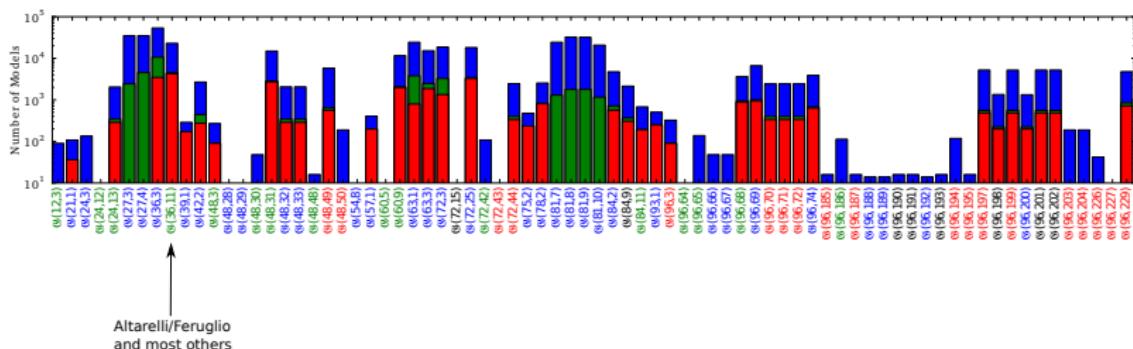
- In a recent publication we had scanned 76 discrete groups
- Most papers concentrate on $A_4 \times \mathbb{Z}_n$, $n \geq 3$
- $\Delta(96)$ gives $\theta_{13} \neq 0^\circ$ but is large
- We identified smallest group with TBM \rightarrow Higher order corrections!



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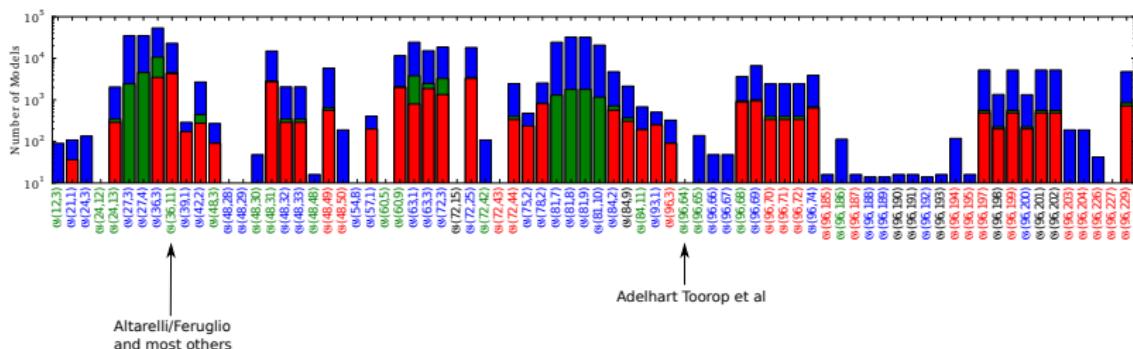
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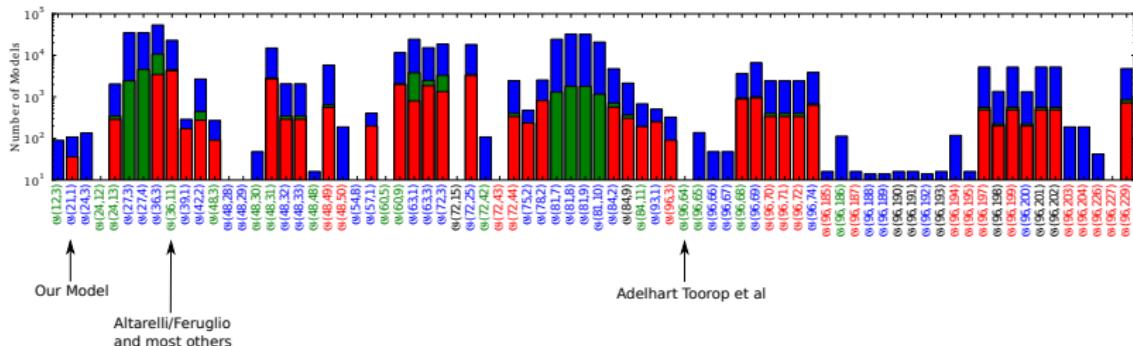
Altarelli/Feruglio
and most others

Adelhart Toorop et al

Discrete Flavor Symmetries

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Discrete Flavor Symmetries

C. Luhn, K. M. Parattu, A. Wingerter, *work in progress*

❶ Symmetries of the model

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times T_7 \times \mathrm{U}(1)_R$$

❷ Particle content and charges

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	T_7	$\mathrm{U}(1)_R$
L	(2, -1)	3	1
e	(1, 2)	1	1
μ	(1, 2)	1'	1
τ	(1, 2)	1''	1
h_u	(2, 1)	1	0
h_d	(2, -1)	1	0
φ	(1, 0)	3	0
$\tilde{\varphi}$	(1, 0)	3'	0

❸ Breaking the family symmetry

$$\langle \varphi \rangle = (v_\varphi, v_\varphi, v_\varphi), \quad \langle \tilde{\varphi} \rangle = (v_{\tilde{\varphi}}, 0, 0)$$

All you need to know about T_7

Tensor Products

$$\mathbf{1} \times \mathbf{1} = \mathbf{1}$$

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Contractions

$$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3},$$

$$z = \begin{pmatrix} \frac{1}{3}\sqrt{3}x_1y_1 + \frac{1}{3}\sqrt{3}x_2y_3 + \frac{1}{3}\sqrt{3}x_3y_2 \\ x_1y_2 \left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}i \right) + x_2y_1 \left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}i \right) + x_3y_3 \left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}i \right) \\ x_1y_3 \left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}i \right) + x_2y_2 \left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}i \right) + x_3y_1 \left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}i \right) \end{pmatrix}$$

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$$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3}',$$

$$z = \begin{pmatrix} -\frac{1}{6}\sqrt{6}x_1y_1 + \frac{1}{3}\sqrt{6}x_2y_3 - \frac{1}{6}\sqrt{6}x_3y_2 \\ -\frac{1}{6}\sqrt{6}x_1y_3 - \frac{1}{6}\sqrt{6}x_2y_2 + \frac{1}{3}\sqrt{6}x_3y_1 \\ \frac{1}{3}\sqrt{6}x_1y_2 - \frac{1}{6}\sqrt{6}x_2y_1 - \frac{1}{6}\sqrt{6}x_3y_3 \end{pmatrix}$$

Our T_7 Model at Leading Order

➤ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\tilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\tilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\tilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\tilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$3' \otimes 3 \otimes 1 \otimes 1 = (1+1'+1''+3+3') \otimes 1 \otimes 1 = 1+1'+1''+3+3'$$

➤ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3} \sqrt{3} L_1 e h_d \tilde{\varphi}_1 + y_e \frac{1}{3} \sqrt{3} L_2 e h_d \tilde{\varphi}_2 + y_e \frac{1}{3} \sqrt{3} L_3 e h_d \tilde{\varphi}_3 \\ & + y_\mu \frac{1}{3} \sqrt{3} L_1 \mu h_d \tilde{\varphi}_3 + y_\mu \frac{1}{3} \sqrt{3} L_2 \mu h_d \tilde{\varphi}_1 + y_\mu \frac{1}{3} \sqrt{3} L_3 \mu h_d \tilde{\varphi}_2 \\ & + y_\tau \frac{1}{3} \sqrt{3} L_1 \tau h_d \tilde{\varphi}_2 + y_\tau \frac{1}{3} \sqrt{3} L_2 \tau h_d \tilde{\varphi}_3 + y_\tau \frac{1}{3} \sqrt{3} L_3 \tau h_d \tilde{\varphi}_1 \end{aligned}$$

➤ Contract $SU(2)_L$ indices and substitute vevs $\langle \tilde{\varphi} \rangle = (v_{\tilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3} \sqrt{3} v_d v_{\tilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3} \sqrt{3} v_d v_{\tilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3} \sqrt{3} v_d v_{\tilde{\varphi}} L_3^{(2)} \tau$$

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Our T_7 Model at Leading Order

➤ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\bar{\nu}}}{\sqrt{6}\Lambda} \times \begin{pmatrix} L_1^{(2)} & e & \mu & \tau \\ L_2^{(2)} & y_e & 0 & 0 \\ L_3^{(2)} & 0 & y_\mu & 0 \\ 0 & 0 & 0 & y_\tau \end{pmatrix}, \quad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{pmatrix} L_1^{(1)} & \sqrt{2}y_2 v_\varphi + 2y_1 v_{\bar{\varphi}} & L_1^{(1)} & L_3^{(1)} \\ L_2^{(1)} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & \sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_{\varphi} + 2y_1 v_{\bar{\varphi}} \\ L_3^{(1)} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\bar{\varphi}} & \sqrt{2}y_2 v_\varphi \end{pmatrix}$$

➤ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➤ Neutrino mixing matrix

$$U_{\text{PMNS}} = D_L U_L^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \stackrel{e,\mu,\tau \rightarrow \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Formdiagonalizable! Mixing matrix does not depend on A , B , masses do!

➤ Mixing angles: $\theta_{12} = 35.26^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$ Tribimaximal ✓

Our T_7 Model at Leading Order

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$$M_{\ell^+} = -\frac{v_d v_{\bar{d}}}{\sqrt{6}\Lambda} \times \begin{pmatrix} e & \mu & \tau \\ L_1^{(2)} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, & M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{pmatrix} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \\ \begin{pmatrix} \sqrt{2}A + 2B & -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A \\ -\frac{1}{2}\sqrt{2}A & \sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B \\ -\frac{1}{2}\sqrt{2}A & -\frac{1}{2}\sqrt{2}A + 2B & \sqrt{2}A \end{pmatrix} \end{pmatrix}$$

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$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

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Next-to-Leading-Order Corrections

- Remember leading-order superpotential:

➤ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\tilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\tilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\tilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\varphi}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$3' \otimes 3 \otimes 1 \otimes 1 = (1+1'+1''+3+3') \otimes 1 \otimes 1 = 1+1'+1''+3+3'$$

- Superpotential with two flavon fields (massdim ≤ 6 or 7)

$$\begin{aligned} & y_e L e h_d \tilde{\varphi} + C_7 L e h_d \varphi \varphi + C_8 L e h_d \varphi \tilde{\varphi} + C_9 L e h_d \tilde{\varphi} \tilde{\varphi} + C_{10} L e h_u h_d h_d \tilde{\varphi} \\ & y_\mu L \mu h_d \tilde{\varphi} + C_{12} L \mu h_d \varphi \varphi + C_{13} L \mu h_d \varphi \tilde{\varphi} + C_{14} L \mu h_d \tilde{\varphi} \tilde{\varphi} + C_{15} L \mu h_u h_d h_d \tilde{\varphi} \\ & y_\tau L \tau h_d \tilde{\varphi} + C_{17} L \tau h_d \varphi \varphi + C_{18} L \tau h_d \varphi \tilde{\varphi} + C_{19} L \tau h_d \tilde{\varphi} \tilde{\varphi} + C_{20} L \tau h_u h_d h_d \tilde{\varphi} \\ & y_2 L L h_u h_u \varphi + y_1 L L h_u h_u \tilde{\varphi} + \underbrace{C_3 L L h_u h_u \varphi \varphi}_{\text{}} + \underbrace{C_4 L L h_u h_u \varphi \tilde{\varphi}}_{\text{}} + \underbrace{C_5 L L h_u h_u \tilde{\varphi} \tilde{\varphi}}_{\text{}} \end{aligned}$$

- Achieved $\theta_{13} \simeq 3.5^\circ$, pushing for higher values
- Pick only contributions that change θ_{13}
→ Introduce N_R and/or Δ for renormalizable UV completion

Vacuum Stabilization

- Mechanism of vacuum stabilization still lurking in dark
- How do we realize

$$\langle \varphi \rangle = (v_\varphi, v_\varphi, v_\varphi), \quad \langle \tilde{\varphi} \rangle = (v_{\tilde{\varphi}}, 0, 0) \quad ?$$

- Use supersymmetry: F -term equations relate flavon vevs
- Not straightforward. Either
 - need to set some terms to zero, or
 - impose a symmetry to do so, or
 - spatially separate flavons in extra dimensions, or
 - introduce more flavons.

Conclusions

- We started from TBM and NLO corrections give $\theta_{13} \neq 0^\circ$
- Model is very economical:
 - Family symmetry T_7 is second-smallest group w/3-dim irreps
 - No extra (shaping) symmetries, i.e. no extra $U(1)$ or \mathbb{Z}_N
 - Only 2 flavon fields
- Model is very predictive:
 - TBM is good prediction for θ_{12}, θ_{23}
 - NLO corrections push θ_{13} into 3σ band and θ_{12}, θ_{23} stable
 - Corrections come from two sources, we need one contribution:
→ Renormalizable realization w/type I and/or type II seesaw
- Vacuum stabilization kind of a headache
→ Either more symmetry or more flavons