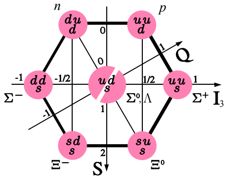

Isospin breaking corrections to light hadron masses

Antonin Portelli

Budapest-Marseille-Wuppertal collaboration

14th of May 2012 – RPP 2012, Montpellier

Centre de Physique Théorique, Marseille, France

Motivations

Isospin symmetry breaking

Isospin symmetry is broken because :

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Mass [PDG 2012]	$2.5\,\binom{+0.6}{-0.8}$	$5.0\,\binom{+0.7}{-0.9}$
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This breaking implies **mass splittings** in isospin multiplets.

Isospin breaking parameters

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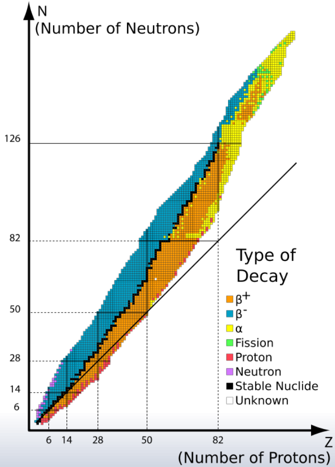
Isospin breaking effects

Sum of two little effects of the same order ($\sim 1\%$), eventually competing.

Nucleon mass splitting



Nucleon mass splitting is experimentally very well known :

$$M_p - M_n = -1.29333214(43)\ \mathrm{MeV}$$

Although it is a $1‰$ effect, it determines through β decay **the stable nuclides spectrum**.

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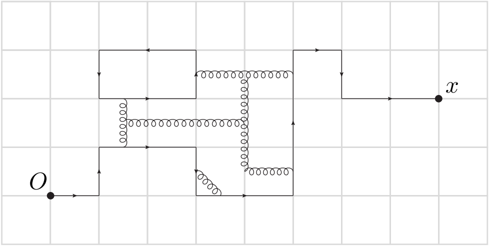
Lattice QCD could give a way to solve numerically this problem.

Predicting a $1‰$ effect through lattice simulation is a **considerable computing challenge**.

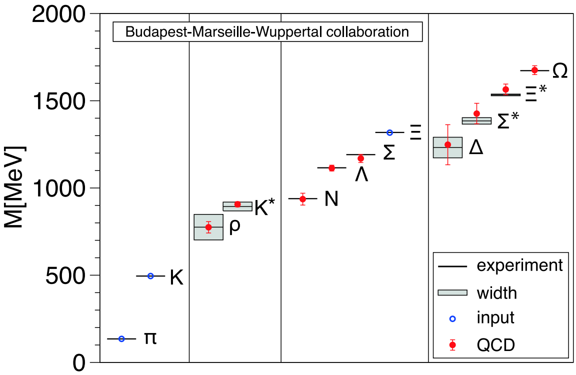
Lattice QCD



Numerical Monte-Carlo evaluation of QCD path integral :

$$\langle O\rangle = \frac{1}{\mathscr{Z}}\int \mathrm{D}U_\mu O_{\mathrm{Wick}}[D^{-1}]\det(D)\exp(-S_{\mathrm{gauge}})$$

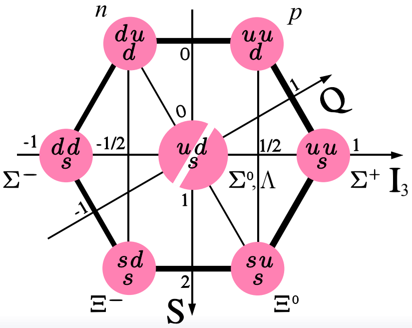
Light QCD spectrum solved



[BMWc 2008, Science, hep-lat/0906.3599]

Goal: octet baryon mass splittings

There are 3 stable baryon multiplets formed with u,d and s quarks :



Mass splittings are experimentally known [PDG 2012] :

$$M_p - M_n = -1.29333214(43)\ \text{MeV}$$
$$M_{\Sigma^+} - M_{\Sigma^-} = -8.08(08)\ \text{MeV}$$
$$M_{\Xi^0} - M_{\Xi^-} = -6.85(21)\ \text{MeV}$$

Simulation setup & methodology

Actions

- $N_f = 2 + 1$ QCD simulations with Lüscher-Weisz gauge action, tree level $\mathrm{O}(a)$-improved Wilson fermions and two steps of HEX smearing.
  [BMWc 2010, JHEP, hep-lat/1011.2711]

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- Thus, simulation is **quenched** in QED (neutral sea quarks).

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- EM fields generated with α set to its physical value.

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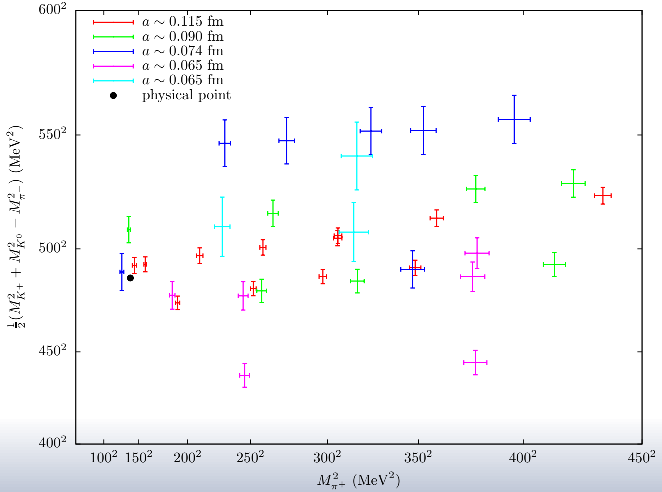
Quark propagators are contracted to form hadron propagators.

Simulation landscape



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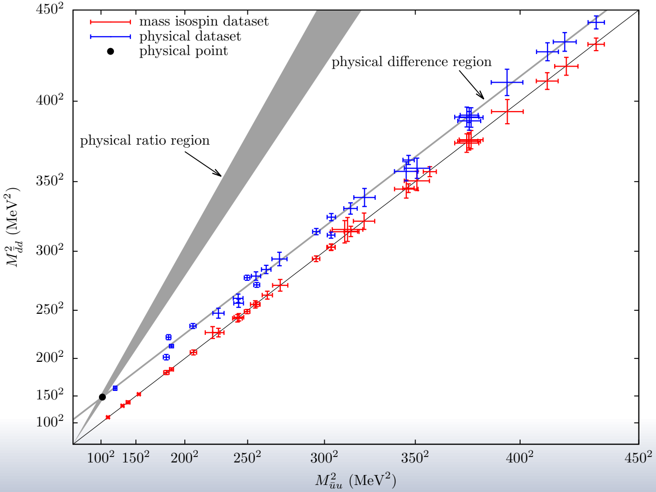

Propagator fits

Zero momentum propagators of two isospin partners X_1 and X_2 are fitted in a combined way with the model :

$$\begin{cases} G_{X_1}(t) = A_1 \exp[-(M_X + \Delta M_X)\frac{t}{2}] \\ G_{X_2}(t) = A_2 \exp[-(M_X - \Delta M_X)\frac{t}{2}] \end{cases}$$

The parameter ΔM_X is the mass splitting $M_{X_1} - M_{X_2}$.

Interpolation model

ΔM_X can be expanded at NLO in isospin breaking parameters :

$$\Delta M_X = \alpha A_{\mathrm{EM}} M_X + (m_u - m_d) A_{\mathrm{QCD}} M_X$$

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- $A_{\mathrm{EM,QCD}}$ still depend on isospin symmetric parameters of QCD+QED : m_{ud},m_s,L and a
- Due to our setup, distinction between sea and valence is a NNLO isospin breaking effect.

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$A_{\mathrm{EM,QCD}}$ can then be expanded in the isospin symmetric parameters:

$$A_{\mathrm{EM,QCD}} = A^{\phi} + P_{ud}(m_{ud} - m_{ud}^{\phi}) + P_s(m_s - m_s^{\phi}) + P_{\mathrm{FV}}\frac{1}{L} + P_D a$$

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- Power finite volume corrections are expected with EM interaction.

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Total : 256 different analyses.

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BMWc error evaluation :

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One additional source of systematics: **QED quenching**.

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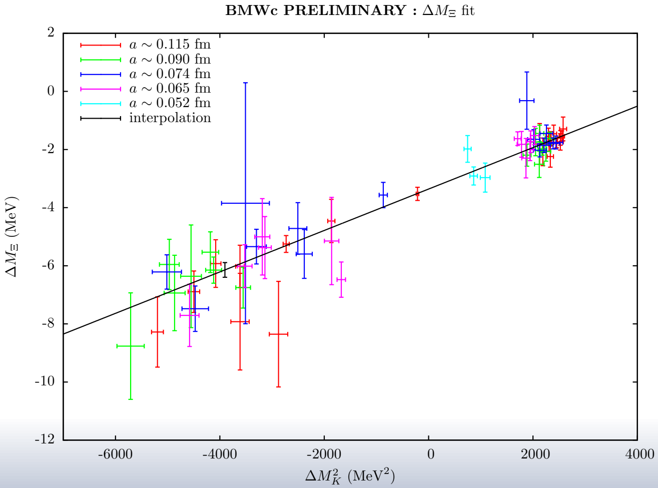
Typical relative error: $\frac{M_\Xi - M_N}{3M_N} \simeq 13\%$.

Preliminary results

Ξ mass splitting



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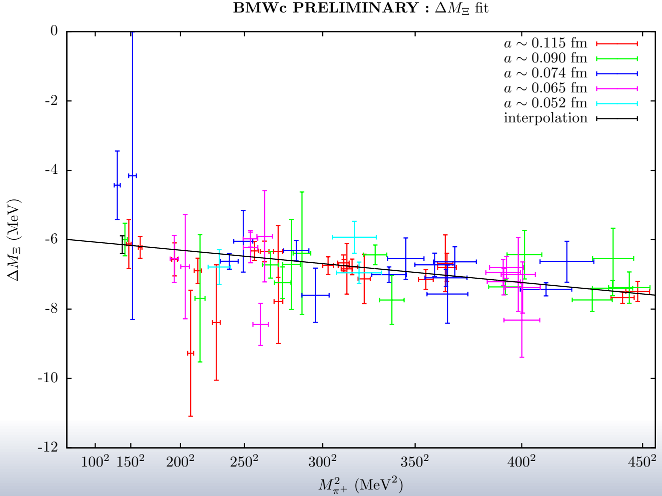

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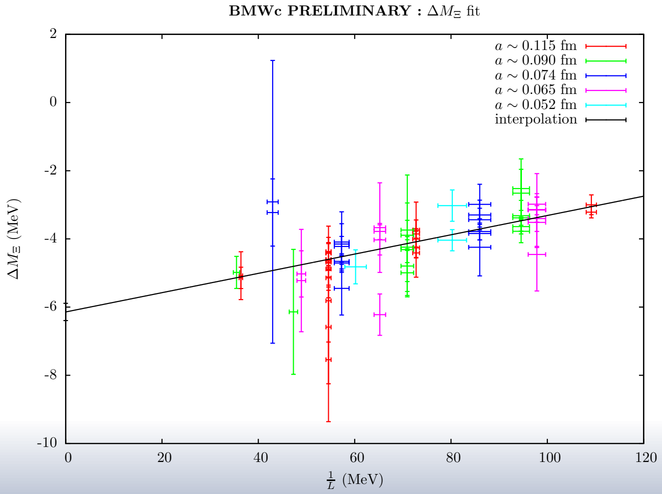

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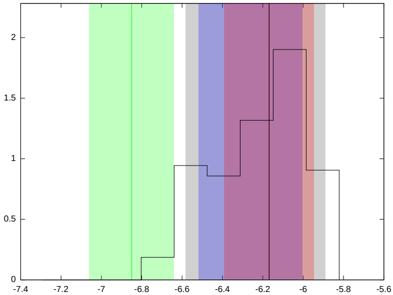

error: systematic statistic systematic + statistic experimental

$$M_{\Xi^0} - M_{\Xi^-} = -6.16(22)_{\text{stat.}} \binom{+16}{-35}_{\text{sys.}} (80)_{\text{quench.}} \binom{+85}{-90}_{\text{total}}$$

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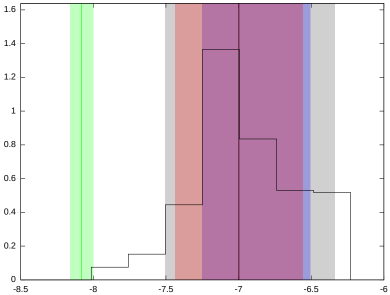

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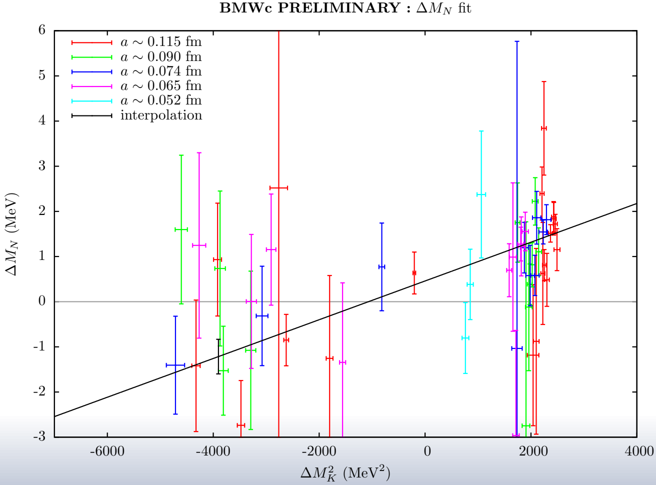
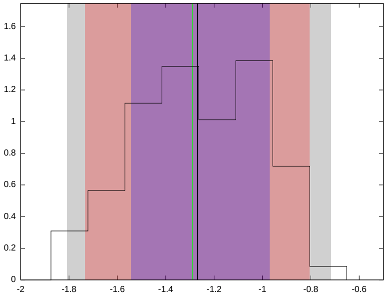
$$M_{\Sigma^+} - M_{\Sigma^-} = -7.00(0.44)_{\text{stat.}} \binom{+0.49}{-0.25}_{\text{sys.}} (0.91)_{\text{quench.}} \binom{+1.12}{-1.04}_{\text{total}}$$

N mass splitting



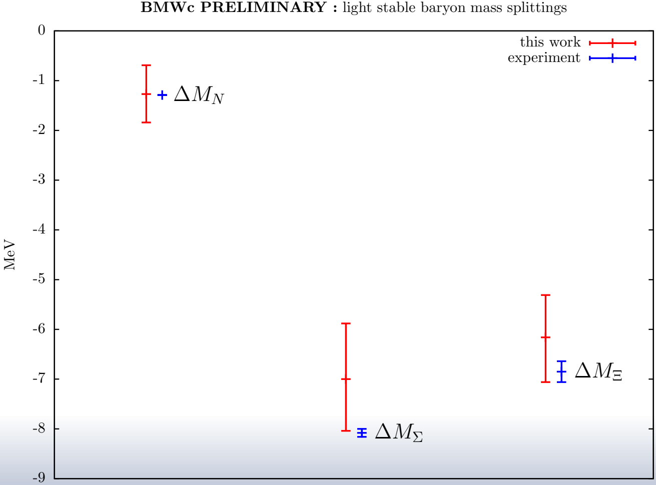
N mass splitting



error: systematic statistic systematic + statistic experimental

$$M_p - M_n = -1.27(46)_{\text{stat.}} \left({}^{+30}_{-28}\right)_{\text{sys.}} (17)_{\text{quench.}} \left({}^{+58}_{-57}\right)_{\text{total}}$$

Summary


BMWc PRELIMINARY : light stable baryon mass splittings
this work
experiment
MeV
0
-1
-2
-3
-4
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-7
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ΔM_N
ΔM_Σ
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Epilogue

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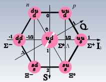

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Lattice QCD can have now **a significative sensivity to $1‰$ corrections** on hadronic energies.

Perspectives

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- Middle term: compute **light quark masses**.
- Long term: Explore ways to perform **full QCD+QED simulations**.

Thank you.

BMWc Collaboration

Budapest (Eötvös University)
S.D. Katz

Marseille (CPT)
J. Frison, L. Lellouch, A. Portelli, A. Sastre

Wuppertal (Bergische Universität)
S. Dürr, Z.I. Fodor, C. Hölbling, S. Krieg, T. Kurth, T. Lippert, C. Mc Neile, K.K. Szabo