TESTING ELECTROMAGNETIC PROTON FFS IN THE ANNIHILATION PROCESS $p + \bar{p} \to \pi^0 + e^- + e^+$

C. Adamuščín,J. Guttmann , F. E. Maas, M. Zambrana

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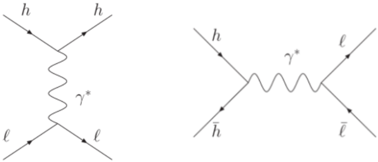
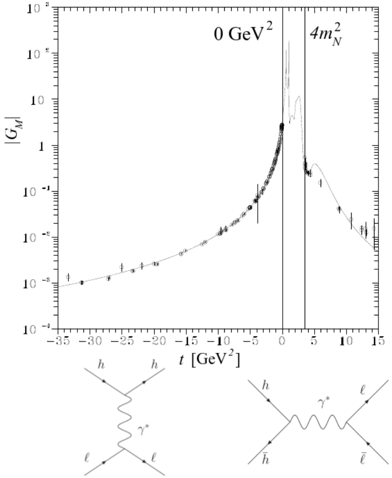
MOTIVATION

- Nucleons are non-point-like particles [*Hofstadter (1958)*].
- Internal electromagnetic (EM) structure of nucleons are described by EM form factors – scalar functions of one variable ($q^2 = t$). Usually the Pauli and Dirac form factors F_1, F_2 or the Sachs form factors G_E, G_M are used.
- The EM form factors of nucleons are usually measured
  - in elastic scattering processes $\ell\ N \to \ell\ N$ (space-like region $t < 0$)
  - in annihilation processes $\ell^- \ell^+ \to N\bar{N}$ or $N\bar{N} \to \ell^- \ell^+$ (time-like region $t > 4m_N^2$)



Motivation

- 'Unphysical' region $t \in (0, 4m_N^2)$ is not kinematically allowed in these processes - no data available.
- According to VMD based models of hadrons EM form factors should have interesting resonance behavior in the 'unphysical' region.
- Electromagnetic form factors are complex functions in the time-like region.



Resume of the old analysis – PRC 75, 045205

- 2 processes were analyzed

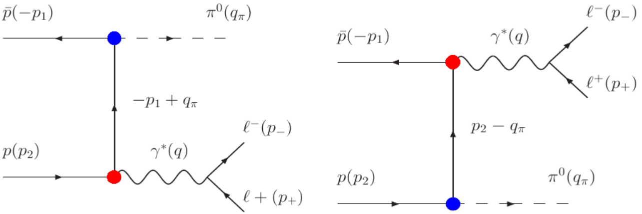
$$\bar{p}p \to e^+e^-\pi^0 \ , \ \bar{p}n \to e^+e^-\pi^-$$

- in an experimental setup where the pion is fully detected
- Compton-type annihilation Feynman amplitudes
- Main drawbacks
  - neglected off-shell effects
  - neglected pion mass $\sim \frac{2m_\pi}{M_N} \approx 7\%$
  - integration over the lepton phase space

$$\int d\Gamma \sum J_\mu J_\mu^* = -\frac{2\pi}{3}(q^2 + 2\mu^2)\beta(g_{\mu\nu} - q_\mu q_\nu / q^2)$$

Formalism

- the matrix elements of the considered reactions are calculated in the framework of a phenomenological approach based on Compton-like Feynman amplitudes



- Feynman diagrams for the reaction $\bar{p} + p \to \pi^0 + \ell^+ + \ell^-$
  - EM nucleon form factors
  - $g_{\pi NN}$ coupling

PARAMETRIZATION OF THE ELECTROMAGNETIC VERTICES

- the electromagnetic vertices $\gamma^* p p^*$ used in the Feynman diagrams involve off shell protons
- the off-shell effects were neglected as in the previous work
- hadron currents are parametrized by standard EM FFs

$$< N(p')|\Gamma_\mu(q)^N|N(p) >= \bar{u}(p') \left[F_1^N(q^2)\gamma_\mu + \frac{F_2^N(q^2)}{4M}[\hat{q},\gamma_\mu] \right] u(p)$$

PION-NUCLEON INTERACTION

- special attention must be devoted to the pion nucleon interaction, in the vertex $\pi N \bar{N}$
- it can be parametrized as

$$\bar{v}(p_1 - q)\gamma_5 u(p_2) g_{\pi NN}(m_\pi^2)$$

- and it can be related to the axial nucleon form factor by Goldberger-Treiman relation [*Fuchs, Scherer (2003)*] as

$$\frac{G_A}{f_\pi} = \frac{g_{\pi NN}}{M}$$

New analysis

- only $\bar{p}p \to e^+e^-\pi^0$ is analyzed
- **differences:**
  - pion mass contribution
  - no integration over the lepton phase space
- **common features:**
  - the same Feynman diagrams
  - neglected off-shell effects
  - therefore the same hadronic current

$$\mathcal{J}_\mu = \bar{v}(p_1)\mathcal{O}_\mu v(p_2)$$

where

$$\mathcal{O}_\mu = \Gamma_\mu^p(q)\frac{\hat{p}_1 - \hat{q} - M}{(p_1-q)^2 - M^2}\gamma_5 g_\pi - \gamma_5 \frac{\hat{p}_2 - \hat{q} + M}{(p_2-q)^2 - M^2}\Gamma_\mu^p(q) g_\pi$$

- to calculate the differential cross section one need to find expression for the amplitude $|\mathcal{M}|^2$ and phase space volume $d\Gamma$

HADRONIC,LEPTONIC TENSOR + AMPLITUDE

- the hadronic tensor

$$\mathcal{H}_{\mu\nu} = \frac{1}{4}\mathrm{Tr}(\hat{p}_1 - M)\mathcal{O}_\mu(\hat{p}_2 + M)(\mathcal{O}_\nu)^*$$

- we have used full leptonic tensor

$$\mathcal{L}_{\mu\nu} = 4q_{+\mu}q_{-\nu} + 4q_{+\nu}q_{-\mu} - 2g_{\mu\nu}q^2$$

- therefore amplitude of the process is calculated as

$$|\mathcal{M}|^2 = \frac{4(2\pi)^2\alpha^2}{(q^2)^2}\mathcal{H}_{\mu\nu}\mathcal{L}_{\mu\nu}$$

- the amplitude

$$|\mathcal{M}|^2 = \frac{e^4 g(m_\pi^2)^2}{q^4 BC}\mathcal{R} = \frac{e^4 g(m_\pi^2)^2}{q^4 BC}\Big[F_1^p F_1^{p*} 4m_\pi^2 \mathcal{X}_1 + (F_1^p - F_2^p)(F_1^p - F_2^p)^* \mathcal{X}_2 + F_2^p F_2^{p*}\mathcal{X}_3\Big]$$

AMPLITUDE

- the calculation was partly done by *Form*

$$
\begin{aligned}
\mathcal{R} \; = \; & F_1^p F_1^{p*} 4m_\pi^2 \Big\{ -M^2 \frac{(B+C)^2}{BC} q^2 - \frac{4\,(Bp_2\cdot q_- - Cp_1\cdot q_-)^2}{BC} + \\
+ \; & \Big[2\frac{B-C}{BC}(Bp_2\cdot q_- - Cp_1\cdot q_-) - (B+C) + m_\pi^2 \Big] q^2 + q^4 \Big\} \\
+ \; & (F_1^p - F_2^p)(F_1^p - F_2^p)^* \Big\{ \Big((B+C)^2 + \Big((B+C) + 4(p_1\cdot q_- + p_2\cdot q_-) \Big)^2 \Big) q^2 - \\
- \; & \Big(4(B+C-m_\pi^2) + 2m_\pi^2 \frac{(B+C)^2}{BC} + 16(p_2\cdot q_- + p_1\cdot q_-) \Big) q^4 + 4q^6 \Big\} \\
+ \; & F_2^p F_2^{p*} \Big\{ \frac{1}{M^2} \Big[-4\Big(Bp_2\cdot q_- - Cp_1\cdot q_- \Big)^2 + \Big[2(B-C)(Bp_2\cdot q_- - Cp_1\cdot q_-) - \\
- \; & (B+C)BC + m_\pi^2 \Big(BC + 4\frac{(Bp_2\cdot q_- - Cp_1\cdot q_-)^2}{BC} \Big) \Big] q^2 + \Big[BC + m_\pi^2 \Big((B+C) - \\
- \; & m_\pi^2 - 2\frac{B-C}{BC}(Bp_2\cdot q_- - Cp_1\cdot q_-) \Big) \Big] q^4 - m_\pi^2 q^6 \Big] + q^2 (B+C)^2 \Big(\frac{m_\pi^2 q^2}{BC} - 1 \Big) \Big\}
\end{aligned}
$$

where

$$B = m_\pi^2 - 2p_2\cdot q_\pi \;\; ; \;\; C = m_\pi^2 - 2p_1\cdot q_\pi$$

Amplitude

- it seems to be complicated but there are only five different terms $(+\, q^2)$

$$B+C\,,\ B-C\,,\ BC\,,\ p_2\cdot q_- + p_1\cdot q_-$$

$$Bp_2\cdot q_- - Cp_1\cdot q_-$$

- in LAB frame

$$B+C = m_\pi^2 - s + q^2$$
$$B-C = s - q^2 + m_\pi^2 - 4M\varepsilon_\pi$$
$$BC = (s - q^2 - 2M\varepsilon_\pi)(2M\varepsilon_\pi - m_\pi^2)$$
$$Bp_2\cdot q_- - Cp_1\cdot q_- = \varepsilon_-(s(E-\varepsilon_\pi) + m_\pi^2 M - q^2 E)$$
$$-\varepsilon_- P(s - q^2 - 2M\varepsilon_\pi)\cos\theta_-$$
$$p_2\cdot q_- + p_1\cdot q_- = (M+E)\varepsilon_- - P\varepsilon_-\cos\theta_-,$$

The phase space volume - LAB frame

- in case of 3 outgoing particles the phase space volume

$$d\Gamma = \frac{(2\pi)^4}{(2\pi)^9}\frac{d^3q_\pi}{2\varepsilon_\pi}\frac{d^3q_-}{2\varepsilon_-}\frac{d^3q_+}{2\varepsilon_+}\delta^4(p_1+p_2-q_\pi-q_--q_+)$$

- which can be rewritten in terms of variables $dq^2, d\varepsilon_\pi, d\varepsilon_-, d\cos\theta_-$ using Kuraev trick(s)

$$d\Gamma = \frac{1}{8(2\pi)^4\sqrt{\mathcal{D}}P|q_\pi|}dq^2 d\varepsilon_\pi d\varepsilon_- d\cos\theta_-$$

where

$$\mathcal{D} = 1-\cos^2\theta_\pi-\cos^2\theta_--\cos^2\theta_{\pi,e^-}+2\cos\theta_\pi\cos\theta_-\cos\theta_{\pi,e^-}$$

PION

- the substitution

$$\frac{d^3q_+}{2\varepsilon_+} = d^4q_+\delta(q_+^2 - m_\ell^2)\theta(\varepsilon_+ - m_\ell)$$

allows to get rid of the positron phase space volume – d^4q_+

$$\int d^4q_+\delta^4(p_1 + p_2 - q_\pi - q_- - q_+) = 1$$

- the phase space volume of the pion in the LAB frame

$$\frac{d^3q_\pi}{2\varepsilon_\pi} = \frac{\varepsilon_\pi d\varepsilon_\pi |q_\pi|}{2\varepsilon_\pi} d\phi_\pi d\cos\theta_\pi = \pi |q_\pi| d\varepsilon_\pi d\cos\theta_\pi$$

- electron

$$\frac{d^3q_-}{2\varepsilon_-} = \frac{1}{2}\varepsilon_- d\varepsilon_- d\Omega_-$$

$d\Omega_-$

- the goal is to remove dependence on $d\phi_-$

$$d\Omega_- = d\cos\theta_- d\phi_- = d\cos\theta_- d\phi_- d\cos\theta_{\pi,e^-} \times$$
$$\times\, \delta(\cos\theta_{\pi,e^-} - \cos\theta_\pi \cos\theta_- - \sin\theta_\pi \sin\theta_- \cos\phi_-)$$

as scalar product of the unit vectors $n_\pi \cdot n_- = \cos\theta_{\pi,e^-}$

$n_\pi = (\sin\theta_\pi, 0, \cos\theta_\pi)$, $n_- = (\cos\phi_- \sin\theta_-, \sin\phi_- \sin\theta_-, \cos\theta_-)$

- now we can integrate over $d\phi_-$

$$d\Omega_- = \frac{d\cos\theta_- d\cos\theta_{\pi,e^-}}{|\sin\theta_\pi \sin\theta_{\pi,e^-} \sin\phi_-|} = \frac{d\cos\theta_- d\cos\theta_{\pi,e^-}}{\sqrt{\mathcal{D}}}$$

$d\cos\theta_{\pi,e^-}$, DIFFERENTIAL CROSS SECTION IN THE LAB

- recall the extra 1-dimensional δ-function

$$\delta(q_+^2 - m_\ell^2) = \delta((p_1 + p_2 - q_\pi - q_-)^2 - m_\ell^2) = \delta(?? + q_\pi \cdot q_-)$$

- the only term dependent on $\cos\theta_{\pi,e^-}$ is $q_\pi \cdot q_-$, so

$$\frac{d((p_1 + p_2 - q_\pi - q_-)^2 - m_\ell^2)}{d\cos\theta_{\pi,e^-}} = \frac{dq_\pi \cdot q_-}{d\cos\theta_{\pi,e^-}} = |q_\pi|\varepsilon_-$$

- therefore the **phase space volume**

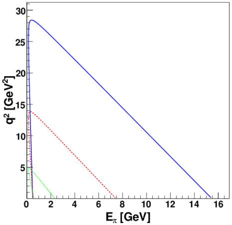
$$d\Gamma = \frac{1}{4(2\pi)^4} d\varepsilon_\pi d\varepsilon_- \frac{d\cos\theta_\pi d\cos\theta_-}{\sqrt{\mathcal{D}}}$$

- **differential cross section in the LAB frame**

$$d^4\sigma = \frac{1}{32(2\pi)^4 MP^2 |q_\pi| \sqrt{\mathcal{D}}} |\mathcal{M}_0|^2 \times dq^2 d\varepsilon_\pi d\varepsilon_- d\cos\theta_-$$

KINEMATICS

- the allowed kinematical region for the q^2, ε_π was calculated in the previous analysis
- taking into account the conservation laws



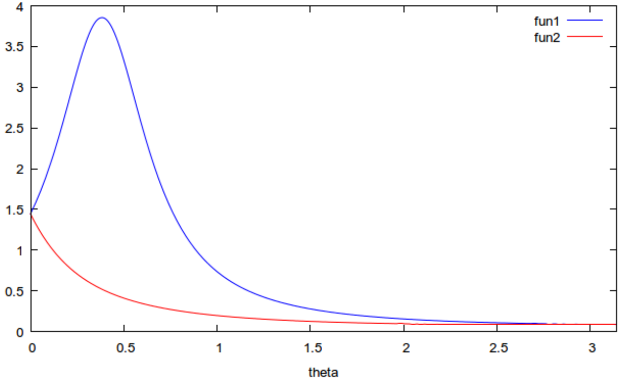
$$E + M = \varepsilon_\pi + \varepsilon_- + \varepsilon_+$$
$$0 = |\vec{q}_\pi| \sin\theta_\pi \pm \varepsilon_- \sin\theta_- \pm \varepsilon_+ \sin\theta_+$$
$$P = |\vec{q}_\pi| \cos\theta_\pi + \varepsilon_- \cos\theta_- + \varepsilon_+ \cos\theta_+$$

we get

$$\varepsilon_- = \frac{M(\varepsilon_\pi - M - E) + \varepsilon_\pi E - P|q_\pi|\cos\theta_\pi - \frac{m_\pi^2}{2}}{\varepsilon_\pi - M - E + P\cos\theta_- \pm |q_\pi|(\sin\theta_\pi \sin\theta_- \mp \cos\theta_\pi \cos\theta_-)}$$

KINEMATICS

- for given values of q^2, ε_π



- the allowed kinematical region for ε_-, θ_-

AN ALTERNATIVE CHOICE OF THE PHASE SPACE

- phase space with trivial allowed kinematical region

$$(A_1, A_2) \times (B_1, B_2) \times ...$$

- an example of such phase space is the pion phase space in the CM frame

$$q^2 \in (4m_\ell^2, (\sqrt{s} - m_\pi)^2), \ \cos\theta_\pi^{cm} \in (-1, 1)$$

- a similar situation can be obtained for the lepton pair phase space in the γ^* rest frame - CM frame of the outgoing lepton pair

$$\cos\theta_-^* \in (-1, 1), \ \phi_-^* \in (0, 2\pi)$$

Phase space volume

- pion:

$$\frac{d^3 q_\pi}{2\varepsilon_\pi} = \pi |q_\pi^{cm}| d\varepsilon_\pi^{cm} d\cos\theta_\pi^{cm}$$

where

$$d\varepsilon_\pi^{cm} = -\frac{dq^2}{4E^{cm}}$$

$$\Rightarrow \frac{d^3 q_\pi}{2\varepsilon_\pi} = \frac{\sqrt{(s+m_\pi^2-q^2)^2 - 16E^{cm\,2} m_\pi^2}}{32(2\pi)^2 E^{cm\,2}} dq^2 d\cos\theta_\pi^{cm}$$

- lepton pair:

$$\frac{d^3 q_-}{2\varepsilon_-}\frac{d^3 q_+}{2\varepsilon_+}\delta^4 = \frac{d^3 q_-}{2\varepsilon_-}\delta^1 = \frac{d\Omega_-^* |q_-^*| \varepsilon_-^* d\varepsilon_-^*}{2\varepsilon_-^*}\delta^1 = \frac{1}{8} d\Omega_-^*$$

- because $\delta^1 = \delta((p_1+p_2-q_\pi-q_-)^2 - m_\ell^2)$

$$\frac{d((p_1+p_2-q_\pi-q_-)^2 - m_\ell^2)}{d\varepsilon_-^*} = -2\sqrt{q^2} = -4|q_-^*|$$

DIFF. CROSS SECTION

$$d^4\sigma = \frac{\sqrt{(s+m_\pi^2-q^2)^2-16E^{cm\,2}m_\pi^2}}{2048(2\pi)^4E^{cm\,3}p^{cm}}|\mathcal{M}_0|^2\times dq^2 d\cos\theta_\pi^{cm} d\cos\theta_-^* d\phi_-^*$$

where $|\mathcal{M}_0|^2$ is Lorentz invariant. However we should be able to calculate it from input variables $q^2, \cos\theta_\pi^{cm}, \cos\theta_-^*, \phi_-^*$

- B,C can be calculated in th CM frame - O.K.
- scalar products $p_1\cdot q_-, p_2\cdot q_-$ are more complicated. Vectors should be boosted tho the same kin. frame and then multiplied. For example

$$p_1\cdot q_- + p_2\cdot q_- = \sqrt{q^2}E^{cm}\gamma(1-\cos\theta_-^*\beta)$$

- γ,β are Lorentz factors of the boost

Conclusions

- detailed calculation of the differential cross section of the $p\bar{p} \to \pi^0 e^- e^+$
- we can give the formula for the diff. cross section
  - in the LAB frame
  - in the 'mixed' CM+γ^* frame with the trivial allowed kinematical region
- extraction of the 3 structure functions $|F_1|^2, |F_2|^2, |F_1 - F_2|^2$
- they can be measured in the 'unphysical' region
- the $|F_1 - F_2|^2$ gives access to $\cos(\phi_1 - \phi_2)$