



TDAs for PANDA processes

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based on work done with

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Phys Rev D76, 2007; Phys Rev D82, 2010; Phys Rev D84, 2011; arXiv 1112.3570

the PANDA@FAIR processes



$$\bar{N}N \to \pi\gamma^* \to \pi e^+ e^-$$

$$\bar{N}N \to \pi\psi \to \pi e^+e^-$$

but also

 $\bar{N}N \to \eta \gamma^* \to \eta e^+ e^-$, $\bar{N}N \to \pi \pi \gamma^* \to \pi \pi e^+ e^-$, ...

Forward and Backward kinematics

J.P. Lansberg, B. Pire, L. Szymanowski'07: πN TDAs arise in the factorized description of

$$N(p_1) + \bar{N}(p_2) \to \gamma^*(q) + \pi(p_\pi) \to l^+(k_1) + l^-(k_2) + \pi(p_\pi)$$



• $W^2 = (p_1 + p_2)^2$ and $q^2 = Q^2$ - large; $(p_1 - p_\pi)^2$ -small ($\theta_\pi \sim 0$ in C.M.S: near forward kinematics)

+ reversed case small u where

 π^0 is forward and γ^* backward in CMS (γ^* almost at rest in lab)

Interpretation of the $(\pi \rightarrow N)or(N \rightarrow \pi)$ TDAs

Develop proton wave function as (schematically) $|qqq > + |qqq\pi > +...$ |qqq > is described by proton DA : $\langle 0 | \epsilon^{ijk} u^i_{\alpha}(z_1) u^j_{\beta}(z_2) d^k_{\gamma}(z_3) | p(p,s) \rangle \Big|_{z^+=0, z_T=0}$

Define matrix elements sensitive to $|qqq \ \pi > part$: the TDAs

$$\left\langle \pi(p') \right| \epsilon^{ijk} u^i_{\alpha}(z_1) u^j_{\beta}(z_2) d^k_{\gamma}(z_3) \left| p(p,s) \right\rangle \Big|_{z^+=0, z_T=0}$$

light cone matrix elements of operators obeying usual RG evolution equations

The $\pi \to N$ TDAs provides information on the next to minimal Fock state in the baryon $p \to p' = p \to \infty \times \begin{bmatrix} 1 & 1 & 1 \\ p' & 1 & 1 \end{bmatrix}^*$

 $Proton = |u \ d \ d \ \pi^+ >$ with small transverse separation for the quark triplet

or how one can find a meson in a proton

Impact parameter interpretation

• As for GPDs Fourier transform $\Delta_T \rightarrow b_T$

$$F(x_i, \xi, u = \Delta^2) \rightarrow \tilde{F}(x_i, \xi, b_T)$$

 \rightarrow Transverse picture of pion cloud $% \left({{\mathbf{T}_{i}}} \right)$ in the proton



if factorization works

Crossing to πN final state

Crossing $\pi N \text{ TDA} \leftrightarrow \pi N \text{ GDA}$ and soft pion theorem

- Crossing relates πN TDA in $\gamma^* N \to \pi N'$ and πN GDAs (light-cone wave function)
- Physical domain in (Δ^2, ξ) -plane (defined by $\Delta_T^2 \leq 0$) in the chiral limit (m = 0):



Soft pion theorem Pobylitsa, Polyakov and Strikman'01 ($Q^2 \gg \Lambda_{\rm QCD}^3/m$) constrains πN GDA at the threshold $\xi = 1$, $\Delta^2 = M^2$.

Soft pion limit

Soft pion theorem for $\pi N~{\rm GDA}$

Soft pion theorem Pobylitsa, Polyakov and Strikman'01 ($Q^2 \gg \Lambda_{QCD}^3/m$):

$$\langle 0|\widehat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(z_1, z_2, z_3)|\pi_a N_\iota\rangle = -\frac{i}{f_\pi} \langle 0| \left[\widehat{Q}^a_5, \,\widehat{O}^{\alpha\beta\gamma}_{\rho\tau\chi}(z_1, z_2, z_3)\right]|N_\iota\rangle\,,$$

with $\left[\widehat{Q}^a_5, \Psi^{\alpha}_{\eta}\right] = -\frac{1}{2} (\sigma_a)^{\alpha}_{\ \delta} \gamma^5_{\eta\tau} \Psi^{\delta}_{\tau};$

- At the pion threshold (ξ = 1, Δ² = M² in the chiral limit) soft pion theorem fixes πN TDAs/GDAs in terms of nucleon DAs V^p, A^p, T^p (see V. Braun, D. Ivanov, A.Lenz, A.Peters'08).
- E.g. soft pion theorem for uud proton to π^0 TDAs:

$$\{V_1^{p\pi^0}, A_1^{p\pi^0}\}(x_1, x_2, x_3, \xi = 1, \Delta^2 = M^2) = -\frac{1}{8}\{V^p, A^p\}(\frac{x_1}{2}, \frac{x_2}{2}, \frac{x_3}{2});$$

$$T_1^{p\pi^0}(x_1, x_2, x_3, \xi = 1, \Delta^2 = M^2) = \frac{3}{8}T^p(\frac{x_1}{2}, \frac{x_2}{2}, \frac{x_3}{2})$$

$$\{V_2^{p\pi^0}, A_2^{p\pi^0}, T_2^{p\pi^0}\} = -\frac{1}{2}\{V_1^{p\pi^0}, A_1^{p\pi^0}, T_1^{p\pi^0}\} \quad T_{3,4}^{p\pi^0} = 0;$$



"Skewing" $\xi=1$ limit for πN TDAs

After suitable change of spectral variables ($\kappa = \alpha_3 + \beta_3$, $\theta = \frac{\alpha_1 + \beta_1 - \alpha_2 - \beta_2}{2}$, $\mu = \alpha_3 - \beta_3$, $\lambda = \frac{\alpha_1 - \beta_1 - \alpha_2 + \beta_2}{2}$) and introduction of "quark-diquark" coordinates $w = x_3 - \xi$; $v = \frac{x_1 - x_2}{2}$:

$$H(w, v, \xi) = \int_{-1}^{1} d\kappa \int_{-\frac{1-\kappa}{2}}^{\frac{1-\kappa}{2}} d\theta \int_{-1}^{1} d\mu_i \int_{-\frac{1-\mu}{2}}^{\frac{1-\mu}{2}} d\lambda \,\delta(w - \frac{\kappa - \mu}{2}(1-\xi) - \kappa\xi)$$
$$\times \delta\left(v - \frac{\theta - \lambda}{2}(1-\xi) - \theta\xi\right) F(\kappa, \theta, \mu, \lambda)$$

• A factorized Ansatz for quadruple distribution F_i :

$$F(\kappa, \theta, \mu, \lambda) = V(\kappa, \theta) h(\mu, \lambda)$$

with the profile $h(\mu, \lambda)$ normalized as $\int d\mu \int d\lambda h(\mu, \lambda) = 1$.

- Since $H(w, v, \xi = 1) = V(w, v)$ for V one may use input from the soft pion theorem
- A possible choice for the profile: $h(\mu, \lambda) = \frac{15}{16} (1 + \mu)((1 \mu)^2 4\lambda^2)$; vanishes at the borders of the definition domain.

From $\xi = 0$ to $\xi = 1$



Nucleon exchange through a TDA

Nucleon pole contribution

• *u*-channel nucleon exchange is complementary to the spectral representation (*D*-term like contributions) non-zero in the ERBL-like region $0 \le x_i \le 2\xi$.

• The effective Hamiltonian for $\pi \bar{N}N$:

$$\mathcal{H}_{\rm eff} = i g_{\pi NN} \bar{N}_{\alpha} (\sigma_a)^{\alpha}_{\ \beta} \gamma_5 N^{\beta} \pi_a$$



$$\langle \pi_a(p_\pi) | \hat{O}^{\alpha \beta \gamma}_{\rho \tau \chi}(\lambda_1 n, \lambda_2 n, \lambda_3 n) | N_\iota(p_1, s_1) \rangle$$

$$= \sum_{s_p} \langle 0 | \hat{O}^{\alpha \beta \gamma}_{\rho \tau \chi}(\lambda_1 n, \lambda_2 n, \lambda_3 n) | N_\kappa(-\Delta, s_p) \rangle (\sigma_a)^{\kappa} \,_{\iota} \frac{i g_{\pi NN} \, \bar{U}_\varrho(-\Delta, s_p)}{\Delta^2 - M^2} \left(\gamma^5 U(p_1, s_1) \right)_\varrho \,.$$

• After decomposition over the Dirac structures:

$$\{V_1, A_1, T_1\}^{(\pi N)}(x_1, x_2, x_3)$$

= $\Theta_{\text{ERBL}}(x_1, x_2, x_3) \times \frac{M f_{\pi} g_{\pi N N}}{\Delta^2 - M^2} \frac{1}{(2\xi)} \{V^p, A^p, T^p\} \left(\frac{x_1}{2\xi}, \frac{x_2}{2\xi}, \frac{x_3}{2\xi}\right);$

 \Rightarrow A spectral representation with input fixed at $\xi = 1$ through soft pion theorem

and deskewing (i.e. $\xi \rightarrow \neq 1$) through an ansatz

 \Rightarrow A nucleon pole exchange in the *u*-channel

These two components are additive and there is no double counting (one may also add a \triangle -pole exchange but small contribution)

→ A model driven by a nucleon DA parametrization
 various existing DAs : CZ, COZ, KS, GS, BLW ...

input dependence

Cross section (for electroproduction now) calculated from the modeled TDA depends much on the DA model



Figure 1: Unpolarized cross section $\frac{d^2\sigma_T}{d\Omega_{\pi}}$ (in nb/sr) for backward $\gamma^* p \to p\pi^0$ (upper panel) and for backward $\gamma^* p \to n\pi^+$ (lower panel) as the function of $x_{\rm B}$ computed in the two component model for πN TDAs for $Q^2 = 10 \,{\rm GeV}^2$, $u = -0.5 \,{\rm GeV}^2$ as a function of $x_{\rm B}$. CZ [7] (red solid lines), COZ [8] (dotted lines), KS [9] (dashed lines), GS [10] (dash-dotted lines) nucleon DAs and BLWNNLO [4] (orange solid lines) were used as inputs for our model.

Conclusions

TDAs explore confinement dynamics of quarks in hadrons .

TDA extraction is crucial to probe meson content of baryons

CLAS $\gamma^* p \rightarrow \pi^+ n$ very preliminary analysis by Kijun Park I

Table: Determination of kinematic bin

variable	unit	num. bin	range	bin size
\overline{W}	GeV	1	> 2.0	0.4
Q^2	${\sf GeV}^2$	5	$1.6\sim 4.5$	various
$ \Delta_T^2 $	${\sf GeV}^2$	1	< 0.5	0.5



→ PANDA @FAIR : timelike channels