## TDAs for PANDA processes

PANDA 2012, ORSAY, January 2012

B. Pire<br>CPhT, École Polytechnique

based on work done with
JP Lansberg, K Semenov-Tian-Shansky, L Szymanowski
Phys Rev D76, 2007 ; Phys Rev D82, 2010 ; Phys Rev D84, 2011 ; arXiv 1112.3570

## the PANDA@FAIR processes


$\bar{N} N \rightarrow \pi \gamma^{*} \rightarrow \pi e^{+} e^{-}$
$\bar{N} N \rightarrow \pi \psi \rightarrow \pi e^{+} e^{-}$

## but also

$$
\bar{N} N \rightarrow \eta \gamma^{*} \rightarrow \eta e^{+} e^{-} \quad, \quad \bar{N} N \rightarrow \pi \pi \gamma^{*} \rightarrow \pi \pi e^{+} e^{-}
$$

## Forward and Backward kinematics

- J.P. Lansberg, B. Pire, L. Szymanowski'07: $\pi N$ TDAs arise in the factorized description of

$$
N\left(p_{1}\right)+\bar{N}\left(p_{2}\right) \rightarrow \gamma^{*}(q)+\pi\left(p_{\pi}\right) \rightarrow l^{+}\left(k_{1}\right)+l^{-}\left(k_{2}\right)+\pi\left(p_{\pi}\right)
$$



- $W^{2}=\left(p_{1}+p_{2}\right)^{2}$ and $q^{2}=Q^{2}$ - large; $\left(p_{1}-p_{\pi}\right)^{2}$-small $\left(\theta_{\pi} \sim 0\right.$ in C.M.S: near forward kinematics)
+ reversed case small $u$ where
$\pi^{0}$ is forward and $\gamma^{*}$ backward in CMS ( $\gamma^{*}$ almost at rest in lab)


## Interpretation of the $(\pi \rightarrow N) \operatorname{or}(N \rightarrow \pi)$ TDAs

Develop proton wave function as (schematically) $|q q q>+| q q q \pi>+\ldots$
$\mid q q q>$ is described by proton DA : $\left.\langle 0| \epsilon^{i j k} u_{\alpha}^{i}\left(z_{1}\right) u_{\beta}^{j}\left(z_{2}\right) d_{\gamma}^{k}\left(z_{3}\right)|p(p, s)\rangle\right|_{z^{+}=0, z_{T}=0}$
Define matrix elements sensitive to $\mid q q q \pi>$ part : the TDAs

$$
\left.\left\langle\pi\left(p^{\prime}\right)\right| \epsilon^{i j k} u_{\alpha}^{i}\left(z_{1}\right) u_{\beta}^{j}\left(z_{2}\right) d_{\gamma}^{k}\left(z_{3}\right)|p(p, s)\rangle\right|_{z^{+}=0, z_{T}=0}
$$

light cone matrix elements of operators obeying usual RG evolution equations
$\Rightarrow$ The $\pi \rightarrow N$ TDAs provides information on the next to minimal Fock state in the baryon


Proton $=\left\lvert\, \begin{array}{ll}u d d & \pi^{+}>\end{array}\right.$with small transverse separation for the quark triplet
or how one can find a meson in a proton

## Impact parameter interpretation

- As for GPDs Fourier transform $\Delta_{T} \rightarrow b_{T}$

$$
F\left(x_{i}, \xi, u=\Delta^{2}\right) \rightarrow \tilde{F}\left(x_{i}, \xi, b_{T}\right)
$$

$\rightarrow$ Transverse picture of pion cloud in the proton

if factorization works

## Crossing to $\pi N$ final state

## Crossing $\pi N$ TDA $\leftrightarrow \pi N$ GDA and soft pion theorem

- Crossing relates $\pi N$ TDA in $\gamma^{*} N \rightarrow \pi N^{\prime}$ and $\pi N$ GDAs (light-cone wave function)
- Physical domain in $\left(\Delta^{2}, \xi\right)$-plane (defined by $\left.\Delta_{T}^{2} \leq 0\right)$ in the chiral limit ( $m=0$ ):

- Soft pion theorem Pobylitsa, Polyakov and Strikman'01 ( $\left.Q^{2} \gg \Lambda_{\mathrm{QCD}}^{3} / m\right)$ constrains $\pi N$ GDA at the threshold $\xi=1, \Delta^{2}=M^{2}$.


## Soft pion limit

Soft pion theorem for $\pi N$ GDA

- Soft pion theorem Pobylitsa, Polyakov and Strikman'01 $\left(Q^{2} \gg \Lambda_{\mathrm{QCD}}^{3} / m\right)$ :

$$
\langle 0| \widehat{O}_{\rho \tau \chi}^{\alpha \beta \gamma}\left(z_{1}, z_{2}, z_{3}\right)\left|\pi_{a} N_{\iota}\right\rangle=-\frac{i}{f_{\pi}}\langle 0|\left[\widehat{Q}_{5}^{a}, \widehat{O}_{\rho \tau \chi}^{\alpha \beta \gamma}\left(z_{1}, z_{2}, z_{3}\right)\right]\left|N_{\iota}\right\rangle,
$$

with $\left[\widehat{Q}_{5}^{a}, \Psi_{\eta}^{\alpha}\right]=-\frac{1}{2}\left(\sigma_{a}\right)_{\delta}^{\alpha} \gamma_{\eta \tau}^{5} \Psi_{\tau}^{\delta} ;$

- At the pion threshold ( $\xi=1, \Delta^{2}=M^{2}$ in the chiral limit) soft pion theorem fixes $\pi N$ TDAs/GDAs in terms of nucleon DAs $V^{p}, A^{p}, T^{p}$ (see V. Braun, D. Ivanov, A.Lenz, A.Peters'08).
- E.g. soft pion theorem for uud proton to $\pi^{0}$ TDAs:

$$
\begin{aligned}
& \left\{V_{1}^{p \pi^{0}}, A_{1}^{p \pi^{0}}\right\}\left(x_{1}, x_{2}, x_{3}, \xi=1, \Delta^{2}=M^{2}\right)=-\frac{1}{8}\left\{V^{p}, A^{p}\right\}\left(\frac{x_{1}}{2}, \frac{x_{2}}{2}, \frac{x_{3}}{2}\right) \\
& T_{1}^{p \pi^{0}}\left(x_{1}, x_{2}, x_{3}, \xi=1, \Delta^{2}=M^{2}\right)=\frac{3}{8} T^{p}\left(\frac{x_{1}}{2}, \frac{x_{2}}{2}, \frac{x_{3}}{2}\right) \\
& \left\{V_{2}^{p \pi^{0}}, A_{2}^{p \pi^{0}}, T_{2}^{p \pi^{0}}\right\}=-\frac{1}{2}\left\{V_{1}^{p \pi^{0}}, A_{1}^{p \pi^{0}}, T_{1}^{p \pi^{0}}\right\} \quad T_{3,4}^{p \pi^{0}}=0
\end{aligned}
$$

## A skewing ansatz

## "Skewing" $\xi=1$ limit for $\pi N$ TDAs

After suitable change of spectral variables $\left(\kappa=\alpha_{3}+\beta_{3}, \theta=\frac{\alpha_{1}+\beta_{1}-\alpha_{2}-\beta_{2}}{2}\right.$, $\mu=\alpha_{3}-\beta_{3}, \lambda=\frac{\alpha_{1}-\beta_{1}-\alpha_{2}+\beta_{2}}{2}$ ) and introduction of "quark-diquark" coordinates $w=x_{3}-\xi ; v=\frac{x_{1}-x_{2}}{2}$ :

$$
\begin{aligned}
& H(w, v, \xi)=\int_{-1}^{1} d \kappa \int_{-\frac{1-\kappa}{2}}^{\frac{1-\kappa}{2}} d \theta \int_{-1}^{1} d \mu_{i} \int_{-\frac{1-\mu}{2}}^{\frac{1-\mu}{2}} d \lambda \delta\left(w-\frac{\kappa-\mu}{2}(1-\xi)-\kappa \xi\right) \\
& \times \delta\left(v-\frac{\theta-\lambda}{2}(1-\xi)-\theta \xi\right) F(\kappa, \theta, \mu, \lambda)
\end{aligned}
$$

- A factorized Ansatz for quadruple distribution $F_{i}$ :

$$
F(\kappa, \theta, \mu, \lambda)=V(\kappa, \theta) h(\mu, \lambda)
$$

with the profile $h(\mu, \lambda)$ normalized as $\int d \mu \int d \lambda h(\mu, \lambda)=1$.
■ Since $H(w, v, \xi=1)=V(w, v)$ for $V$ one may use input from the soft pion theorem

- A possible choice for the profile: $h(\mu, \lambda)=\frac{15}{16}(1+\mu)\left((1-\mu)^{2}-4 \lambda^{2}\right)$; vanishes at the borders of the definition domain.

From $\xi=0$ to $\xi=1$


## Nucleon exchange through a TDA

Nucleon pole contribution

- u-channel nucleon exchange is complementary to the spectral representation ( $D$-term like contributions) non-zero in the ERBL-like region $0 \leq x_{i} \leq 2 \xi$.
- The effective Hamiltonian for $\pi \bar{N} N$ :

$$
\mathcal{H}_{\mathrm{eff}}=i g_{\pi N N} \bar{N}_{\alpha}\left(\sigma_{a}\right)_{\beta}^{\alpha} \gamma_{5} N^{\beta} \pi_{a}
$$



$$
\begin{aligned}
& \left\langle\pi_{a}\left(p_{\pi}\right)\right| \widehat{O}_{\rho \tau \chi}^{\alpha \beta \gamma}\left(\lambda_{1} n, \lambda_{2} n, \lambda_{3} n\right)\left|N_{\iota}\left(p_{1}, s_{1}\right)\right\rangle \\
& =\sum_{s_{p}}\langle 0| \widehat{O}_{\rho \tau \chi}^{\alpha \beta \gamma}\left(\lambda_{1} n, \lambda_{2} n, \lambda_{3} n\right)\left|N_{\kappa}\left(-\Delta, s_{p}\right)\right\rangle\left(\sigma_{a}\right)_{\iota}^{\kappa} \frac{i g_{\pi N N} \bar{U}_{\varrho}\left(-\Delta, s_{p}\right)}{\Delta^{2}-M^{2}}\left(\gamma^{5} U\left(p_{1}, s_{1}\right)\right)_{\varrho}
\end{aligned}
$$

- After decomposition over the Dirac structures:

$$
\begin{aligned}
& \left\{V_{1}, A_{1}, T_{1}\right\}^{(\pi N)}\left(x_{1}, x_{2}, x_{3}\right) \\
& =\Theta_{\mathrm{ERBL}}\left(x_{1}, x_{2}, x_{3}\right) \times \frac{M f_{\pi} g_{\pi N N}}{\Delta^{2}-M^{2}} \frac{1}{(2 \xi)}\left\{V^{p}, A^{p}, T^{p}\right\}\left(\frac{x_{1}}{2 \xi}, \frac{x_{2}}{2 \underline{\xi}}, \frac{x_{3}}{2 \xi}\right)
\end{aligned}
$$

## a 2 - component model for TDA

$\Rightarrow$ A spectral representation with input fixed at $\xi=1$ through soft pion theorem

```
and deskewing (i.e. }\xi->\not=1\mathrm{ ) through an ansatz
```

$\leadsto$ A nucleon pole exchange in the $u$-channel

These two components are additive and there is no double counting
(one may also add a $\Delta$-pole exchange but small contribution)
$\Rightarrow$ A model driven by a nucleon DA parametrization various existing DAs : CZ, COZ, KS, GS, BLW ...

## input dependence

## Cross section ( for electroproduction now) calculated from the modeled TDA depends much on the DA model



Figure 1: Unpolarized cross section $\frac{d^{2} \sigma_{T}}{d \Omega_{\pi}}$ (in $\mathrm{nb} / \mathrm{sr}$ ) for backward $\gamma^{*} p \rightarrow p \pi^{0}$ (upper panel) and for backward $\gamma^{*} p \rightarrow n \pi^{+}$(lower panel) as the function of $x_{\mathrm{B}}$ computed in the two component model for $\pi N$ TDAs for $Q^{2}=10 \mathrm{GeV}^{2}, u=-0.5 \mathrm{GeV}^{2}$ as a function of $x_{\mathrm{B}}$. CZ [7] (red solid lines), COZ [8] (dotted lines), KS [9] (dashed lines), GS [10] (dash-dotted lines) nucleon DAs and BLWNNLO [4] (orange solid lines) were used as inputs for our model

## Conclusions

TDAs explore confinement dynamics of quarks in hadrons.
TDA extraction is crucial to probe meson content of baryons
$\rightleftharpoons$ First signals at JLab at $6 \mathrm{GeV}+$ CLAS12 : spacelike channels

CLAS $\gamma^{*} p \rightarrow \pi^{+} n$ very preliminary analysis by Kijun Park I

Table: Determination of kinematic bin

| variable | unit | num. bin | range | bin size |
| :--- | :---: | :---: | :---: | :--- |
| $W$ | GeV | 1 | $>2.0$ | 0.4 |
| $Q^{2}$ | $\mathrm{GeV}^{2}$ | 5 | $1.6 \sim 4.5$ | various |
| $\left\|\Delta_{T}^{2}\right\|$ | $\mathrm{GeV}^{2}$ | 1 | $<0.5$ | 0.5 |


$\leadsto$ PANDA @FAIR : timelike channels

