The annihilation reaction $\bar{p} + p \to e^+ + e^- + \pi^0$

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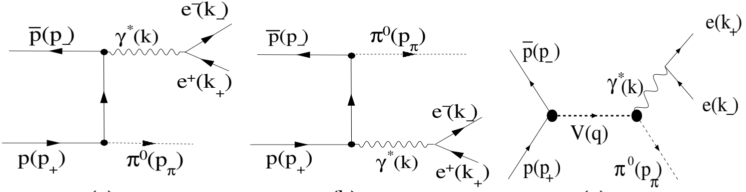
Motivations

- First 'inelastic' reaction in $\bar{p}p$ annihilation into leptons ;
- Investigate the kinematical region below the $\bar{p}p$ threshold, $s = 4M_p^2$, the 'unphysical' region ;
- Learn about
  - the reaction mechanism
  - electromagnetic form factors
  - axial form factors
- Investigate open questions :
  - $\bar{N}N$ resonance ? Protonium state ?
  - Point-like baryons at threshold ?
- Crossing symmetry

History

- 1965 M.P. Rekalo proposed the reaction $\pi + N \to N + \ell^+ + \ell^-$ to investigate time-like form factors below threshold [*Sov.J.Nucl.Phys.* **1** *(1965) 760*].
- 1996 A.Z. Dubnickova, S. Dubnicka, and M.P. Rekalo, [*Z.Phys.* **C70** *(1996) 473*] investigated the reaction $\bar{p} + p \to e^+ + e^- + \pi^0$ near treshold, based on one diagram.
- 2008 C. Adamuscin, E. A. Kuraev, E. Tomasi-Gustafsson and F. E. Maas, *Testing axial and electromagnetic nucleon form factors in time-like region in the processes $\bar{p} + n \to \pi^- + \ell^- + \ell^+$ and $\bar{p} + p \to \pi^0 + \ell^- + \ell^+$,* $\ell = e, \mu$ [PRC **75** (2007) 045205].
- 2010 E.A. Kuraev, Yu.M. Bystritskiy, V.V. Bytev, E. Tomasi-Gustafsson, S-channel ω exchange [arXiv :1012.5720 hep-ph].
- 21-XII-2011 J. Boucher, PhD - Feasibility study in PANDA.

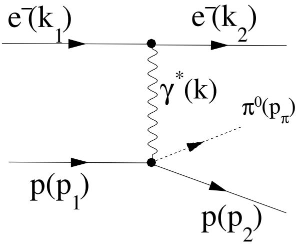
Crossing symmetry : $\bar{p} + p \rightarrow \pi^0 + \ell^- + \ell^+$

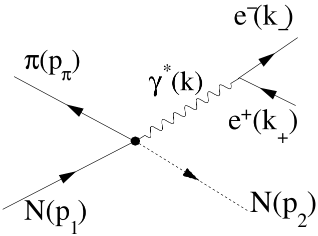


(a) (b) (c)

$e^- + p \rightarrow e^- + p + \pi^0$

$\pi + N \rightarrow e^+ + e^- + N$





Cross section for $\bar{p} + p \to \pi^0 + \ell^- + \ell^+$

$$d\sigma = \frac{1}{2 \cdot 2 \cdot 4I} \sum_{spins} |\mathcal{M}|^2 d\Phi_3$$

- $4I$ is the incident flux :
  $I = \sqrt{(p_+ p_-)^2 - M_p^4} = (1/2)\sqrt{s(s - 4M_p^2)}$,
  - M_p is the proton mass,
  - $s = (p_+ + p_-)^2$ is the total invariant mass,
- The factor $[1/(2 \cdot 2)]$: averaging over initial particles polarizations,
- $d\Phi_3$ is the phase volume for 3-body process
- $\mathcal{M}$: the matrix element : s, t, u channels and their interference

The Phase Volume for a 3-body process

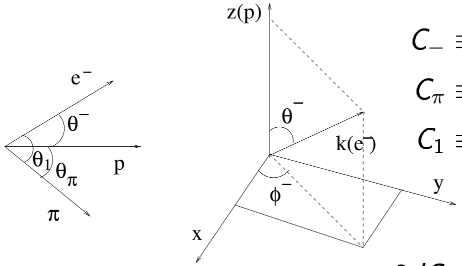
$$d\Phi_3 = (2\pi)^4 \delta(p_+ + p_- - q_+ - q_- - q_\pi) \frac{d^3\vec{q_+}}{(2\pi)^3 2E_+} \frac{d^3\vec{q_-}}{(2\pi)^3 2E_-} \frac{d^3\vec{q_\pi}}{(2\pi)^3 2E_\pi}.$$

- Integrate over q_+ using δ function

$$d\Phi_3 = \frac{1}{2^7\pi^5}\delta(q_+^2 - m_e^2)\ \beta_- E_- dE_-\ \beta_\pi E_\pi dE_\pi\ d\Omega_- d\Omega_\pi,$$

- Manage angular variables
  $d\Omega_- d\Omega_\pi = dC_-\ d\phi_-\ dC_\pi\ d\phi_\pi$
- Integrate over $\phi_- (\to E_-, E_\pi, C_-, C_\pi)$

The phase-space : Euler parametrization



$$C_- \equiv \cos\theta_- = \cos(\widehat{\vec{p}, \vec{q}_-})$$
$$C_\pi \equiv \cos\theta_\pi = \cos(\widehat{\vec{p}, \vec{q}_\pi})$$
$$C_1 \equiv \cos\theta_1 = \cos(\widehat{\vec{q}_-, \vec{q}_\pi})$$

$$dC_-\ dC_\pi\ d\phi_\pi = \frac{2dC_-dC_\pi dC_1}{\sqrt{D}},$$

$$D = 1 - C_-^2 - C_\pi^2 - C_1^2 + 2C_-C_\pi C_1.$$

Calculate all variables as function of E_-, E_π, C_-, C_π.

$$d\Phi_3 = \frac{1}{2^6\pi^4}\frac{dE_-\ dE_\pi\ dC_-\ dC_\pi}{\sqrt{D^{(0)}}},\ C_1 \rightarrow C_1^{(0)}$$

The positivity of $D^{(0)}$ defines the allowed kinematical region.

Root Structure and Class

- struct KinematicPoint

```
{       public :
                double s, Em, Cm, Epi, Cpi ; } ;
```

- class CrossSection

```
{       public :
                CrossSection() ;
                bool SetKinematics(KinematicPoint p) ;
                double Value(KinematicPoint p) ; }
```

- CrossSection : :SetKinematics(KinematicPoint _p)

```
{                s = _p.s ;
                Em = _p.Em ;
                Cm = _p.Cm ;
                Epi = _p.Epi ;
                Cpi = _p.Cpi ;
         if ..../return false ;
         if(D < 0){/* cout << "D < 0 : D = " << D <<
endl; */return false; }
         return true ; }
```

How to use ?

- double CrossSection : :Value(KinematicPoint p) {
  define traces
  OUTPUT-cross section : I, dΦ3, $|\mathcal{M}_s|^2$, $|\mathcal{M}_t|^2$, $|\mathcal{M}_u|^2$, $\mathcal{M}_{st}$, $\mathcal{M}_{su}$, $\mathcal{M}_{tu}$, ... }
- int main() {
  CrossSection cs ;
  KinematicPoint p ;
  bool inside = cs.SetKinematics(p) ;
  if (inside) { toto=cs.Value(p) ; }
  build n-tuples : s, qq, E_π, C_π, E_-, C_-,toto
  }

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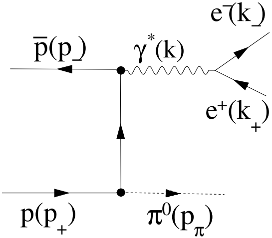
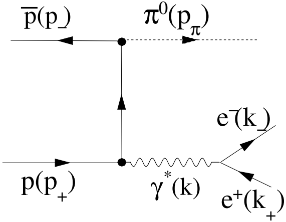
t and u channels

$$|\mathcal{M}_t|^2 = -\left[\frac{e^2 g_{\pi pp}}{k^2(t - M_p^2)}\right]^2 S_t;\ |\mathcal{M}_u|^2 = -\left[\frac{e^2 g_{\pi pp}}{k^2(u - M_p^2)}\right]^2 S_u,$$





where $S_{t,u}$ are expressed in terms of traces of Dirac matrices :

$$S_t = \mathrm{Tr}[\hat{q}_- \gamma^\mu \hat{q}_+ \gamma^\nu]\, \mathrm{Tr}[(\hat{p}_- - M_p)\, \gamma_5\, (-\hat{p}_- + \hat{q}_\pi + M_p)\, \Gamma_\mu \left(k^2\right) (\hat{p}_+ + M_p)\, \tilde{\Gamma}_\nu \left(k^2\right) (-\hat{p}_- + \hat{q}_\pi + M_p)\, \gamma_5],$$

$$S_u = \mathrm{Tr}[\hat{q}_- \gamma^\mu \hat{q}_+ \gamma^\nu]\, \mathrm{Tr}[(\hat{p}_- - M_p)\, \Gamma_\mu \left(k^2\right) \left(-\hat{p}_- + \hat{k} + M_p\right) \gamma_5\, (\hat{p}_+ + M_p)\, \gamma_5 \left(-\hat{p}_- + \hat{k} + M_p\right) \tilde{\Gamma}_\nu \left(k^2\right)]$$

Regge factors

- include the effects of initial state strong interaction from exchange of vector, scalar, pseudoscalar mesons $\to$ Regge form of the amplitudes
- introduce Regge factors to include infinite number of resonances

$$R(t) = \left(\frac{s}{s_0}\right)^{2[\alpha(t)-1]}, \quad \alpha_p(t) = \frac{1}{2} + r\frac{\alpha_s}{\pi}\frac{t - M^2}{M^2}$$

$$R(u) = \left(\frac{s}{s_0}\right)^{2[\alpha(u)-1]}, \quad \alpha_p(u) = \frac{1}{2} + r\frac{\alpha_s}{\pi}\frac{u - M^2}{M^2}$$

where $s_0 \simeq 1$ GeV^2 and $r\alpha_s/\pi \simeq 0.7$ fitting parameters

A. B. Kaidalov, arXiv :hep-ph/0103011

- neglect the tu interference : t and u diagrams dominate in different kinematical regions : forward/backward

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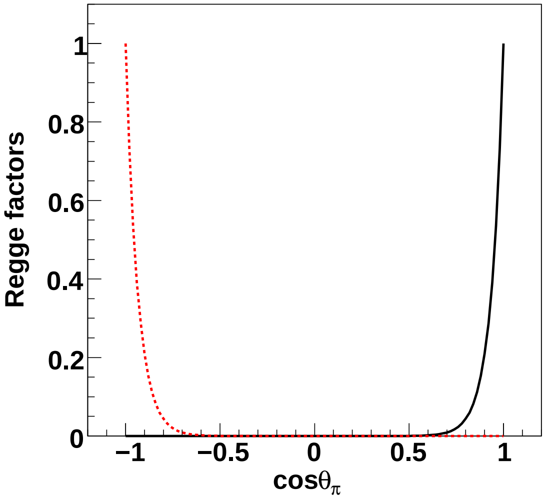
Illustration of Regge factors

$R(u)$ $R(t)$

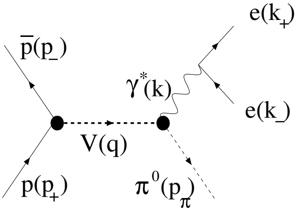


The s-channel - Why ω-exchange ?

- At $\theta_{CM} = 90^\circ$ the cross section is dominated by s-channel vector meson exchange
- This includes $p\bar{p}$ bound state, (heavy) vector and scalar mesons, radially excited meson states..
- The present considerations are outside the kinematics of narrow resonances ;
- $\omega(1450)$, $\omega(1650)$ are important in a limited kinematical region (total energy close to the mass), outside,they are suppressed by form factors, being more extended objects.
- The largest anomalous vertex is $\rho\omega\pi \to$ largest quark coupling

  E. Witten, Nucl. Phys. B223 (1983) 422, O. Kaymakcalan, S. Rajeev, J. Schechter, PRD30 (1984) 594.

S-channel cross section



$$\mathcal{M}_s = -e g_{\omega pp} \frac{G_{\pi\omega\gamma}\left(q^2, k^2\right)}{M_\omega k^2 \left(q^2 - M_\omega^2 + i M_\omega \Gamma_\omega\left(q^2\right)\right)} \epsilon^{\mu\nu qk} J_\mu^{(\omega)}\left(p_+, p_-\right) j_\nu\left(q_+, q_-\right),$$

- $g_{\omega pp}$ is the coupling constant of ω-meson with the proton,
- M_ω and $\Gamma_\omega\left(q^2\right)$ are the mass and the total decay width of ω-meson (constant) [PdG] :

$$\Gamma_\omega\left(q^2\right) \approx \Gamma_\omega\left(M_\omega^2\right) = 8.49 \text{ MeV}.$$

- $k = q_+ + q_-$ is the intermediate photon momentum,
- $j^\mu\left(q_+, q_-\right) = \bar{u}\left(q_-\right)\gamma^\mu v\left(q_+\right)$ is the final lepton pair current.

S-channel cross section

The square of matrix element $\mathcal{M}_s$ has the form :

$$\sum_{spins} |\mathcal{M}_s|^2 = e^2 g^2_{\omega pp} \frac{\left|G_{\pi\omega\gamma}\left(q^2,k^2\right)\right|^2 S_s}{M^2_\omega \left(k^2\right)^2 \left|q^2 - M^2_\omega + iM_\omega\Gamma_\omega\left(q^2\right)\right|^2},$$

where S_s is the following trace :

$$\begin{aligned} S_s &= \epsilon^{\mu\nu qk}\epsilon^{\alpha\beta qk} \sum_{spins} J^{(\omega)}_\mu\left(p_+,p_-\right) J^{(\omega)*}_\alpha\left(p_+,p_-\right) \\ &\quad j_\nu\left(q_+,q_-\right) j^*_\beta\left(q_+,q_-\right) = \\ &= \epsilon^{\mu\nu qk}\epsilon^{\alpha\beta qk} \,\mathrm{Tr}\left[\left(\hat{p}_- - M_p\right)\Gamma^{(\omega)}_\mu(q)\left(\hat{p}_+ + M_p\right)\tilde{\Gamma}^{(\omega)}_\alpha(q)\right] \\ &\quad \mathrm{Tr}\left[\hat{q}_-\gamma_\nu\hat{q}_+\gamma_\beta\right], \quad \tilde{O} = \gamma_0 O^+ \gamma_0 \end{aligned}$$

The ωpp-vertex $J_\mu^{(\omega)}(p_+, p_-)$

$$J_\mu^{(\omega)}(p_+, p_-) = \bar{v}(p_-)\Gamma_\mu^{(\omega)}(q)u(p_+), \;\; q = p_+ + p_-$$

- parametrization of $\Gamma_\mu^{(\omega)}(q)$ via form factors :

$$\Gamma_\mu^{(\omega)}(q) = F_1^{(\omega)}(q^2)\gamma_\mu + \frac{i}{2M_p}F_2^{(\omega)}(q^2)\sigma_{\mu\nu}q^\nu$$

where $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$,

- Approximations :
  - $F_2^{(\omega)} \approx 0$ *R. Machleidt, PRC63 (2001) 024001.*
  - $F_1^{(\omega)}$ real

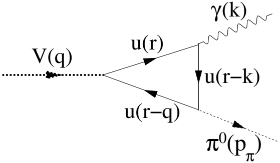
$$F_1^\omega(q^2) = \left[\frac{\Lambda_\omega^4}{\Lambda_\omega^4 + (q^2 - M_\omega^2)^2}\right]^{4/3},$$

  - Normalization $F_1^\omega(M_\omega^2) = 1$.
  - $\Lambda_\omega = 1.25$ GeV is an empirical cut-off.

C. Fernandez-Ramirez, et al., Annals Phys. **321** *(2006) 1408.*

The $\omega\pi\gamma^*$-vertex

Following *M.K. Volkov, V.N. Pervushin, Atomizdat, (1978)*



$$\left|G_{\omega\pi\gamma}\left(q^2,k^2\right)\right|^2 = 9\frac{\alpha}{\pi^3}\frac{g^2_{\omega uu}M^2_\omega}{F^2_\pi}\left|I\left(q^2,k^2\right)\right|^2,$$

- $g_{\pi uu}/m_u = 1/F_\pi$, $F_\pi = 93$ MeV [PDG] is the pion decay constant.
- $g_{\omega uu} = 5.94$ and $g_{\pi uu} = 2.9$ are the couplings of ω and π mesons with the light u-quarks in the loop *M. K. Volkov, E. A. Kuraev, and Y. M. Bystritskiy, Phys. Atom. Nucl.* **72** *(2009) 1513*
- $I\left(q^2,k^2\right)$ is the internal quark loop integral (real)

$$I(q^2,k^2) = \frac{m_u^2}{2\left(k^2-s\right)}\left[\ln^2\left(\frac{q^2}{m_u^2}\right) - \ln^2\left(\frac{k^2}{m_u^2}\right)\right],$$

$k^2 \gg m_u^2$, $q^2 \gg m_u^2$, m_u is the constituent quark mass.

The $\omega\pi\gamma^*$-vertex

Width of radiative decay $\omega \to \pi^0\gamma$:

$$\Gamma\left(\omega \to \pi^0\gamma\right) = \frac{\alpha}{192}\frac{M_\omega^3}{F_\pi^2}\frac{g_{\omega uu}^2}{\pi^4}\left(1-\frac{M_\pi^2}{M_\omega^2}\right)^3 \approx 550 \text{ keV}.$$

The decay branching is equal to :

$$BR\left(\omega \to \pi^0\gamma\right) = \frac{\Gamma\left(\omega \to \pi^0\gamma\right)}{\Gamma_\omega} = 6.5\%,$$

to be compared with :
$BR^{exp.}(\omega \to \pi^0\gamma) = (8.28 \pm 0.28)\%$ [PDG].
Alternatively, one can choose a phenomenological parametrization (monopole on q^2 and k^2) :

$$G_{\omega\pi\gamma^*}\left(q^2,k^2\right) = \frac{G_{\omega\pi\gamma}\left(0,0\right)}{(1+q^2/M_\omega^2)(1+k^2/M_\omega^2)},$$

where, the constant $G_{\omega\pi\gamma}\left(0,0\right)$ is derived from the radiative decay $\omega \to \pi^0\gamma$

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The cross section

The five-fold cross section

$$d\sigma = \frac{1}{2 \cdot 2 \cdot 4I} \sum_{spins} |\mathcal{M}|^2 d\Phi_3$$

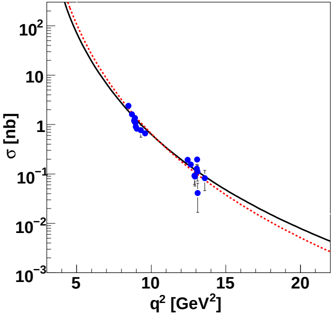
is calculated with the total matrix element squared

$$\begin{aligned} \sum_{spins} |\mathcal{M}|^2 \quad = \quad & \sum_{spins} \Big\{ |\mathcal{M}_s|^2 + |\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 + \\ + \quad & \left(\mathcal{M}_s \mathcal{M}_t^+ + \mathcal{M}_s^+ \mathcal{M}_t\right) + \\ + \quad & \left(\mathcal{M}_s \mathcal{M}_u^+ + \mathcal{M}_s^+ \mathcal{M}_u\right) + \\ + \quad & \left(\mathcal{M}_t \mathcal{M}_u^+ + \mathcal{M}_t^+ \mathcal{M}_u\right) \Big\} , \end{aligned}$$

where the terms are expressed as result of traces.

The S-channel cross section

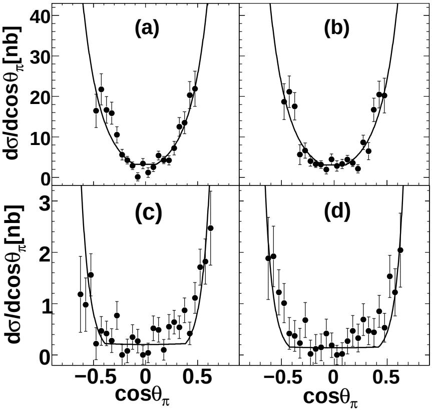
Experimental data for 2.911 GeV $\leq \sqrt{s} \leq$ 3.686 GeV integrated in the range $|\cos\theta_\pi| < 0.2$ [*T. A. Armstrong et al. [Fermilab E760 Collaboration PRD56 (1997) 2509]*



$\omega\pi\gamma$ vertex :

- monopole parametrization
- Triangle diagram

The angular distributions



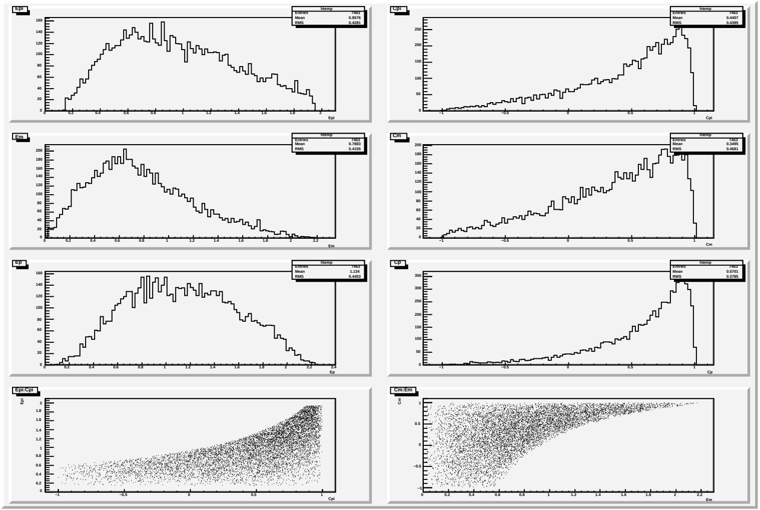
(a) $\sqrt{s} = 2.975$ GeV
(b) $\sqrt{s} = 2.985$ GeV
(c) $\sqrt{s} = 3.591$ GeV
(d) $\sqrt{s} = 3.686$ GeV

Good agreement fitting with

- energy dependent range for s-channel ;
- no interference each mechanism acts in a different kinematical region.

Some spectra in Laboratory frame

S-channel - p=1.7 GeV



Conclusions and Plans

- *Phenomenological model based on s, t and u channels*
- *Few parameters with physical meaning (to be fixed on future data ?)*
- *Built 'home-made' Montecarlo. Euler angular variables, easy to handle*
- *Built 5-dim function ready for use in PANDAROOT generator*
- *Built PANDAROOT generators for t- u- and s- channels separately*
- run PANDAROOT at MonteCarlo level and test.
- **run PANDAROOT → full simulation.**
- find formally simpler analytical formulas