Nucleon Form Factor Processes at Panda: Theoretical Analysis

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January 2012

Outline

- Introduction:

  Electromagnetic Form Factors of the Nucleon and Two-Photon Exchange

- Two-Photon Exchange in the Timelike Region: $p\bar{p} \to e^+e^-$

  in coll. with: N. Kivel, M. Vanderhaeghen

- Form Factors in the Unphysical Region: $p\bar{p} \to \pi^0 e^+e^-$

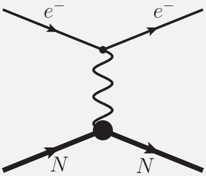
  in coll. with: C. Adamuščín, F. Maas, M. Vanderhaeghen, M. Zambrana

- Summary

Electromagnetic Form Factors of the Nucleon

Spacelike Region ($q^2 < 0$)

Elastic eN-scattering:



Electromagnetic current:

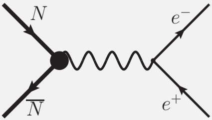
$$\langle N(p')|J^\mu_{em}|N(p)\rangle =$$

$$\bar{u}(p')\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Form factors are real functions of $Q^2 = -q^2$

Timelike Region ($q^2 > 0$)

$p\bar{p}$-Annihilation:



Crossing symmetry:

$$\langle N(p')|J^\mu_{em}|N(p)\rangle \to \langle 0|J^\mu_{em}|N(p)\overline{N}(p')\rangle$$

$$\langle 0|J^\mu_{em}|N(p)\overline{N}(p')\rangle =$$

$$\bar{v}(p')\left[F_1(q^2)\gamma^\mu - F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Form factors are complex functions of q^2

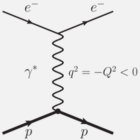
Spacelike Electromagnetic Form Factors

e-p-Scattering: Rosenbluth separation

Obtain form factors from ε-dependence of unpolarized cross section

Rosenbluth cross section in 1γ-exchange approximation:

$$d\sigma = \mathcal{C}(Q^2, \epsilon) \underbrace{\left[G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \right]}_{\text{reduced cross section } \sigma_R}$$



with $\tau = \frac{Q^2}{4m^2}$ and $\varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)^{-1}$

e-p-Scattering: Polarization transfer

Scatter polarized electron beam & measure outgoing proton polarization:

$$\vec{e} + p \to e + \vec{p}$$

Polarization ratio: $$\frac{P_t}{P_l} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}} \frac{G_E(Q^2)}{G_M(Q^2)}$$

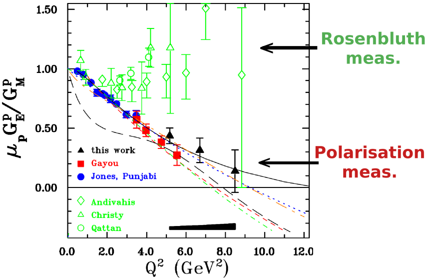
Spacelike Electromagnetic Form Factors

Results:

- The two methods give different results:

  Rosenbluth exp:
  $\frac{\mu_p G_E}{G_M} \approx 1$

  Polarization exp.:
  linear decrease of $\frac{G_E}{G_M}$ with Q^2

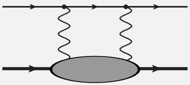


(Puckett et al., PRL (2010))

A possible explanation for the discrepancy:

Two-photon exchange corrections

(Guichon, Vanderhaeghen (2003), Blunden et al. (2003), ...)



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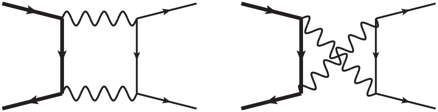
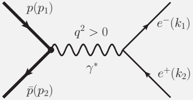
Two-Photon Exchange in the Timelike Region



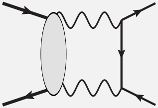
Two-Photon Exchange in Timelike Processes?



$p\,\overline{p} \longrightarrow e^+\,e^-$

Cross section in Born approximation:

$$d\sigma_{1\gamma} = \mathcal{C}(q^2)\left[|G_M|^2(1+\cos^2\vartheta) + \frac{1}{\tau}|G_E|^2\sin^2\vartheta\right]$$



$$\widetilde{G_M}(q^2,t) = G_M(q^2) + \delta\widetilde{G_M}(q^2,t)$$

$$\widetilde{F_2}(q^2,t) = F_2(q^2) + \delta\widetilde{F_2}(q^2,t)$$

$$\widetilde{F_3}(q^2,t) = 0 + \delta\widetilde{F_3}(q^2,t)$$

$\Uparrow$ proton FF $\quad \Uparrow$ 2γ amplitude

$$\mathcal{M} = \frac{e^2}{q^2}\,\overline{u}(k_1)\,\gamma_\mu\,v(k_2) \times \overline{N}(p_2)\left[\widetilde{G_M}\gamma^\mu - \widetilde{F_2}\frac{P^\mu}{M} + \widetilde{F_3}\frac{\not{K}P^\mu}{M^2}\right]N(p_1)$$

Leading pQCD Analysis of 2γ-Exchange

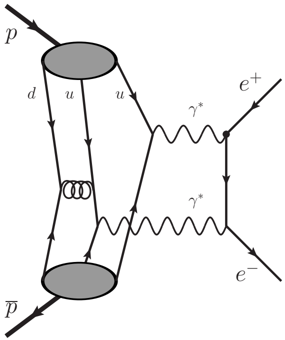
2γ-exchange at large q^2

$\Rightarrow$ consider factorization approach:

2γ-exchange correction as convolution of:

- Hard amplitude (calculable in pQCD)
- nonperturbative contribution:
  Nucleon **D**istribution **A**mplitudes φ_N

$$\varphi_N(x_i') * T_H(q^2, \varepsilon) * \varphi_N(x_i)$$



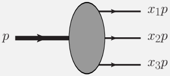
2γ-exchange amplitudes:

$$\delta\widetilde{G_M} \sim \frac{s}{m^2}\widetilde{F_3} \sim \frac{\alpha_{em}\alpha_s}{q^4}$$

$$\delta\widetilde{F_2} \sim 1/q^6 \quad \text{(suppressed)}$$

Nucleon Distribution Amplitudes

Distribution Amplitude φ_N:



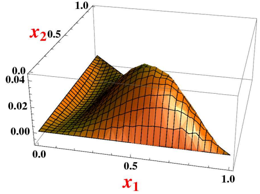
describes how the longitudinal momentum is shared between the constituents

Model for asymptotic behavior of the DAs and the first corrections:

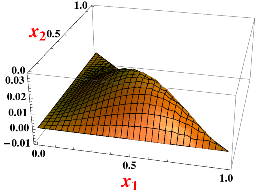
$$\varphi_N(x_i) \simeq 120\, f_N\, x_1 x_2 x_3\, (1 + r_-\,(x_1 - x_2) + r_+\,(1 - 3x_3) + \ldots)$$

DAs include 3 parameters: f_N, r_-, r_+

	f_N (10^{-3} GeV^2)	r_-	r_+
COZ[1]	5.0	4.0	1.1
BLW[2]	5.0	1.37	0.35

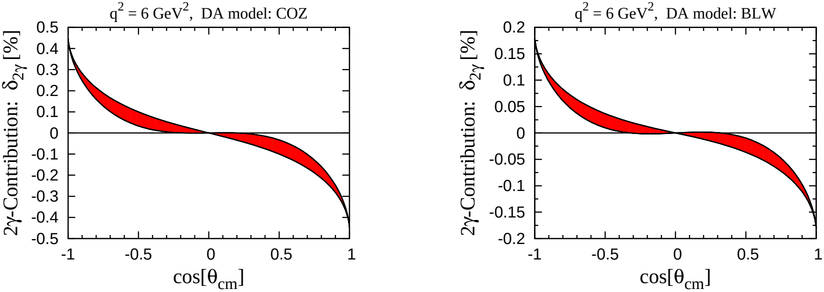


[1] Chernyak et al., Z.Phys C (1989)



[2] Braun et al., PRD (2006)

Results

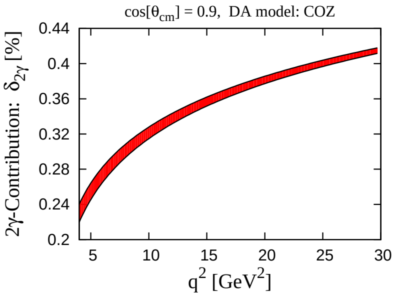


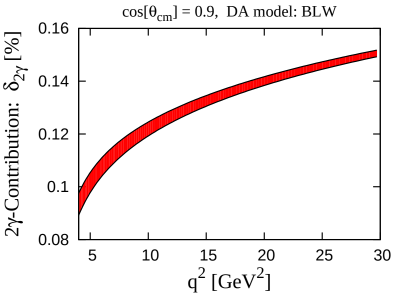
(JG, Kivel, Vanderhaeghen, PRD(2011))

Two-Photon Contribution $\delta_{2\gamma}$: $d\sigma_{1\gamma+2\gamma} = d\sigma_{1\gamma}\left(1 + \boldsymbol{\delta_{2\gamma}}\right)$

- Form factor from QCD-fit: $|G_M(q^2)| = \frac{C}{q^4 \log^2\left(\frac{q^2}{\Lambda^2}\right)}$ (Lepage, Brodsky PRL43)
- Assumptions: $|G_M| = |G_E|$, $G_M = G_M^*$
- $\delta\widetilde{G}_E = \lambda\, \delta\widetilde{G}_M$, with $-1 < \lambda < 1$ (bands)

Results





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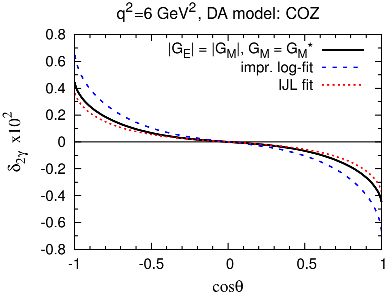

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Two-Photon Contribution $\delta_{2\gamma}$: $d\sigma_{1\gamma+2\gamma} = d\sigma_{1\gamma}\left(1 + \boldsymbol{\delta_{2\gamma}}\right)$

3 parametrizations give similiar results:

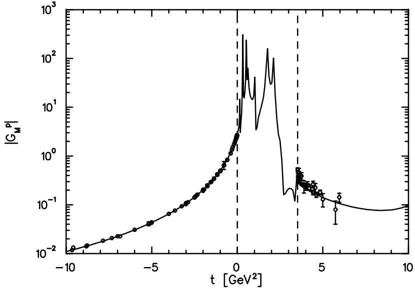
- Form factor from QCD-fit using $|G_M| = |G_E|$, $G_M = G_M^*$
- Fit incl. logarithmic corrections, Brodsky et al. (2004)
- VMD model, Iachello et al. (2004)

Nucleon Form Factors in the Unphysical Region

Form Factors in the Unphysical Region

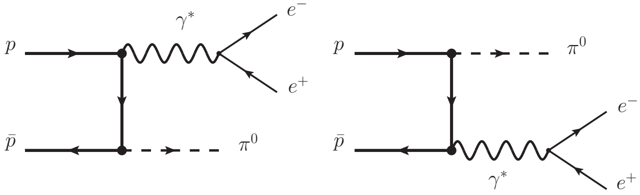
- Unphysical region:
  $0 < q^2 < 4m^2$
- Not accessible by process
  $p + \bar{p} \rightarrow e^+ + e^-$
- Idea: Consider process
  $\boldsymbol{p + \bar{p} \rightarrow \pi^0 + \gamma^*}$
  $\boldsymbol{\rightarrow \pi^0 + e^+ + e^-}$

  (Dubničková et al., ZPhys. (1996), Adamuščín et al., PRC (2007))



Process $p + \bar{p} \to \pi^0 + e^+ + e^-$

Phenomenological Approach:
(Adamuščín et al., PRC (2007))



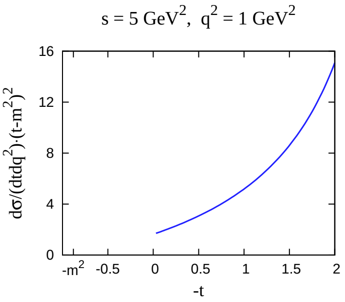
$\bar{p}(p_1) + p(p_2) \to \pi^0(q_\pi) + e^-(k_1) + e^+(k_2)$

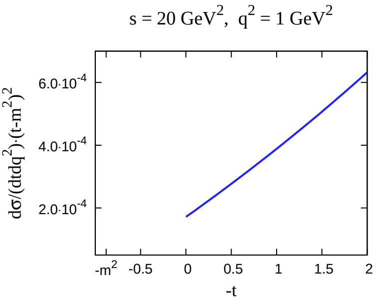
- Decribed by nucleon exchange
- Neglecting off shell effects $\to \gamma^* NN$-vertices parametrized by:

$$\left[F_1(q^2)\gamma^\mu - \frac{i}{2m}F_2(q^2)\sigma^{\mu\nu}q_\nu\right]$$

Process $p + \bar{p} \rightarrow \pi^0 + e^+ + e^-$

Cross Section (within Born model): $\frac{d\sigma}{dtdq^2} \cdot (t - m^2)^2 \quad [nb]$





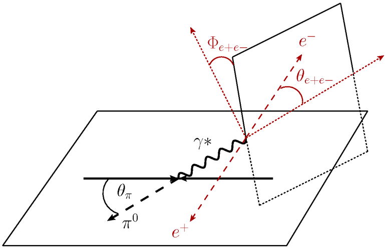
$p\bar{p} \to \pi^0 e^+ e^-$: Model Independent Calculation

Amplitude:

Separate amplitude of the process:

$$|\mathcal{T}|^2 = \sum_\lambda \Big| \underbrace{\left(\mathcal{M}^\mu \cdot \varepsilon^*_\mu(q,\lambda)\right)}_{\text{evaluate in } (p\bar{p})\text{-cm frame}} \frac{1}{q^2} \underbrace{\left(\varepsilon_\nu(q,\lambda)\bar{u}(k_1)\, e\gamma^\nu\, v(k_2)\right)}_{\text{evaluate in } \gamma^*\text{-rest frame}} \Big|^2$$

$\mathcal{M}^\mu$: $p\bar{p} \to \pi^0\gamma^*$ amplitude



Variables:

s, $t \leftrightarrow \theta_\pi$ $\theta_{e^+e^-}$, $\Phi_{e^+e^-}$

Photon-virtuality q^2

$p\bar{p} \rightarrow \pi^0 e^+ e^-$: Model Independent Calculation

Cross Section

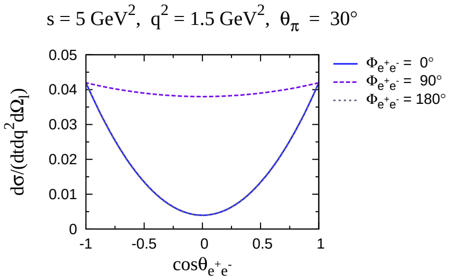
$$\frac{d\sigma}{dt\, dq^2\, d\Omega_l} = \frac{1}{16\pi^2 s(s-4m^2)} \frac{e^2}{(4\pi)^2\, 2\, q^2} \frac{4\pi}{3} \cdot \mathcal{W}(\theta_{e^+e^-}, \Phi_{e^+e^-})$$

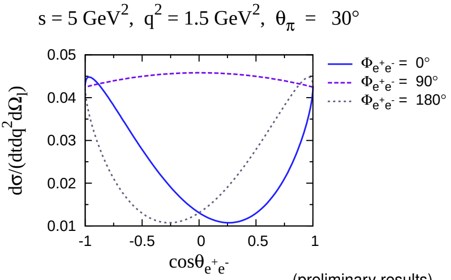
$$\begin{aligned} \mathcal{W}(\theta_{e^+e^-}, \Phi_{e^+e^-}) \quad = \quad & \frac{3}{4\pi} \Big[\sin^2\theta_{e^+e^-}\, \rho_{00} + (1+\cos^2\theta_{e^+e^-})\rho_{11} \\ & + \sqrt{2}\sin 2\theta_{e^+e^-}\, \cos\Phi_{e^+e^-}\, \mathrm{Re}[\rho_{10}] \\ & + \sin^2\theta_{e^+e^-}\, \cos 2\Phi_{e^+e^-}\, \mathrm{Re}[\rho_{1-1}] \Big] \end{aligned}$$

Density matrix: $\rho_{\lambda\lambda'} = \left(\mathcal{M}^\mu \varepsilon^*_\mu(\boldsymbol{q}, \lambda)\right) \cdot \left(\mathcal{M}^\mu \varepsilon^*_\mu(\boldsymbol{q}, \lambda')\right)^*$

Results: (within Born-Model)

Cross Section: $d\sigma/(dt\,dq^2\,d\Omega_l)$ $[nb/GeV^4 sr]$





(preliminary results)

- Nucleon form factor:

  $|G_M(q^2)| = C/q^4 \log^2\left(\frac{q^2}{\Lambda^2}\right)$

  $G_M = G_M^* = G_E$

- Nucleon form factor:

  VMD model

  (Iachello et al. (2004))

- $\sin 2\theta_{e^+e^-} \cos \Phi_{e^+e^-}$ structure: basically proportional to $(G_E - G_M) \sim F_2$

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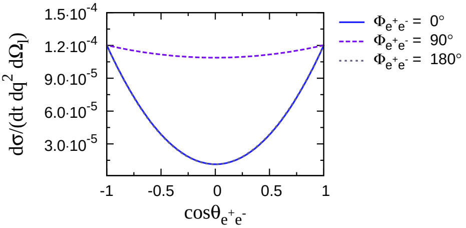
Cross Section: $d\sigma/(dt\,dq^2\,d\Omega_l)$ $\left[nb/GeV^4 sr\right]$

$s = 10\ GeV^2,\ q^2 = 3\ GeV^2,\ \theta_\pi = 30°$



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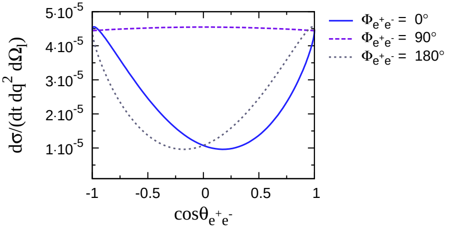

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Summary

2γ-exchange in the **timelike region**:

- Estimate of 2γ-exchange using a pQCD factorization approach
- Contribution $\delta_{2\gamma} \lesssim 1\%$

Proton form factor in the **unphysical region**:

- Accessible in the process $p\bar{p} \rightarrow \pi^0 e^+ e^-$
- General form for decay angular distribution
- Measurement necessitates fixed q^2, different t values
- Form factor extraction: requires model
  $\rightarrow$ can be tested through t dependence