18th-20th June, 2012
17th Rencontres Itzykson @ IPhT

Precision Power Spectrum Calculations for Large-scale Structure Observations

Atsushi Taruya
RESearch **C**enter for the **E**arly **U**niverse (RESCEU), Univ. Tokyo

In collaboration with
Francis Bernardeau, Takahiro Nishimichi, Codis Sandrine

Plan of this talk

Perturbation theory of large-scale structure
from practical point-of-view

Recent progress on perturbation theory

Accelerated power spectrum calculation

Summary

Large-scale structure (LSS)

Fundamental observable: galaxy clustering patterns
(and weak lensing)

Statistical nature of LSS contains valuable cosmological info.

Power spectrum P(k), or correlation function ξ(r)

Shape & amplitude

Historical record of the primordial Universe
(Initial condition & late-time evolution)

Further, additional observational effects give much more benefit:

- Alcock-Paczynski effect ······· cosmic expansion
- Redshift distortion effect ······· growth of structure

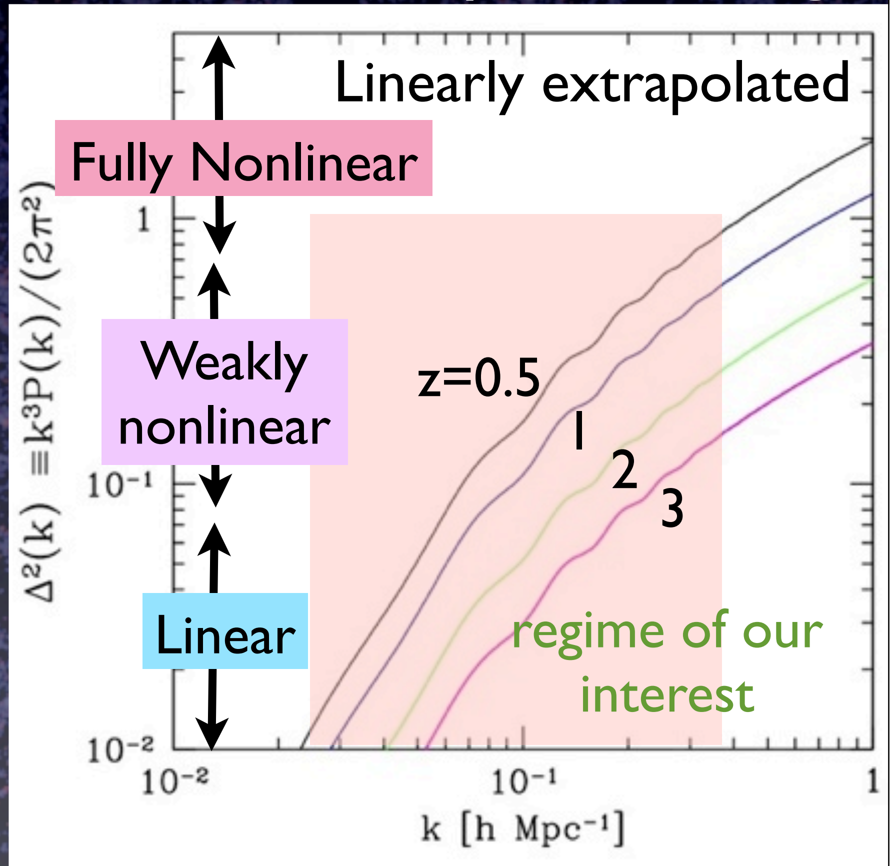
With BAOs as standard ruler,

measurements of these are now top priority in future surveys

Confronting theory with obs.

High-precision <u>theoretical template</u> of P(k) & ξ(r)

is observationally demanding



Taking account of nonlinear systematics:

- gravity clustering
- Redshift distortions
- galaxy biasing

Small, but non-negligible at ~1% precision

In weakly nonlinear regime,

Perturbation theory approach is viable

(though role of N-body is still crucial)

Perturbation theory approach

Large-scale structure formation based on gravitational instability

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86),
Suto & Sasaki ('91), Jain & Bertschinger ('94), ...

Cold dark matter + baryons = pressureless & irrotational fluid

Basic eqs. (GR w/o ν)

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\left[(1+\delta)\vec{v}\right] = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{a}\vec{\nabla}\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi = 4\pi G\,\overline{\rho}_{\rm m}\,\delta$$

standard PT
$|\delta| \ll 1$

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots \qquad \langle\delta(\boldsymbol{k};t)\delta(\boldsymbol{k}';t)\rangle = (2\pi)^3\,\delta_{\rm D}(\boldsymbol{k}+\boldsymbol{k}')\,P(|\boldsymbol{k}|;t)$$

Perturbation theory approach

Large-scale structure formation based on gravitational instability

Juszkiewicz ('81), Vishniac ('83), Goroff et al. ('86), Suto & Sasaki ('91), Jain & Bertschinger ('94), ...

Cold dark matter + baryons = pressureless & irrotational fluid

Modern description

Doublet $\Psi_a(\boldsymbol{k};\eta) = \begin{pmatrix} \delta_{\rm m}(\boldsymbol{k};\eta) \\ \theta(\boldsymbol{k};\eta)/f(\eta) \end{pmatrix}$

in Fourier space

Linear growth factor

$\eta \equiv \ln D_+(t)$ $\quad f = \dfrac{d\ln D_+}{d\ln a}$

$$\frac{\partial}{\partial\eta}\Psi_a(\boldsymbol{k};\eta) + \Omega_{ab}(\eta)\Psi_b(\boldsymbol{k};\eta) = \int\frac{d^3\boldsymbol{k}_1 d^3\boldsymbol{k}_2}{(2\pi)^3}\,\delta_{\rm D}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2)\,\gamma_{abc}(\boldsymbol{k}_1,\boldsymbol{k}_2)\,\Psi_b(\boldsymbol{k}_1;\eta)\Psi_c(\boldsymbol{k}_2;\eta)$$

standard PT

$|\delta| \ll 1$

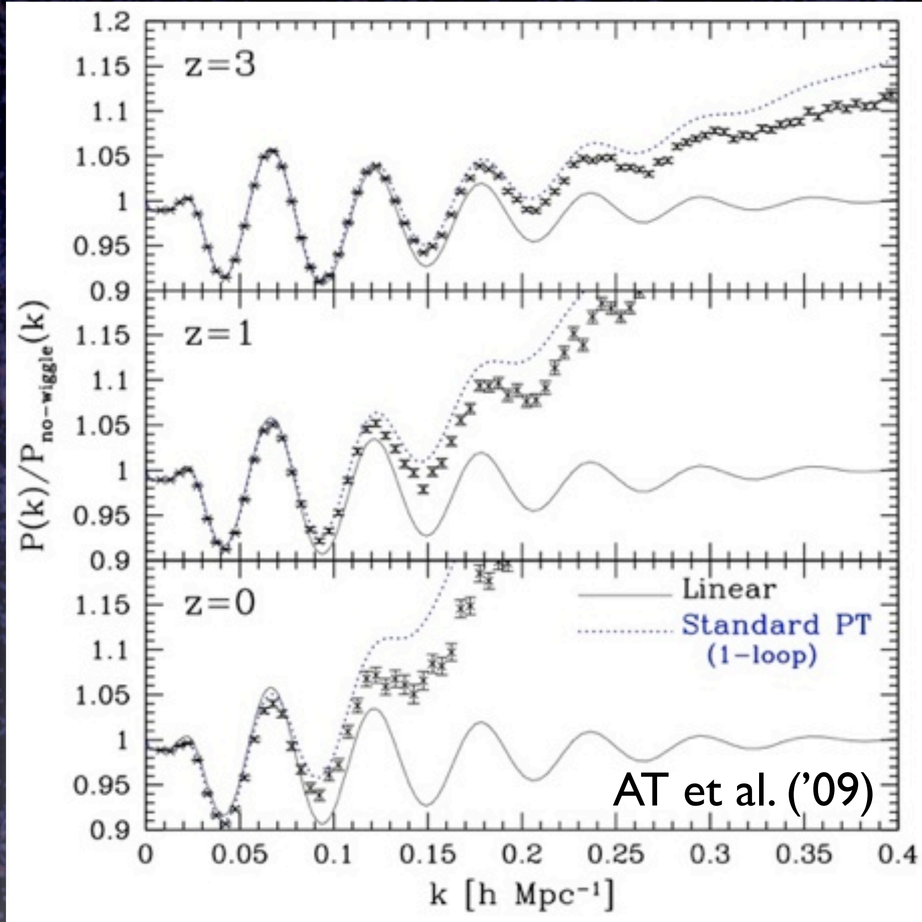
$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots \qquad \langle\delta(\boldsymbol{k};t)\delta(\boldsymbol{k}';t)\rangle = (2\pi)^3\,\delta_{\rm D}(\boldsymbol{k}+\boldsymbol{k}')\,P(|\boldsymbol{k}|;t)$$

Perturbation theory : revolution

Standard PT turns out to have a poor convergence

Improved PT ('06~'08)

- RPT Crocce & Scoccimarro ('06ab, '08)
- Large-N Valageas ('07)
- Closure theory AT & Hiramatsu ('08)
- LRT Matsubara ('08ab), Okamura et al. ('11)
- RegPT(Γ-expansion) Bernardeau et al. ('08,'11)
- Time-RG Pietroni ('08)



Good convergence of improved PT is ensured by re-organizing standard PT expansion by means of non-perturbative quantities

Perturbation theory : revolution

Standard PT turns out to have a poor convergence

Improved PT ('06~'08)

RPT Crocce & Scoccimarro ('06ab, '08)

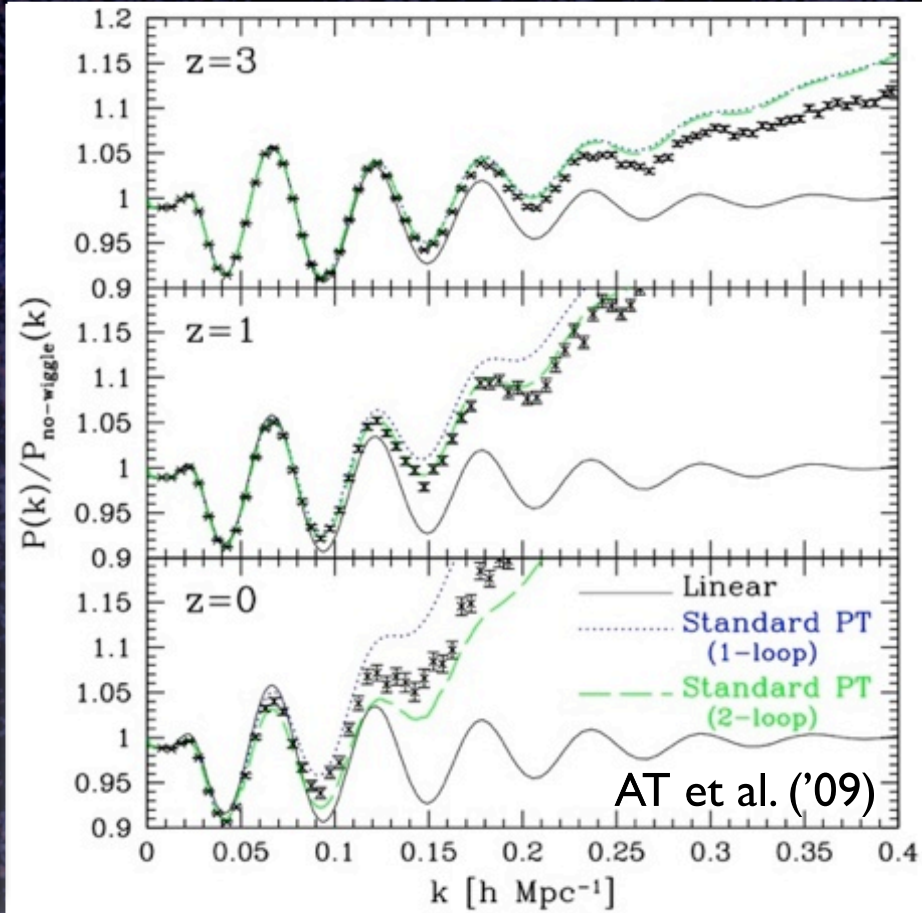
Large-N Valageas ('07)

Closure theory AT & Hiramatsu ('08)

LRT Matsubara ('08ab), Okamura et al. ('11)

RegPT(Γ-expansion) Bernardeau et al. ('08,'11)

Time-RG Pietroni ('08)



AT et al. ('09)

Good convergence of improved PT is ensured by re-organizing standard PT expansion by means of non-perturbative quantities

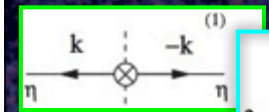
Standard PT vs. improved PT

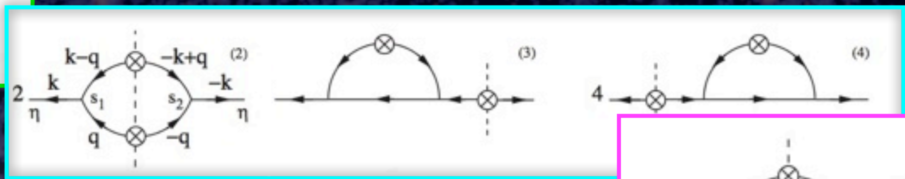
Standard PT

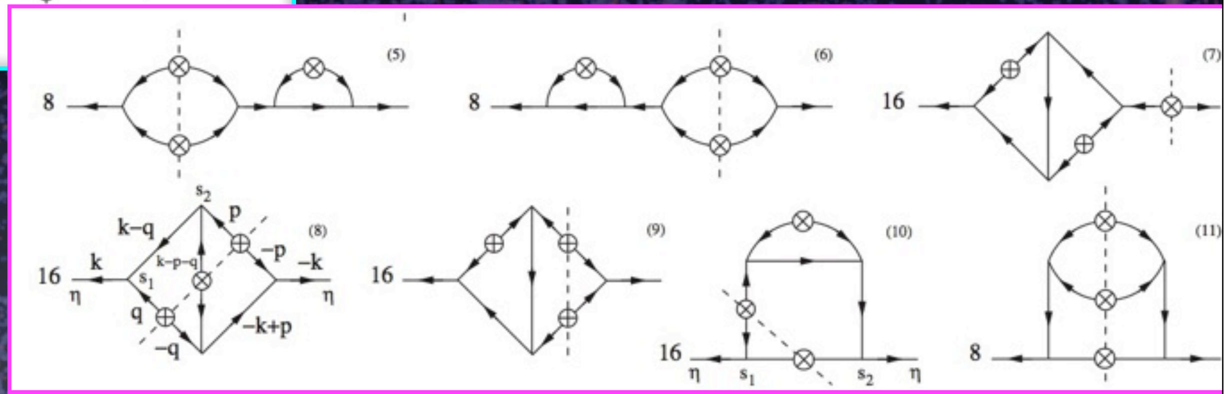
$$P^{(mn)} \simeq \langle \delta^{(m)} \delta^{(n)} \rangle$$

$$P(k) = \underbrace{P^{(11)}(k)}_{\text{Linear (tree)}} + \underbrace{\left(P^{(22)}(k) + P^{(13)}(k)\right)}_{\text{1-loop}} + \underbrace{\left(P^{(33)}(k) + P^{(24)}(k) + P^{(15)}(k)\right)}_{\text{2-loop}} + \cdots$$

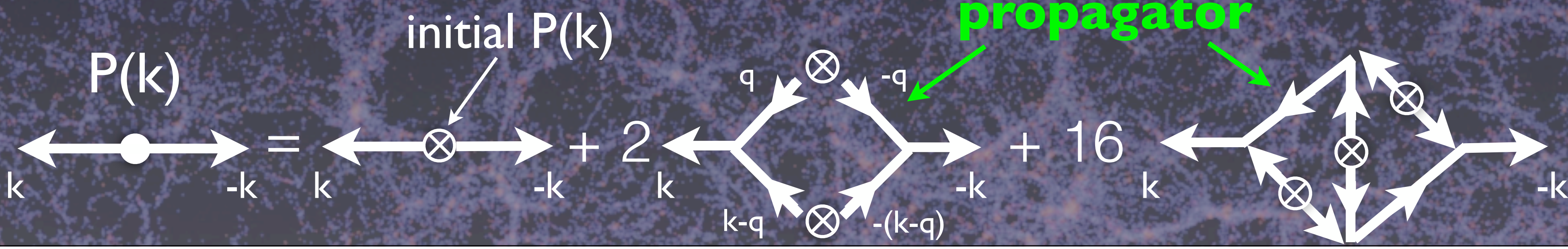
Linear (tree) 1-loop 2-loop







RPT



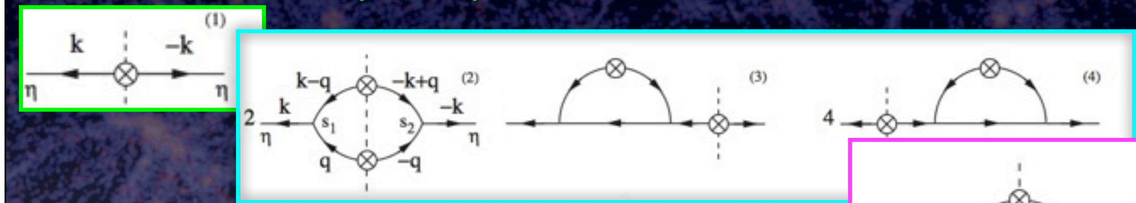
Standard PT vs. improved PT

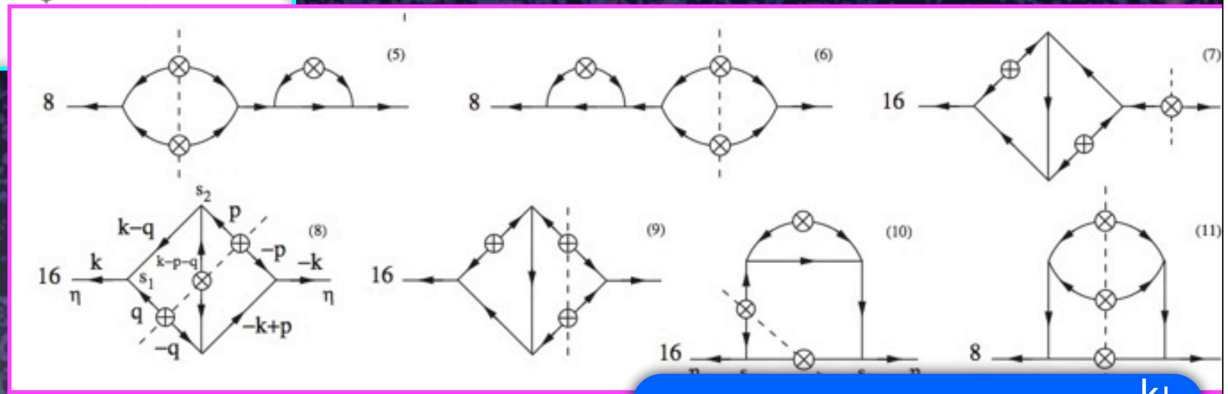
Standard PT

$$P^{(mn)} \simeq \langle \delta^{(m)} \delta^{(n)} \rangle$$

$$P(k) = \underline{P^{(11)}(k)} + \underline{\left(P^{(22)}(k) + P^{(13)}(k)\right)} + \underline{\left(P^{(33)}(k) + P^{(24)}(k) + P^{(15)}(k)\right)} + \cdots$$

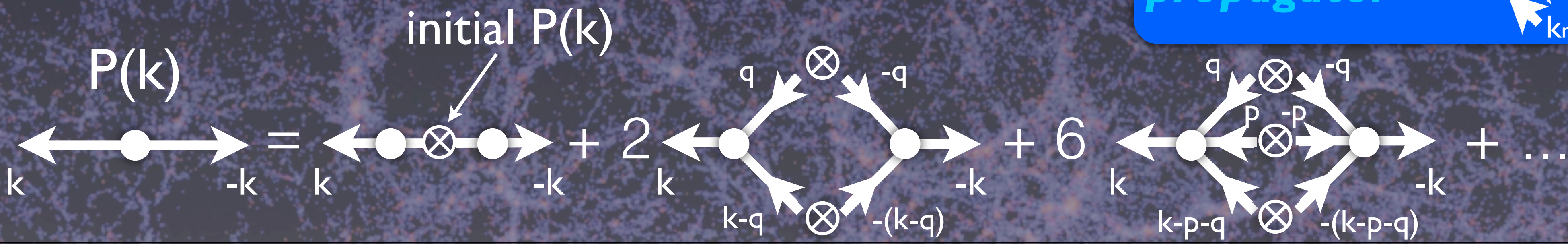
Linear (tree) — 1-loop — 2-loop





RegPT (Γ-expansion)

see Francis' talk in detail



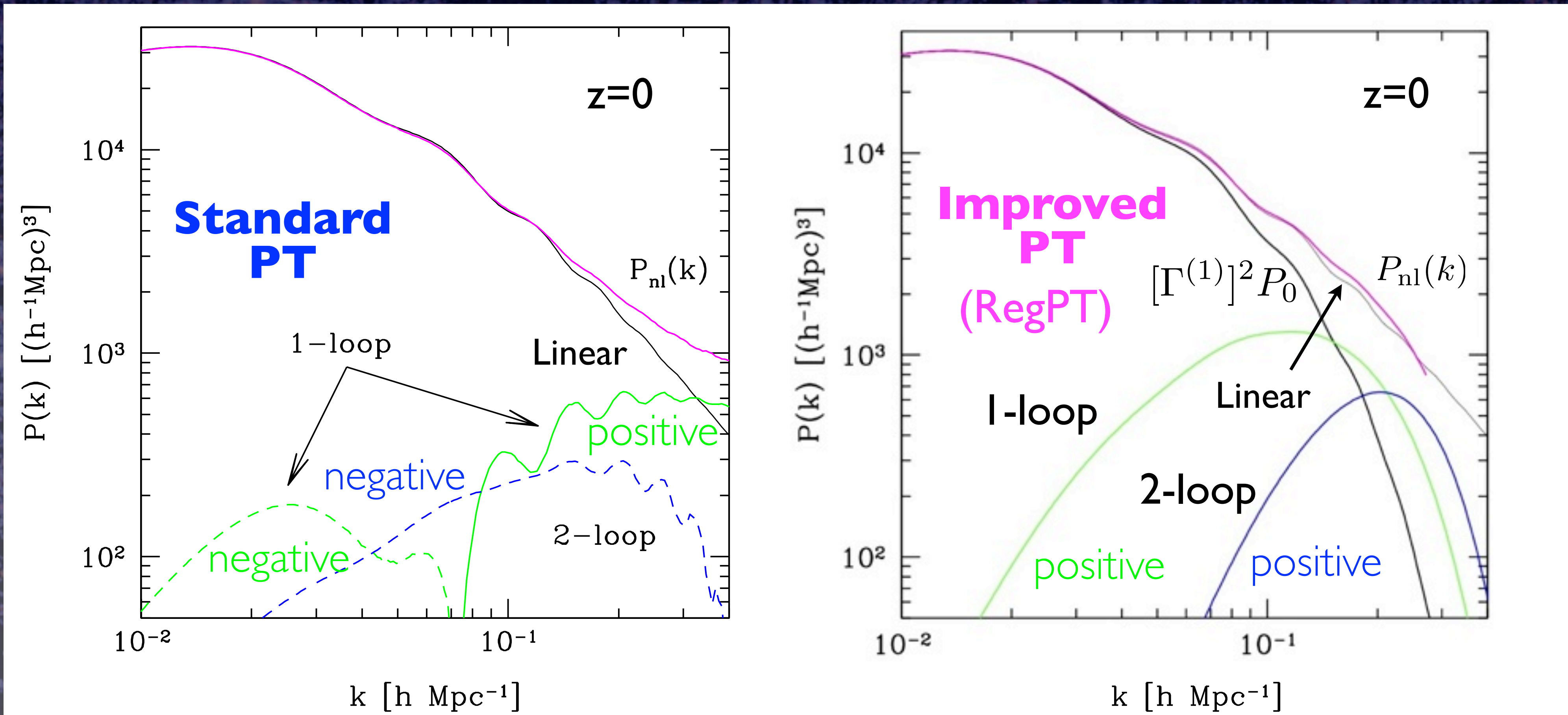
Convergence of PT expansion

AT, Bernardeau, Nishimichi, Codis (in prep.)

AT et al. ('09)

- All corrections become comparable at low-z.
- Positivity is not guaranteed.

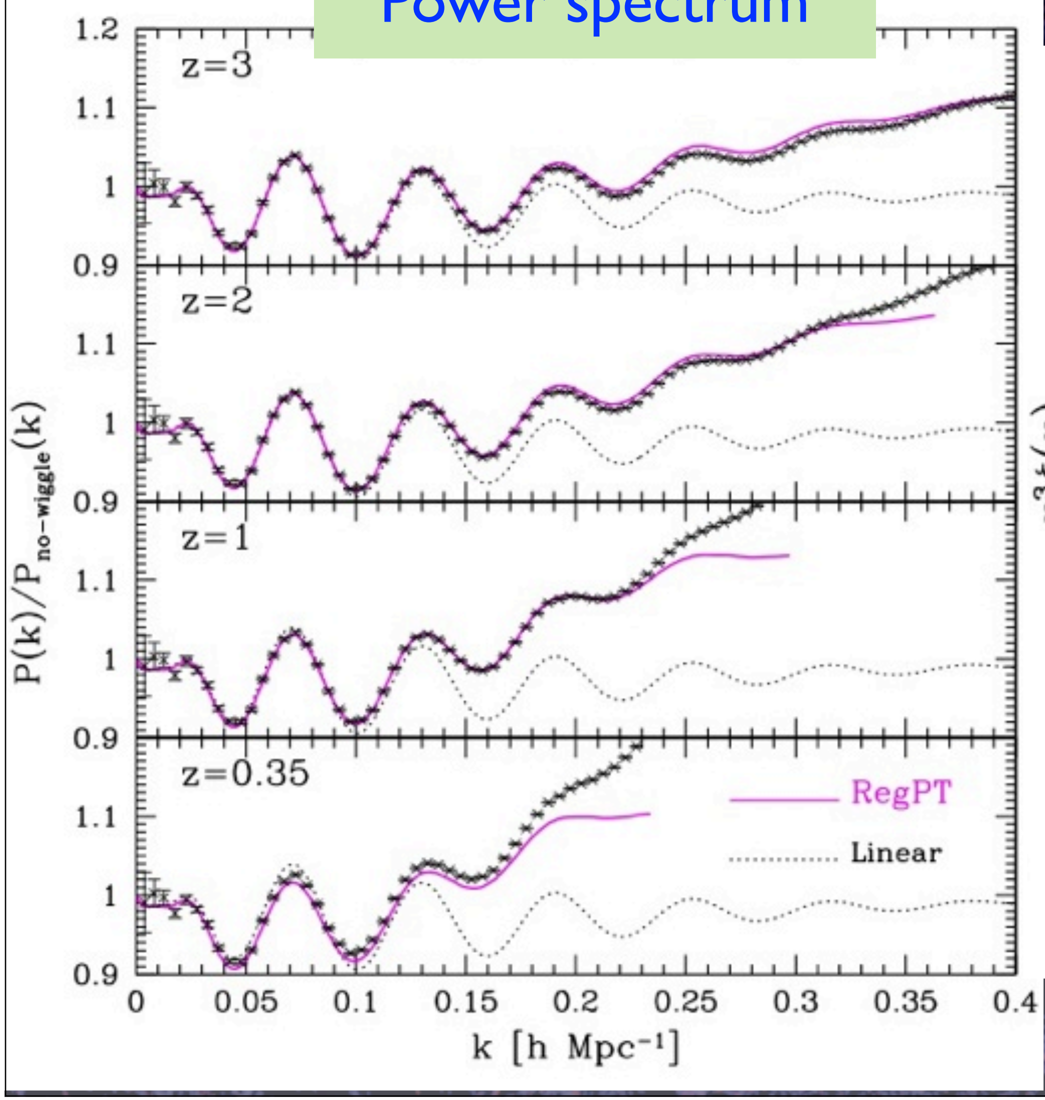
Corrections are positive and localized, and shifted to higher-k as increasing the order of PT expansion



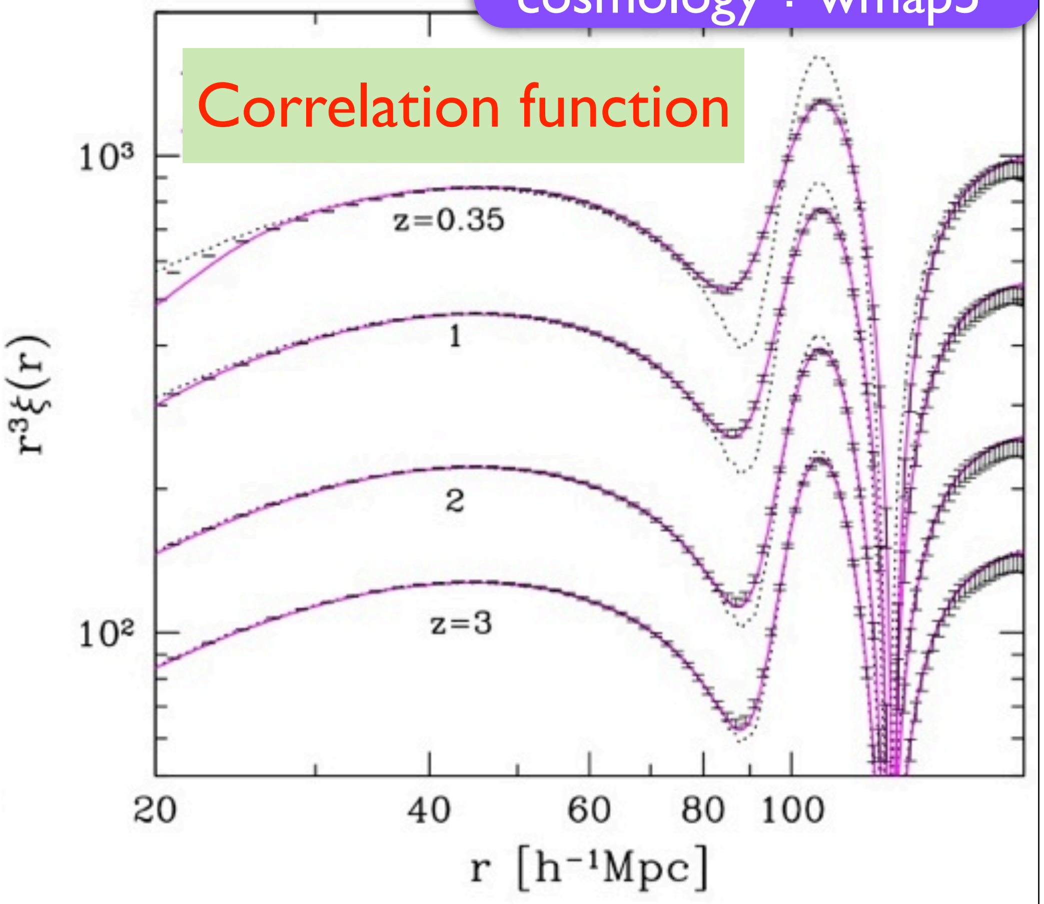
Check with N-body simulations

$L_{\rm box} = 2,048\,h^{-1}\,{\rm Mpc}$
# of particles : $1,024^3$
# of runs : 45
cosmology : wmap5

Power spectrum



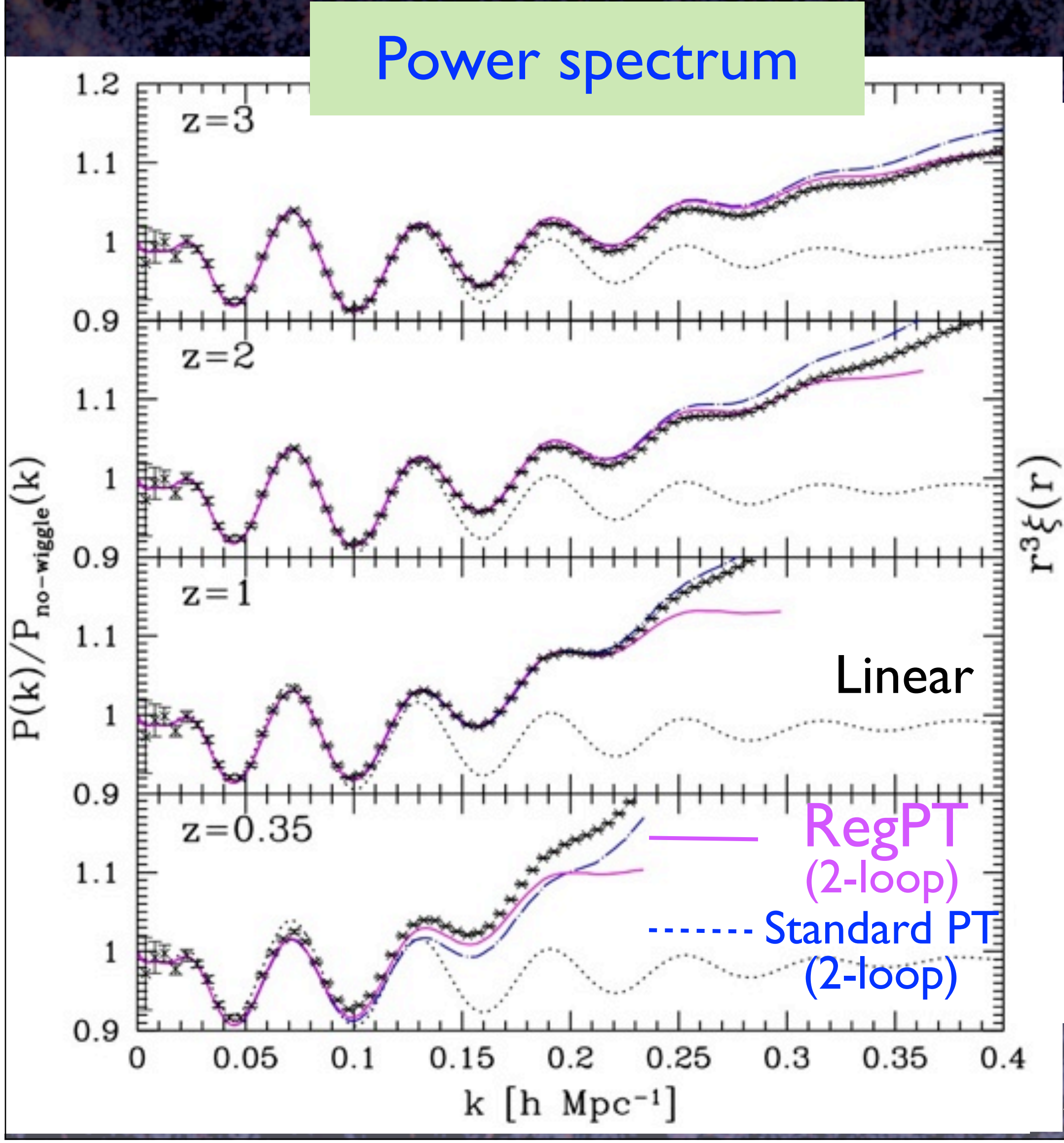
Correlation function

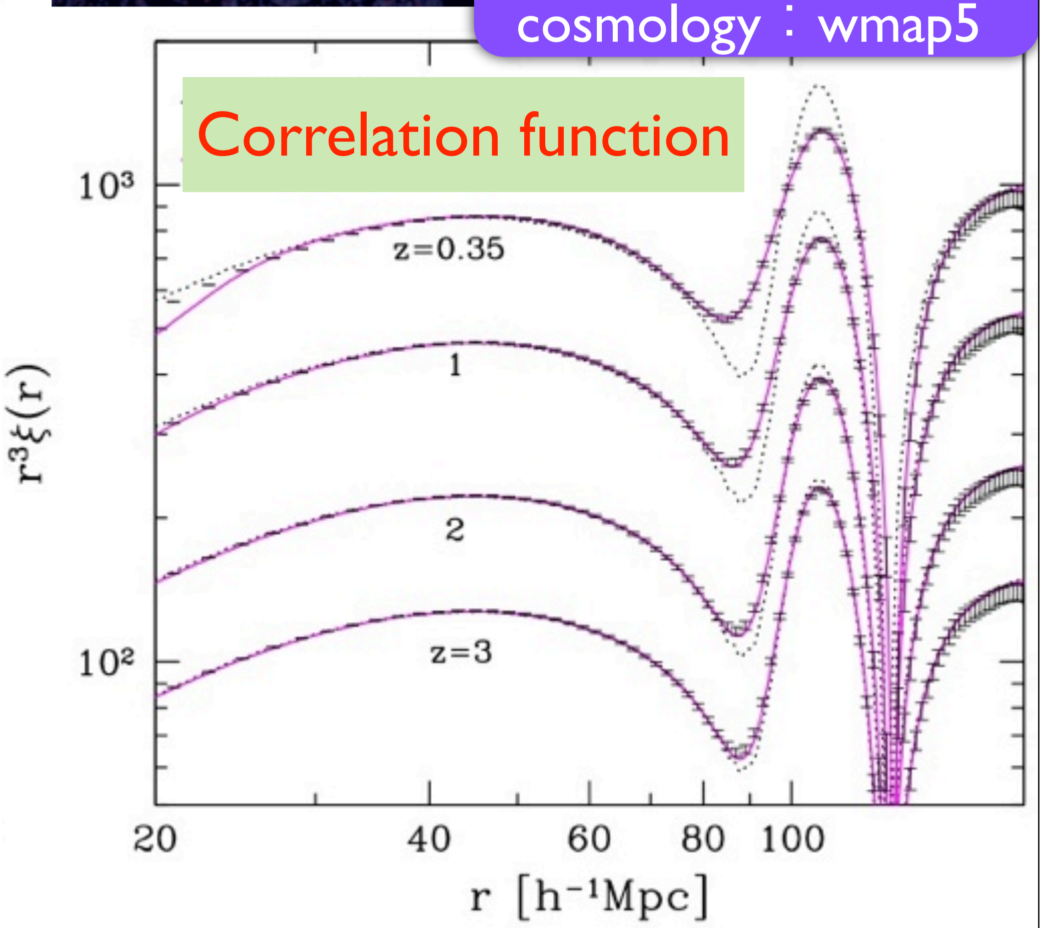


AT, Bernardeau, Nishimichi, Codis (in prep.)

Check with N-body simulations

$L_{\rm box} = 2,048\, h^{-1}$ Mpc
# of particles : $1,024^3$
# of runs : 45
cosmology : wmap5





AT, Bernardeau, Nishimichi, Codis (in prep.)

Extension: improved PT in redshift space

With the sophisticated modeling, redshift-space distortions are mostly under control in weakly nonlinear regime

$$P^{(\mathrm{S})}(k,\mu) = \underline{e^{-(k\mu f\sigma_{\mathrm{v}})^2}} \quad \text{Damping func.}$$
$$\times\left[P_{\delta\delta}(k) - 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) + A(k,\mu) + B(k,\mu)\right]$$

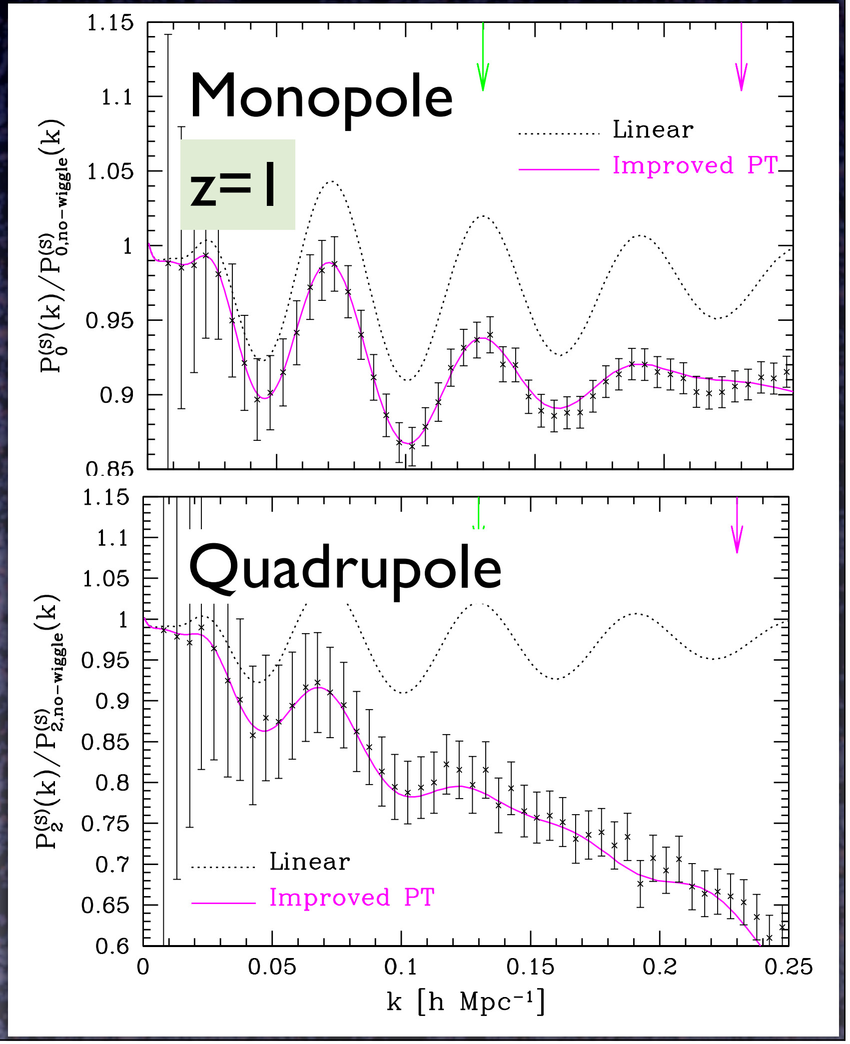
AT et al. ('10)

Nishimichi & AT ('11)

see also

Reid & White ('11)

Seljak & McDonald ('11)



Still, Galaxy biasing is pain in the neck

Extension: improved PT in redshift space

With the sophisticated modeling, redshift-space distortions are mostly under control in weakly nonlinear regime

$$P^{(\mathrm{S})}(k,\mu) = \underline{e^{-(k\mu f\sigma_{\mathrm{v}})^2}} \text{ Damping func.}$$
$$\times\left[P_{\delta\delta}(k) - 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) + A(k,\mu) + B(k,\mu)\right]$$

AT et al. ('10)

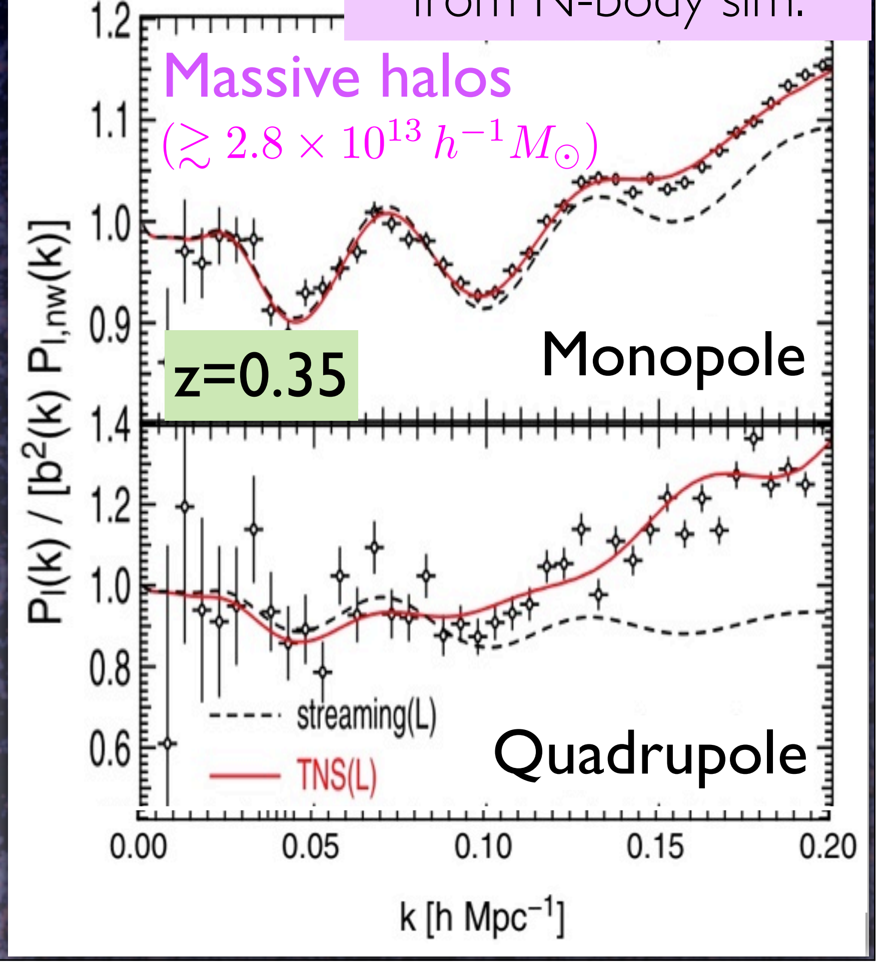
Nishimichi & AT ('11)

see also

Reid & White ('11)

Seljak & McDonald ('11)

Halo bias is calibrated from N-body sim.



Still, Galaxy biasing is pain in the neck

Yet another issue

"computational cost"

Even with improved PT, higher-order corrections (i.e., 2-loop) need to be computed for a better prediction, but they require a time-consuming calculation

multi-dimensional integration (5D in 2-loop)

······ *typically, ~hours* (c.f. 2D in 1-loop)

$P_0(k)$: initial P(k)

ex)

$$\int \frac{d^3\boldsymbol{p}\, d^3\boldsymbol{q}}{(2\pi)^6} F_n(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{q})\, P_0(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|)\, P_0(p)\, P_0(q)$$

independent of cosmology — sensitive to cosmology

·······still impractical for (global) cosmological parameter search

How to accelerate PT calculations

General strategies

- Find a '*sophisticated*' treatment at 1-loop level calculations

  no need for higher-dimensional integration

  Audren & Lesgourgues '12
  Anselmi & Pietroni '12

  but need a trick to effectively improve predictions

- Exploit a clever numerical scheme at 2-loop order

  not necessarily force to improve predictions

  but need to reduce higher-dimensional integrals

For the rest of this talk,

based on improved PT by means of 'regularized' multi-point propagators (RegPT), we present a method to *reduce any integrals to 1D integrals*

→ *Amazingly fast calculation* (few sec.) *is possible !!* ***RegPTfast***

Accelerated calculation: *RegPTfast*

Given the data set for RegPT calculations in a fiducial cosmology,

General idea

Suppose that linear P(k) in target model is close to the one in the fiducial model:

$$P_{0,\mathrm{target}}(k) = P_{0,\mathrm{fid}}(k) + \delta P_0(k); \quad \delta P_0(k) \ll P_{0,\mathrm{fid}}(k)$$

perturbation

$$P(k) \longrightarrow P_{\text{un-pert}}[k, z; P_{0,\mathrm{fid}}] + P_{\mathrm{corr}}[k, z; \delta P_0]$$

use prepared data set

Corrections needs to be newly evaluated, but with just *1D integration*

(quickly done with just few sec. !!)

Accelerated calculation: *RegPTfast*

Given the data set for RegPT calculations in a fiducial cosmology,

General idea

Suppose that linear P(k) in target model is close to the one in the fiducial model:

$$P_{0,\rm target}(k) = P_{0,\rm fid}(k) + \delta P_0(k)\,; \quad \delta P_0(k) \ll P_{0,\rm fid}(k)$$

perturbation

$$P(k) \longrightarrow P_{\text{un-pert}}[k,z;P_{0,\rm fid}] + P_{\rm corr}[k,z;\delta P_0]$$

use prepared data set

Corrections needs to be newly evaluated, but with just ***1D integration***

(quickly done with just few sec. !!)

ex) $$\int \frac{d^3\boldsymbol{p}\,d^3\boldsymbol{q}}{(2\pi)^6} F_n(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q},\boldsymbol{p},\boldsymbol{q})\,P_0(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|)\,P_0(p)\,P_0(q)$$

symmetric kernel

perturbation ⟶

$$3\int \frac{d^3\boldsymbol{p}\,d^3\boldsymbol{q}}{(2\pi)^6} F_n(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q},\boldsymbol{p},\boldsymbol{q})\,P_{0,\rm fid}(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|)\,P_{0,\rm fid}(p)\,\delta P_0(q)$$

Accelerated calculation: *RegPTfast*

Given the data set for RegPT calculations in a fiducial cosmology,

General idea

Suppose that linear P(k) in target model is close to the one in the fiducial model:

$$P_{0,\rm target}(k) = P_{0,\rm fid}(k) + \delta P_0(k); \quad \delta P_0(k) \ll P_{0,\rm fid}(k)$$

perturbation

$$P(k) \longrightarrow P_{\text{un-pert}}[k,z;P_{0,\rm fid}] + P_{\rm corr}[k,z;\delta P_0]$$

use prepared data set

Corrections needs to be newly evaluated, but with just *1D integration*

(quickly done with just few sec. !!)

ex) $$\int \frac{d^3\boldsymbol{p}\,d^3\boldsymbol{q}}{(2\pi)^6} F_n(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q},\boldsymbol{p},\boldsymbol{q})\,P_0(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|)\,P_0(p)\,P_0(q)$$

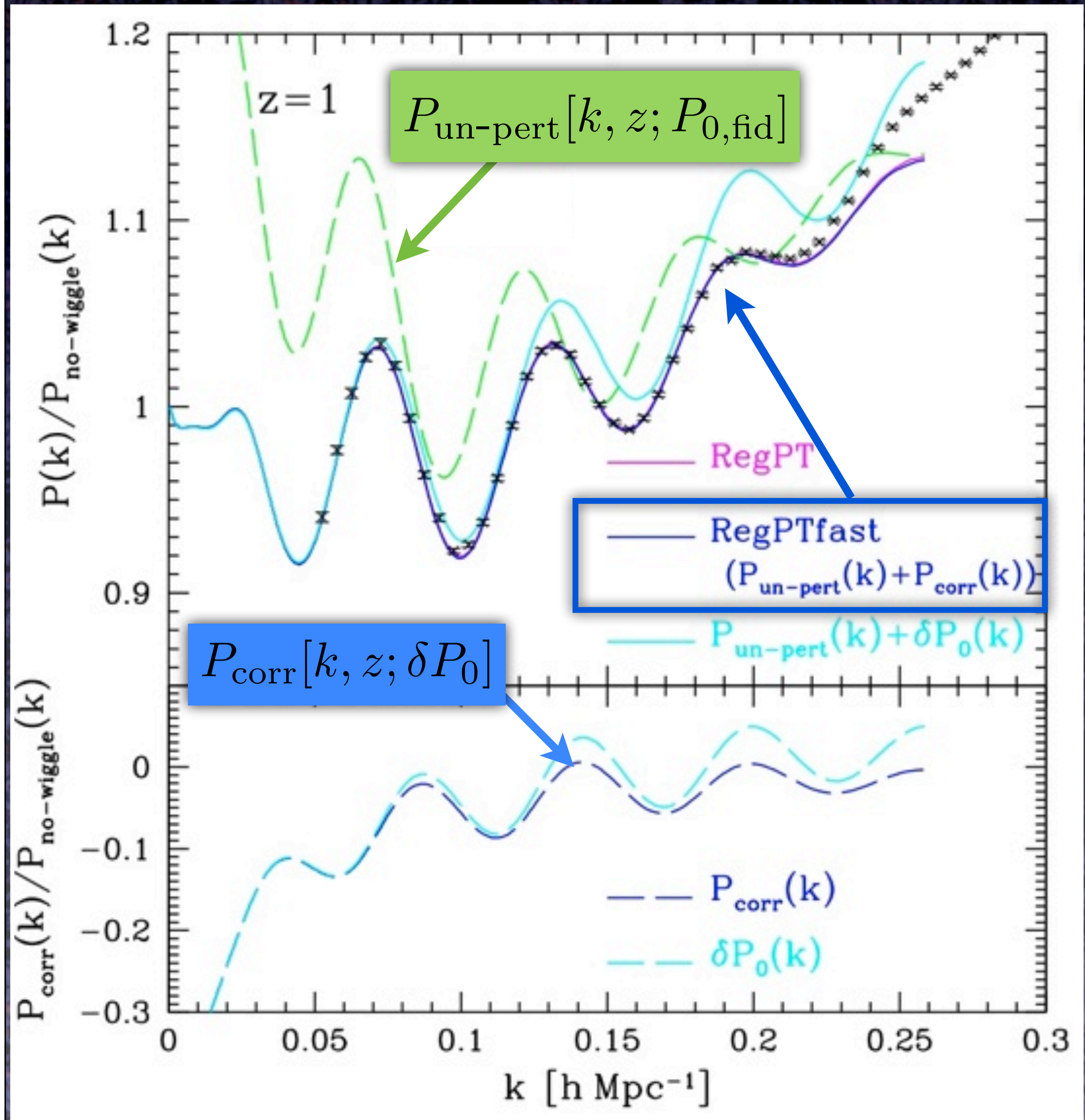
symmetric kernel

perturbation ⟶ $$\int \frac{dq\,q^2}{2\pi^2} K_n(q,k)\,\delta P_0(q)$$

$$K_n(q,k) = 3\int \frac{d^2\Omega_q\,d^3\boldsymbol{p}}{(2\pi)^6} F_n(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q},\boldsymbol{p},\boldsymbol{q})\,P_{0,\rm fid}(|\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q}|)\,P_{0,\rm fid}(p)$$

Demonstration

AT, Bernardeau, Nishimichi, Codis (in prep.)



Target (N-body)

wmap5 cosmological model

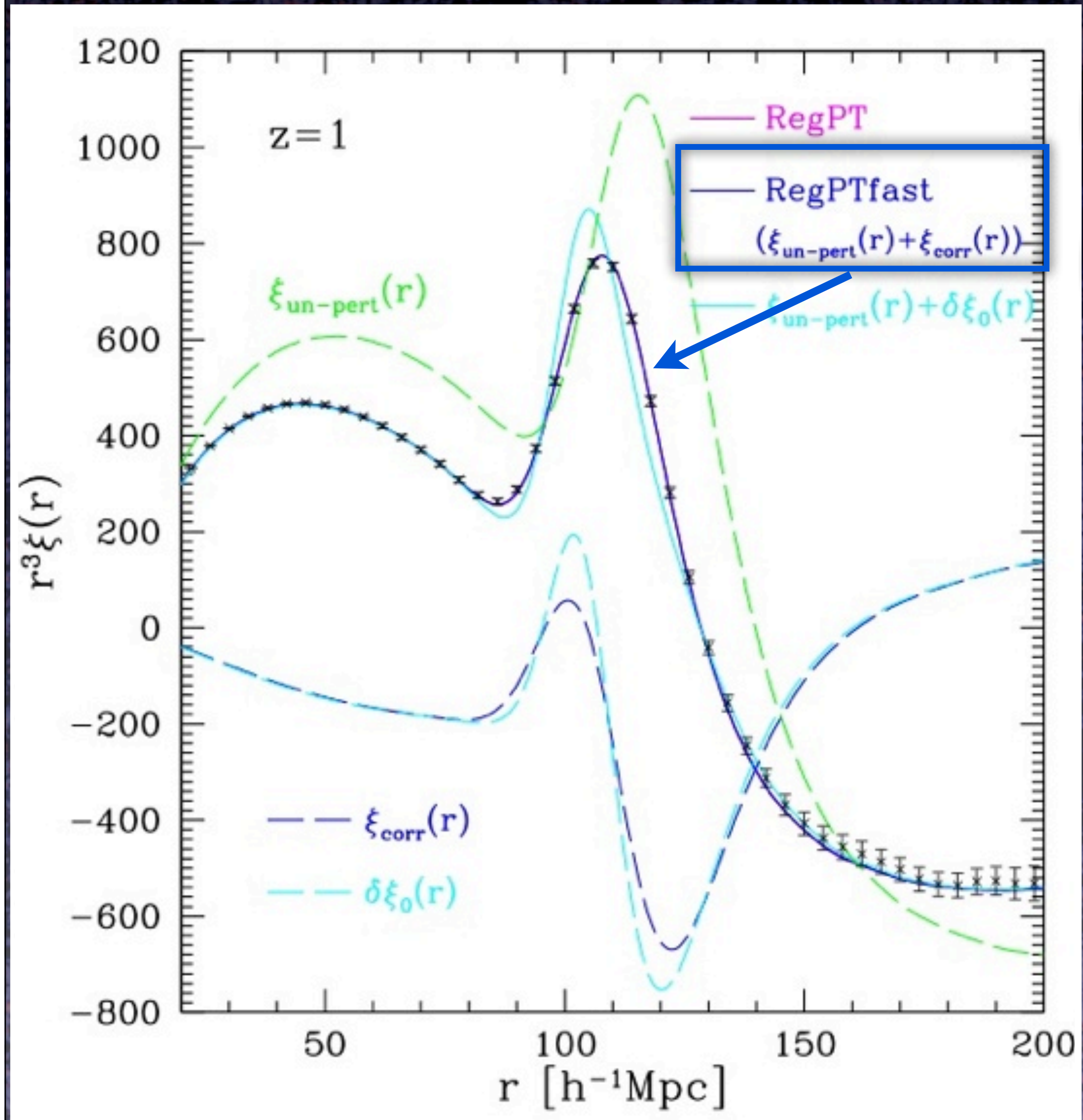
Fiducial

wmap3 cosmological model

Fiducial (wmap3)	Target (wmap5)
$\Omega_m = 0.234$	$\Omega_m = 0.279$
$\Omega_\Lambda = 0.766$	$\Omega_\Lambda = 0.721$
$\Omega_b/\Omega_m = 0.175$	$\Omega_b/\Omega_m = 0.165$
$\sigma_8 = 0.76$	$\sigma_8 = 0.817$

Demonstration

AT, Bernardeau, Nishimichi, Codis (in prep.)



Target (N-body)

wmap5 cosmological model

Fiducial

wmap3 cosmological model

Fiducial (wmap3)	Target (wmap5)
$\Omega_{\rm m} = 0.234$	$\Omega_{\rm m} = 0.279$
$\Omega_\Lambda = 0.766$	$\Omega_\Lambda = 0.721$
$\Omega_{\rm b}/\Omega_{\rm m} = 0.175$	$\Omega_{\rm b}/\Omega_{\rm m} = 0.165$
$\sigma_8 = 0.76$	$\sigma_8 = 0.817$

Re-scaling the power spectrum

Re-scaling the amplitude in fiducial model $(P_{0,\rm fid} \to \alpha\, P_{0,\rm fid})$,

$$\delta\, P_0(k) = P_{0,\rm target}(k) - \alpha\, P_{0,\rm fid}(k)$$

can be small so that the perturbative analysis works well.

→ enlarge the applicability of the present method

(the assumption $|(P_{0,\rm target} - P_{0,\rm fid})/P_{0,\rm fid}| \ll 1$ is *not always necessary*)

$P_{\rm un\text{-}pert}[k, z; P_{0,\rm fid}]$ $P_{\rm corr}[k, z; \delta P_0]$

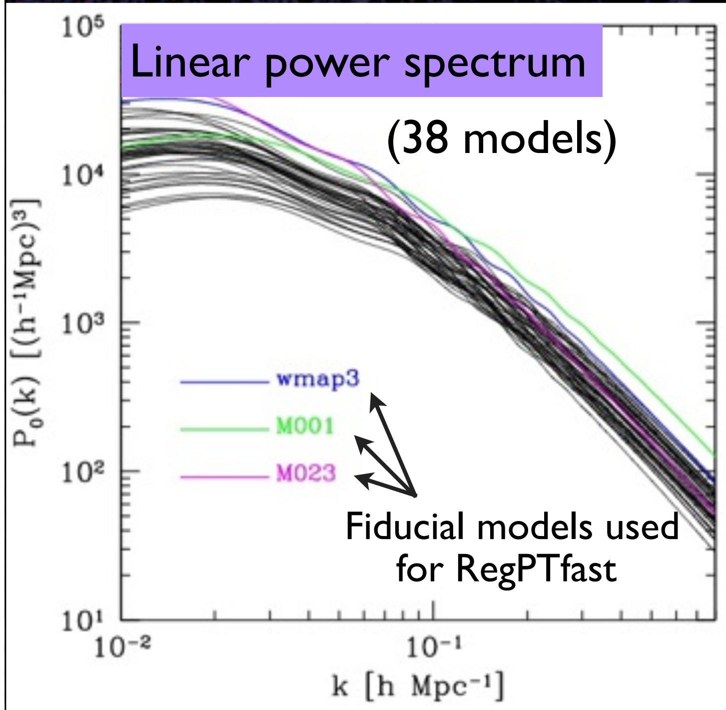
Since we know how the resultant PT predictions are re-scaled, we can easily get a re-scaled power spectrum w/o extra effort.

Then, to what extent the RegPTfast treatment is valid and accurate ?

- convergence btw. RegPTfast and rigorous RegPT calculations
- validity range of RegPT(fast) predictions

Convergence of RegPTfast

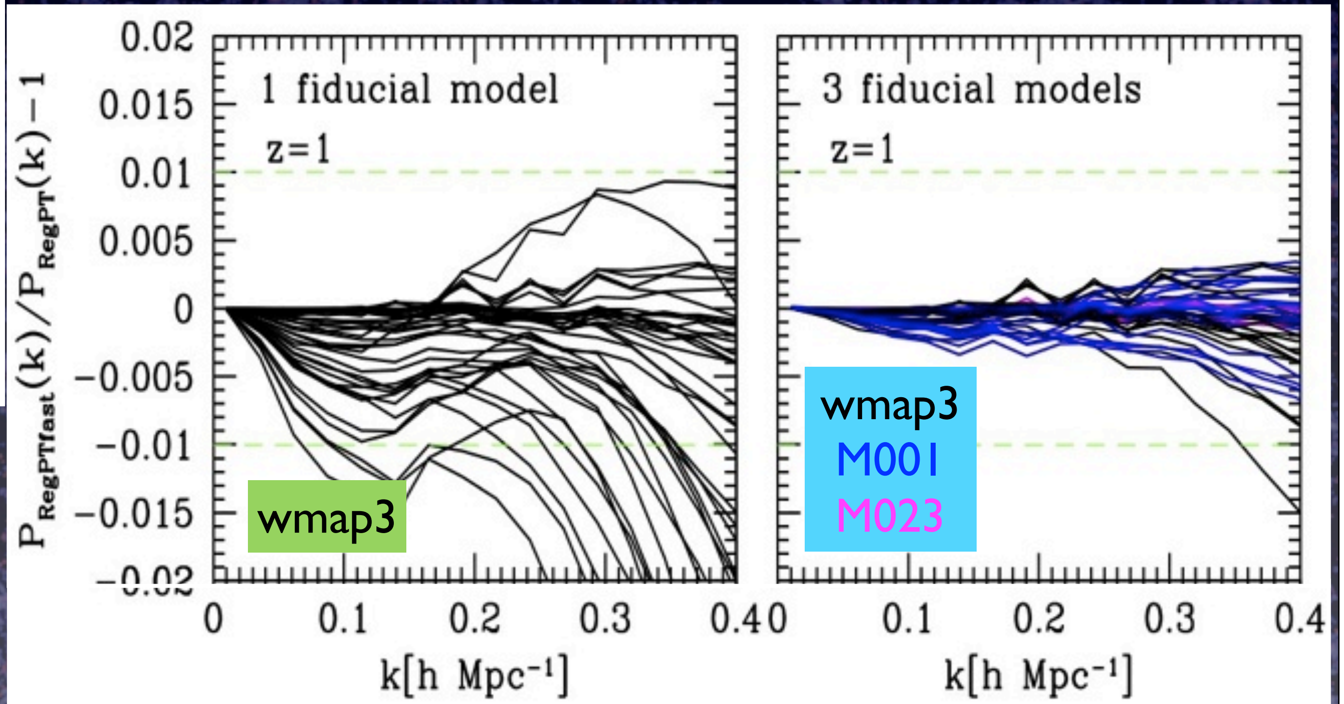
AT, Bernardeau, Nishimichi, Codis (in prep.)



Cosmological parameters in wCDM model

$$0.120 < \Omega_{\rm m}h^2 < 0.155$$
$$0.0215 < \Omega_{\rm b}h^2 < 0.0235$$
$$0.85 < n_s < 1.05$$
$$-1.30 < w < -0.70$$
$$0.616 < \sigma_8 < 0.9$$

Ratio of RegPTfast to (rigorous) RegPT results

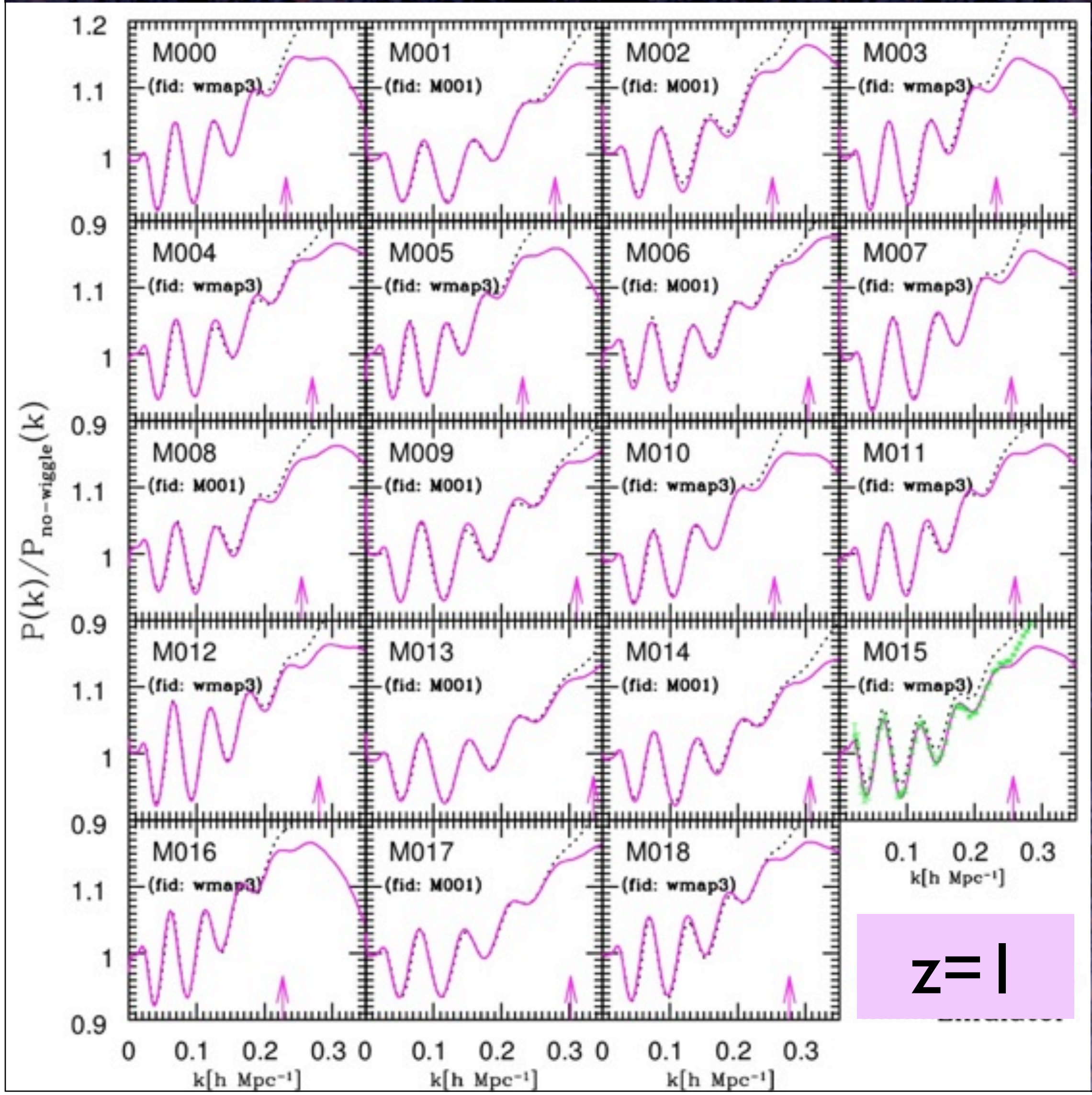


With only 3 fiducial models, one can cover a wide range of cosmological models.

Accuracy of RegPT

AT, Bernardeau, Nishimichi, Codis (in prep.)

Testing accuracy of RegPTfast in 38 cosmological models



Cosmic emulator Lawrence et al. ('10)

gives interpolated result of power spectrum from N-body simulations for 38 models

(restricted to *z*<1 & *k*<1 h/Mpc)

: RegPTfast

: Cosmic Emulator

Both results agree with each other with 1% level at k<k_crit:

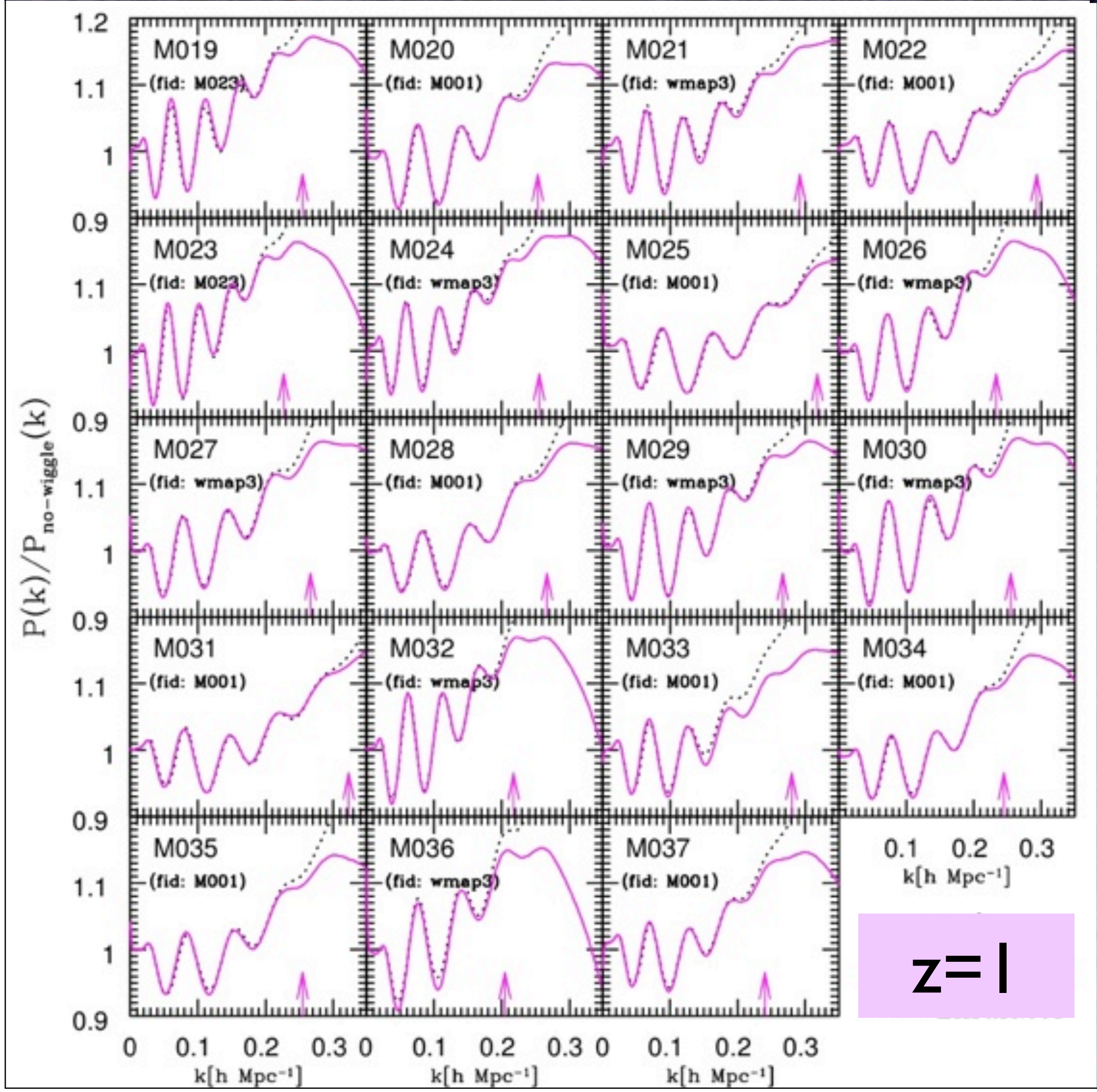
$$\frac{k_{\rm crit}^2}{6\pi^2}\int_0^{k_{\rm crit}} dq\, P_{\rm lin}(q;z) = 0.7$$

$$(k \lesssim 0.2 - 0.3\, h\,{\rm Mpc}^{-1} @ z = 1)$$

Accuracy of RegPT

AT, Bernardeau, Nishimichi, Codis (in prep.)

Testing accuracy of RegPTfast in 38 cosmological models



Cosmic emulator Lawrence et al. ('10)

gives interpolated result of power spectrum from N-body simulations for 38 models

(restricted to z<1 & k<1 h/Mpc)

: RegPTfast

: Cosmic Emulator

Both results agree with each other with 1% level at k<k_crit:

$$\frac{k_{\rm crit}^2}{6\pi^2}\int_0^{k_{\rm crit}} dq\, P_{\rm lin}(q;z) = 0.7$$

$$(k \lesssim 0.2 - 0.3\, h\,{\rm Mpc}^{-1} \,@\, z=1)$$

Summary

PT approach to precision power spectrum calculation for LSS now moves on to the 2nd stage (practical phase)

Though applicability is restricted to weakly non-linear regime,

- gravitational clustering
- redshift-space distortions

are now mostly under control.

In addition,

proposed accelerated calculation method is very powerful

······· few sec. on (my) laptop, no parallelization required

publicly available code: *RegPT*

- fast, exact-modes
- not only P(k) , but also ξ(r)

major release will be soon