MODIFICATIONS OF GRAVITY IN THE RADIATION-DOMINATED EPOCH

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Outline:

1. Scalar-tensor theories as models for modified gravity
2. Cosmological Perturbations
3. CMB observables
4. Extensions
5. Summary and Outlook

I. SCALAR TENSOR THEORIES AS MODELS FOR MODIFIED GRAVITY

- Scalar-tensor theories are *one* class of models for describing modifications to General Relativity. In these theories there is, in addition to metric tensor, an additional scalar degree of freedom in the gravitational sector, which couples to matter.
- In the following, we will use the Einstein frame description, in which the action takes the form:

$$\mathcal{S} = \int \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right] + S_{\mathrm{matter}}\left(\tilde{g}^{(i)}_{\mu\nu}, \chi_i\right)$$

where

$$\tilde{g}^{(i)}_{\mu\nu} = C^{(i)}(\phi)g_{\mu\nu}$$

This is the simplest form, we consider extensions later.

The field equations are

$$G_{\mu\nu} = \kappa \left(T^{(\phi)}_{\mu\nu} + T^{\mathrm{matter}}_{\mu\nu} \right)$$

$$\Box\phi = V_{,\phi} + \sum_i \frac{C^{(i)}_{,\phi}}{2C^{(i)}} g^{\mu\nu} T^{(i)}_{\mu\nu}$$

$$\nabla^\mu T^{(i)}_{\mu\nu} = -\frac{C^{(i)}}{2C^{(i)}} g^{\alpha\beta} T^{(i)}_{\alpha\beta} \nabla_\nu \phi$$

We will consider the case that the field is coupled to all matter species. In order for the theory to be consistent with observations, we either need the couplings

$$\beta = \frac{C_{,\phi}}{2C}$$

to the individual species are small or that the force mediated by the field is screened (e.g. chameleon mechanism or symmetron mechanism).

In this talk

- scalar field is specified by its mass $m^2 = V_{,\phi\phi}$
- the coupling function $\beta = \frac{C_{,\phi}}{2C}$

2. COSMOLOGICAL PERTURBATIONS

Modifications to Einstein's theory will, in general, result in

a) different expansion history
b) modified growth of perturbations in the different matter species.

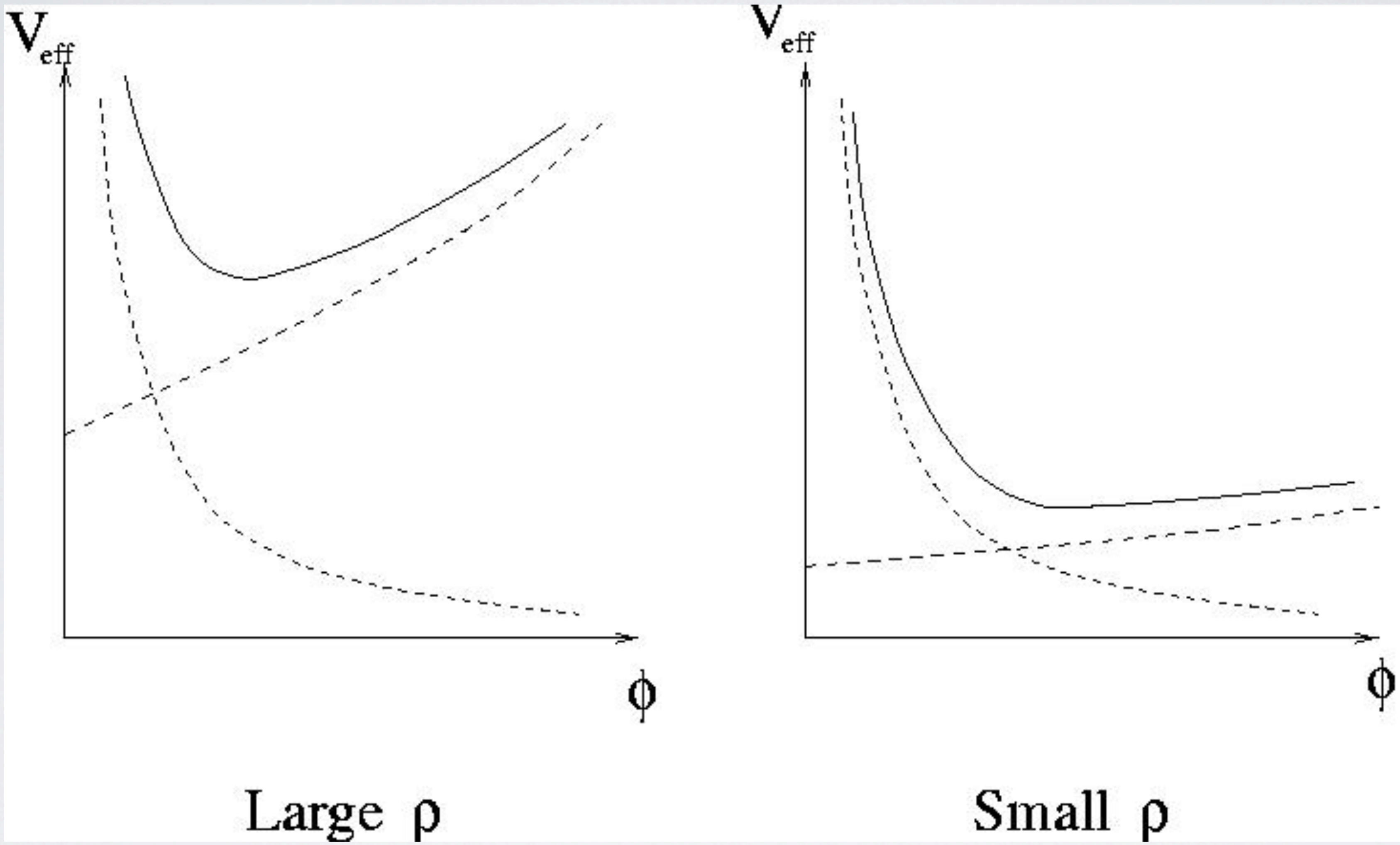
Here will will consider the case that the mass of the field is much heavier than the expansion rate, resulting in the field sitting in the minimum of an effective potential (if minimum exists). We will assume for most of the talk that

$$m^2 > H^2$$

This ensures that the field evolves slowly and that the background evolution mimics ΛCDM (just like in the original Chameleon setup). We will assume that this is the case, but it is not an crucial assumption. We will consider the case that baryons and cold dark matter are coupled to the scalar field, with couplings

$$\beta_b \ , \ \beta_c$$

Scalar field governed by an effective potential, which consists of two parts:



Not all choices of the bare potential allow for the chameleon mechanism to operate.

Evolution equations of cosmological perturbations:

$$(\delta\phi)^{\cdot\cdot} + 2\mathcal{H}(\delta\phi)^{\cdot} + \left(k^2 + a^2\frac{d^2V}{d\phi^2}\right)\delta\phi = \left(3\dot{\Phi} + \dot{\Psi}\right)\dot{\phi} - 2\Psi\left(\frac{dV_{\rm eff}}{d\phi}\right) - a^2\sum_i \beta_i\delta\rho_i$$

$$\dot{\delta}_b = -\theta_b + 3\dot{\Phi} + \frac{d\beta_b}{d\phi}\dot{\phi}\delta\phi + \beta(\delta\phi)^{\cdot}$$

$$\theta_b = -\mathcal{H}\theta_b + k^2\Phi + \frac{an_e\sigma_T}{R}\left(\theta_\gamma - \theta_b\right) + \beta_b k^2\delta\phi - \beta_b\dot{\phi}\theta_b$$

$$\dot{\delta}_\gamma = -\frac{4}{3}\theta_\gamma$$

$$\theta_\gamma = \frac{k^2}{4}\delta_\gamma + k^2\Phi - an_e\sigma_T\left(\theta_\gamma - \theta_b\right)$$

Subhorizon evolution is standard oscillatory behaviour. However, presence of scalar field modifies the sound speed (Brax & Davis (2011)):

$$\delta_\gamma \propto \exp\left(-\frac{k^2}{k_D^2}\right)\exp\left(\pm ik\tilde{r}_s\right)$$

$$k_D^{-2} = \frac{1}{6}\int_0^\eta \frac{1}{an_e\sigma_T}\left(\frac{1}{1+R}\left(\frac{16}{15}+\frac{R^2}{1+R}\right)\right)d\eta'$$

$$\tilde{r}_s = \int_0^\eta \tilde{c}_s d\eta'$$

$$\tilde{c}_s^2 = c_s^2\left(1-\frac{9\Omega_b\beta_b^2 R\mathcal{H}^2}{k^2+a^2m^2}\right) \qquad c_s^2 = \frac{1}{3}\frac{1}{1+R} \qquad R = \frac{3}{4}\frac{\rho_b}{\rho_\gamma}$$

Sound speed: $$\tilde{c}_s^2 = c_s^2 \left(1 - \frac{9\Omega_b \beta_b^2 R \mathcal{H}^2}{k^2 + a^2 m^2}\right)$$

Sound speed can be negative for large enough couplings.
I am going to ignore this issue here. Similar issue in chameleon-type models (discussed e.g. in Bean et al (2008), Corasaniti (2008)).

Note that $$9\Omega_b \beta_b^2 R \mathcal{H}^2 \approx 1.5 \beta_b^2 10^{-5} \mathrm{Mpc}^{-2}$$

and therefore large enough coupling needed to see significant modification of sound speed.

3. CMB OBSERVABLES

Modified sound speed changes sound horzion:

$$\tilde{r}_s = \int_0^\eta \tilde{c}_s d\eta \qquad\qquad \tilde{c}_s^2 = c_s^2 \left(1 - \frac{9\Omega_b \beta_b^2 R \mathcal{H}^2}{k^2 + a^2 m^2}\right)$$

Position of first peak is modified since $l \propto \dfrac{1}{\tilde{r}_s}$

Position of first peak well known (220.1±0.8), so sound horizon can vary not by much.

Of course, the whole CMB anisotropy power spectrum contains information about couplings to matter.

CMB distortions due to dissipation of acoustic waves:

- Injection of energy into the baryon-photon fluid (e.g. due to dissipation of the energy stored in acoustic oscillatons) potentially leads to a modification of the Planck spectrum.
- At high enough redshifts ($z \gg 10^6$) photons are quickly thermalized and therefore CMB spectrum remains blackbody
- Below $z \approx 2\times10^6$ thermalization due to double Compton scattering becomes less effective.
- Two types of distortions: μ-type distortion and y-type distortion.

$$\frac{1}{e^{\frac{h\nu}{kT}}-1} \rightarrow \frac{1}{e^{\frac{h\nu}{kT}+\mu(\nu)}-1}$$

Chemical potential μ has been shown to be constant, with deviations from a constant to be expected for very small frequencies. Processes relevant between $z \approx 2\times10^6$ and $z \approx 5\times10^4$.

Time evolution of **μ** governed by (Hu et al (1992,1994))

$$\frac{d\mu}{dt} = -\frac{\mu}{t_{\rm DC}(z)} + 1.4\frac{dQ/dt}{\rho_\gamma}$$

with (Y_p is helium mass fraction)

$$t_{\rm DC} = 2.06 \times 10^3 3 \left(1 - \frac{Y_p}{2}\right)^{-1} (\Omega_b h^2)^{-1} z^{-9/2} s$$

Solution to equation above:

$$\mu = 1.4 \int_{z_f}^{z_i} dz \frac{dQ/dz}{\rho_\gamma} e^{-(z/z_{\rm DC})^{5/2}}$$

with

$$z_{\rm DC} = 1.97 \times 10^6 \left(1 - \frac{1}{2}\frac{Y_p}{0.24}\right)^{-5/2} \left(\frac{\Omega_b h^2}{0.0224}\right)^{-2/5}$$

Energy density of acoustic wave in standard picture:

$$Q = \rho_\gamma c_s^2 \left\langle \delta_\gamma^2(\mathbf{x}) \right\rangle$$

with

$$\left\langle \delta_\gamma^2(\mathbf{x}) \right\rangle = \int \frac{d^3k}{(2\pi)^3} P_\gamma(k)$$

Relate power spectrum to initial power spectrum via transfer function (Chluba et al (2011)):

$$P_\gamma(k) = \Delta_\gamma^2(k) P_\gamma^i(k)$$ with $$\Delta_\gamma(k) \approx 3\cos(kr_s) e^{-(k/k_D)^2}$$

and (k_0 = 0.002 Mpc^{-1})

$$P_\gamma^i = 1.45 \frac{2\pi^2 A_\zeta}{k^3} \left(\frac{k}{k_0}\right)^{n_s - 1 + \frac{1}{2}\ln(k/k_0)\alpha} \qquad \alpha = \frac{dn_s}{d\ln k}$$

In the case of modified gravity with k-dependent sound speed:

(vdB & Sculthorpe (2012))

$$Q = \rho_\gamma \int \frac{d^3k}{(2\pi)^3} \tilde{c}_s^2(k) P_\gamma(k)$$

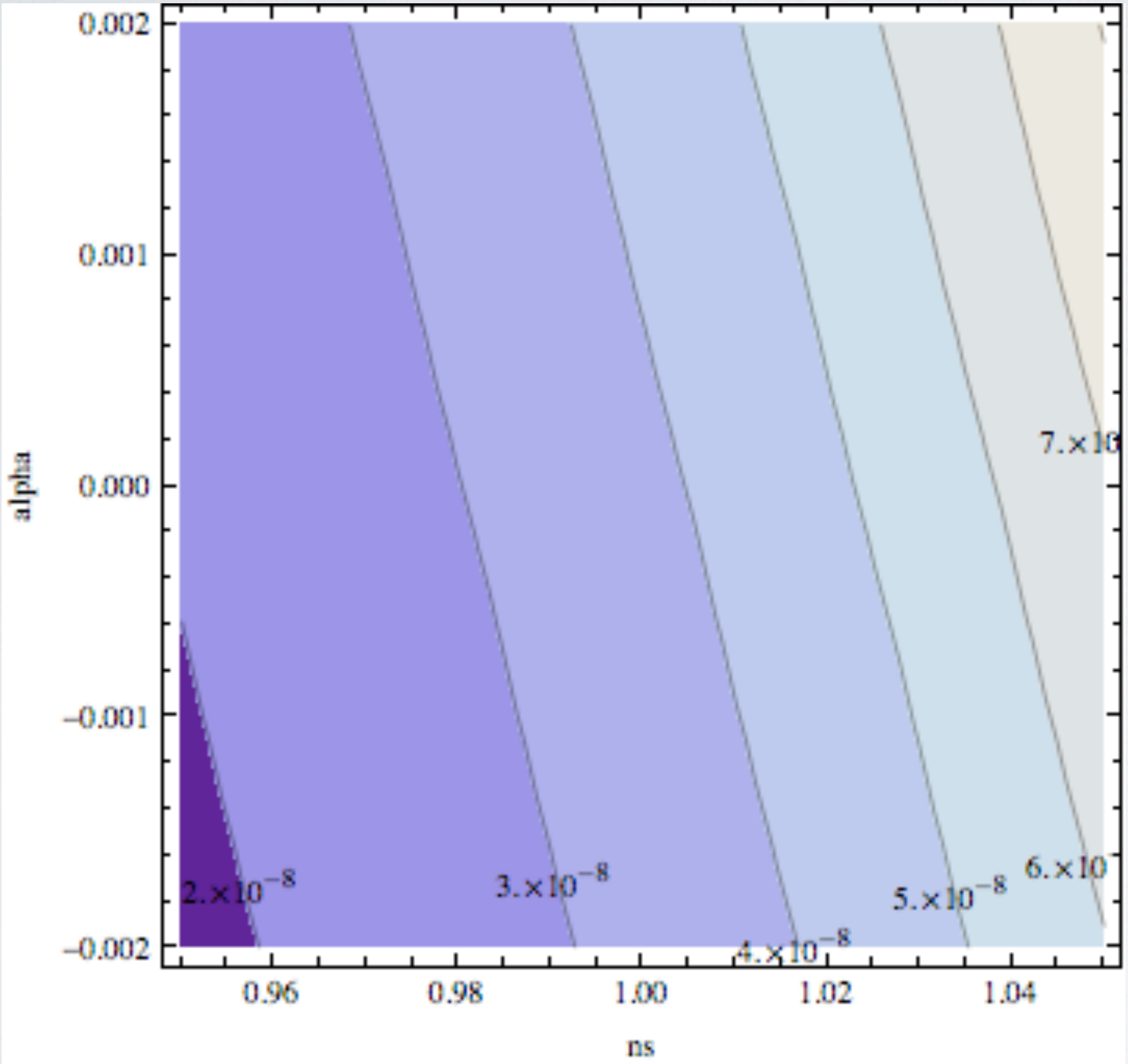
Strictly speaking for a generic modified gravity theory the transfer functions will have to be modified. However, since we are assuming

$$m^2 > H^2$$

the modifications of gravity are suppressed on large scales. So, we expect that the transfer functions have the same form, but with the modified sound speed instead of the standard one.

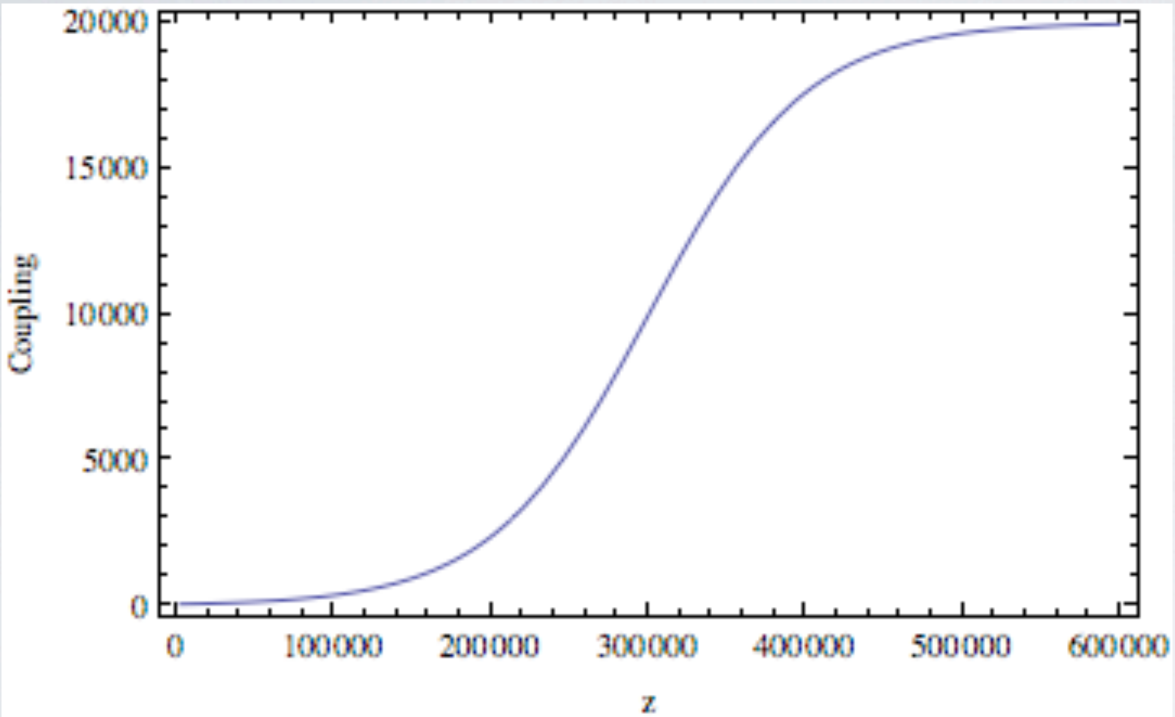
Standard case:

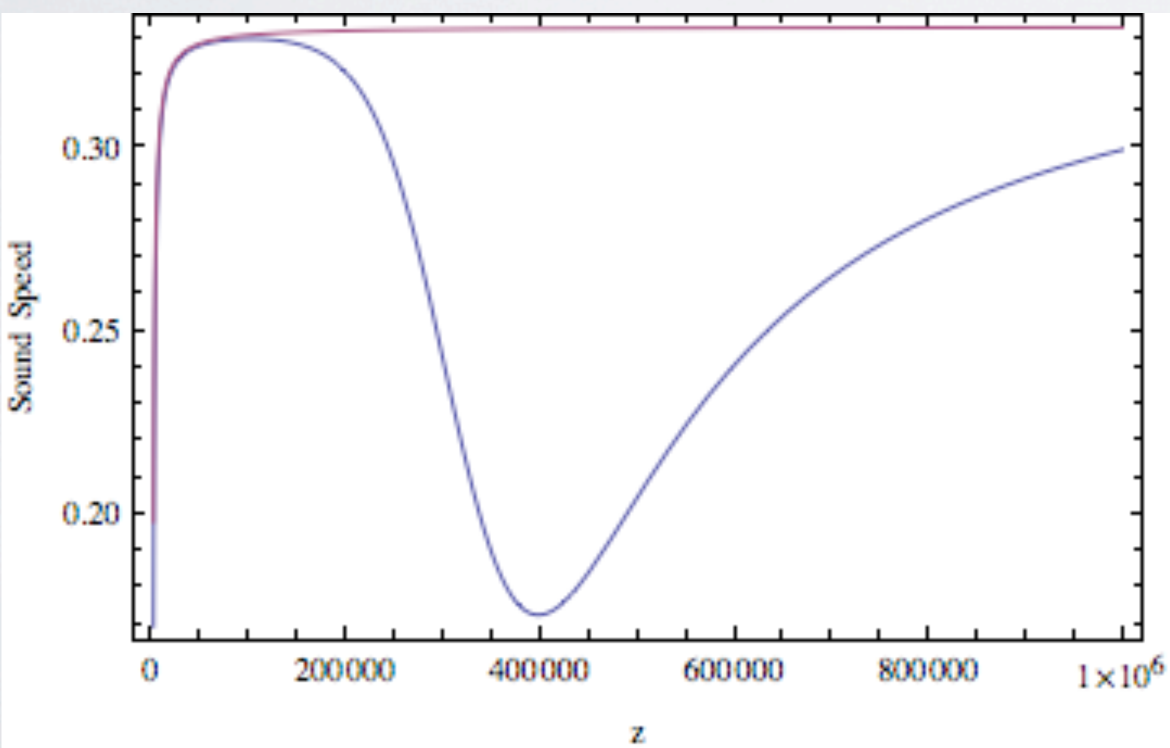
Dent et al (2012), Chluba et al (2012)



Isocurvature perturbations can influence these results.

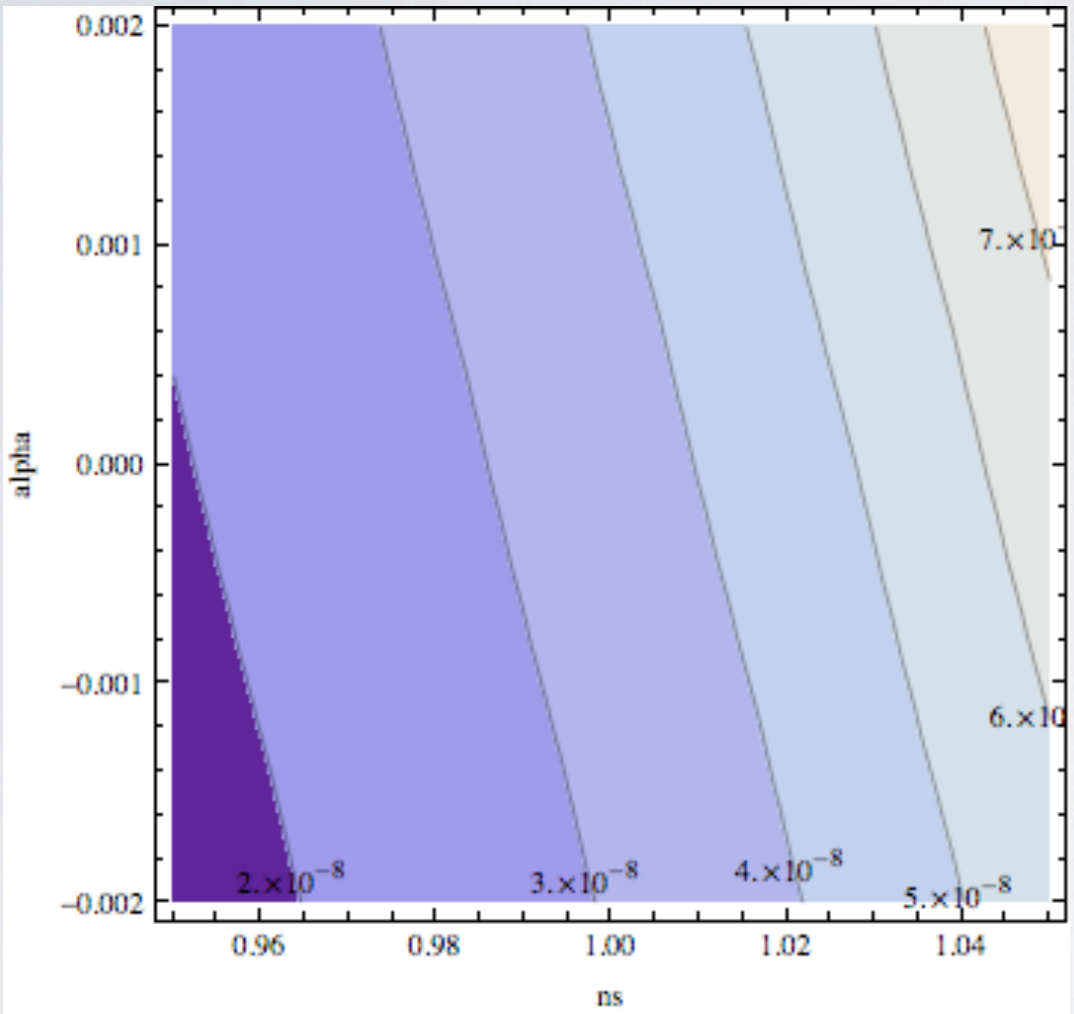
Example of modified gravity



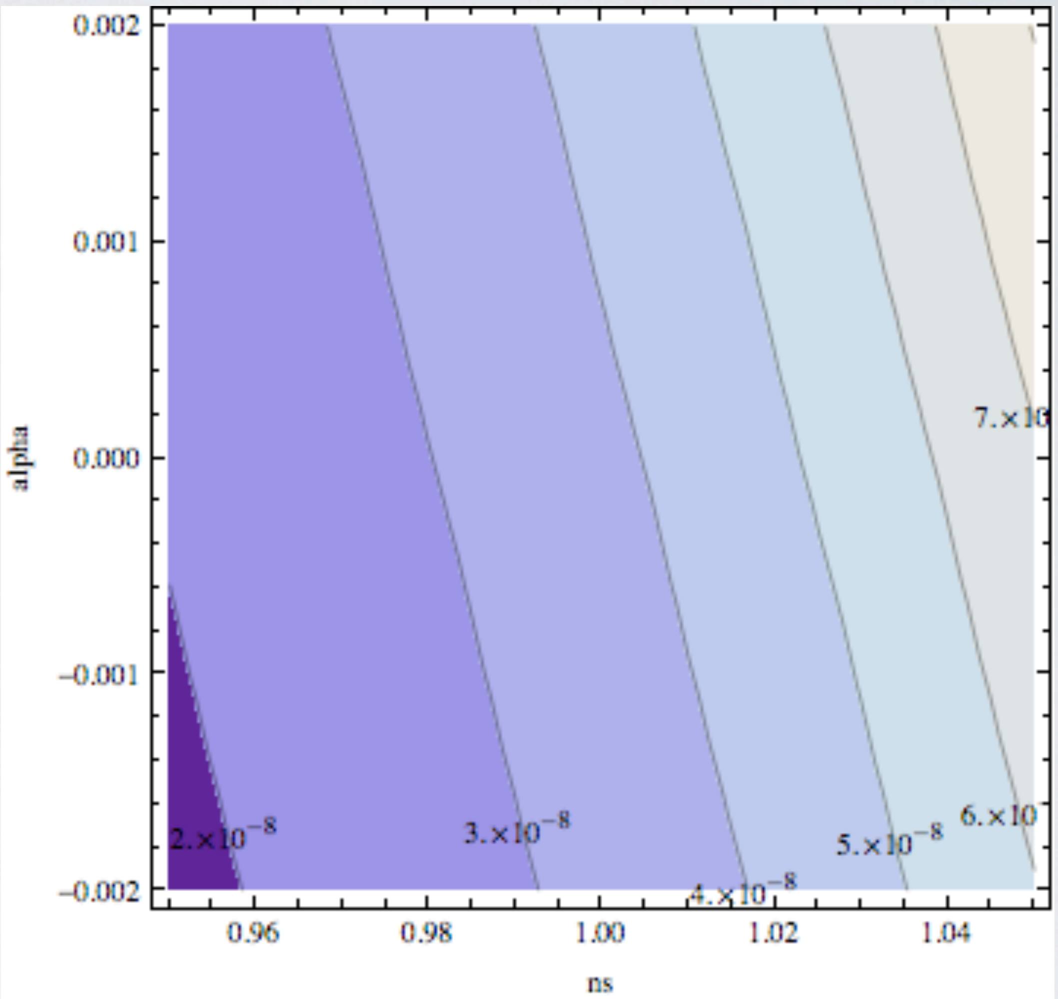


$$m(z) = 150H(z)$$

Modified Gravity



Standard Einstein



Modified gravity contribution could potentially important, same order of magnitude than other processes (Chluba et al 2011, 2012). Details depending on potential and coupling.

4.EXTENSIONS

work in progress with Greg Sculthorpe

We are extending the theory by considering disformal couplings, i.e.

$$\mathcal{S} = \int \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right] + S_{\rm matter}\left(\tilde{g}^{(i)}_{\mu\nu}, \chi_i\right)$$

with

$$\tilde{g}^{(i)}_{\mu\nu} = C^{(i)}(\phi)g_{\mu\nu} + D^{(i)}(\phi)\phi_{,\mu}\phi_{,\nu}$$

Theories of this kind are motivated from theories of massive gravity, DBI-Galileon, etc. In the context of local experiments, they have been considered by Noller (2012), Koivisto et al (2012) and Brax et al (2012). In particular, local fifth forces do not constrain disformal coupled fields (Noller (2012)). Optical experiments do a bit better, but still not that strong constraints. Consider here the case of **one** species (baryons in practice) coupled to the scalar.

Field equations:

Koivsoto et al (2012), vdB & Sculthorpe (2012)

$$G_{\mu\nu} = \kappa \left(T^{(\phi)}_{\mu\nu} + T^{\rm matter}_{\mu\nu} \right)$$

$$\Box\phi - \frac{dV}{d\phi} + Q = 0$$

$$\nabla^{\mu} T_{\mu\nu} = Q \nabla_{\nu}\phi$$

with

$$Q = \frac{C_{,\phi}}{2C} g^{\mu\nu} T_{\mu\nu} + \frac{D_{,\phi}}{2C} \phi_{,\mu}\phi_{,\nu} T^{\mu\nu} - \nabla_{\nu} \left[\frac{D}{C} \phi_{,\mu} T^{\mu\nu} \right]$$

FRW, conformal time:

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2\frac{dV}{d\phi} = a^2\bar{Q}$$

$$a^2\bar{Q} = -\frac{\rho}{2(C + D(\rho - \dot{\phi}/a^2))}\left[a^2\frac{dC}{d\phi}(1-3w) - 2D\left(3\mathcal{H}\dot{\phi}(1+w) + a^2\frac{dV}{d\phi} + \frac{C_{,\phi}}{C}\dot{\phi}^2\right) + D_{,\phi}\dot{\phi}^2\right]$$

So, effective coupling (at background level) is

$$\beta_{\rm eff} = -\frac{Q}{\rho}$$

Perturbation equations:

$$\dot{\delta} = -(1+w)\left(\theta - 3\dot{\Phi}\right) - 3\mathcal{H}(c_s^2 - w)\delta - \frac{\bar{Q}}{\rho}\dot{\phi}\delta + \frac{\delta Q}{\rho}\dot{\phi} - \frac{\bar{Q}}{\rho}(\delta\phi)^{\cdot}$$

$$\dot{\theta} = -\mathcal{H}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{k^2}{1+w}c_s^2\delta - k^2\sigma + k^2\Psi - k^2\frac{\bar{Q}}{\rho(1+w)}\delta\phi + \frac{\bar{Q}}{\rho}\dot{\phi}\theta$$

$$(\delta\phi)^{\cdot\cdot} + 2\mathcal{H}(\delta\phi)^{\cdot} + \left(k^2 + a^2\frac{d^2V}{d\phi^2}\right)\delta\phi = (3\dot{\Phi} + \dot{\Psi})\dot{\phi} - 2\Psi\left(\frac{dV}{d\phi} + \bar{Q}\right) + a^2\delta Q$$

Perturbed coupling

$$\delta Q = -\frac{\rho}{a^2 C + D(a^2\rho - \dot{\phi}^2)}[\mathcal{B}_1\delta + \mathcal{B}_2\dot{\Phi} + \mathcal{B}_3\Psi + \mathcal{B}_4(\delta\phi)^{\cdot} + \mathcal{B}_5\delta\phi]$$

$$\mathcal{B}_1 = \frac{a^2 C'}{2}\left(1 - 3\frac{\delta P}{\delta\rho}\right) - 3D\mathcal{H}\dot{\phi}\left(1 + \frac{\delta P}{\delta\rho}\right) - Da^2(V' - \bar{Q}) - D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D}\right)$$

$$\mathcal{B}_2 = 3D\dot{\phi}(1+w),$$

$$\mathcal{B}_3 = 6D\mathcal{H}\dot{\phi}(1+w) + 2D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D} + \frac{\bar{Q}}{\rho}\right),$$

$$\mathcal{B}_4 = -3D\mathcal{H}\dot{\phi}(1+w) - 2D\dot{\phi}\left(\frac{C'}{C} - \frac{D'}{2D} + \frac{\bar{Q}}{\rho}\right),$$

$$\mathcal{B}_5 = \frac{a^2 C''(1-3w)}{2} - Dk^2(1+w) - Da^2V'' - D'a^2V' - 3D'\mathcal{H}\dot{\phi}(1+w)$$
$$- D\dot{\phi}^2\left(\frac{C''}{C} - \left(\frac{C'}{C}\right)^2 + \frac{C'D'}{CD} - \frac{D''}{2D}\right) + (a^2C' + D'a^2\rho - D'\dot{\phi}^2)\frac{\bar{Q}}{\rho}.$$

Coupling function and its perturbation have a rather rich structure (translation: they are very complicated). But, in the Newtonian limit and for pressureless matter, the form of sound speed takes the following form:

$$\tilde{c}^2_{\gamma,s} = c^2_{\gamma,s}\left(1 - \frac{9\Omega_b \beta_b^2 R \mathcal{H}^2}{k^2 + a^2 m^2}\right)$$

but now $\beta_b = -\frac{\bar{Q}}{\rho_b}$ and the mass is modified too.

$$a^2\bar{Q} = -\frac{\rho_b}{2(C + D(\rho_b - \dot{\phi}^2/a^2))}\left[a^2\frac{dC}{d\phi} - 2D\left(3\mathcal{H}\dot{\phi} + a^2\frac{dV}{d\phi} + \frac{C_{,\phi}}{C}\dot{\phi}^2\right) + D_{,\phi}\dot{\phi}^2\right]$$

The evolution of the effective coupling function (and effective mass) can be much more complicated if *D* is non-vanishing.

5. SUMMARY AND OUTLOOK

- Some modified gravity theories predict modification of dynamics of coupled baryon-photon plasma, with sound speed modifications depending on the coupling of scalar field to baryons and mass of scalar degree of freedom.
- Depending on how coupling varies in time, sound horizon at decoupling is modified only little, but CMB distortion can different from GR case. Need to know all processes leading to CMB distortion to constrain modified gravity (also dependent on inflationary model). Mod Grav has the tendency to lower CMB distortions.
- In light of recent results in constraining disformal coupled fields, however, CMB constraints are *complementary*. Depends on potential and evolution of scalar field. Certainly will put additional constrains on disformal coupling.
- In case of purely conformally coupled fields, coupling needs to be strong ($\gtrsim 10^3$) in very early universe to see effect on CMB distortions and then either coupling drops to smaller values *or* mass of fields becomes suddenly very large otherwise position of first peak shifted significantly.
- Similar for disformal couplings, but not constrained by local experiments. Evolution of effective coupling model dependent.

5. SUMMARY AND OUTLOOK

- Work needs to be done on transfer functions and check under which circumstances effects of modified gravity are small.
- Models need to be studied in detail, e.g.

$$D(\phi) \propto M^{-4} = \mathrm{const}$$

with conformal coupling simplest (exponential) form.

- Multiple couplings to diverse species needs to be investigated too.

$$\Box\phi - \frac{dV}{d\phi} + Q = 0 \qquad\qquad Q = \frac{C_{,\phi}}{2C} g^{\mu\nu} T_{\mu\nu} + \frac{D_{,\phi}}{2C} \phi_{,\mu}\phi_{,\nu} T^{\mu\nu} - \nabla_\nu \left[\frac{D}{C}\phi_{,\mu} T^{\mu\nu}\right]$$

in particular, effects of couplings during structure formation.

This is work in progress.