The Cosmic Microwave Background and Dark Matter

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Itzykson meeting, Saclay, 19 June 2012

(Cold) Dark Matter as a model

Dark Matter: $\nearrow$ Particle (microphysics)

$\searrow$ Dust fluid (macrophysics)

Particle (PDM) $\Longrightarrow$ Non-gravitational detection (perhaps, perhaps, perhaps)

DAMA, CoGeNt, Crest

$\Downarrow$ $\Uparrow$?

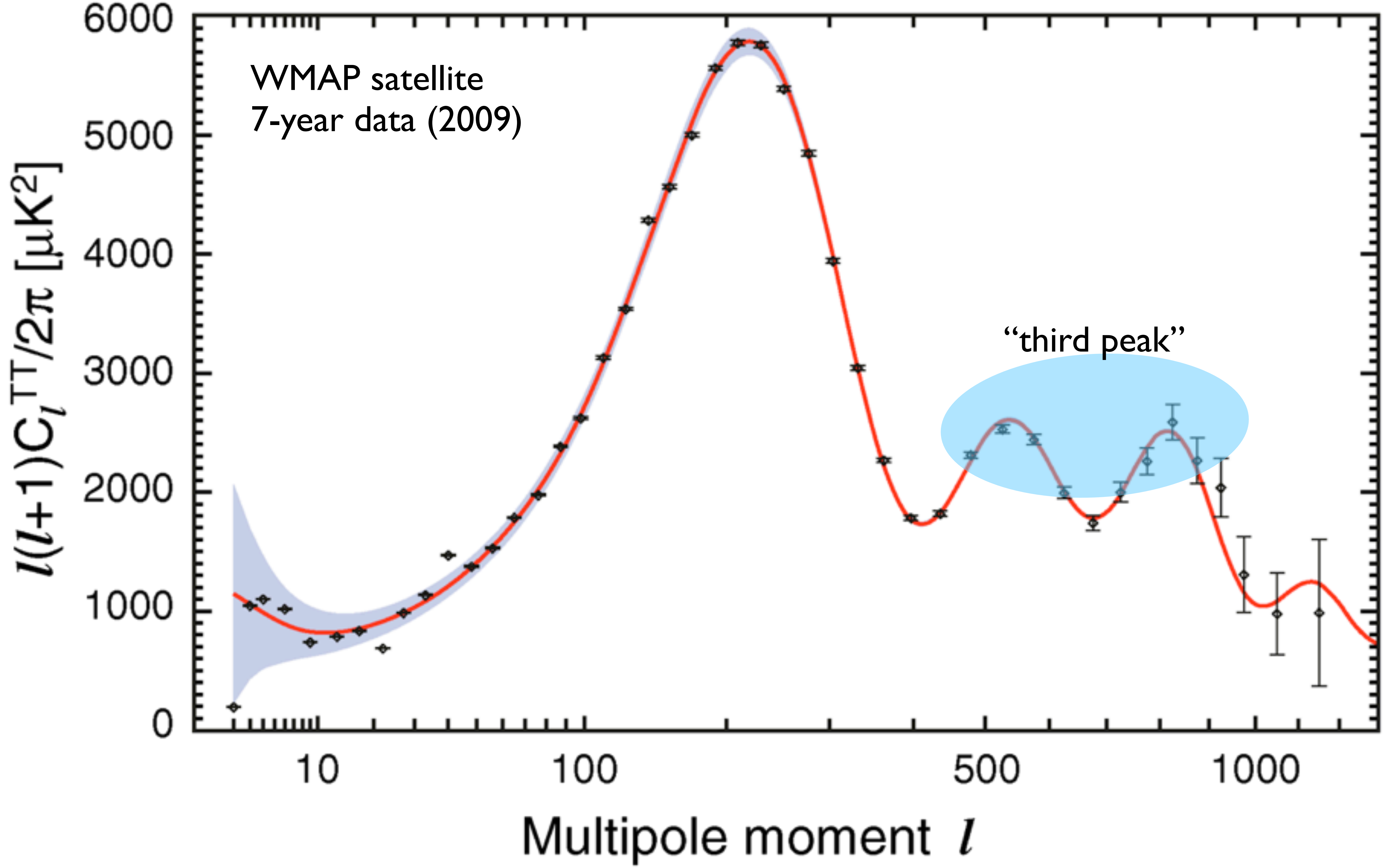
Dust fluid (ΛCDM)

- $\nearrow$ Extremely successful for LSS, CMB, clusters
- $\rightarrow$ Potential problems for galaxies (cuspy halos..., satellites...,)
- $\searrow$ Need Dark Energy $\Rightarrow$ Λ

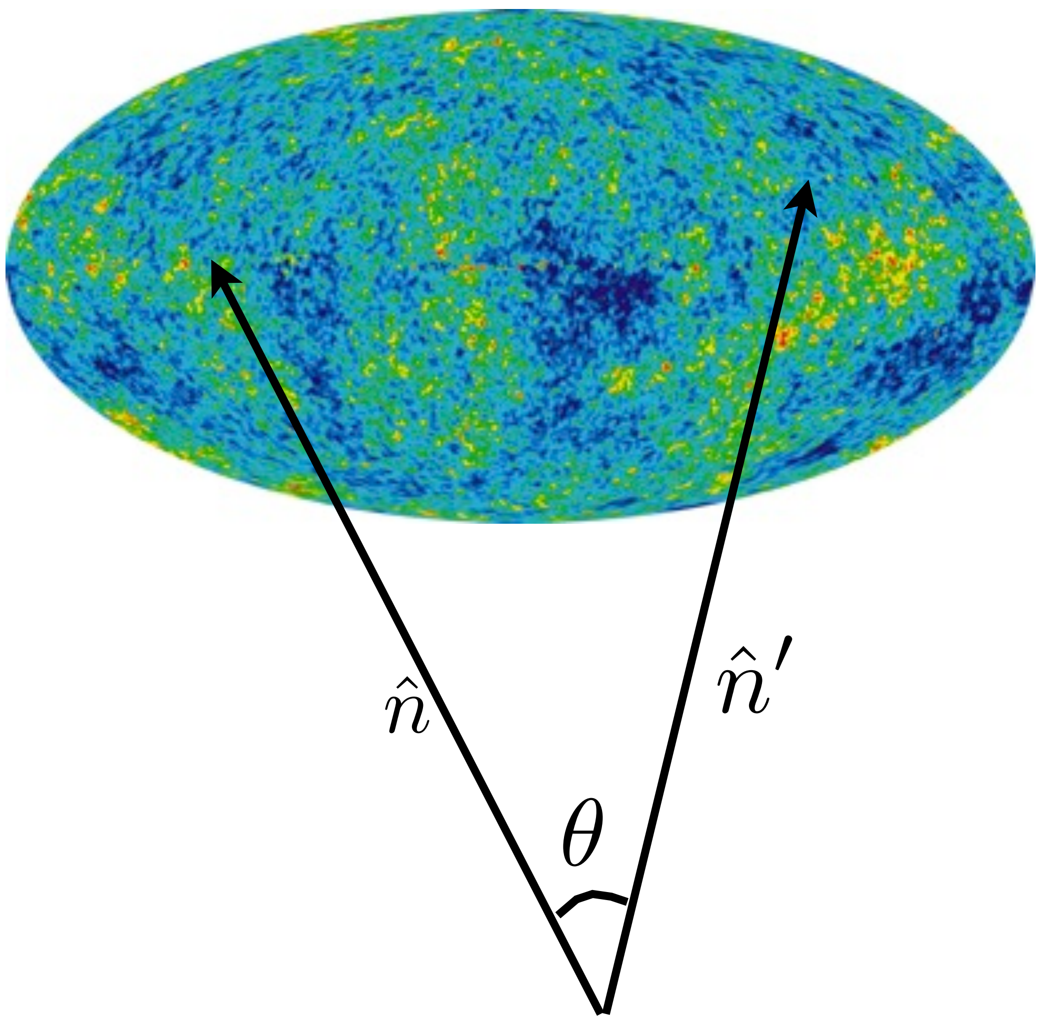
Motivation

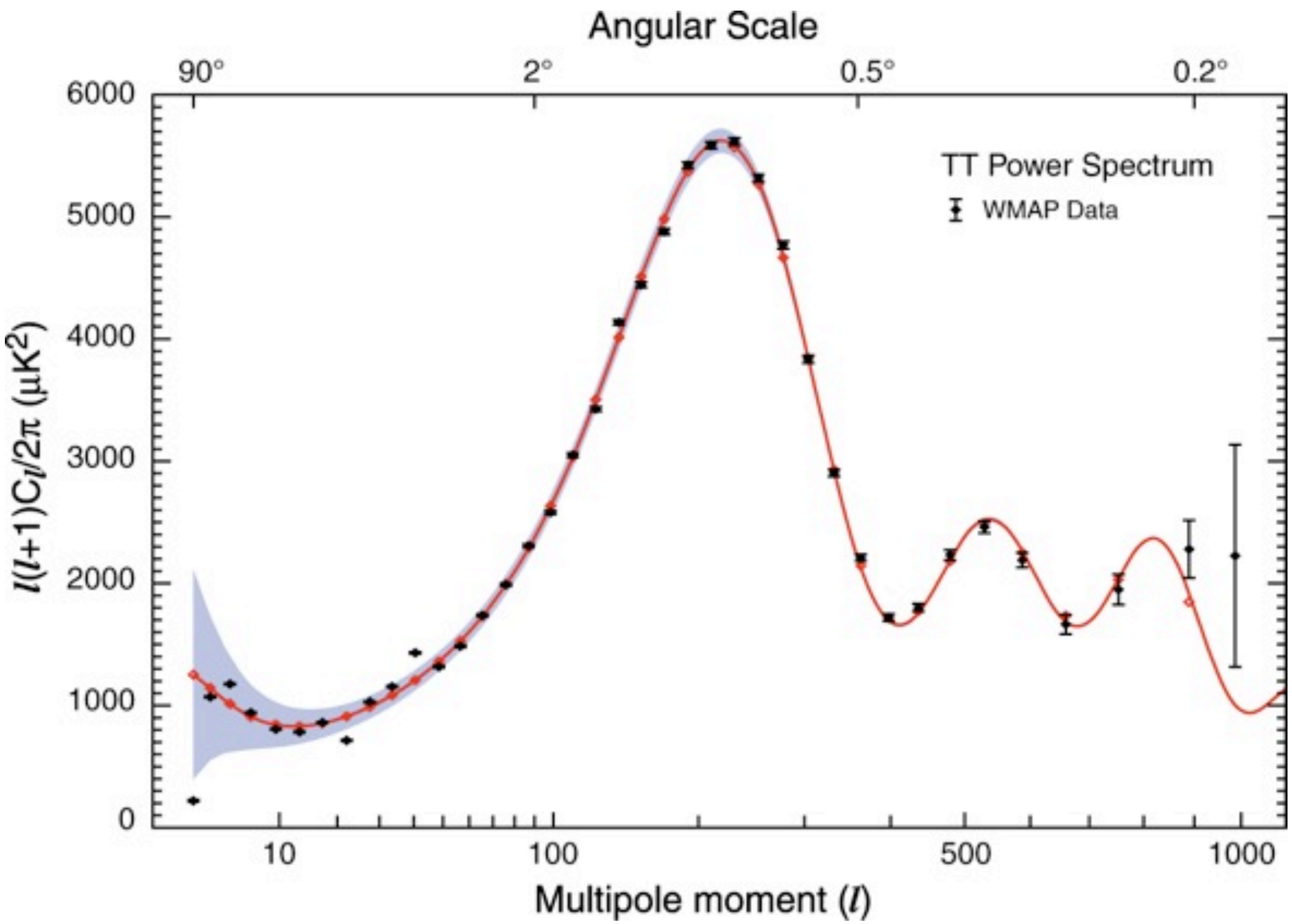
- Best evidence for CDM comes from CMB
  - CMB described by linear theory
  - No messy astrophysics
  - Very accurate observations
- Effects of CDM on CMB
  - Pedagogical
  - Can help to test properties of dark matter
  - Can help to test GR
- Does a "dust fluid" imply particle dark matter?
- Can a modification of gravity produce similar effects

Why CMB indicates CDM



CMB angular power spectrum





$$\langle \Theta(\hat{n})\Theta(\hat{n}')\rangle = \sum_l (2l+1) C_l P_l(cos\theta)$$

C_l :Angular power spectrum

$P_l(\mu)$: Legendre polynomials

"angular eigenfunctions" $\mu = [-1, 1]$

Functions defined in $[-1, 1]$
can be expanded in terms of $P_l(\mu)$

Fluctuations in the Universe

Metric fluctuations $$ds^2 = a^2\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\vec{x}\cdot d\vec{x}\right]$$

Two "Newtonian" potentials: $\Phi(\tau,k)$ $\Psi(\tau,k)$ (in Fourier space)

Fluids:

Density contrast: $\delta \equiv \dfrac{\delta\rho}{\bar{\rho}}$

Velocity $u_i = a\vec{\nabla}_i\theta$

(adiabatic) Pressure fluctuation: $\delta P = c_a^2\delta\rho$

Shear σ

$\Longrightarrow$ $\delta(\tau,k)$ $\theta(\tau,k)$

Adiabatic speed of sound

- Baryons and Cold Dark Matter $c_a^2 = 0$
- Photons and massless neutrinos $c_a^2 = \frac{1}{3}$
- Massive neutrinos $0 \leq c_a^2 \leq \frac{1}{3}$

CMB description

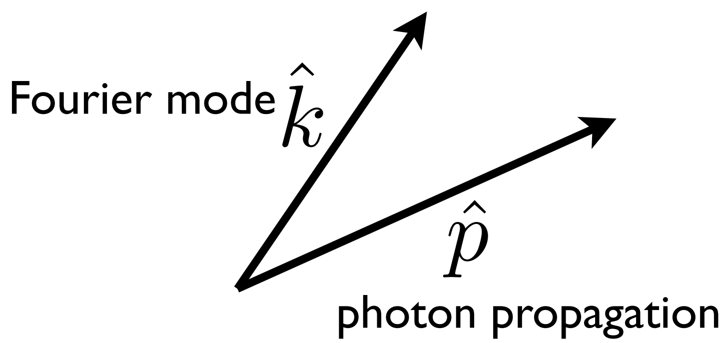
(ignoring polarization effects)

CMB Temperature contrast: $\Theta(\tau, \vec{k}, \hat{p})$

Boltzmann equation

$$\dot{\Theta} + ik\mu\Theta - \dot{\Phi} + ik\mu\Psi = an_e\sigma_T\left[\Theta_0 - \Theta + ik\mu\theta_b\right]$$

Compton scattering



$$\mu = \hat{k} \cdot \hat{p}$$

$$\Rightarrow \Theta(\tau, \vec{k}, \hat{p}) = \Theta(\tau, k, \mu)$$

Expand in multipole moments:

$$\Theta(\tau, k, \mu) = \sum_\ell (2\ell + 1)\Theta_\ell(\tau, k)P_\ell(\mu)$$

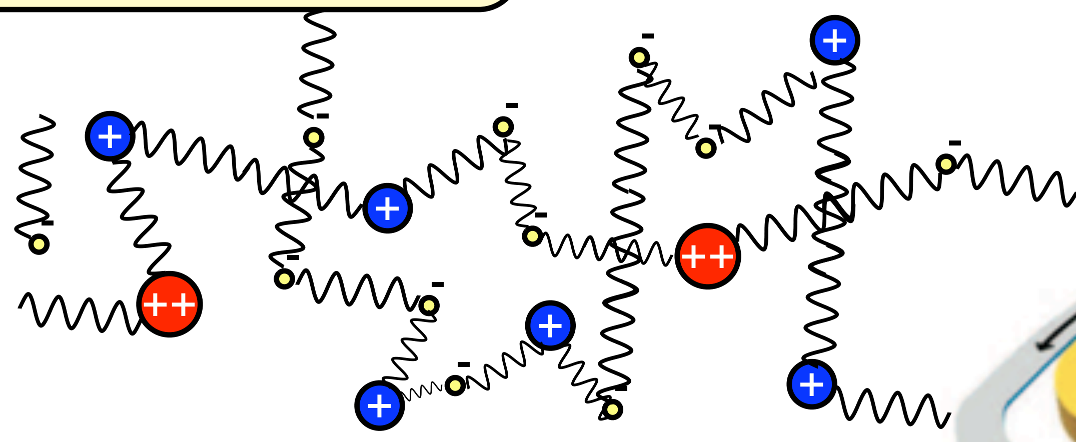
Angular Power Spectrum

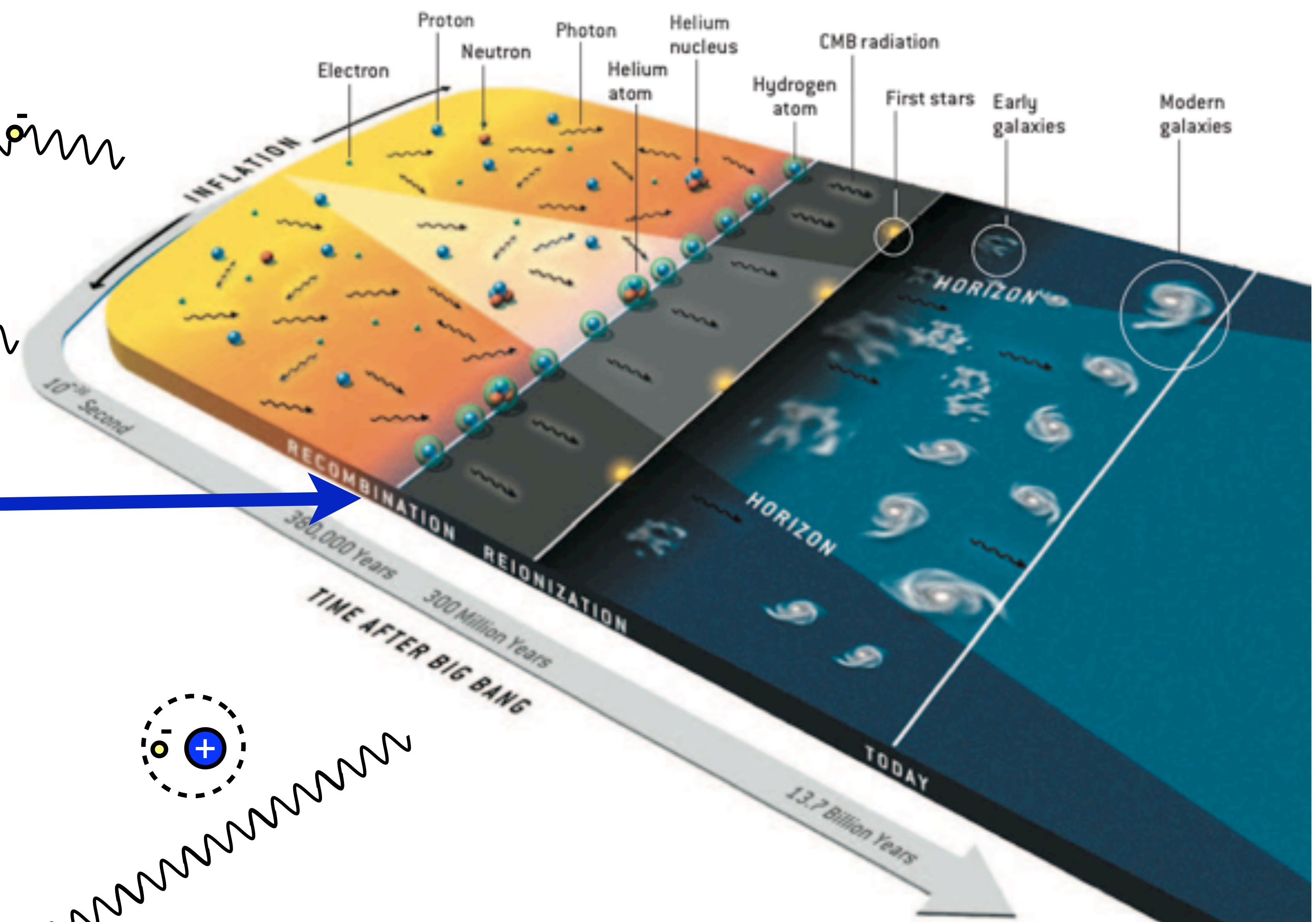
$$C_\ell = \frac{2}{\pi}\int dk k^2 P_0(k)|\Theta_\ell(\tau_0, k)|^2$$

Tight-coupling and Free-streaming

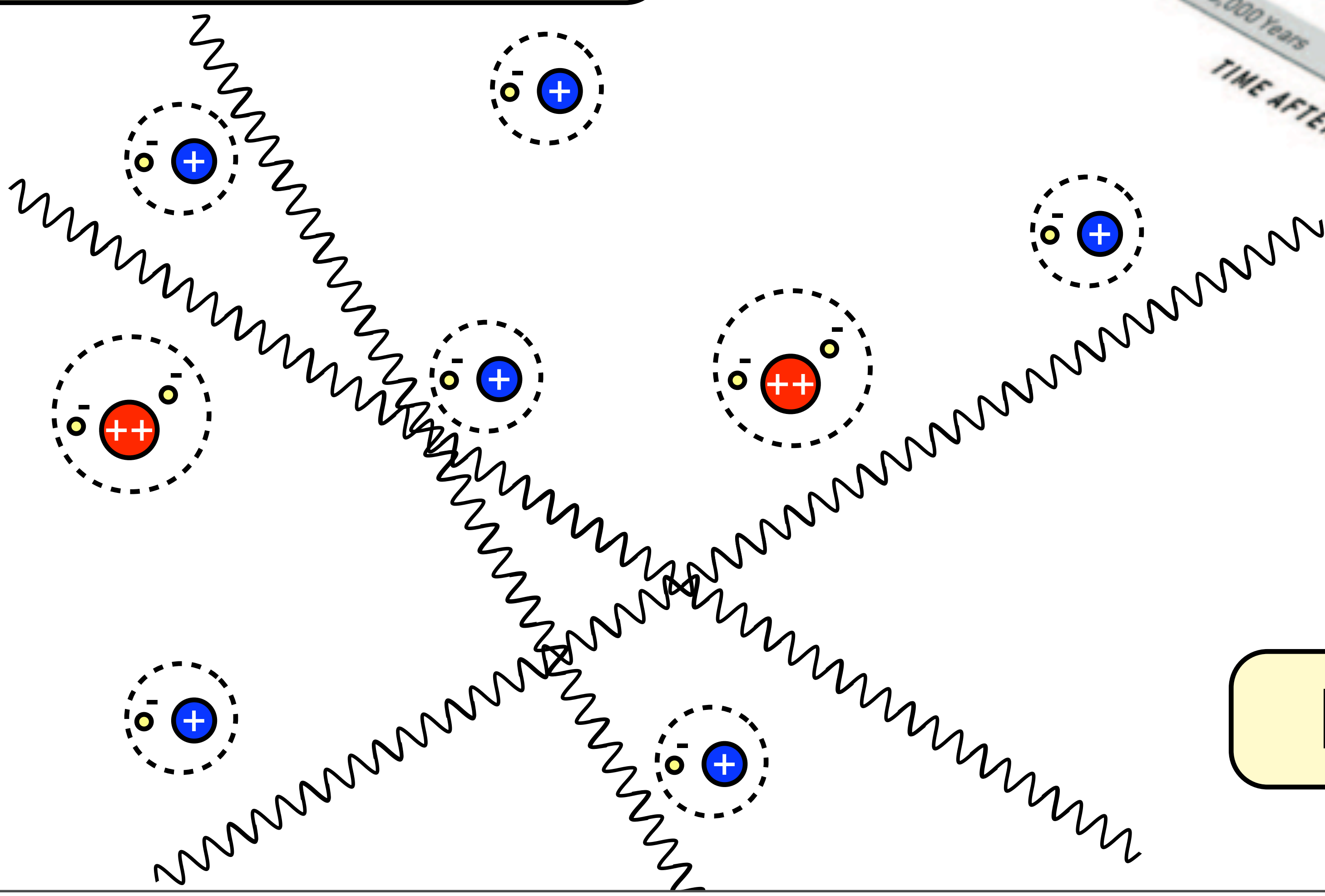
Tight-coupling:

Compton scattering





τ_* **Recombination**



Free-streaming

Solving the Boltzmann equation

(Hu & Sugiyama 1996)

$\tau < \tau_*$ Tight-coupling:

$$\ddot{\Theta}_0 + \frac{R}{1+R}\frac{\dot{a}}{a}\dot{\Theta}_0 + k^2 c_s^2 \Theta_0 = -\frac{k^2}{3}\Psi + \ddot{\Phi} + \frac{R}{1+R}\frac{\dot{a}}{a}\dot{\Phi}$$

Damped harmonic oscillator — forced by gravity Φ , Ψ

$\Theta_0(\tau, k)$

$R = \frac{3\rho_b}{4\rho_\gamma}$: Baryon-photon ratio

$c_s^2 = \frac{1}{3(1+R)}$: Baryon-photon fluid sound speed

$\tau > \tau_*$ Free-streaming:

$$\dot{\Theta} + ik\mu\Theta - \dot{\Phi} + ik\mu\Psi = 0$$

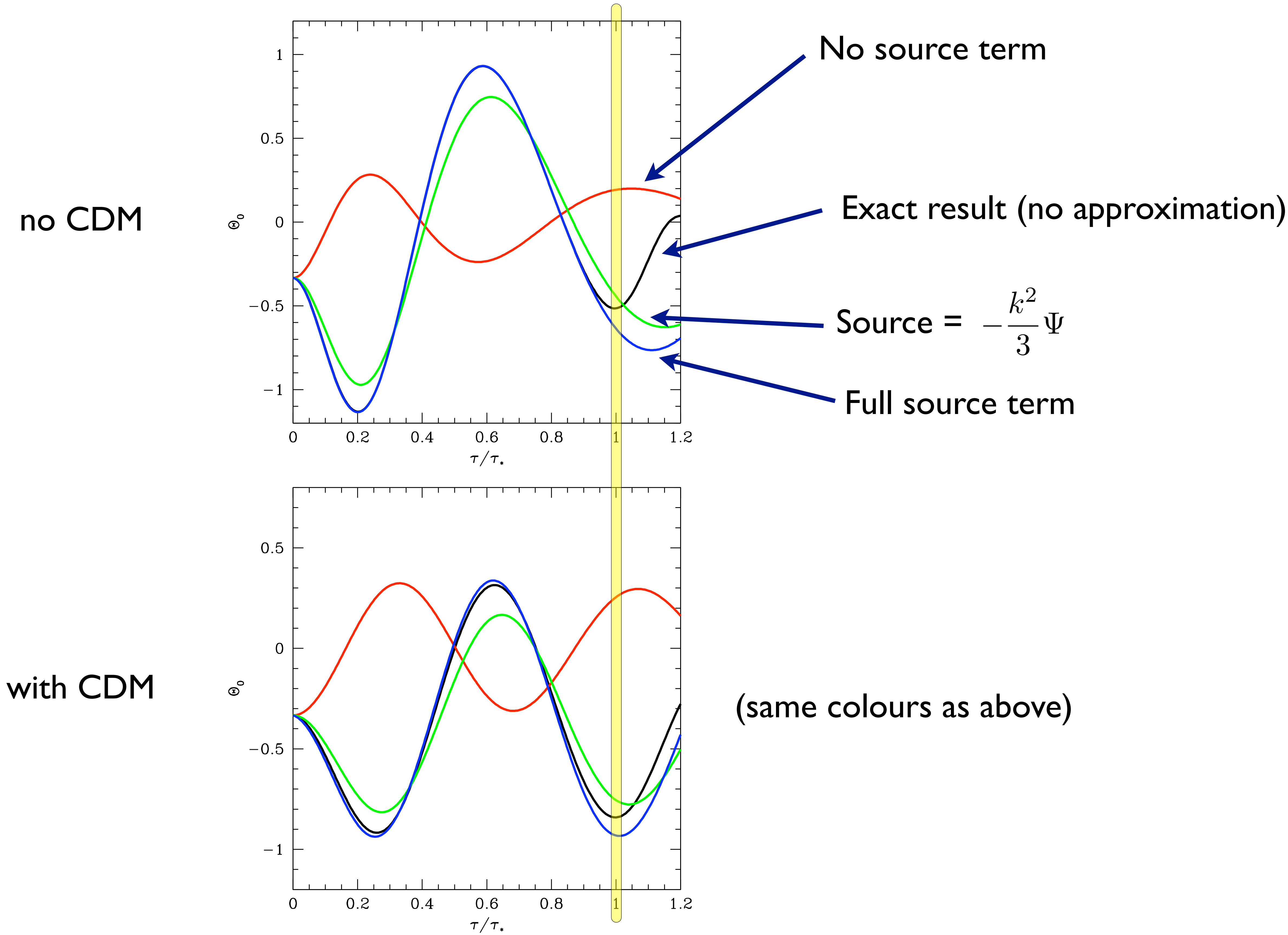
$$\Rightarrow \quad \Theta(\tau_0, k, \mu) = e^{ik\mu(\tau_* - \tau_0)}\left[\Theta_0 + \Psi - ik\mu\theta_b\right] + \int_{\tau_*}^{\tau_0} d\tau e^{ik\mu(\tau-\tau_0)}\left(\dot{\Phi} + \dot{\Psi}\right)$$

Primary anisotropies — Integrated Sachs-Wolfe effect (ISW)

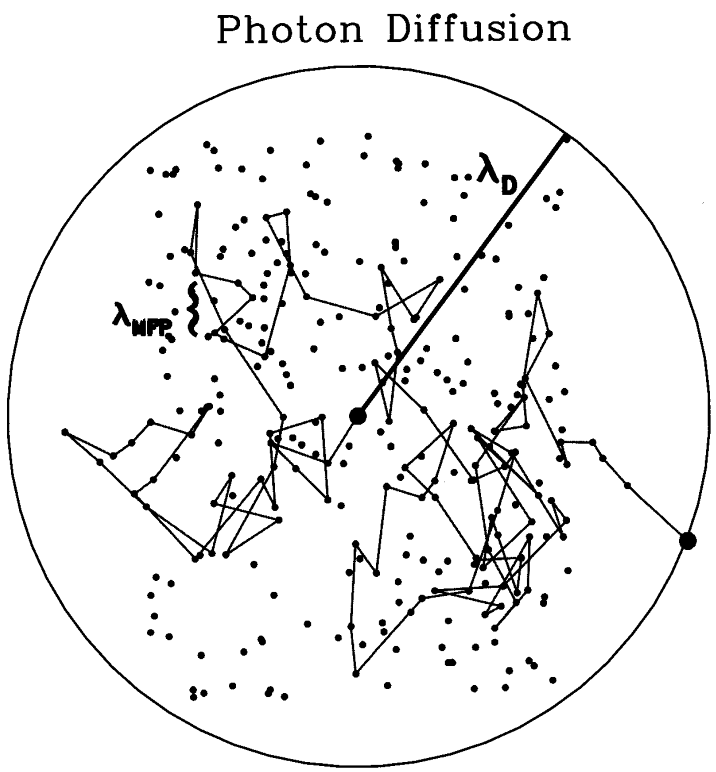
Effective temperature $\Theta_0 + \Psi$

Doppler $ik\mu\theta_b$ (ignore from now on)

Goodness of the approximation



Silk damping

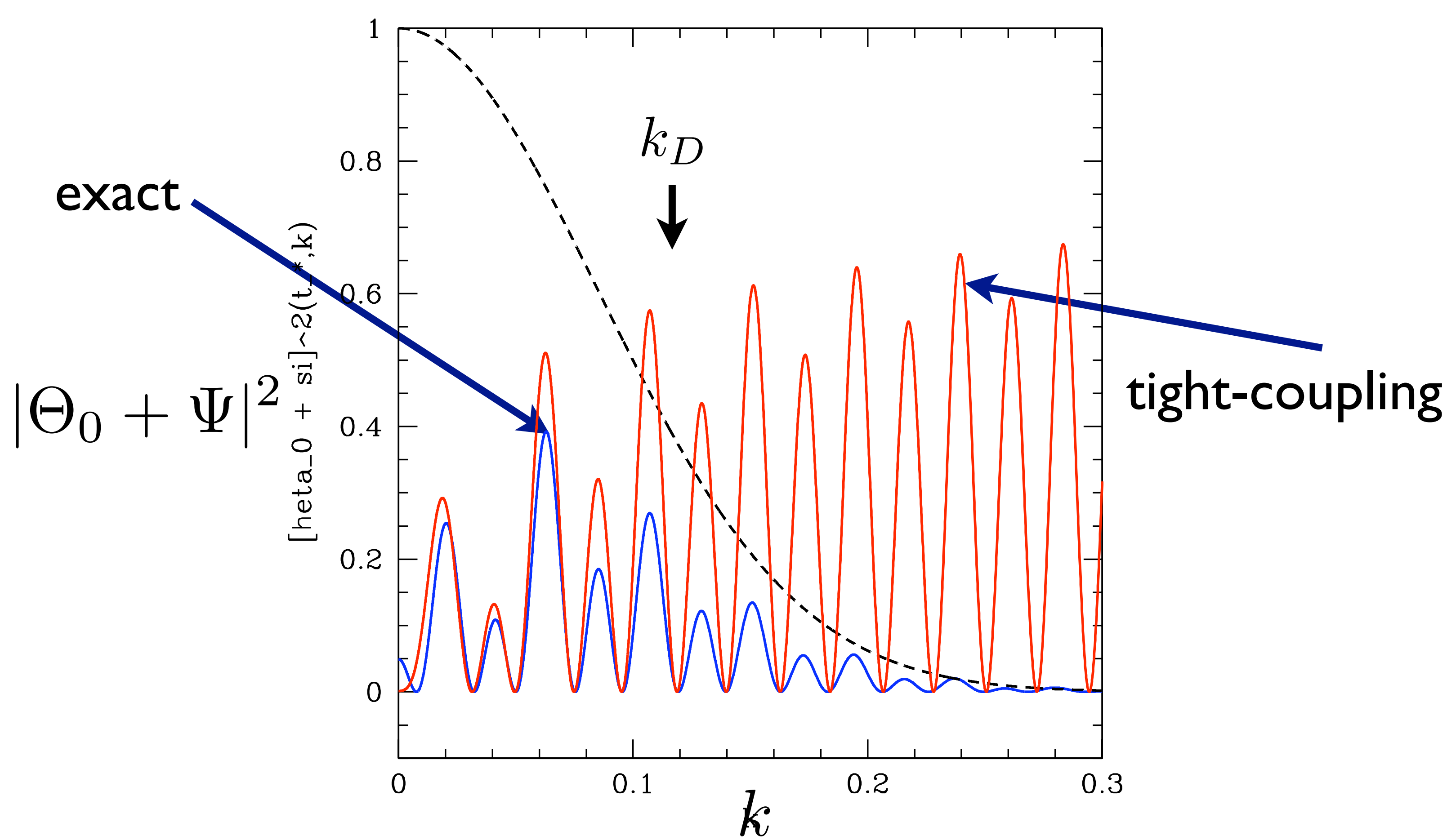


Breakdown of tight-coupling

Random walk: $\lambda_D \sim \lambda_{MFP}\sqrt{N} \sim \frac{1}{\sqrt{n_e \sigma_T H}}$

$$\Theta_0 + \Psi \rightarrow e^{-k/k_D}(\Theta_0 + \Psi)$$

Typical values: $k_D \sim 0.12 Mpc^{-1}$ for $\Omega_b h^2 = 0.02$
$k_D \sim 0.45 Mpc^{-1}$ for $\Omega_b h^2 = 0.22$



not very relevant for first 3 peaks

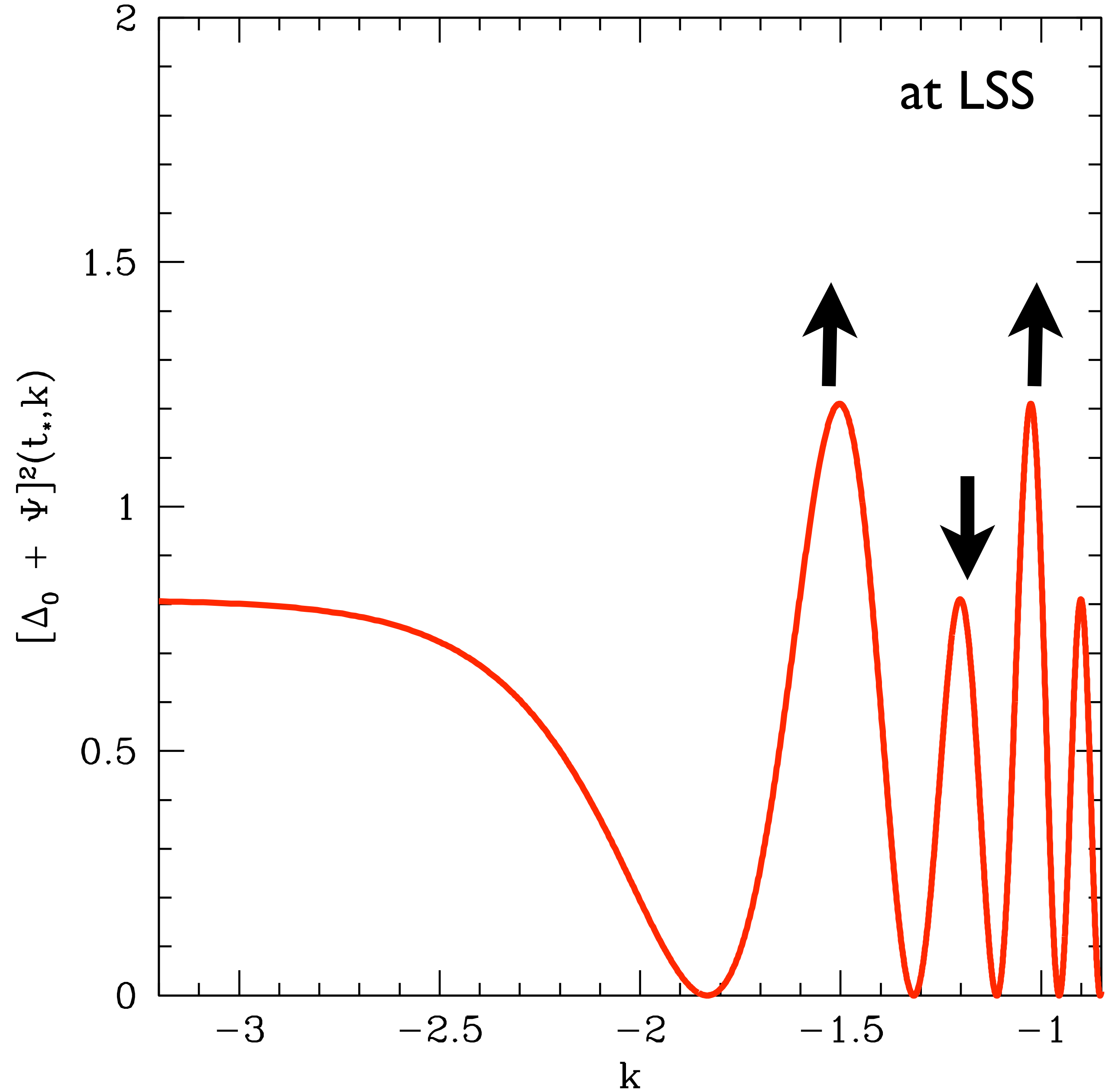
Acoustic peaks

(Hu & Sugiyama 1996)

Assume constant potentials, WKB solution:

$$\Theta_0 + \Psi = -R\Psi + A\cos(kr_s)$$

effective temperature = baryon drag + adiabatic mode oscillation



Oscillation frequency given by sound horizon

$$r_s = \int c_s d\tau$$

Oscillation zero-point displaced by gravity

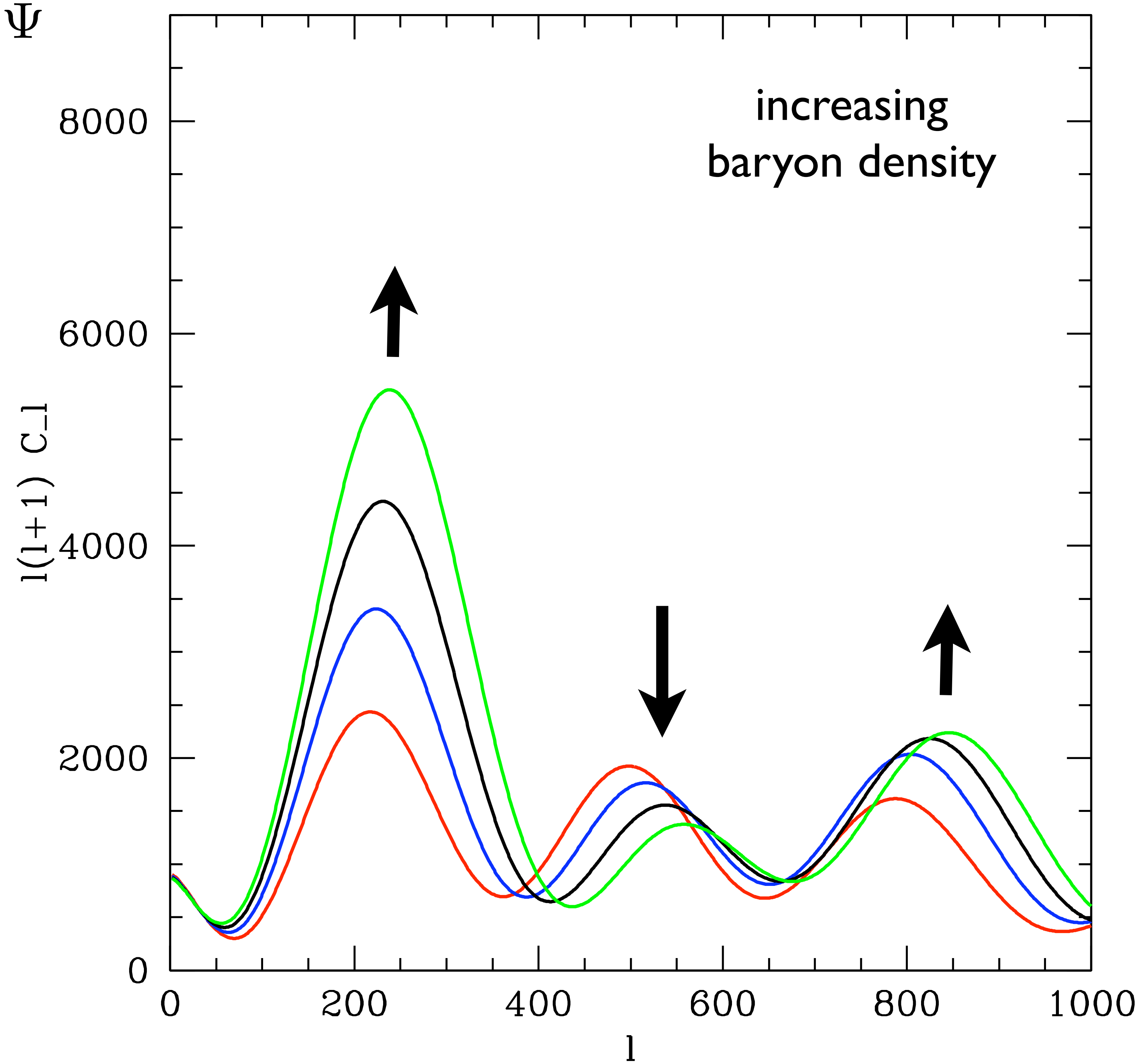
$$-R\Psi$$

The baryon drag

(Hu & Sugiyama 1996)

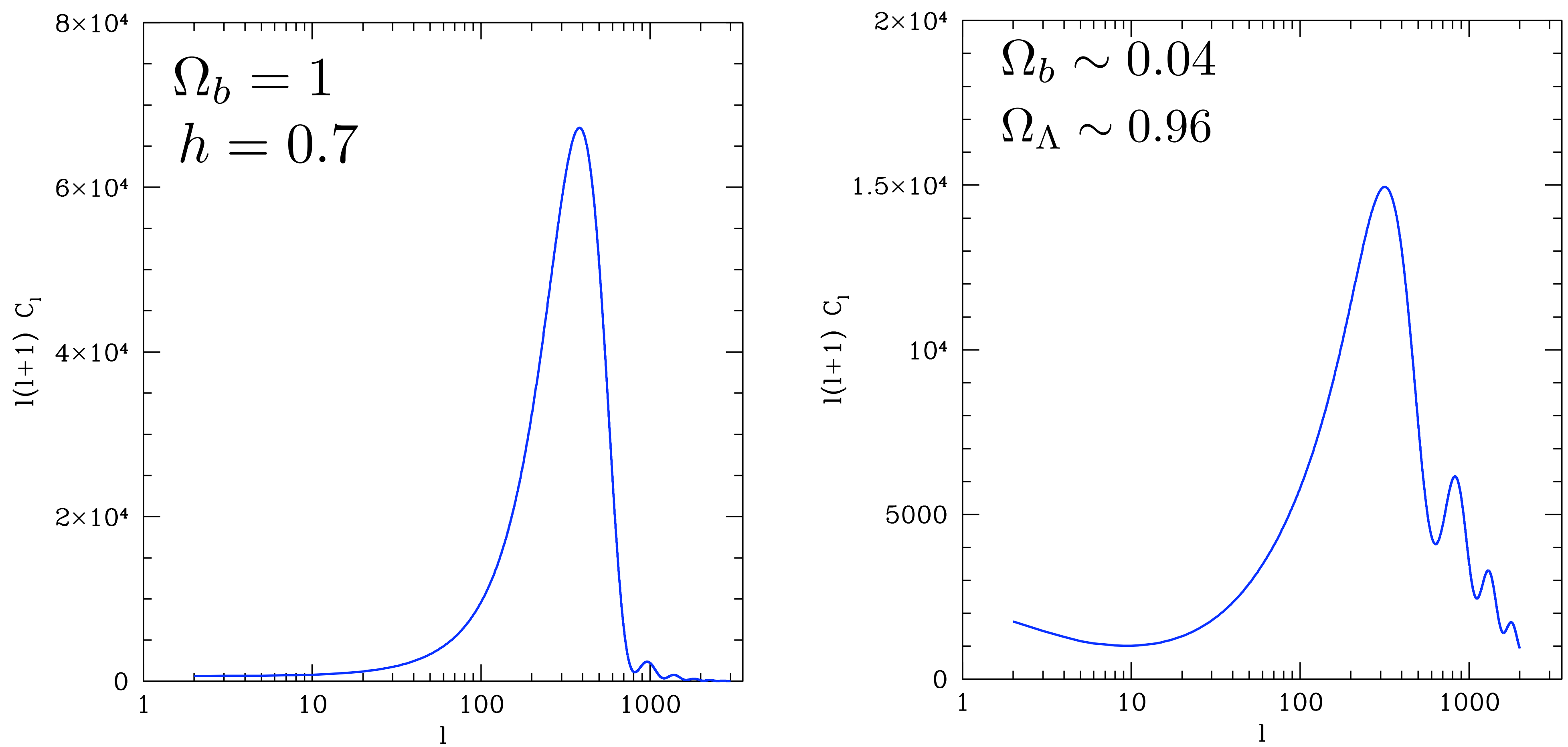
$$\Theta_0 + \Psi = \boxed{-R\Psi} + A\cos(kr_s)$$

$C_\ell^{\Theta_0+\Psi}$



$$R = \frac{3\rho_b}{4\rho_\gamma}$$

CMB for Baryon-only Universe

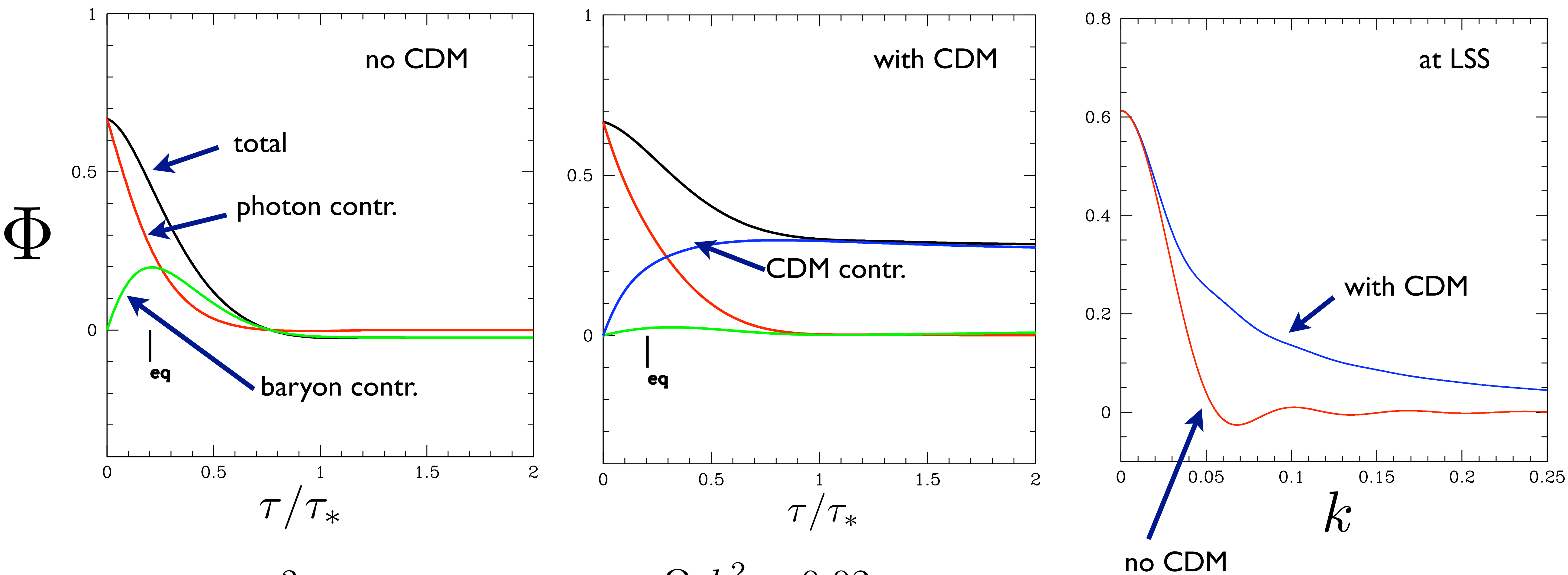


Why aren't peak heights alternating?

Potential evolution

Radiation era $\ddot{\Phi} + 4\frac{\dot{a}}{a}\dot{\Phi} + \frac{k^2}{3}\Phi = 0$ $\Rightarrow \Phi = \frac{\frac{\sin(k\tau/\sqrt{3})}{k\tau} - \cos(k\tau/\sqrt{3})}{k^2\tau^2}$ oscillatory decay

Matter era $\ddot{\Phi} + 3\frac{\dot{a}}{a}\Phi = 0$ $\Rightarrow \Phi = const$

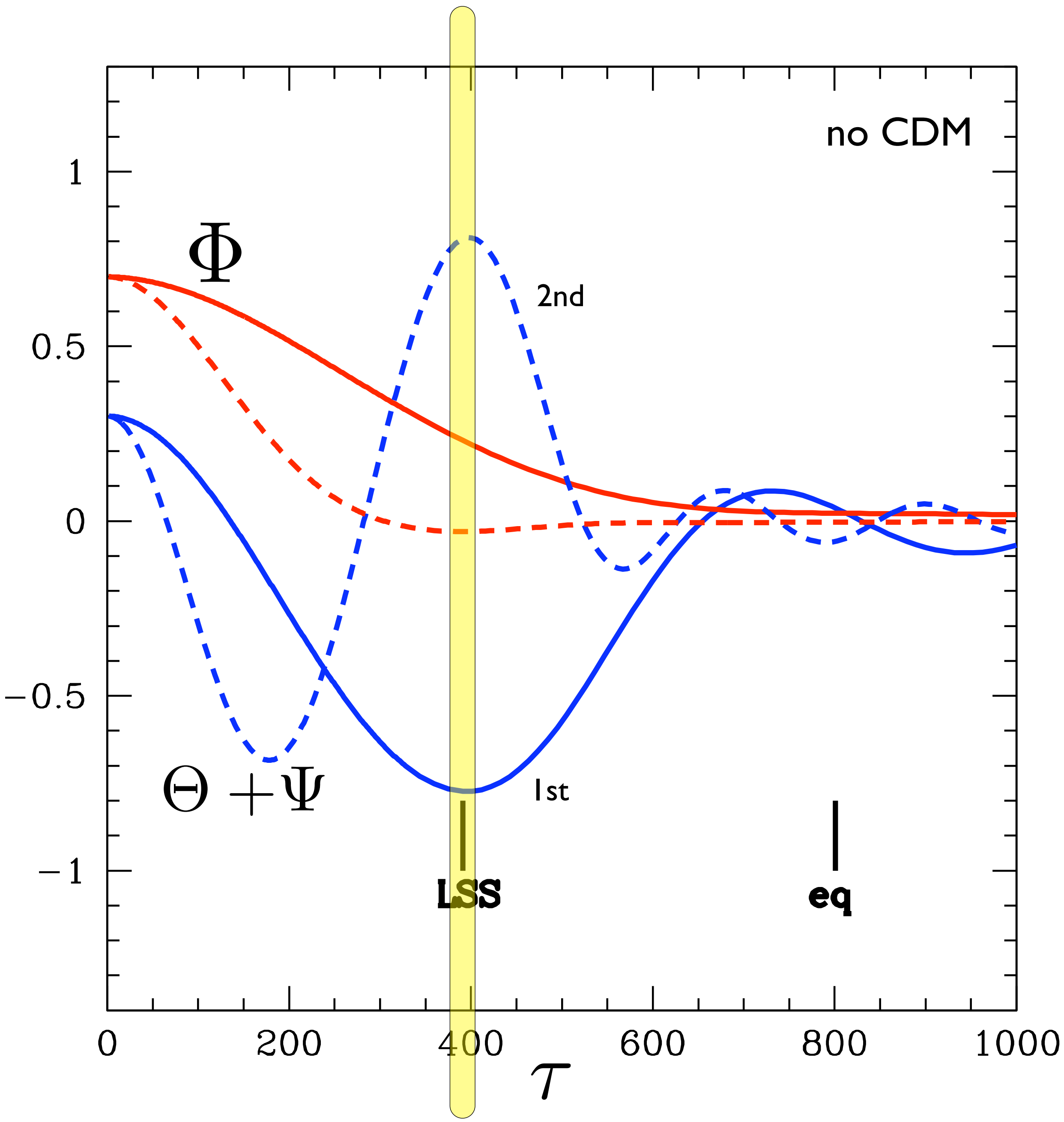


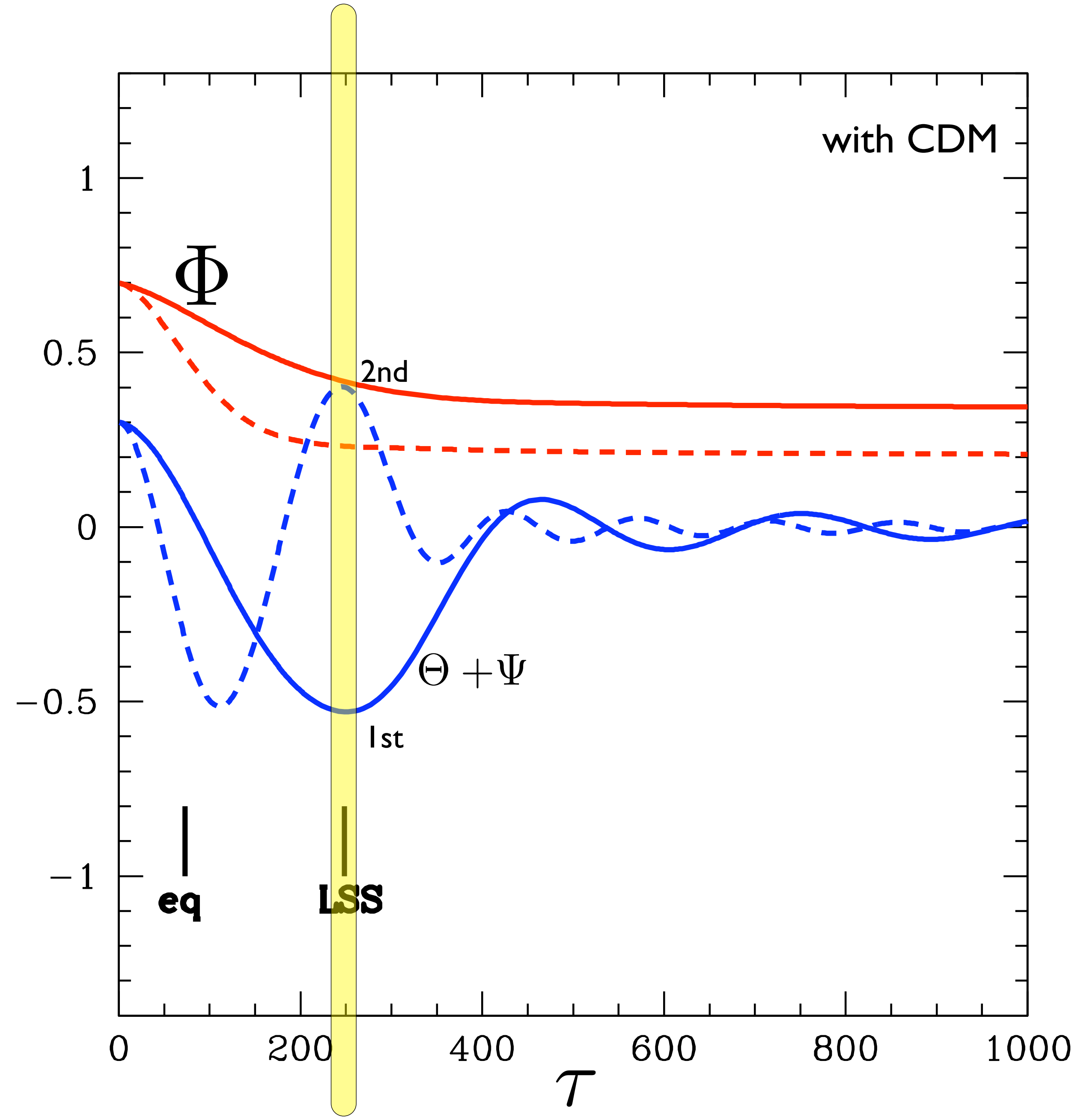
$\Omega_b h^2 = 0.22$

$\Omega_b h^2 = 0.02$

$\Omega_c h^2 = 0.2$

Potential decay





$$\Omega_b h^2 = 0.02$$
$$\Omega_c h^2 = 0$$

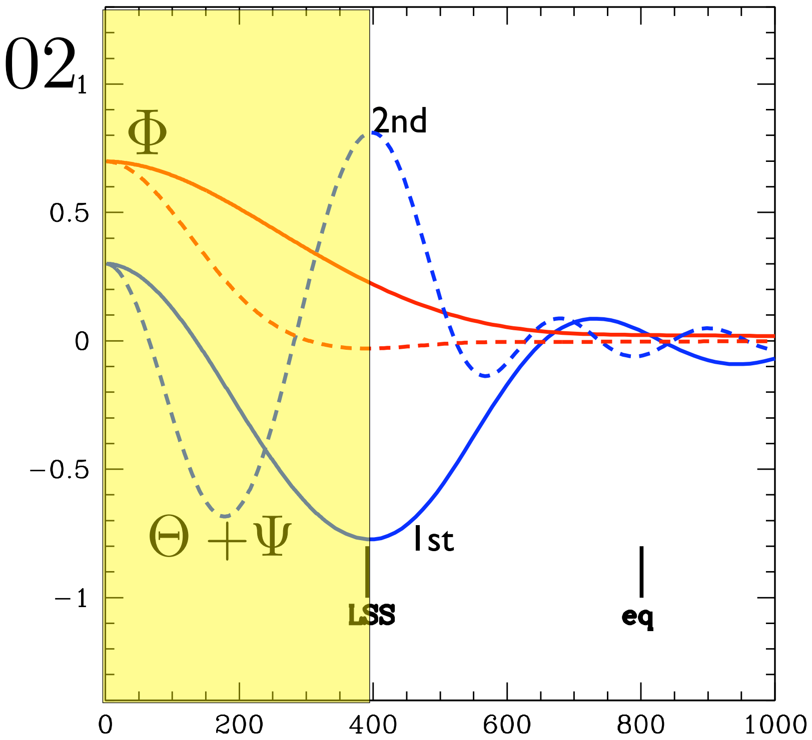
$$\Omega_b h^2 = 0.02$$
$$\Omega_c h^2 = 0.2$$

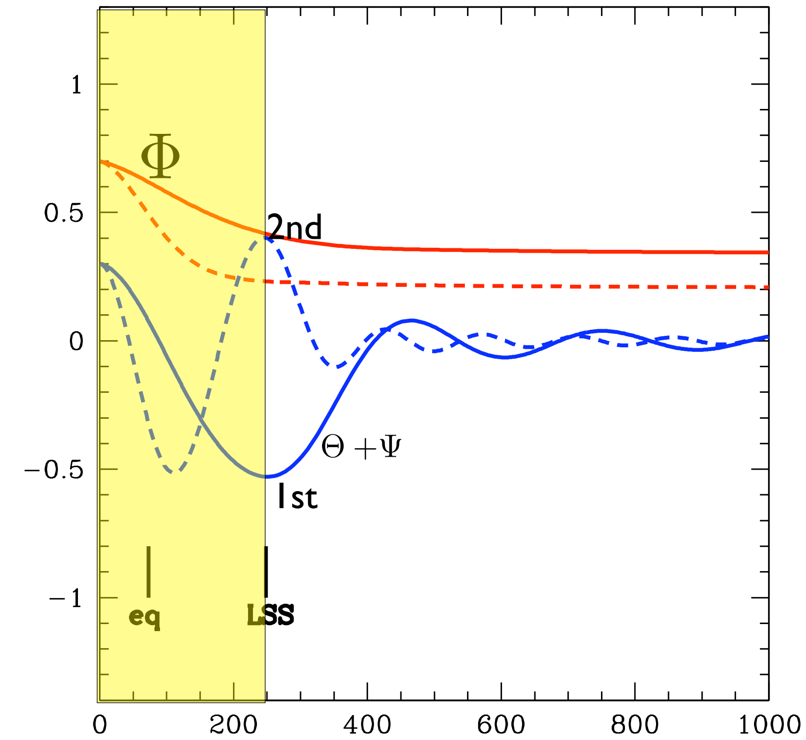
Acoustic driving

(Hu & Sugiyama 1996)

$\Omega_b h^2 = 0.02$

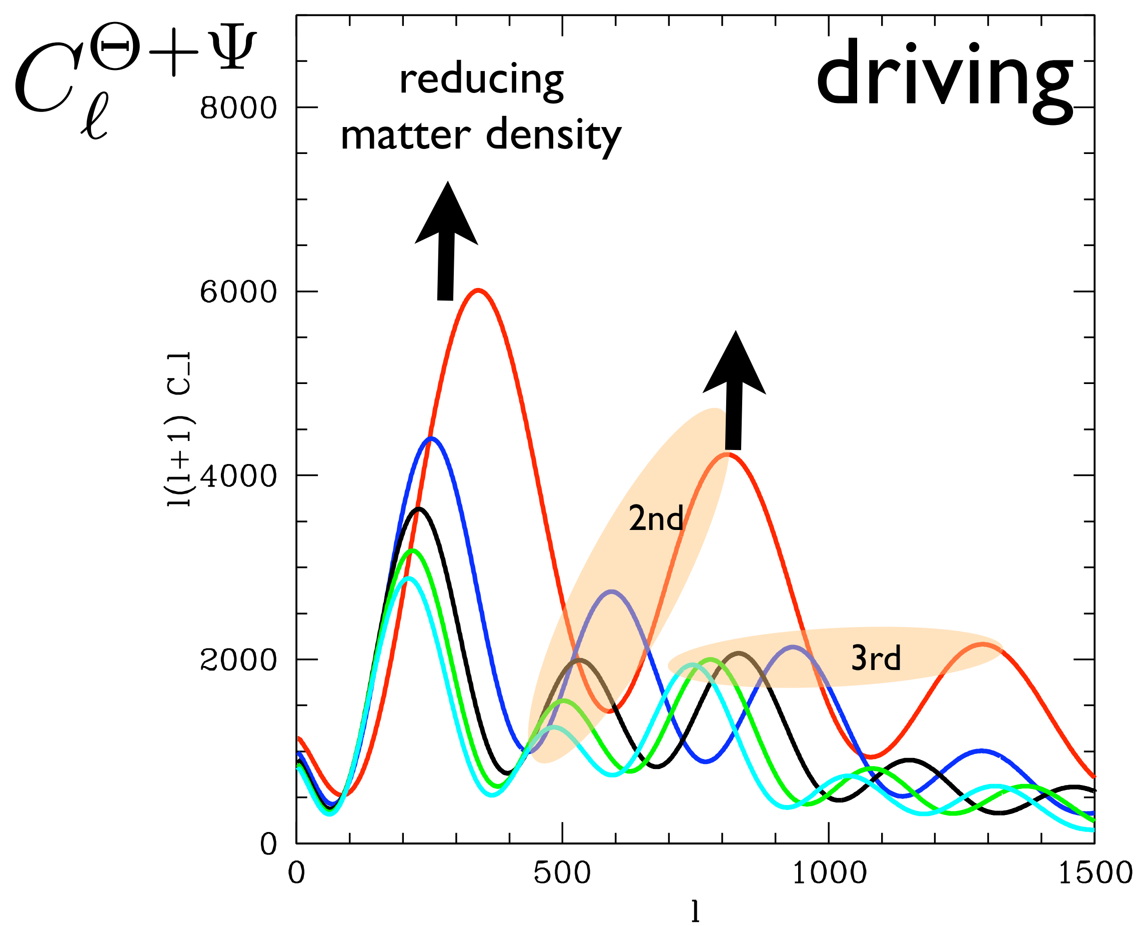
$\Omega_c h^2 = 0$





$\Omega_b h^2 = 0.02$

$\Omega_c h^2 = 0.2$

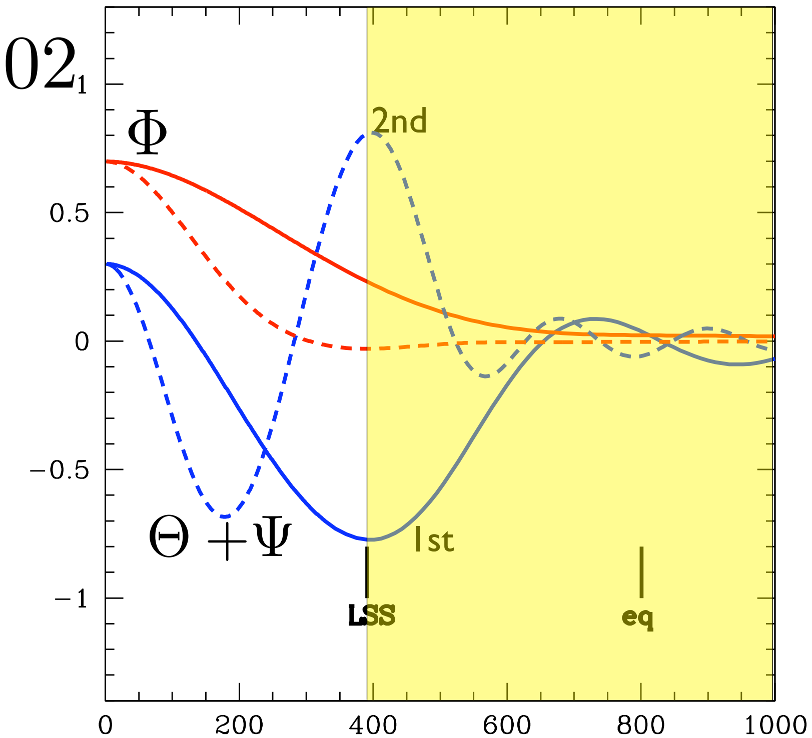


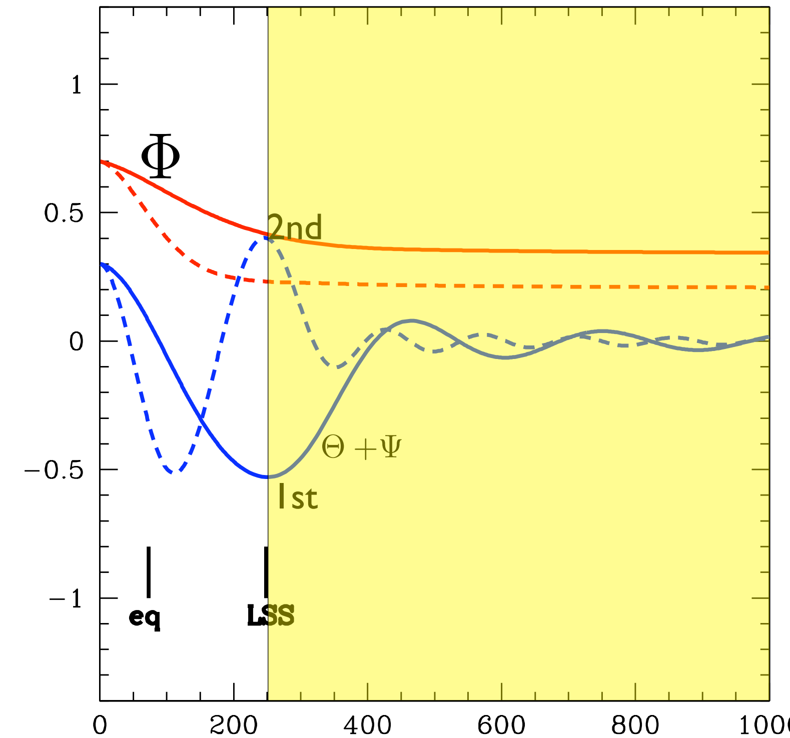
due to potential evolution during tight-coupling

$$\ddot{\Theta}_0 + \frac{R}{1+R}\frac{\dot{a}}{a}\dot{\Theta}_0 + k^2 c_s^2 \Theta_0 = -\frac{k^2}{3}\Psi + \ddot{\Phi} + \frac{R}{1+R}\frac{\dot{a}}{a}\dot{\Phi}$$

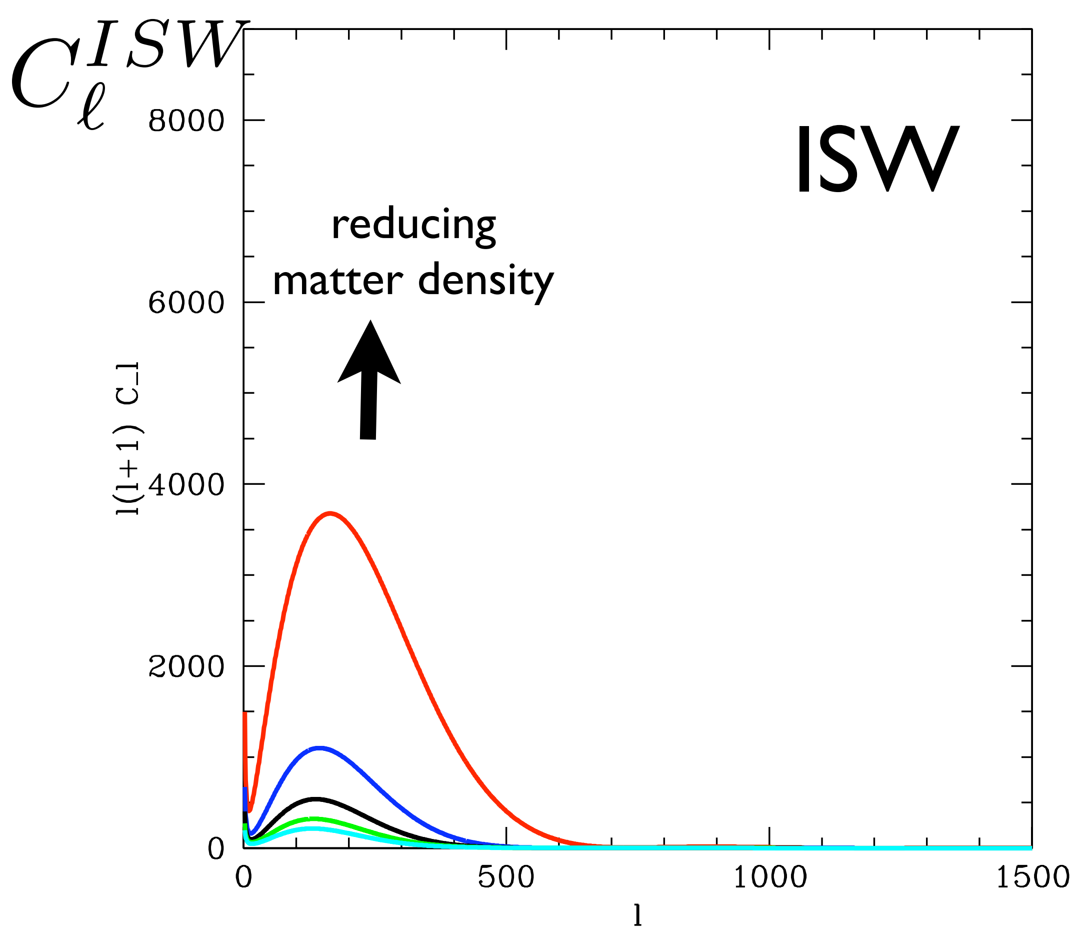
ISW effect

$$\Omega_b h^2 = 0.02$$
$$\Omega_c h^2 = 0$$





$$\Omega_b h^2 = 0.02$$
$$\Omega_c h^2 = 0.2$$



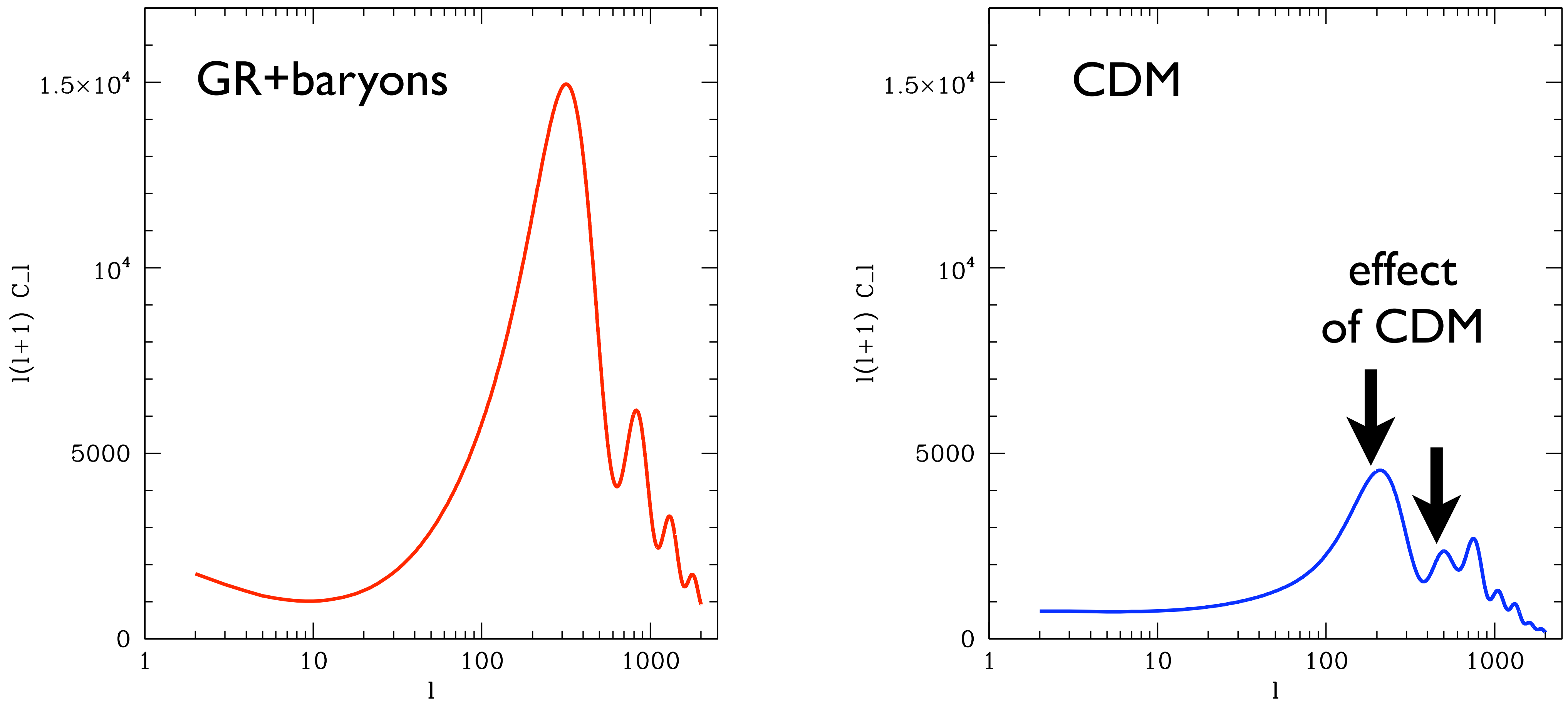
Due to potential evolution during free-streaming

$$\Theta_\ell^{ISW}(\tau_0, k) = \int_{\tau_*}^{\tau_0} d\tau j_\ell[k(\tau_0 - \tau)] \left(\dot{\Phi} + \dot{\Psi}\right)$$

- this is early ISW
- occurs just after recombination
- nothing to do with dark energy

CDM effect on CMB

- Baryons raise odd peaks relative to even peaks
- Increasing CDM density, moves equality forward in time
- Potentials decay during radiation era, constant in matter era
- Potential decay during tight-coupling (before recombination) drives the anisotropies
- Potential decay after recombination boosts anisotropies due to the Integrated Sachs-Wolfe effect



Beyond CDM

- Properties of dark matter: Generalized Dark Matter (W. Hu 1998)
- Assume CDM but test GR
- Try to replace CDM with a modification of GR

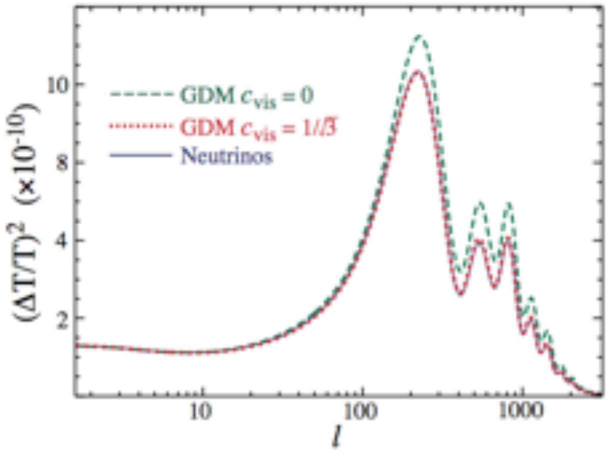
Generalized Dark Matter

(Hu 1998)

- Background equation of state $w \neq 0$, may also be time-varying
- Non-adiabatic speed of sound: $\delta P = c_s^2 \delta\rho + 3\frac{\dot{a}}{a}(c_s^2 - c_a^2)(\rho + P)\theta$

  $c_s \neq c_a$
- Shear viscosity c_{vis} obtained from effective shear σ (modelled after neutrinos)

Hu et al. 1998



Li et al. 2008:

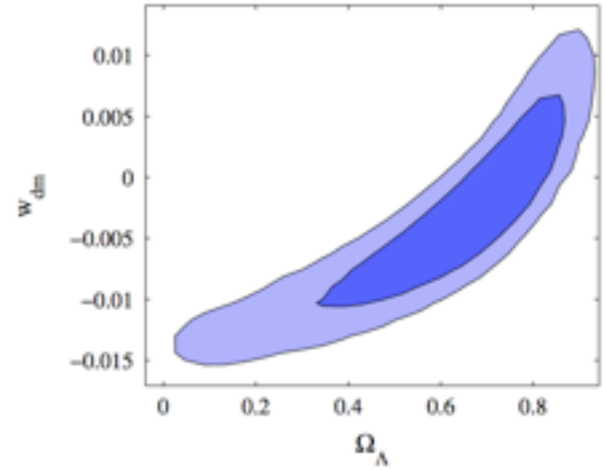
•assume background CDM, but general pressure perturbation and shear.

•parameterize growing mode of CDM

•certain engineered δP and σ after LSS give identical spectra to standard ΛCDM

Calabrese et al. 2009

Experiment	Limits on w_{dm}
WMAP	$-0.35^{+0.56+1.17}_{-0.58-0.98} \cdot 10^{-2}$
WMAP ($\Omega_\Lambda = 0$)	$-1.39^{+0.16+0.34}_{-0.54-0.95} \cdot 10^{-2}$
All CMB + SDSS + SNLS	$0.07^{+0.21+0.41}_{-0.21-0.42} \cdot 10^{-2}$



Alternative to ΛCDM?

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Abstract

In this review we present a thoroughly comprehensive survey of recent work on modified theories of gravity and their cosmological consequences. Amongst other things, we cover General Relativity, Scalar-Tensor, Einstein-Aether, and Bimetric theories, as well as TeVeS, $f(R)$, general higher-order theories, Hořava-Lifschitz gravity, Galileons, Ghost Condensates, and models of extra dimensions including Kaluza-Klein, Randall-Sundrum, DGP, and higher co-dimension braneworlds. We also review attempts to construct a Parameterised Post-Friedmannian formalism, that can be used to constrain deviations from General Relativity in cosmology, and that is suitable for comparison with data on the largest scales. These subjects have been intensively studied over the past decade, largely motivated by rapid progress in the field of observational cosmology that now allows, for the first time, precision tests of fundamental physics on the scale of the observable Universe. The purpose of this review is to provide a reference tool for researchers and students in cosmology and gravitational physics, as well as a self-contained, comprehensive and up-to-date introduction to the subject as a whole.

Keywords: General Relativity, Gravitational Physics, Cosmology, Modified Gravity

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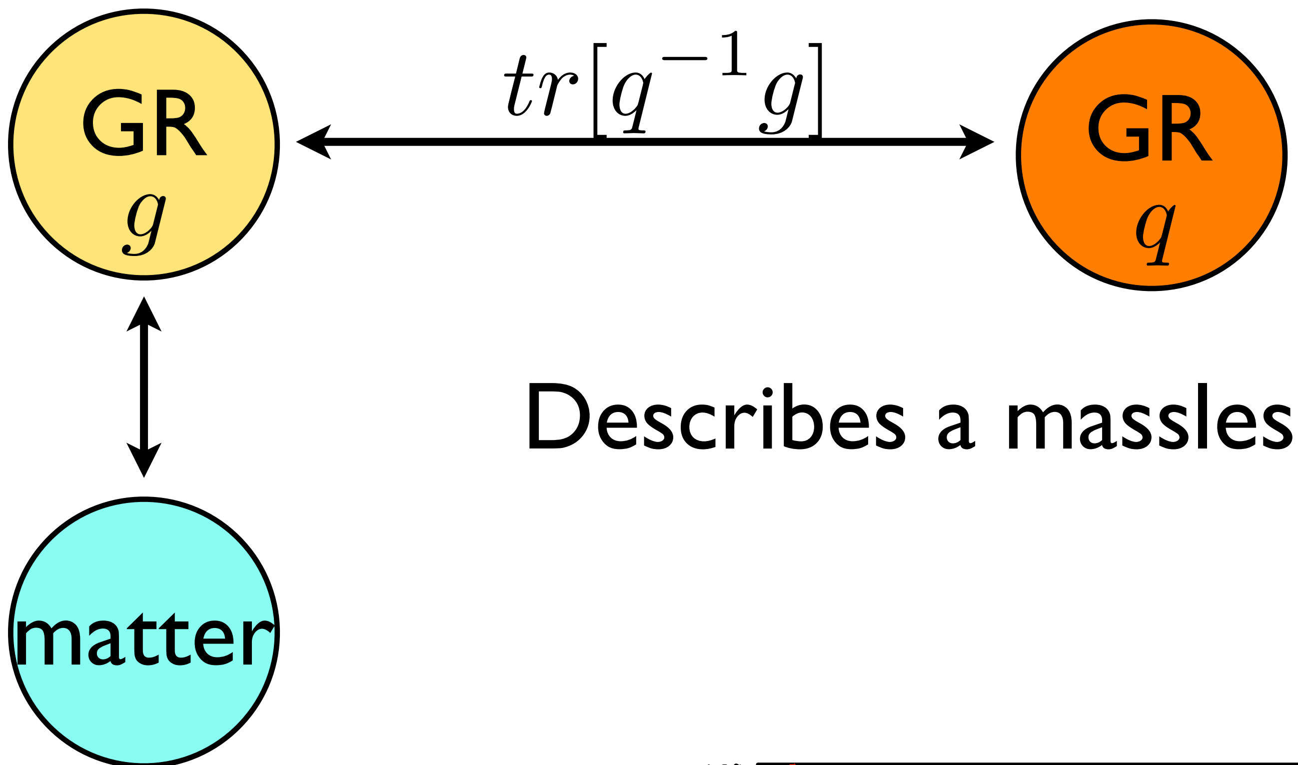
Modified gravity and the CMB

- Equality: New dof may change the background
- Potential evolution depends on Modified Einstein equations.
- Any potential decay during tight-coupling (before recombination) drives the anisotropies
- Any potential decay after recombination boosts anisotropies due to the Integrated Sachs-Wolfe effect

Eddington-Born-Infeld theory

(Banados 2009)

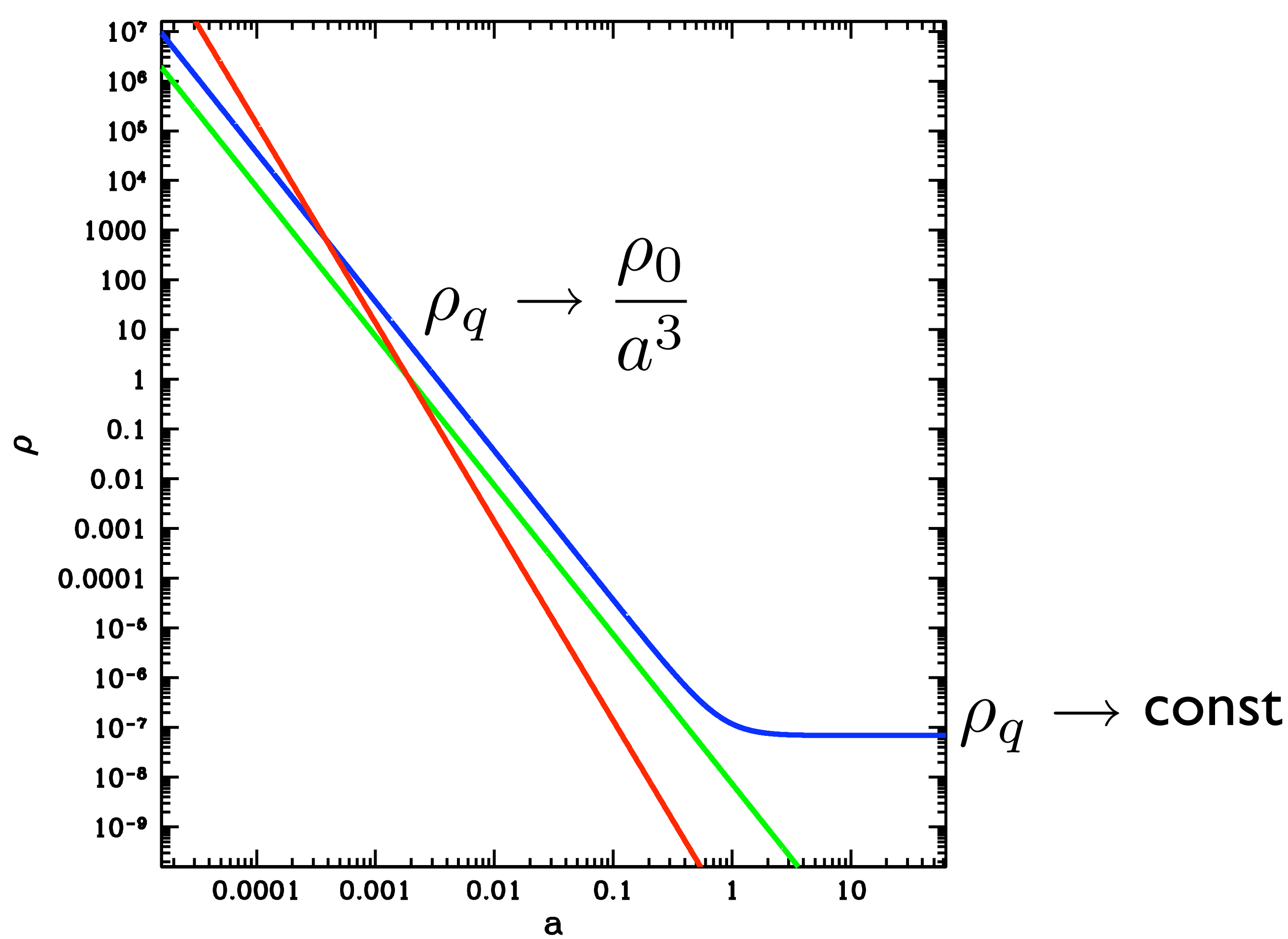


two metrics

two sectors of GR

Describes a massless and a massive graviton

$$m \sim H_0 \sim 10^{-33} eV$$

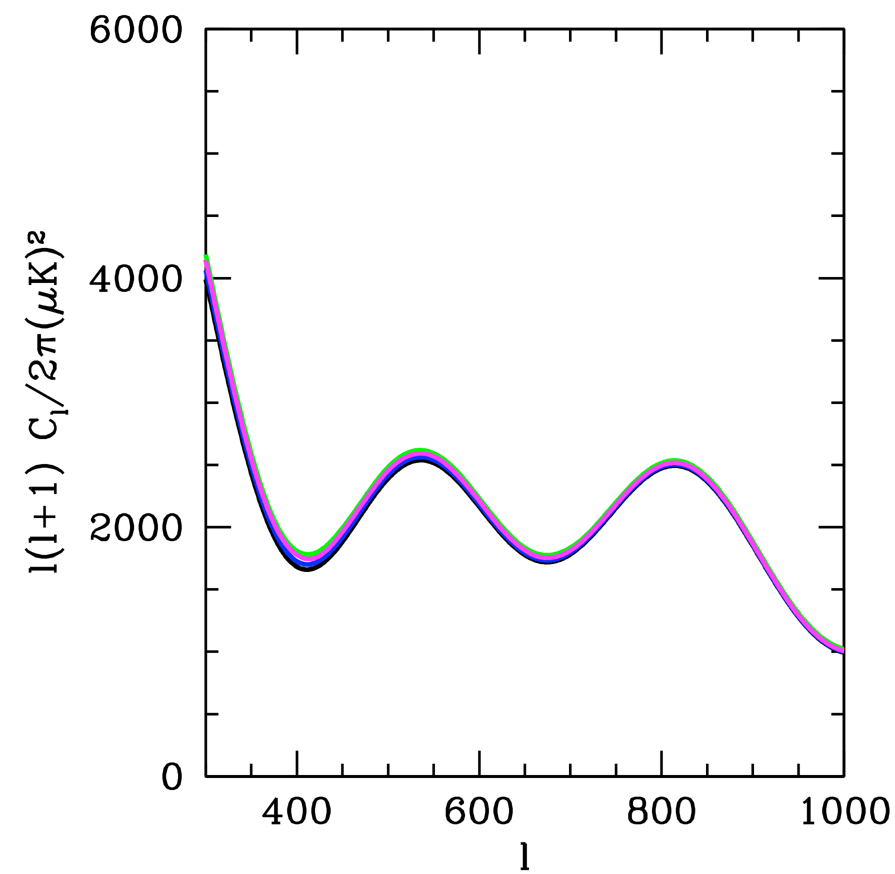


Perturbations: DM with stress

(Bañados, Ferreira, C.S. 2009)

2nd metric has 4 new dof

map them into $\delta, \theta, \delta P, \sigma$

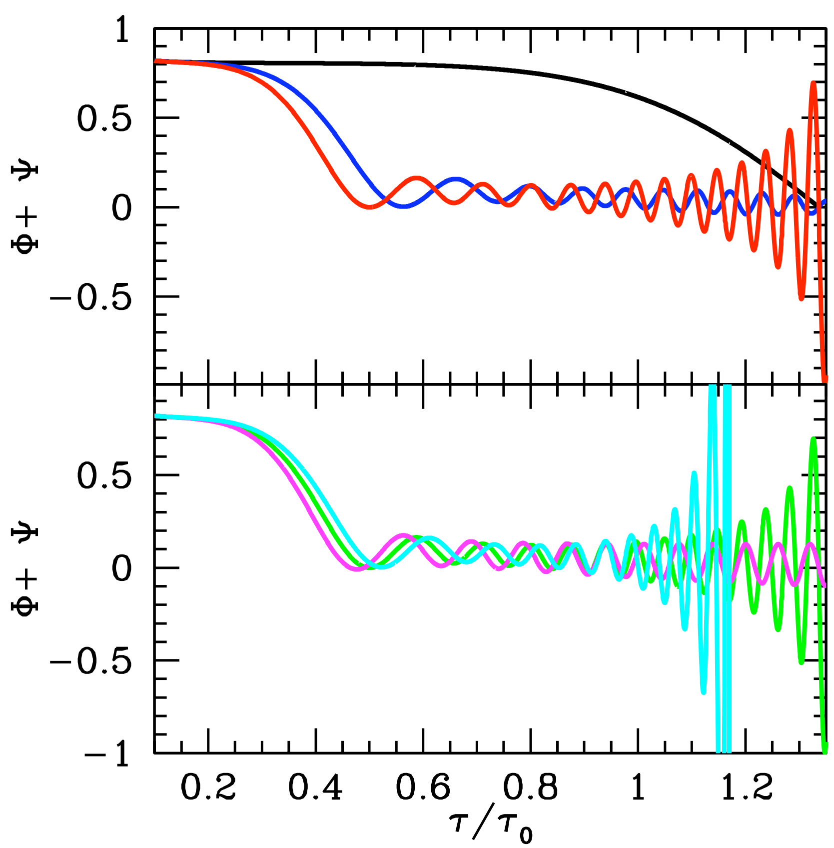


Perturbations behave like CDM on small scales

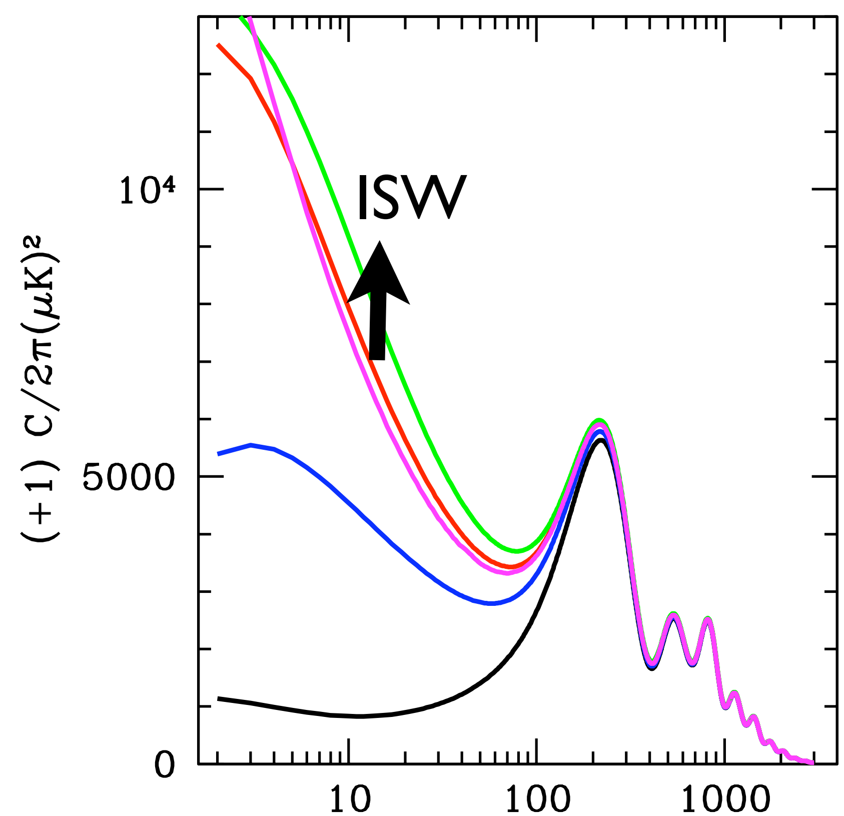
pressure perturbation $\delta P \sim 0$

shear $\sigma \sim 0$

Instability



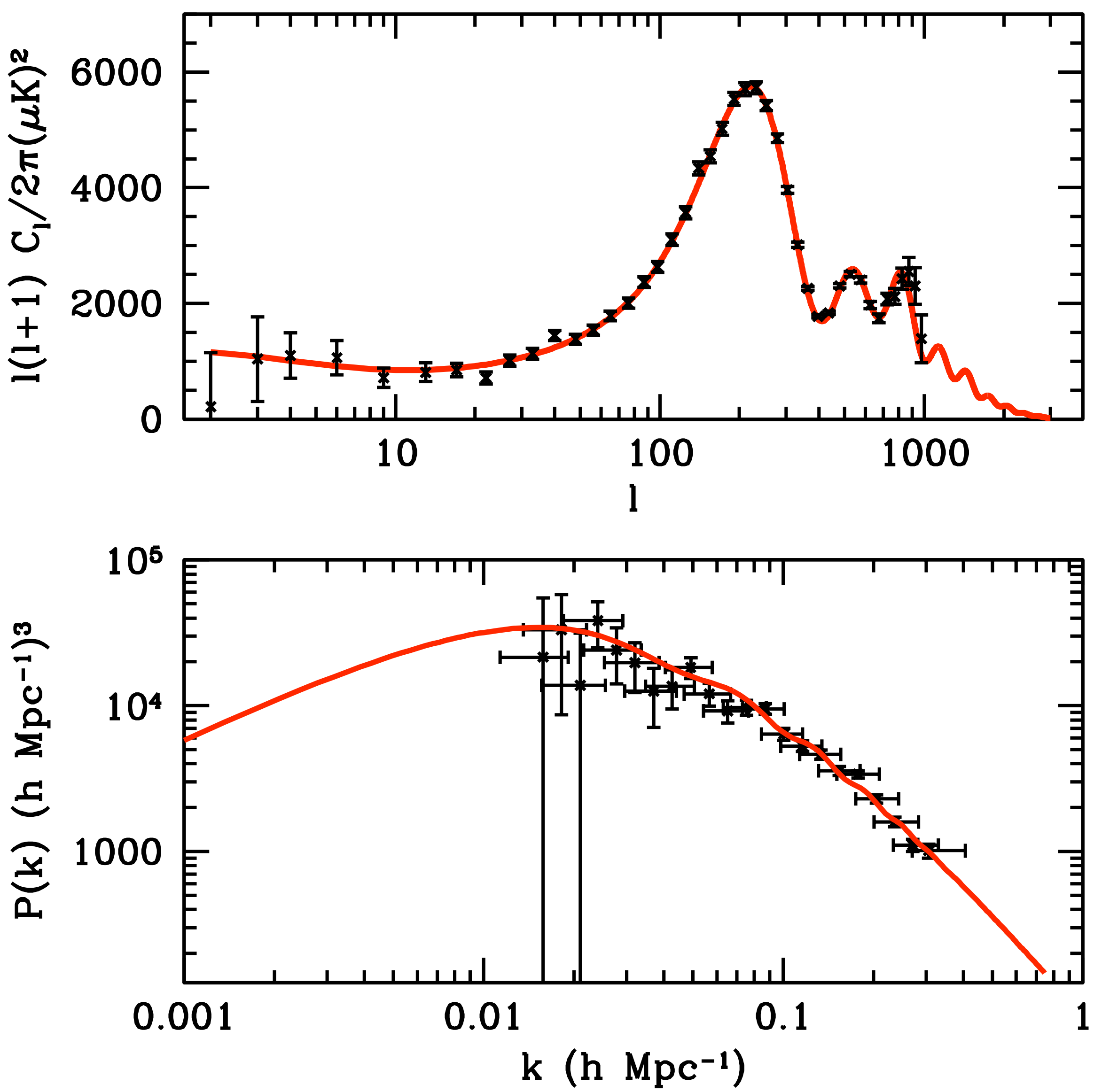
$\Rightarrow$



Posing as ΛCDM

(Bañados, Ferreira, C.S. 2009)

- Instability due to Boulware-Deser ghost
- Can be curbed by re-introducing bare Λ



but ghost remains
theory is not good

Parameterizing deviations from GR

C.S. (2009)
T. Baker, P. Ferreira, C.S. , J. Zuntz(2011)
(similar to Batteye & Pearson 2012)

- Inspired by PPN
- Add terms to Einstein equations involving new dof and potentials

we can always re-shufle any set of field equations to look like :

$$\delta G_{\mu\nu} = 8\pi G \delta T^{(known)}_{\mu\nu} + \delta U_{\mu\nu}$$

Parameterize $\delta U_{\mu\nu}$ by requiring diffeo inv. and up-to two time derivatives

Metric GI variables: $\hat{\Phi}$ and $\hat{\Gamma} = \frac{1}{k}\left(\hat{\Phi} + \frac{\dot{a}}{a}\hat{\Psi}\right)$ contain only 1 time derivative

New dof: $\hat{\chi}$ and $\dot{\hat{\chi}}$

- Bianchi identity determines field equations for new dof
- Solve to get CMB spectrum + P(k)

Parameterization

Required by diff. inv. (no freedom)

(0,0) $$U_\Delta = k^2\left[A_0\hat\Phi + F_0\hat\Gamma + \alpha_0\hat\chi + \frac{1}{k}\alpha_1\dot{\hat\chi} + kM_\Delta V\right]$$

(0,i) $$U_\Theta = k\left[B_0\hat\Phi + I_0\hat\Gamma + \beta_0\hat\chi + \frac{1}{k}\beta_1\dot{\hat\chi} + kM_\Theta V\right]$$

traced (i,i) $$U_P = k^2C_0\hat\Phi + kC_1\dot{\hat\Phi} + k^2J_0\hat\Gamma + kJ_1\dot{\hat\Gamma} + k^2\gamma_0\hat\chi + k\gamma_1\dot{\hat\chi} + \gamma_2\ddot{\hat\chi} + k^3M_PV$$

traceless (i,j) $$U_\Sigma = D_0\hat\Phi + \frac{1}{k}D_1\dot{\hat\Phi} + K_0\hat\Gamma + \frac{1}{k}K_1\dot{\hat\Gamma} + \epsilon_0\hat\chi + \frac{1}{k}\epsilon_1\dot{\hat\chi} + \frac{1}{k^2}\epsilon_2\dot{\hat\chi}$$

	Definition	GR	Horndeski	Aether	DGP
Q_1	$\frac{F_0+3\mathcal{H}_kI_0}{I_0-2}$	0	$3\mathcal{H}_k\left(1-\frac{\Theta}{\mathcal{H}\tilde{\mathcal{G}}_T}\right)$	0	0
Q_2	$1+\frac{A_0}{2}$	1	$\tilde{\mathcal{G}}_T$	1	$1-\frac{3}{2r_c^2X}$
Q_3	$-\frac{I_0}{I_0-2}$	0	$\frac{1}{\tilde{\mathcal{G}}_T}-1$	$\frac{\alpha}{2-\alpha}$	$-\frac{1}{1-\frac{2}{3}r_c^2X}$
Q_4	D_0-1	-1	$-\tilde{\mathcal{F}}_T$	-1	$-\left(1+\frac{3}{r_c^2X(1+3\omega)}\right)$
Q_5	$D_1\mathcal{H}_k-1$	-1	$-\tilde{\mathcal{G}}_T$	$-(1+c_1+c_3)$	$-\left(1+\frac{3}{r_c^2X(1+3\omega)}\right)$
Q_6	$1+Q_4-\frac{C_0}{2}$	0	0	0	$-\frac{3}{4r_c^2X}\frac{(\omega+1)(3\omega+7)}{(1+3\omega)}$

Toy case: no extra fields

Assume background unchanged

(0,0) $$U_\Delta = k^2\left[A_0\hat{\Phi} + \cancel{E_0}\hat{\Gamma} + \cancel{\alpha_0}\hat{\chi} + \frac{1}{\cancel{k}}\cancel{\alpha_1}\dot{\hat{\chi}} + kM_\Delta V\right]$$

(0,i) $$U_\Theta = k\left[B_0\hat{\Phi} + \cancel{I_0}\hat{\Gamma} + \beta_0\cancel{\hat{\chi}} + \frac{1}{k}\cancel{\beta_1}\hat{\chi} + kM_\Theta V\right]$$

traced (i,i) $$U_P = k^2C_0\hat{\Phi} + kC_1\dot{\hat{\Phi}} + k^2\cancel{J_0}\hat{\Gamma} + k\cancel{J_1}\dot{\hat{\Gamma}} + k^2\cancel{\gamma_0}\hat{\chi} + k\cancel{\gamma_1}\dot{\hat{\chi}} + \cancel{\gamma_2}\ddot{\hat{\chi}} + k^3M_PV$$

traceless (i,j) $$U_\Sigma = D_0\hat{\Phi} + \frac{1}{k}D_1\dot{\hat{\Phi}} + K_0\hat{\Gamma} + \frac{1}{k}\cancel{K_1}\dot{\hat{\Gamma}} + \cancel{\epsilon_0}\hat{\chi} + \frac{1}{k}\cancel{\epsilon_1}\hat{\chi} + \frac{1}{k^2}\cancel{\epsilon_2}\hat{\chi}$$

Further condition to avoid instability: $D_1 + K_0 = 0$

$$\begin{aligned}
A_0 &= 2\mathcal{H}_k^2P_0\\
B_0 &= -\frac{2}{3}\mathcal{H}_k(P_1+P_0)\\
C_0 &= -\frac{2\mathcal{H}_k}{k}\dot{P}_0 + \frac{2}{3}P_1 - 2\left[\frac{2}{k}\dot{\mathcal{H}}_k + \mathcal{H}_k^2 - \frac{1}{3}\right]P_0\\
C_1 &= -2\mathcal{H}_kP_0 + \frac{3a^2(X+Y)}{k\mathcal{H}} = \mathcal{H}_k\left(-2P_0 + 9\Omega_X\right)\\
D_0 &= \frac{\mathcal{H}_k}{k}\dot{P}_1 + \left(\frac{1}{k}\dot{\mathcal{H}}_k + 2\mathcal{H}_k^2 + \frac{1}{3}\right)P_1 - \left[\frac{1}{k}\dot{\mathcal{H}}_k - \mathcal{H}_k^2 - \frac{1}{3}\right]P_0\\
D_1 &= \mathcal{H}_kP_1\\
K_0 &= \frac{3a^2(X+Y)}{2k\mathcal{H}} = \frac{9}{2}\mathcal{H}_k\Omega_X
\end{aligned}$$

One function of time remains

$$P_0$$

Potential engineering

Potentials constant during matter era: $P_0 = \frac{3}{2}\frac{(1-3\Omega_X)}{1-\Omega_X}\Omega_X$

Two parameters: $P_0 = \frac{3}{2}\frac{(1-3\Omega_X)}{1-\Omega_X}\Omega_X + 3\alpha_{(i)}\Omega_{(i)} + 3\beta\Omega_r\Omega_X$

departures from constancy during matter era

radiation era modification

$$\Phi = -\frac{8\pi G a^2}{2k^2\left(1+\frac{9}{2}\mathcal{H}_k^2\Omega_X\right)}\left[\rho\delta + 3\mathcal{H}(1+w)\theta\right]$$

$$\dot{\Phi} = 4\pi G a^2\rho(1+w)\theta + \frac{k}{2}B_0\Phi - \mathcal{H}\Psi$$

$$\Psi = \frac{2k^2}{2k^2+9\mathcal{H}^2\Omega_X}\left\{-8\pi G a^2(\rho+P)\Sigma + [1-D_0]\,\Phi\right\}$$

- Initial conditions different from LCDM
- Work in progress...

Conclusion

- 3rd peak in CMB spectrum appears raised: indication for CDM
- Why?
  - Potentials decay during radiation domination
  - Potentials stay constant during matter domination
  - Potential decay enhances anisotropies through acoustic driving (tight-coupling) and ISW effect (free-streaming)
  - CDM puts CMB into matter era least potential decay
  - suppresses 1st and 2nd peak so that 3rd peak "appears" raised.
- Can be used to test non-standard properties of CDM and test GR
- Linear "Effective CDM" equations do not imply particle DM.
- Difficult to have radical departures from CDM (anything non-CDM leads to potential evolution), but need to be quantified.