Ghost-free Interacting Spin-2's

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Based on work with Rachel Rosen arXiv:1203.5783

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The cosmological constant problem

One motivation: the cosmological constant problem:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu} \qquad \frac{\Lambda}{M_P^2} \sim 10^{-122}$$

Really small

Two aspects to the problem:

- existence of the small number (naturalness)
- stability under quantum corrections (technical naturalness)

Two roads to take:

- Take GR and the CC seriously ($\rightarrow$ anthropics, landscape)
- Modify things

Conservative modification of gravity

• Lorentz-Invariance $\rightarrow$ degrees of freedom are classified by mass and spin/helicity

• Should be an infrared modification, to say something about the cosmological constant without messing up solar system tests of gravity

• GR is the unique theory of an interacting massless helicity-2 at low energies $\rightarrow$ to modify gravity is to change the degrees of freedom

First thought: make the graviton massive

IR modification scale

$$V(r) \sim \frac{M}{M_P^2} \frac{1}{r} e^{-mr}, \quad m \sim H$$

Extra DOF: 5 massive spin states as opposed to 2 helicity states

Other motivations

1) It is an interesting field theoretic question: is it possible to have a consistent theory of an interacting massive spin-2 particle (or multiple spin 2's)?

2) It shows us new mechanisms: massive gravity is a deformation of GR $\rightarrow$ pathologies should go away as mass term goes to zero $\rightarrow$ new mechanisms for curing pathologies

Massive gravity

KH Massive gravity review: arXiv:1105.3735

$$\frac{M_P^2}{2}\int d^4x\ \left[(\sqrt{-g}R)-\sqrt{-g}\frac{1}{4}m^2V(g,h)\right], \qquad g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$

$$V(g,h)=V_2(g,h)+V_3(g,h)+V_4(g,h)+V_5(g,h)+\cdots\ ,$$

Fierz-Pauli tuning, 5 linear D.O.F.

Fierz, Pauli (1939)

$$\begin{aligned}
V_2(g,h) &= \langle h^2\rangle-\langle h\rangle^2, \\
V_3(g,h) &= +c_1\langle h^3\rangle+c_2\langle h^2\rangle\langle h\rangle+c_3\langle h\rangle^3, \\
V_4(g,h) &= +d_1\langle h^4\rangle+d_2\langle h^3\rangle\langle h\rangle+d_3\langle h^2\rangle^2+d_4\langle h^2\rangle\langle h\rangle^2+d_5\langle h\rangle^4, \\
V_5(g,h) &= +f_1\langle h^5\rangle+f_2\langle h^4\rangle\langle h\rangle+f_3\langle h^3\rangle\langle h\rangle^2+f_4\langle h^3\rangle\langle h^2\rangle+f_5\langle h^2\rangle^2\langle h\rangle \\
&\quad +f_6\langle h^2\rangle\langle h\rangle^3+f_7\langle h\rangle^5, \\
&\ \vdots
\end{aligned}$$

The Boulware-Deser ghost

Boulware, Deser (1972)

ADM variables:
$$\begin{aligned} g_{00} &= -N^2 + g^{ij} N_i N_j, \\ g_{0i} &= N_i, \\ g_{ij} &= g_{ij}. \end{aligned}$$

Hamiltonian:

$$S = \frac{M_P^2}{2} \int d^4x p^{ab} \dot{g}_{ab} - N\mathcal{C} - N_i \mathcal{C}^i$$

In GR, lapse and shift are lagrange multipliers enforcing gauge constraints

In massive GR, they are auxiliary variables

Phase space DOF = 6 spatial metric + 6 canonical momentum - 0 constraints = 12 $\rightarrow$ 6 real space DOF

Extra non-linear D.O.F. is the <u>Boulware-Deser ghost</u>

Hamiltonian is unbounded.

The effective field theory: longitudinal mode

Deffayet, Rombouts (2005)
Creminelli, Nicolis, Papucci, Trincherini (2005)

After extracting longitudinal mode, $h_{\mu\nu} \to h_{\mu\nu} + 2\partial_\mu \partial_\nu \phi + \cdots$

The smallest scale is carried by a cubic scalar interaction:

$$-3(\partial\hat{\phi})^2 + \frac{2}{\Lambda_5^5}\left[(\Box\hat{\phi})^3 - (\Box\hat{\phi})(\partial_\mu\partial_\nu\hat{\phi})^2\right] + \frac{1}{M_P}\hat{\phi}T$$

$\Lambda_5 \equiv (M_P m^4)^{1/5}$ This is the (UV) strong coupling scale of the theory

Higher-derivative interaction $\to$ fourth order equations $\to$ extra ghost degree of freedom

Expand around the spherical background: $\phi = \Phi(r) + \varphi$

$$\sim -(\partial\varphi)^2 + \frac{(\partial^2\Phi)}{\Lambda_5^5}(\partial^2\varphi)^2 + \text{interactions}$$

$$m^2_{\text{ghost}}(r) \sim \frac{\Lambda_5^5}{\partial^2\Phi(r)}$$

Tuning interactions to raise the cutoff (dRGT theory)

$$\begin{aligned}
V_2(g,h) &= \langle h^2\rangle - \langle h\rangle^2, \\
V_3(g,h) &= +c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3, \\
V_4(g,h) &= +d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4, \\
V_5(g,h) &= +f_1\langle h^5\rangle + f_2\langle h^4\rangle\langle h\rangle + f_3\langle h^3\rangle\langle h\rangle^2 + f_4\langle h^3\rangle\langle h^2\rangle + f_5\langle h^2\rangle^2\langle h\rangle \\
&\quad + f_6\langle h^2\rangle\langle h\rangle^3 + f_7\langle h\rangle^5, \\
&\ \vdots
\end{aligned}$$

After extracting longitudinal mode, $h_{\mu\nu} \to h_{\mu\nu} + 2\partial_\mu\partial_\nu\phi + \cdots$

the bad terms, those with cutoffs $< \Lambda_3 \equiv (m^2 M_P)^{1/3}$ are the scalar self-interactions

$$(\partial^2\phi)^n$$

Can choose the interactions, order by order in h, so that the scalar self-interactions cancel.

Arkani-Hamed, Georgi and Schwartz (2003)

Creminelli, Nicolis, Papucci, Trincherini (2005)

Once this is done, the cutoff of the theory will be $\Lambda_3 = (m^2 M_P)^{1/3}$

Galileons

Diagonalize: $\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \hat{\phi}\hat{h}_{\mu\nu} + \frac{2(6c_3-1)}{\Lambda_3^3}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}$

de Rham, Gabadadze (2010)

$$\frac{1}{2}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta}$$

$$-3(\partial\hat{\phi})^2 + \frac{6(6c_3-1)}{\Lambda_3^3}(\partial\hat{\phi})^2\Box\hat{\phi} - 4\frac{(6c_3-1)^2-4(8d_5+c_3)}{\Lambda_3^6}(\partial\hat{\phi})^2\left([\hat{\Pi}]^2-[\hat{\Pi}^2]\right)$$

$$-\frac{40(6c_3-1)(8d_5+c_3)}{\Lambda_3^9}(\partial\hat{\phi})^2\left([\hat{\Pi}]^3-3[\hat{\Pi}^2][\hat{\Pi}]+2[\hat{\Pi}^3]\right)$$

Longitudinal mode is described by Galileon interactions:

Nicolis, Rattazzi, Trincherini (2008)

$$\mathcal{L}_2 = -\frac{1}{2}(\partial\phi)^2\ ,$$

$$\mathcal{L}_3 = -\frac{1}{2}(\partial\phi)^2[\Pi]\ ,$$

$$\mathcal{L}_4 = -\frac{1}{2}(\partial\phi)^2\left([\Pi]^2-[\Pi^2]\right)\ ,$$

$$\mathcal{L}_5 = -\frac{1}{2}(\partial\phi)^2\left([\Pi]^3-3[\Pi][\Pi^2]+2[\Pi^3]\right)$$

- Equations of motion are second order (no ghost)
- Symmetry under shifts of the field and its derivative $\phi(x) \to \phi(x) + c + c_\mu x^\mu$
- Not renormalized at any loop (no quantum corrections in the decoupling limit)

Vainshtein Mechanism in dRGT theory

$$\mathcal{L} = -3(\partial\hat{\phi})^2 - \frac{1}{\Lambda_3^3}(\partial\hat{\phi})^2\Box\hat{\phi} + \frac{1}{M_4}\hat{\phi}T$$

Studied In DGP context by: Nicolis, Rattazzi (2004)

Solution around point source of mass M:

$$\hat{\phi}(r) \sim \begin{cases} \Lambda_3^3 r_V^{(3)\,2}\left(\frac{r}{r_V^{(3)}}\right)^{1/2} & r \ll r_V^{(3)}, \\ \Lambda_3^3 r_V^{(3)\,2}\left(\frac{r_V^{(3)}}{r}\right) & r \gg r_V^{(3)}. \end{cases}$$

Vainshtein radius: $r_V^{(3)} \equiv \left(\frac{M}{M_{Pl}}\right)^{1/3}\frac{1}{\Lambda_3}$

5-th force on a test particle, relative to gravity:

$$\frac{F_\phi}{F_{\text{Newton}}} = \frac{\hat{\phi}'(r)/M_P}{M/(M_P^2 r^2)} = \begin{cases} \sim\left(\frac{r}{r_V^{(3)}}\right)^{3/2} & r \ll r_V^{(3)}, \\ \sim 1 & r \gg r_V^{(3)}. \end{cases}$$

$\hat{\phi} = \Phi + \varphi, \quad T = T_0 + \delta T$

$$-3(\partial\varphi)^2 + \underbrace{\boxed{\frac{2}{\Lambda^3}\left(\partial_\mu\partial_\nu\Phi - \eta_{\mu\nu}\Box\Phi\right)}}_{\sim\left(\frac{r_V^{(3)}}{r}\right)^{3/2}}\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{\Lambda^3}(\partial\varphi)^2\Box\varphi + \frac{1}{M_4}\varphi\delta T$$

Kinetic terms are enhanced, which means that, after canonical normalization, the coupling to δT is suppressed. The non-linear coupling scale is also raised.

This is known as a Screening mechanism

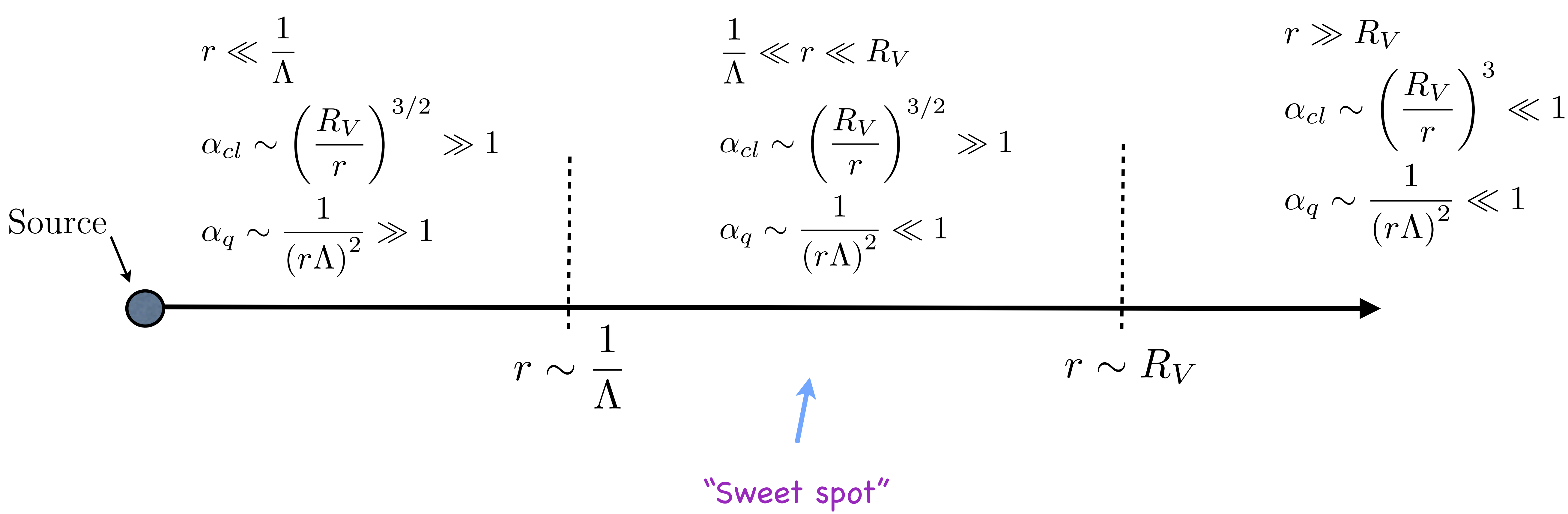
Quantum corrections and the effective field theory

Non-renormalizable effective theory with a cutoff Λ. Must include all terms compatible with galilean symmetry, suppressed by powers of the cutoff

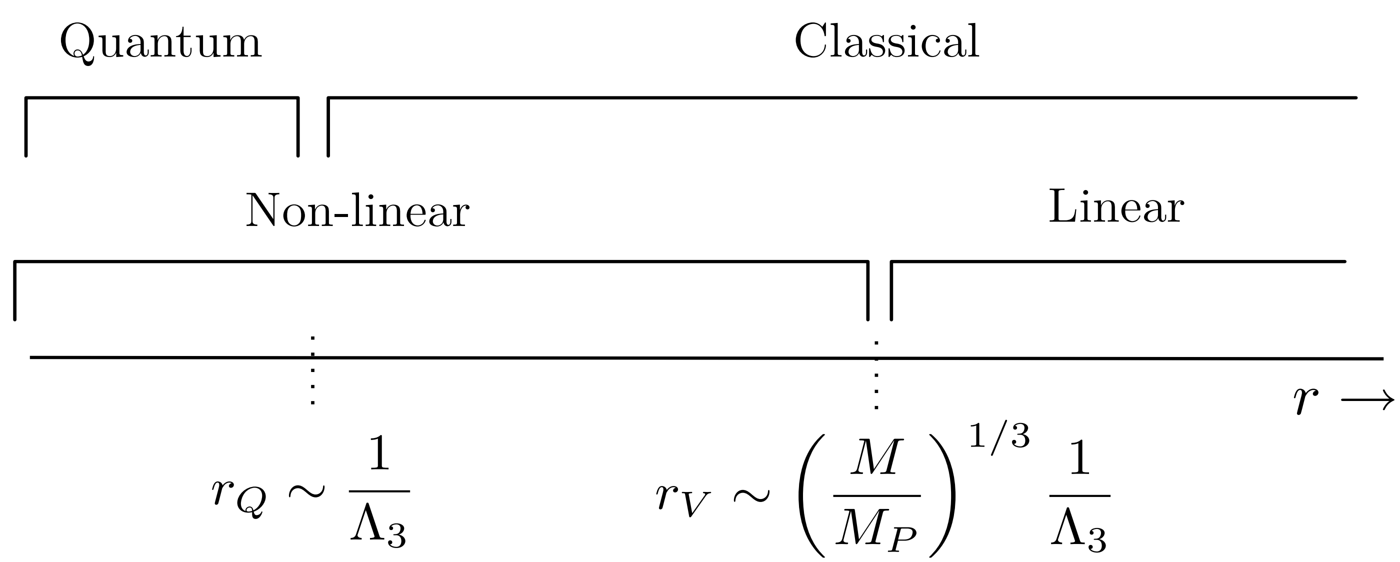
$$\mathcal{L} \sim (\partial\pi)^2 + \frac{1}{\Lambda^{3n}}(\partial\pi)^2(\partial\partial\pi)^n + \frac{1}{\Lambda^{m+3n-4}}\partial^m(\partial\partial\pi)^n$$

Galileon terms $\alpha_{cl} \equiv \dfrac{\partial\partial\pi}{\Lambda^3}$

Terms with at least two derivatives per field $\alpha_q \equiv \dfrac{\partial^2}{\Lambda^2}$



“Good” massive gravity: The Λ_3 theory (dRGT gravity)



- Higher cutoff

- Ghost free in the decoupling limit

- Possesses a screening mechanism in the non-linear regime, which is under control quantum mechanically, and restores continuity with GR as m approaches 0.

“Good” massive gravity: The Λ_3 theory (dRGT gravity)

The theory with this choice can be re-summed   de Rham, Gabadadze, Tolley (2011)

$$\frac{M_P^2}{2}\int d^4x\,\sqrt{-g}\left[R - m^2\sum_{n=0}^{4}\beta_n S_n\left(\sqrt{g^{-1}\eta}\right)\right]$$

Symmetric Polynomials

$$\begin{aligned}
S_0(M) &= 1\,, \\
S_1(M) &= [M]\,, \\
S_2(M) &= \frac{1}{2!}\left([M]^2 - [M^2]\right)\,, \\
S_3(M) &= \frac{1}{3!}\left([M]^3 - 3[M][M^2] + 2[M^3]\right)\,,
\end{aligned}$$

$$S_4(M) = Det(M)$$

- Free of the Boulware-Deser ghost, to all orders in interactions and beyond the decoupling limit

Hassan, Rosen (2011)

Vielbein formulation of ghost-free massive gravity

Or in terms of vierbeins $g_{\mu\nu} = e_\mu{}^A e_\nu{}^B \eta_{AB}$

KH, Rachel Rosen (2012)

$$\frac{M_P^{D-2}}{2} \int d^D x \; |e| R[e] - m^2 \sum_n a_n \int \epsilon_{A_1 \cdots A_D} e^{A_1} \wedge \cdots \wedge e^{A_n} \wedge 1^{A_{n+1}} \wedge \cdots \wedge 1^{A_n}$$

Ghost-free mass terms are simply all possible ways of wedging a vierbein and background vierbein:

$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge e^{A_4}$$
$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge 1^{A_4}$$
$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge 1^{A_3} \wedge 1^{A_4}$$
$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge 1^{A_2} \wedge 1^{A_3} \wedge 1^{A_4}$$
$$\epsilon_{A_1 A_2 A_3 A_4} 1^{A_1} \wedge 1^{A_2} \wedge 1^{A_3} \wedge 1^{A_4}$$

Vielbein formulation of massive gravity

KH, Rachel Rosen (2012)

Vielbein formulation makes it easy to see that the theory is ghost free:

Parametrize vierbeins as an upper triangular vierbein times a boost

$$\hat{E}_\mu{}^A = \begin{pmatrix} N & N^i e_i{}^a \\ 0 & e_i{}^a \end{pmatrix} \qquad \Lambda(p)^A{}_B = \begin{pmatrix} \gamma & p^a \\ p_b & \delta^a{}_b + \frac{1}{\gamma+1} p^a p_b \end{pmatrix}$$

$$E_\mu{}^A = \Lambda(p)^A{}_B \hat{E}_\mu{}^B = \begin{pmatrix} N\gamma + N^i e_i{}^a p_a & Np^a + N^i e_i{}^b(\delta_b{}^a + \frac{1}{\gamma+1} p_b p^a) \\ e_i{}^a p_a & e_i{}^b(\delta_b{}^a + \frac{1}{\gamma+1} p_b p^a) \end{pmatrix}$$

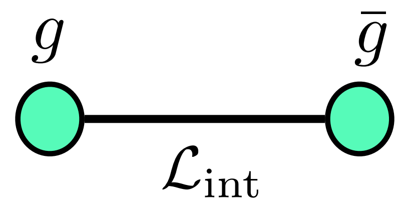
Due to structure of epsilons in the wedge product, mass terms are manifestly linear in lapse and shift:

$$N\mathcal{C}^{\mathrm{m}}(e,p) + N^i \mathcal{C}^{\mathrm{m}}_i(e,p) + \mathcal{H}(e,p)$$

Ghost free bi-gravity

Hassan, Rosen (2011)

Two-site model: bi-gravity



$$\frac{M_g^2}{2}\sqrt{-g}R[g] + \frac{M_{\bar{g}}^2}{2}\sqrt{-\bar{g}}R[\bar{g}] - \sqrt{-g}\frac{1}{4}m^2 M_{\rm eff}^2 \sum_n \mathcal{L}_n^{\rm TD}(\sqrt{g^{-1}\bar{g}})$$

$$M_{\rm eff}^2 \equiv \left(\frac{1}{M_g^2} + \frac{1}{M_{\bar{g}}^2}\right)^{-1}$$

- Linear theory: massless graviton + massive graviton of mass m ($=$ 7 DOF).
- One diff. invariance $\rightarrow$ generically 12 - 4 $=$ 8 DOF non-linearly
- Special constraint from absence of DB ghost $\rightarrow$ 7 DOF non-linearly

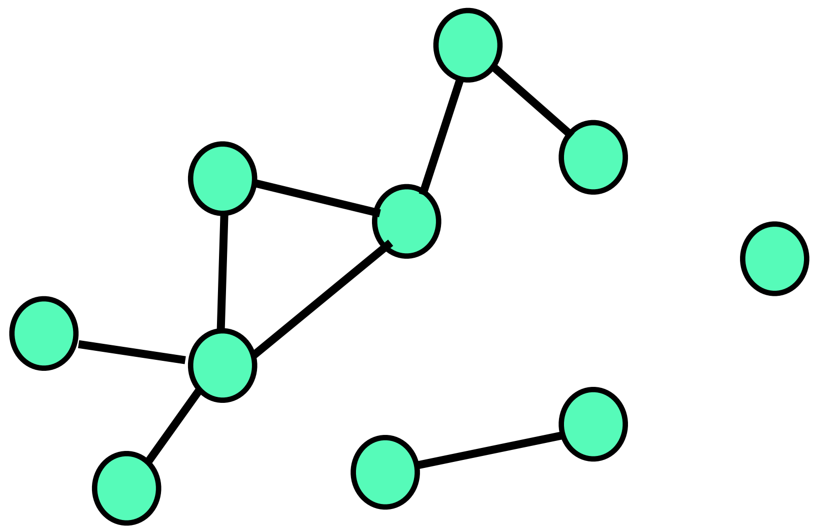
Vierbein formulation:

$$\sim \sum_n a_n \epsilon_{A_1 \cdots A_D} e_{(1)}^{A_1} \wedge \cdots \wedge e_{(1)}^{A_n} \wedge e_{(2)}^{A_{n+1}} \wedge \cdots \wedge e_{(2)}^{A_D}$$

Ghost free bi-gravity

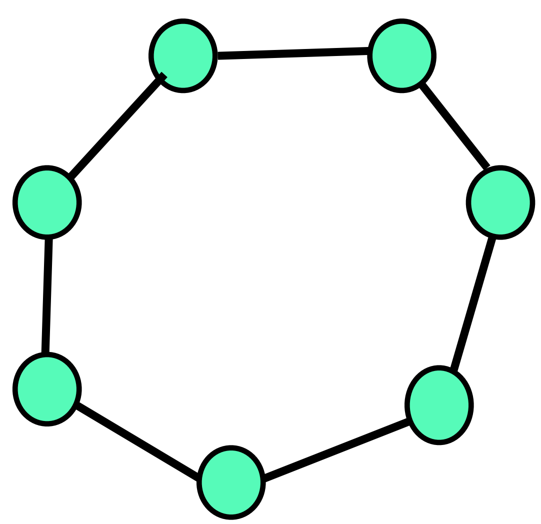
Multi-metric theory graph: one massless graviton per connected component + tower of massive gravitons

KH, Rachel Rosen (2012)



Ghost-free deconstructed gravitational dimensions

Arkani-Hamed, Georgi and Schwartz (2003)



Ghost free multi-gravity

KH, Rachel Rosen (to appear soon)

Most general ghost-free potential interaction of multiple gravitons

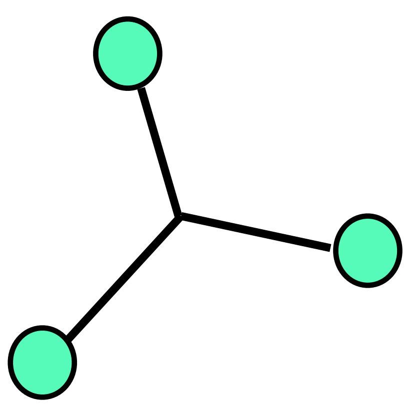
$$\sim T^{I_1 I_2 \cdots I_D} \epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(I_1)} \wedge^{A_2}_{(I_2)} \wedge \cdots \wedge e^{A_D}_{(I_D)}$$

New ghost-free multi-metric interactions in 4-dimensions:

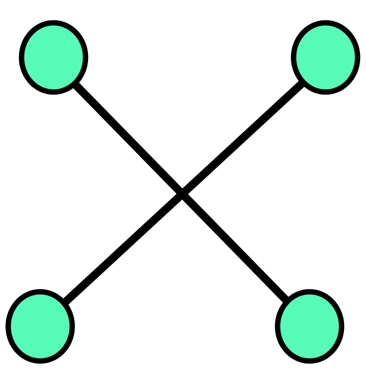
$$\epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(1)} \wedge e^{A_2}_{(1)} \wedge e^{A_3}_{(2)} \wedge e^{4}_{(3)}$$

$$\epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(1)} \wedge e^{A_2}_{(2)} \wedge e^{A_3}_{(2)} \wedge e^{4}_{(3)}$$

$$\epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(1)} \wedge e^{A_2}_{(2)} \wedge e^{A_3}_{(3)} \wedge e^{4}_{(3)}$$



$$\epsilon_{A_1 A_2 \cdots A_D} e^{A_1}_{(1)} \wedge e^{A_2}_{(2)} \wedge e^{A_3}_{(3)} \wedge e^{4}_{(4)}$$



Interaction of longitudinal modes $\rightarrow$ multi-galileon interactions

UV completion issues

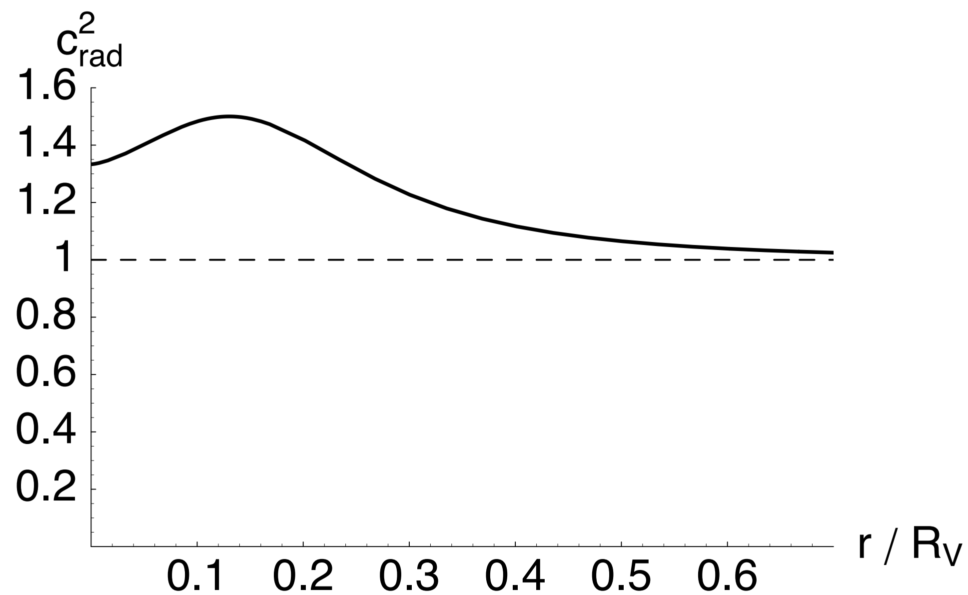
Seeming low scale cutoff: $\Lambda_3 \sim (m^2 M_P)^{1/3}$

Classical duals can be found: linear *inside* the Vainshtein radius, non-linear outside

Grregory Gabadadze, KH, David Pirtskhalava arXiv:1202.6364

Superluminality:

Cubic galileon: Speed of radial perturbations around a spherical solution $\pi = \pi_0(r) + \varphi$



Adams, Arkani-Hamed, Dubovsky, Nicolis Rattazzi (2006)

UV completion not a relativistic theory: $[\phi(x), \phi(y)] = 0, \quad (x-y)^2$ spacelike

Summary and open issues

- Λ_3 massive gravity and its extensions are the best behaved IR modification of gravity proposed so far, with potential to address the CC naturalness problem

- ~ 40 year old problem of the Boulware-Deser ghost has been solved

- Generic appearance of galileons, scalar theories with interesting and promising properties

- Multi-metric theories provide more parameter space in which to address superluminality/strong coupling. More room for model building.

- Still the issue of UV completion/duality