Recovering Cosmology in Massive Gravity

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Overview

- Why Massive Gravity?
- Problems with FRW
- FRW on de Sitter Massive gravity/bigravity
- Higuchi and Vainshtein
- Resolution - Inhomogenities?

Why Massive Gravity?

Massive Gravity Theories are a remarkably a constrained modification of general relativity at large distance scales - graviton is assumed to acquire a mass

In present talk I shall only be concerned with models where this occurs without breaking Lorentz or de Sitter symmetries

They are interesting in that as in GR, there are a finite number of consistent allowed terms in the Lagrangian that do not give rise to ghosts

By Massive Gravity we mean a nonlinear completion of Fierz-Pauli coupled to matter

Markus Fierz and Wolfgang Pauli, 1939

$$\Box h_{\mu\nu} + \cdots = m^2(h_{\mu\nu} - 1\,\eta_{\mu\nu} h)$$

$$5 = 2s + 1$$

Fierz-Pauli mass term

guarantees 5 rather than 6 propagating degrees of freedom

Massless spin-two in Minkowski makes sense!

Why Massive Gravity?

Adding a mass to gravity weakens the strength of gravity at large (cosmological) distances

$$V_{Yukawa} \sim \frac{e^{-mr}}{r}$$

But thats not all!

Screening mechanism

Self-acceleration?

Degravitation mechanism?

Why Massive Gravity?

Self-acceleration?

Gravitons can condense to form a condensate whose energy density **sources** self-acceleration

$$\rho_{\mathrm{matter}} \sim 0 \qquad\qquad H \sim m \neq 0$$

Analogous to well-known mechanism in Dvali-Gabadadze-Porrati model (DGP), however here it seems possible to remove the DGP ghost??

Deffayet 2000

Koyama 2005
Charmousis 2006

Why Massive Gravity?

Gravitons can condense to form a condensate whose energy density **compensates** the cosmological constant

Screening mechanism - The Cosmological Constant can be LARGE with the cosmic acceleration SMALL

In a Massive Theory - the c.c. is a `redundant' operator

Why Massive Gravity?

mass term

$$G_{\mu\nu} + m^2 \frac{\partial L_M}{\partial g_{\mu\nu}} = -\Lambda g_{\mu\nu}$$

Graviton condensate:

Spacetime is Minkowski in presence of an arbitrary large Λ

$$g_{\mu\nu} = \left(1 + f\left(\frac{\Lambda}{m^2}\right)\right)\eta_{\mu\nu} \qquad G_{\mu\nu} = 0 \qquad m^2 \frac{\partial L_M}{\partial g_{\mu\nu}} = -\Lambda g_{\mu\nu}$$

Equivalent Statement: The cosmological constant can be reabsorbed into a **redefinition** of the metric and coupling constants - and is hence a **redundant** operator

Why Massive Gravity?

Screening $\longrightarrow$ Degravitation

One strong motivation for considering Massive Gravity is as a toy model of higher dimensional gravity models (eg Cascading Gravity) that potentially exhibit degravitation

de Rham et al 2007

Degravitation = Dynamical Evolution to a Screened Solution from generic initial conditions

Dvali, Hofmann, Khoury 2007

so far it is safe to say that this idea has not YET been fully realized

Why Massive Gravity?

Departure from GR is governed by essentially a single parameter - Graviton Mass

Vainshtein Screening mechanism ensures recovery of GR in limit $m \to 0$

This ensures massive gravity can be easily made to be consistent with most tests of GR (effectively placing an upper bound on m) without spoiling its role as an IR modification

Why Massive Gravity?

Massive Gravity is a natural Infrared Completion of Galileon Theories

Galileon: Nicolis, Rattazzi Trincherini 2010

Decoupling limit of Massive Gravity on Minkowski is a Galileon Theory

de Rham and Gabadadze 2010

Decoupling limit of Massive Gravity on de Sitter is a Galileon Theory (with slightly different coefficients)

de Rham and Renaux-Petel 2012 - today!

The allowed Galileon Interactions are in direct correspondence with the allowed MG interactions

Why Massive Gravity?

Massive Gravity models share many nice features in common with extra dimensional models such as DGP and Cascading Gravity .....

e.g. Vainshtein mechanism, Galileon limit, self-acceleration, possible screening

.... however without the difficulty of having to solve fundamentally higher dimensional equations

Ghost-free Massive Gravity

$$\mathcal{L} = M_{\rm Pl}^2 \sqrt{-\,^{(4)}g} \left(^{(4)}R + 2m^2 \mathcal{U}(g,f) \right) + \mathcal{L}_M$$

$$\mathcal{K}^\mu_\nu(g,f) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}} \qquad \mathcal{U}(g,H) = \mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4$$

$$\begin{aligned}
\mathcal{U}_2 &= \left([\mathcal{K}]^2 - [\mathcal{K}^2]\right), \\
\mathcal{U}_3 &= \left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right), \\
\mathcal{U}_4 &= \left([\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\right)
\end{aligned}$$

de Rham, Gabadadze, Tolley, PRL, 106, 231101 (2011)

Proven fully ghost free in ADM formalism: Hassan and Rosen 2011

Result reconfirmed in Stueckelberg decomposition:
de Rham, Gabadadze, Tolley 2011
Hassan, Schmidt-May, von Strauss 2012
Kluson 2012

Result reconfirmed in helicity decomposition:
de Rham, Gabadadze, Tolley 2011

Now several other proofs: Mehrdad Mirbaryi 2011, AJT to appear

dRGT model: allowed mass terms

de Rham, Gabadadze, Tolley 2011

Build out of unique combination

$$K^\mu{}_\nu = \delta_{\mu\nu} - \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

$$U(g,f) = \sum_i \beta_i U_i(K)$$

Mass terms are *characteristic polynomials*

$$det(\delta^\mu{}_\nu + \lambda K^\mu{}_\nu) = \sum_{n=0}^{n=d} \lambda^n U_n(K)$$

Finite number of allowed interactions in any dimension

Interactions protected by a Nonrenormalization theorem

Generalized to arbitrary (dynamical - bigravity) reference metrics by Hassan, Rosen 2011

A No-Go

The simplest model (dRGT model - Massive Gravity in Minkowski) does not support spatially flat (or closed) cosmological solutions which are FRW meaning homogeneous and isotropic

Argument is simple: as in GR we have Friedman equation and Raychaudhuri equation - the 2nd follows from 1st by diff invariance

But in MG diff invariance is broken and so 2nd does not follow from 2st - consistency of two imposes condition on scale factor

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\vec{x}^2$$

$$\mathcal{L} = 3M_{\rm Pl}^2\left(-\frac{a\dot{a}^2}{N} - m^2(a^3 - a^2) + m^2N(2a^3 - 3a^2 + a)\right)$$

D'Amico et al 2011

$$m^2\partial_0(a^3 - a^2) = 0$$

A No-Go?

It **is possible** to find exact solutions in which the metric takes the form ...

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$$

in which a(t) satisfies a Friedman type equation

D'Amico et al 2011
Volkov 2011
Koyama et al 2011
Gratia et al 2012
Kobayashi et al 2012

But this is achieved by introducing *Stuckelberg* fields which carry the inhomgeneities meaning that these solutions are not truly FRW!!!

Two paths

Accept inhomogeneities:

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtskhalava, Tolley
`Massive Cosmologies' 2011

Not as bad as it sounds! Vainshtein mechanism should guarantee inhomogeneities unobservable before late times

We shall see that very probably this is the `correct' solution

Inhomogenities only appear on scale set by inverse graviton mass

Two paths

Or modify assumptions to allow FRW:

Open Universe solutions: Gumruckcuogli et al 2011

Anisotropic solutions: Gumruckcuogli et al 2012

Felice et al 2012

* Make reference metric **de Sitter** - AJT and Fasiello - tomorrow (for decoupling limit see de Rham, Renaux-Petel 2012 (today) also Berg et al 2012 (today) Alberte 2011)

* Make reference metric **dynamical** - Bigravity/Bimetric

von Strauss et al 2011

Comelli et al 2011

Crisostomi et al 2012

de Sitter MG and bigravity

Qualitatively for the present discussion there is no distinction between bigravity and de Sitter Massive gravity

This is because the second metric may not directly couple to our observable matter (absence of ghosts) other than having its own cosmological constant

AND observational consistency demands its Planck Mass is typically much higher

Thus for suitably low energies bigravity looks like MG on de Sitter (or Minkowski/AdS)

For pedagogy reasons I shall present arguments for de Sitter massive gravity

Crux of problem

Although we can obtain FRW like solutions, number of issues ...

The `mass' of a graviton gets dressed by the background
Generically the mass grows with increasing H

Thus the Vainshtein mechanism is more subtle!! We must send $m \to 0$ in away that compensates growth with H

Generalized Higuchi bound implies $m^2_{\rm dressed}(H) > 2H^2$

Successful Vainshtein mechanism (recovery of GR at large H) and Higuchi bound are incompatible for FRW solutions

Tolley and Fasiello (to appear tomorrow)

Generalized Higuchi bound

Tolley and Fasiello - tomorrow

Previous Work:

Higuchi 1987, Deser and Waldron 2001 (de Sitter) $m^2 \geq 2H^2$

Grisa, Sorbo 2009 Generalized to FRW

Berkhahn et al 2010 (Similar results to above)

Grisa and Sorbo obtain: $m^2 > 2(H^2 + \dot{H})$

seemingly no problem in deccelerating universe ?!?!

However! These authors assumed the equivalence of the *background* FRW metric and *reference* metric - this is inconsistent with known behaviour of dRGT and de Sitter/ Bigravity generalization

Necessary to use **correct** nonlinear theory to obtain result!

Sketch of argument

Starting point

$$\mathcal{L} = M_{\rm Pl}^2 \sqrt{-\,^{(4)}g}\left(\,^{(4)}R + 2m^2\mathcal{U}(g,f)\right) + \mathcal{L}_M$$

$$\mathcal{U}(g,H) = \mathcal{U}_2 + \alpha_3\mathcal{U}_3 + \alpha_4\mathcal{U}_4$$

$$\mathcal{K}^\mu_\nu(g,f) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$$

$f_{\mu\nu}$ – de Sitter spacetime metric

$$\mathcal{U}_2 = \left([\mathcal{K}]^2 - [\mathcal{K}^2]\right),$$
$$\mathcal{U}_3 = \left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),$$
$$\mathcal{U}_4 = \left([\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\right)$$

For experts U1 is removed by tadpole condition and U0 is a c.c. which can be absorbed into definition of matter

Friedman equation

$$H^2 = \frac{1}{3}\rho + m^2(\Gamma - \Gamma^2) + \frac{1}{3}m^2\alpha_3(3\Gamma^2 - \Gamma^3) + \frac{1}{3}\alpha_4 m^2 \Gamma^3$$

$$\Gamma = \frac{H}{H_0} - 1$$

$$\rho_{\text{dark energy}}$$

H_0 is Hubble parameter of reference metric

Dressed Mass and Higuchi

$$m^2_{\text{dressed}}(H) = m^2(1+\Gamma)\left(1-\Gamma(2+\alpha_3(\Gamma-2)-\alpha_4\Gamma)\right)$$

Generalized Higuchi bound is $\quad m^2_{\text{dressed}}(H) > 2H^2$

$$\Gamma = \frac{H}{H_0} - 1$$

arises from coefficient of kinetic term for helicity zero mode

$$\mathcal{L}_{\text{helicity zero}} \propto -m^2_{\text{dressed}}(m^2_{\text{dressed}} - 2H^2)(\partial\pi)^2$$

This is a similar polynomial to what arises in the Friedman equation

Higuchi versus Vainshtein

$$\Gamma = \frac{H}{H_0} - 1$$

$$m^2_{\text{dressed}}(H) = m^2(1+\Gamma)\left(1 - \Gamma(2 + \alpha_3(\Gamma - 2) - \alpha_4\Gamma)\right)$$

$$\rho_{\text{dark energy}} = 3m^2(\Gamma - \Gamma^2) + m^2\alpha_3(3\Gamma^2 - \Gamma^3) + \alpha_4 m^2\Gamma^3$$

Higuchi	Vainshtein
$m^2_{\text{dressed}}(H) > 2H^2$	$\frac{d}{dt}\rho_{\text{dark energy}} \ll \frac{d}{dt}H^2$

If $\alpha_3 - \alpha_4 \neq 0$ $\quad m^2_{\text{dressed}} \sim \rho_{\text{dark energy}} \sim \frac{m^2}{H_0^3}H^3$

$\alpha_3 - \alpha_4 = 0$ (generic) $\quad m^2_{\text{dressed}} \sim \rho_{\text{dark energy}} \sim \frac{m^2}{H_0^2}H^2$

$\alpha_3 = \alpha_4 = 1$ $\quad m^2_{\text{dressed}} \sim \rho_{\text{dark energy}} \sim \frac{m^2}{H_0}H$

Higuchi versus Vainshtein

$$m^2_{\rm dressed}(H) - 2H^2 = m^2(1+\Gamma)\left(1-\Gamma(2+\alpha_3(\Gamma-2)-\alpha_4\Gamma)\right) - 2H_0^2(1+\Gamma)^2 \geq 0$$

Remarkably $\dot{H}$ drops our of generalized bound!!!!

Likely a direct consequence of the ghost-free form (action expressible with only first derivatives - coefficient of helicity zero mode kinetic term is just a function of first derivatives of metric in Stueckelberg analysis)

so the window found by Grisa and Sorbo for deccelerating solutions $m^2 > 2(H^2 + \dot{H})$ is not present

Higuchi versus Vainshtein

$$m^2_{\text{dressed}}(H) - 2H^2 = m^2(1+\Gamma)\left(1 - \Gamma(2 + \alpha_3(\Gamma - 2) - \alpha_4\Gamma)\right) - 2H_0^2(1+\Gamma)^2 \geq 0$$

the qualitative form of these results goes through in the case of bigravity where H_0 is dynamical

similar statement in todays paper Berg et al - today - however our result more general (does not require close to de Sitter)

Their is no regime for the de Sitter MG/bigravity spatially flat cosmologies which is simultaneously obervationally acceptable and ghost-free as long as the helicity zero mode is present

Partially massless case is not included in this statement

Resolution?

The most likely resolution to realise something like out universe in Massive Gravity and Bigravity models is to return to the inhomogenous solutions

D'Amico et al 2011

Higuchi constraint is implied by representation theory of de Sitter group. Introducing inhomogenity in the metric *breaks* this relation

Known exact solutions are self-accelerating type and sit in different branches than the generic solution - as yet the general solution - the one with all 5 degrees of freedom propagating which is continuously connected with the normal Minkowski vacuum is *not known*

Reasons to be hopeful?

We can see the presence of the FRW solutions in the famous decoupling limit $M_P \to \infty \quad \Lambda_3^3 = m^2 M_P$ held fixed

de Rham et al 2010

$$\mathrm{d}s^2 = -[1 - (\dot{H} + H^2)\mathbf{x}^2]\mathrm{d}t^2 + \left[1 - \frac{1}{2}H^2\mathbf{x}^2\right]\mathrm{d}\mathbf{x}^2 = \left(\eta_{\mu\nu} + h^{\mathrm{FRW}}_{\mu\nu}\right)\mathrm{d}x^\mu \mathrm{d}x^\nu$$

The generic solution form for the helicity zero mode near x=0 which is isotropic in this limit is

$$\pi \sim A(t) + B(t)\mathbf{x}^2$$

Equations of motion fix A and B - for example for pure cc source B=constant

$$A = -Bt^2$$

Reasons to be hopeful?

de Rham, Gabadadze, Heisenberg, Pirtzkhalava 2010 - decoupling limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Pi] + \frac{1}{M_{\rm Pl}}h^{\mu\nu}T_{\mu\nu}$$

$$\begin{aligned} \pi &= \frac{1}{2}q_{\rm dS}\,\Lambda_3^3\,x^2 + \phi\,, \\ h_{\mu\nu} &= -\frac{1}{2}H^2_{\rm dS}\,x^2\,\eta_{\mu\nu} + \chi_{\mu\nu} \\ T_{\mu\nu} &= -\lambda\eta_{\mu\nu} + \tau_{\mu\nu}\,. \end{aligned}$$

$$a_1 + 2a_2 q_{\rm dS} + 3a_3 q^2_{\rm dS} = 0\,,$$

$$H^2_{\rm dS} = \frac{\lambda}{3M^2_{\rm Pl}} + \frac{2\Lambda_3^3}{M_{\rm Pl}}\left(a_1 q_{\rm dS} + a_2 q^2_{\rm dS} + a_3 q^3_{\rm dS}\right)$$

background plus perturbations split

$$\mathcal{L}^{(2)} = -\frac{1}{2}\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta} + \frac{6H^2_{\rm dS}M_{\rm Pl}}{\Lambda_3^3}(a_2 + 3a_3 q_{\rm dS})\phi\Box\phi + \frac{1}{M_{\rm Pl}}\chi^{\mu\nu}\tau_{\mu\nu}$$

coefficient of helicity zero simple function of α_3 α_4

Reasons to be hopeful?

$$\mathcal{L}^{(2)} = -\frac{1}{2}\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta} + \frac{6H^2_{\rm dS}M_{\rm Pl}}{\Lambda_3^3}(a_2 + 3a_3 q_{\rm dS})\phi\Box\phi + \frac{1}{M_{\rm Pl}}\chi^{\mu\nu}\tau_{\mu\nu}$$

Decoupling limit implies existence of inhomogenous cosmological solutions for massive gravity in Minkowski (dRGT) which for suitable range of parameters of free from Higuchi bound

Remarkable helicity zero does not couple to matter perts - no vDVZ discontinuity

Absence of Higuchi bound frees up possibility for background Vainshtein effect - consistency with known cosmology

Summary

- FRW (fully homogeneous and isotropic) solutions are a problem in Massive Gravity and Bigravity
- Even when they exist - conflict between requirements of Vainshtein effect in background and Higuchi bound
- Generalized Higuchi bound is insensitive to equation of state for matter i.e. $\dot{H}$ making it more stringent than previously expected
- Most plausible resolution is inhomogenities - even in the case of bigravity - decoupling limit analysis already suggests OK for range of parameters