Modified gravity and the cosmological constant problem

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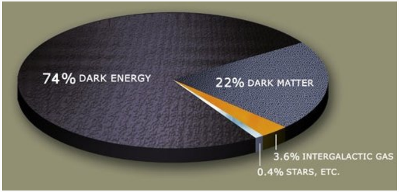
1. Introduction-motivation
   - Self-tuning
2. Horndeski's theory
3. The self-tuning filter
4. The Fab Four
5. Conclusions

Q: What is the matter content of the Universe today?

Assuming homogeinity-isotropy and GR

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

cosmological and astrophysical observations dictate the matter content of the Universe today:



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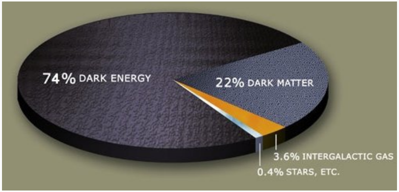

A: -Only a 4% of matter has been discovered in the laboratory. We hope to see more at LHC. But even then...

If we assume only ordinary sources of matter (DM included) there is disagreement between local, astrophysical and cosmological data.

Universe is accelerating $\longrightarrow$ Enter the cosmological constant

Easiest way out: Assume a tiny cosmological constant $\rho_\Lambda = \frac{\Lambda}{8\pi G} = (10^{-3} eV)^4$, ie modify Einstein's equation by,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Cosmological constant introduces $\sqrt{\Lambda}$ and generates a cosmological horizon
- $\sqrt{\Lambda}$ is as tiny as the inverse size of the Universe today, $r_0 = H_0^{-1}$
- Note that $\frac{\text{Solar system scales}}{\text{Cosmological Scales}} \sim \frac{10\text{ A.U.}}{H_0^{-1}} = 10^{-14}$
- But things get worse...
- Theoretically, the size of the Universe would not even include the moon!

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- Vacuum energy fluctuations are at the UV cutoff of the QFT $\Lambda_{vac}/8\pi G \sim m_{Pl}^4 \ldots$
- Vacuum potential energy from spontaneous symmetry breaking $\Lambda_{EW} \sim (200 GeV)^4$
- Bare gravitational cosmological constant Λ_{bare}

$$\Lambda_{obs} \sim \Lambda_{vac} + \Lambda_{pot} + \Lambda_{bare}$$

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Self-Tuning: general idea

Question: What if we break Poincaré invariance at the level of the scalar field?
Keep $g_{\mu\nu} = \eta_{\mu\nu}$ locally but allow for $\phi \neq constant$.
Can we have a portion of flat spacetime whatever the value of the cosmological constant and without fine-tuning any of the parameters of the theory?
Toy model theory of self-adjusting scalar field.

- Beyond leading order $O(\Lambda^4)$, radiative corrections $O(\Lambda^8/M_{pl})$ may spoil self-tuning.

We need:

1. A cosmological background
2. A sufficiently general theory to work with

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A general scalar tensor theory

- Consider ϕ and $g_{\mu\nu}$ as gravitational DoF.
- Consider $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, g_{\mu\nu,i_1}, \dots, g_{\mu\nu,i_1\dots i_p}, \phi, \phi_{,i_1}, \dots, \phi_{,i_1\dots i_q})$ with $p, q \geq 2$ but finite
- $\mathcal{L}$ has higher than second derivatives

What is the most general scalar-tensor theory giving second order field equations?

Similar to Lovelock's theorem but for the presence of higher derivatives in $\mathcal{L}$.
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\mathcal{L} &= \kappa_1(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla^\mu\nabla_i\phi R_{jk}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,\rho}(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla^\mu\nabla_i\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\
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&+F(\phi,\rho)\delta^{ij}_{\mu\nu}R_{ij}{}^{\mu\nu} - 4F(\phi,\rho)_{,\rho}\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
&-3[2F(\phi,\rho)_{,\phi} + \rho\kappa_8(\phi,\rho)]\nabla_\mu\nabla^\mu\phi + 2\kappa_8(\phi,\rho)\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
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where $\kappa_i(\phi,\rho)$, $i = 1,3,8,9$ are 4 arbitrary functions of the scalar field ϕ and its kinetic term denoted as ρ and

$$F_{,\rho} = \kappa_{1,\phi} - \kappa_3 - 2\rho\kappa_{3,\rho}$$

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\mathcal{L} &= \kappa_1(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla^\mu\nabla_i\phi R_{jk}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,\rho}(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla^\mu\nabla_i\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\
&+\kappa_3(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla_i\phi\nabla^\mu\phi R_{jk}{}^{\nu\sigma} - 4\kappa_{3,\rho}(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\
&+F(\phi,\rho)\delta^{ij}_{\mu\nu}R_{ij}{}^{\mu\nu} - 4F(\phi,\rho)_{,\rho}\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
&-3[2F(\phi,\rho)_{,\phi} + \rho\kappa_8(\phi,\rho)]\nabla_\mu\nabla^\mu\phi + 2\kappa_8(\phi,\rho)\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
&+\kappa_9(\phi,\rho), \\
\rho &= \nabla_\mu\phi\nabla^\mu\phi,
\end{aligned}$$

where $\kappa_i(\phi,\rho)$, $i = 1,3,8,9$ are 4 arbitrary functions of the scalar field ϕ and its kinetic term denoted as ρ and

$$F_{,\rho} = \kappa_{1,\phi} - \kappa_3 - 2\rho\kappa_{3,\rho}$$

$$\delta^{i_1\ldots i_h}_{j_1\ldots j_h} = h!\delta^{i_1}_{[j_1}\ldots\delta^{i_h}_{j_h]}$$

Field equations are second order in metric $g_{\mu\nu}$ and ϕ and theory is unique. Most general galileon theory

Cosmological field equations

Consider cosmological background:

1. Assume, $ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$, $\phi = \phi(t)$
2. Modified Friedmann eq (with some matter source).

$$\mathcal{H}(a, \dot{a}, \phi, \dot{\phi}) = -\rho_m$$

Third order polynomial in $H = \frac{\dot{a}}{a}$ with coeffs depending on the Horndeski functionals. Up to first derivatives present.

3. Scalar eq.

$$\mathcal{E}(a, \dot{a}, \ddot{a}, \phi, \dot{\phi}, \ddot{\phi}) = 0$$

$$\ddot{\phi} f(\phi, \dot{\phi}, a, \dot{a}) + g(\phi, \dot{\phi}, a, \dot{a}, \ddot{a}) = 0$$

Linear in $\ddot{\phi}$ and $\ddot{a}$.
Also have 2nd Friedmann equation or usual energy conservation.

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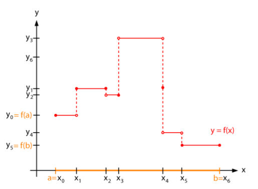
Main Assumptions

- Vacuum energy does not gravitate.
- Assume that $\rho_m = \rho_\Lambda$, a piecewise discontinuous step function of time t. Discontinuous points, $t = t_\star$, are phase transitions which are point like and arbitrary in time.
- 

  $x = time$, and $y = \rho_\Lambda$.
- Assume that spacetime is flat or a flat portion for all t
- $H^2 + \frac{\kappa}{a^2} = 0$, with $\kappa = 0$, or $\kappa = -1$ Milne spacetime $(a(t) = t)$
- ϕ not constant but in principle a function of time t!

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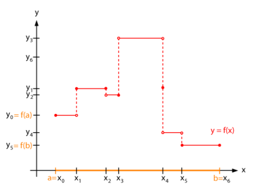
- 

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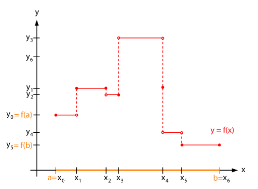

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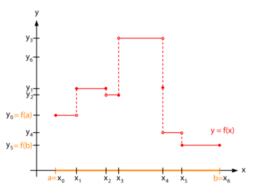

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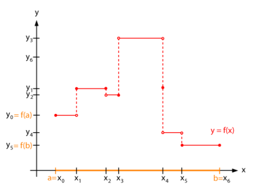

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The self tuning filter

Mathematical regularity imposed by a distributional source

1. We are going to set $H^2 + \frac{\kappa}{a^2}=0$, with $\rho(\Lambda)$ piecewise discontinuous. Then
2. $$\mathcal{H}(a, \phi, \dot{\phi}) = -\rho_\Lambda$$
   $a(t)$, $\dot{a}$ and $\phi(t)$ are continuous whereas $\dot{\phi}$ is discontinuous at $t = t_\star$. $\mathcal{H}$ has to depend on $\dot{\phi}$
3. Scalar eq. on shell is
   $$\mathcal{E}(a, \phi, \dot{\phi}, \ddot{\phi}) = \ddot{\phi} f(\phi, \dot{\phi}, a) + g(\phi, \dot{\phi}, a) = 0$$
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Applying self-tuning filter to cosmological Horndesky

- Using the form of Horndeski cosmological equations:
- We obtain

$$
\begin{aligned}
\kappa_7 &= \frac{1}{36}V_{ringo}''(\phi)\ln|\rho| + \frac{1}{12}V_{paul}'(\phi)\rho - \frac{1}{12}\beta'(\phi) + \lambda(\phi) - \frac{1}{2}V_{john}(\phi)(1-\ln|\rho|) \\
\kappa_8 &= 2p'(\phi) + V_{john}'(\phi)\ln|\rho| - \lambda(\phi) \\
\kappa_9 &= c_0 + \frac{1}{2}V_{george}''(\phi)\rho + \lambda'(\phi)\rho^2 \\
F &= -\frac{1}{12}V_{george}(\phi) - p(\phi)\rho - \frac{1}{2}V_{john}(\phi)\rho\ln|\rho|
\end{aligned}
$$

Applying self-tuning filter to cosmological Horndesky

- Using the form of Horndeski cosmological equations:
  -linearity of second order terms in a and ϕ
  -polynomial form of $\mathcal{H}$
- We obtain

$$\begin{aligned}
\kappa_1 &= \frac{1}{8}{V_{ringo}}'(\phi)\left(1+\frac{1}{2}\ln|\rho|\right)+\frac{1}{4}V_{paul}(\phi)\rho-\frac{1}{12}B(\phi)\\
\kappa_3 &= \frac{1}{16}{V_{ringo}}''(\phi)\ln|\rho|+\frac{1}{12}V'_{paul}(\phi)\rho-\frac{1}{12}B'(\phi)+p(\phi)-\frac{1}{2}V_{john}(\phi)(1-\ln|\rho|)\\
\kappa_8 &= 2p'(\phi)+V'_{john}(\phi)\ln|\rho|-\lambda(\phi)\\
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- All ρ dependance integrated out.

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- Free functions V_{fab4}, c_0 cosmological constant

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\end{aligned}
$$

Relevant and irelevant terms

- Remember the Horndeski action

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \kappa_1(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla^\mu\nabla_i\phi R_{jk}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,\rho}(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla^\mu\nabla_i\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\
& +\kappa_3(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla_i\phi\nabla^\mu\phi R_{jk}{}^{\nu\sigma} - 4\kappa_{3,\rho}(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\
& +F(\phi,\rho)\delta^{ij}_{\mu\nu}R_{ij}{}^{\mu\nu} - 4F(\phi,\rho)_{,\rho}\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
& -3[2F(\phi,\rho)_{,\phi} + \rho\kappa_8(\phi\,\rho)]\nabla_\mu\nabla^\mu\phi + 2\kappa_8\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
& +\kappa_9(\phi,\rho)
\end{aligned}
$$

- The self-tuning filter gave,

$$
\begin{aligned}
\kappa_1 &= \frac{1}{8}V'_{ringo}(\phi)\left(1+\frac{1}{2}\ln|\rho|\right) + \frac{1}{4}V_{paul}(\phi)\rho \\
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\end{aligned}
$$

Relevant and irelevant terms

- Remember the Horndeski action

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \kappa_1(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla^\mu\nabla_i\phi R_{jk}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,\rho}(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla^\mu\nabla_i\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\
& + \kappa_3(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla_i\phi\nabla^\mu\phi R_{jk}{}^{\nu\sigma} - 4\kappa_{3,\rho}(\phi,\rho)\delta^{ijk}_{\mu\nu\sigma}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\
& + F(\phi,\rho)\delta^{ij}_{\mu\nu}R_{ij}{}^{\mu\nu} - 4F(\phi,\rho)_{,\rho}\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
& - 3[2F(\phi,\rho)_{,\phi} + \rho\kappa_8(\phi\,\rho)]\nabla_\mu\nabla^\mu\phi + 2\kappa_8\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
& + \kappa_9(\phi,\rho)
\end{aligned}
$$

- The self-tuning filter gave,

$$
\begin{aligned}
\kappa_1 \quad &= \quad \frac{1}{8}V'_{ringo}(\phi)\left(1 + \frac{1}{2}\ln|\rho|\right) + \frac{1}{4}V_{paul}(\phi)\rho \\
\kappa_3 \quad &= \quad \frac{1}{16}V''_{ringo}(\phi)\ln|\rho| + \frac{1}{12}V'_{paul}(\phi)\rho - \frac{1}{2}V_{john}(\phi)(1 - \ln|\rho|) \\
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$$

- Are these terms recognisable geometric quantities?

1. Switch-on individually each term in the Langrangian then,
2. Use Langrangian and integrate by parts, use Ricci identities, or,
3. Recognise equations of motion

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George is easy

- Start with $\mathcal{L}_{George}$
- Set everybody else to zero
- $$\kappa_9 = \frac{1}{2} V''_{george} \rho, \qquad F = -\frac{1}{12} V_{george}$$
- $$\mathcal{L}_{george} = -\frac{1}{6} V_{george}(\phi) R + \frac{1}{2} \nabla_\mu \left[V'_{george} \partial^\mu \phi \right] . \cong -\frac{1}{6} V_{george}(\phi) R$$
- Einstein-Hilbert non-minimally coupled with a free scalar field

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EoM help for Ringo and John

- Switch on only V_{Ringo} in EoM. We find,
- $$K_1 = \frac{1}{16} V'_{ringo}, \qquad K_3 = \frac{1}{16} V''_{ringo}$$
- The equation of motion reads,

$$\begin{aligned}
\mathcal{E}^{ik}_{ringo} &= \sqrt{-g} K_1(\phi,\rho) \delta^{aijk}_{\lambda\mu\nu\sigma} g^{\lambda b} \nabla^{\mu}\nabla_{i}\phi R_{jk}{}^{\nu\sigma} + K_3(\phi,\rho) \delta^{aijk}_{\lambda\mu\nu\sigma} g^{\lambda b} \nabla_{i}\phi \nabla^{\mu}\phi R_{jk}{}^{\nu\sigma} \\
&= \sqrt{-g} (*R*)^{ijkl} \left(4K_1 \nabla_l \nabla_j \phi + 4K_3 \nabla_l \phi \nabla_j \phi\right)
\end{aligned}$$

- While at the same time we have,

$$\begin{aligned}
&\delta\left[\int_{\mathcal{M}} d^4x \sqrt{-g}\, V(\phi)\hat{\mathcal{G}}\right] \\
&= \int_{\mathcal{M}} d^4x \sqrt{-g}\, \delta g^{ij} \left[4(*R*)_{ikjl} \nabla^l \nabla^k V(\phi)\right] + \delta\phi[\partial_\phi V(\phi)\hat{\mathcal{G}}]
\end{aligned}$$

- Hence $\mathcal{L}_{Ringo} = V_{Ringo}(\phi)\hat{\mathcal{G}}$
- Similarly $\mathcal{L}_{John} = V_{john} G_{ij} \nabla^i\phi\nabla^j\phi$.

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&= \sqrt{-g} (*R*)^{ijkl} \left(4K_1 \nabla_l \nabla_j \phi + 4K_3 \nabla_l \phi \nabla_j \phi \right) \\
&= \frac{1}{4} \sqrt{-g} (*R*)^{ijkl} \nabla_l \nabla_j V_{ringo}(\phi)
\end{aligned}$$

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$$= \int_{\mathcal{M}} d^4x \sqrt{-g}\, \delta g^{ij} \left[2\phi H_{ij} + 4(*R*)_{ikjl} \nabla^l \nabla^k V(\phi) \right] + \delta\phi [\partial_\phi V(\phi) \hat{\mathcal{G}}]$$

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All three $\mathcal{L}_{George}, \mathcal{L}_{Ringo}, \mathcal{L}_{John}$ are KK Lovelock densities

The double dual tensor

- In 4 dimensions we can define a dual of the curvature tensor by dualising each pair of indices
- Double Dual $(*R*)$

$$(*R*)_{\mu\nu\sigma\lambda} = -\frac{1}{4}\varepsilon_{\mu\nu}{}^{ij}\, R_{ijkl}\, \varepsilon_{\sigma\lambda}{}^{kl} = \tfrac{1}{4}\delta^{ijkl}_{\mu\nu\sigma\lambda}\, R_{ijkl}$$

1. Same index properties as R-tensor
2. Divergence free:

$$\nabla_i (*R*)_{jkl}{}^{i} = 0$$

3. Simple trace is Einstein

$$(*R*)^{ik}{}_{jk} = -G^i_j,$$

The double dual tensor

- In 4 dimensions we can define a dual of the curvature tensor by dualising each pair of indices much like the Faraday tensor in EM

$$*F^{ab} = \frac{1}{2}\varepsilon^{abcd}\ F_{cd}$$

- Double Dual $(*R*)$

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As appearing in the Horndeski action

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Paul

- Last term is not recognisable. However, numerous Padilla tricks bring it to the form,
-

$$
\begin{aligned}
\mathcal{L}_{paul} &= \sqrt{-g}V_{Paul}(\phi)\left[R^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi+\right.\\
&\quad +G^{\mu\nu}(\nabla_\mu\phi\nabla_\alpha\phi-g_{\mu\alpha}(\nabla\phi)^2)\nabla^\alpha\nabla_\nu\phi\\
&\quad \left.+R^{\mu\nu}(\nabla_\mu\nabla_\alpha\phi-g_{\mu\alpha}\Box\phi)\nabla^\alpha\phi\nabla_\nu\phi\right]
\end{aligned}
$$

- Therefore

$$\mathcal{L}_{paul}=\sqrt{-g}V_{paul}(\phi)(*R*)^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi$$

- Also a higher KK Lovelock density [K V Akoleyen]

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$$\mathcal{L}_{paul} = \sqrt{-g}V_{Paul}(\phi)\left[R^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi + \right. \\ +G^{\mu\nu}(\nabla_\mu\phi\nabla_\alpha\phi - g_{\mu\alpha}(\nabla\phi)^2)\nabla^\alpha\nabla_\nu\phi \\ \left. +R^{\mu\nu}(\nabla_\mu\nabla_\alpha\phi - g_{\mu\alpha}\Box\phi)\nabla^\alpha\phi\nabla_\nu\phi\right]$$

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$$\mathcal{L}_{paul} = \sqrt{-g}V_{paul}(\phi)(*R*)^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi$$

- Also a higher KK Lovelock density [K V Akoleyen]

Paul

- Last term is not recognisable. However, numerous Padilla tricks bring it to the form,
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Fab 4

Putting it all together

from Horndeski s general action,

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& +F(\phi,\rho)\delta^{ij}_{\mu\nu}R_{ij}{}^{\mu\nu} - 4F(\phi,\rho)_{,\rho}\delta^{ij}_{\mu\nu}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi \\
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$$

Self-tuning filter

Fab 4

Putting it all together

from Horndeski s general action, Self-tuning filter

$$
\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g} V_{john}(\phi) G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \\
\mathcal{L}_{paul} &= \sqrt{-g} V_{paul}(\phi) (*R*)^{\mu\nu\alpha\beta} \nabla_\mu \phi \nabla_\alpha \phi \nabla_\nu \nabla_\beta \phi \\
\mathcal{L}_{george} &= \sqrt{-g} V_{george}(\phi) R \\
\mathcal{L}_{ringo} &= \sqrt{-g} V_{ringo}(\phi) \hat{G}
\end{aligned}
$$

- All are scalar-tensor interaction terms. No kinetic or potential scalar terms
- All related to Lovelock densities via KK reduction.
- divergence freedom keeps order of PDE s down.

Cosmology equations and self tuning

- Friedmann equation reads $\mathcal{H} = -\rho_\Lambda$
-

$$
\begin{aligned}
\mathcal{H}_{john} &= 3V_{john}(\phi)\dot{\phi}^2\left(H^2+\frac{\kappa}{a^2}\right)+6V_{john}(\phi)\dot{\phi}^2H^2 \\
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\mathcal{H}_{ringo} &= -24V'_{ringo}(\phi)\dot{\phi}H\left(H^2+\frac{\kappa}{a^2}\right)
\end{aligned}
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- First find self tuning vacuum setting $H^2+\frac{\kappa}{a^2}=0$
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Cosmology equations and self tuning

- Scalar equation, $E_\phi = E_{john} + E_{paul} + E_{george} + E_{ringo} = 0$
-
$$E_{john} = 6\frac{d}{dt}\left[a^3 V_{john}(\phi)\dot{\phi}\Delta_2\right] - 3a^3 V'_{john}(\phi)\dot{\phi}^2\Delta_2$$
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$$E_{ringo} = -24 V'_{ringo}(\phi)\frac{d}{dt}\left[a^3\left(\frac{\kappa}{a^2}\Delta_1 + \frac{1}{3}\Delta_3\right)\right]$$
- where
$$\Delta_n = H^n - \left(\frac{\sqrt{-\kappa}}{a}\right)^n$$
- which vanishes on shell as it should
- For non trivial cosmology need
$$\{V_{john}, V_{paul}, V_{george}, V_{ringo}\} \neq \{0, 0, constant, constant\}$$

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Conclusions

- Starting from a general scalar tensor theory (Horndeski)
- We have filtered out the theory with self-tuning properties
- Theory has enchanting geometrical properties which we need to understand
- Still have 4 free functions which parametrise the theory. These need to be fixed by cosmology, stability and local constraints.

Many questions unanswered:

1. What is the Fab 4 cosmology? In other words for which of the potentials do we get usual Hot Big Bang cosmology?
2. Usually to escape solar system constraints we take refuge in Veinshtein of chameleon mechanisms...
3. Maybe we can do better by redoing solar system tests from scratch for the self-tuned background in the spirit of [gr-qc/08014339]
4. Black hole solutions of such theories could really help. Also self tuning in different backgrounds.

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Sketch of proof

Consider gravity action including all contributions of cosmological constant in the scalar potential term V,

$$S[\pi, g_{\mu\nu}] = \int d^4x\sqrt{-g}R + L(\pi, g_{\mu\nu}, \partial^m, V)$$

Assume $g_{\mu\nu} = \eta_{\mu\nu}, \pi = constant$. Then

On-shell $L_0 = -V_0\sqrt{-g}$ where $L_0 = L(\eta_{\mu\nu}, constant, \Lambda)$

with EoM,

$\frac{\partial L}{\partial g_{\mu\nu}}_{|0} = \frac{\partial L}{\partial \pi}_{|0} = 0$

scalar EoM is related to the trace of gravity equation

Then Lagrangian has remnant symmetry,

$\delta g_{\mu\nu} = \epsilon g_{\mu\nu}$ and $\delta\pi = -\epsilon$

and hence

$L = \sqrt{-\hat{g}}\mathcal{L}(\hat{g}_{\mu\nu}, \partial)$ with $\hat{g}_{\mu\nu} = e^{\pi}g_{\mu\nu}$

All dependance in π has dropped out.

So,on-shell for vacuum we have

$\frac{\partial L}{\partial g_{\mu\nu}}_{|0} = \frac{1}{2}g^{\mu\nu}L_0$

Hence $V_0(\Lambda) = 0$ and thus the cosmological constant is fine tuned

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- Start with $\mathcal{L}_{George}$
- Set everybody else to zero
- $$\kappa_9 = \frac{1}{2} V''_{george}\rho, \qquad F = -\frac{1}{12} V_{george}$$
- $$\mathcal{L}_{george} \quad = \quad -\frac{1}{6} V_{george}(\phi) R + \frac{1}{2} \nabla_\mu \left[V'_{george} \partial^\mu \phi \right] . \cong -\frac{1}{6} V_{george}(\phi) R$$
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The double dual tensor and Lovelock theory

- In 4 dimensions we can define a dual of the curvature tensor by dualising each pair of indices
- Double Dual $(*R*)$

$$(*R*)_{\mu\nu\sigma\lambda} = -\frac{1}{4}\varepsilon_{\mu\nu}{}^{ij}\ R_{ijkl}\ \varepsilon_{\sigma\lambda}{}^{kl} = \tfrac{1}{4}\delta^{ijkl}_{\mu\nu\sigma\lambda}\ R_{ijkl}$$

1. Same index properties as R-tensor
2. Divergence free:

$$\nabla_i(*R*)_{jkl}{}^{i} = 0$$

3. Simple trace is Einstein

$$(*R*)^{ik}{}_{jk} = -G^i_j\,,$$

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6. Finally the 2nd order Lovelock tensor originating from variation of $\hat{\mathcal{G}}$ is:

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The double dual tensor and Lovelock theory

- In 4 dimensions we can define a dual of the curvature tensor by dualising each pair of indices much like the Faraday tensor in EM

$$*F^{ab} = \frac{1}{2}\varepsilon^{abcd}\, F_{cd}$$

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In $D = 4$ $H_{ij} = 0$ hence $(*R*)_i{}^{klm} R_{jklm} = \frac{1}{4} g_{ij}\hat{\mathcal{G}}$

With a little help from my friends

- Switch on only V_{Ringo} in EoM. We find,
- $$K_1 = \frac{1}{16} V'_{ringo}, \qquad K_3 = \frac{1}{16} V''_{ringo}$$
- The equation of motion reads,

$$\begin{aligned}
\mathcal{E}^{ik}_{ringo} &= \sqrt{-g} K_1(\phi, \rho)\delta^{aijk}_{\lambda\mu\nu\sigma} g^{\lambda b} \nabla^\mu \nabla_i \phi R_{jk}{}^{\nu\sigma} + K_3(\phi, \rho)\delta^{aijk}_{\lambda\mu\nu\sigma} g^{\lambda b} \nabla_i \phi \nabla^\mu \phi R_{jk}{}^{\nu\sigma} \\
&= \sqrt{-g}(*R*)^{ijkl} \left(4K_1 \nabla_l \nabla_j \phi + 4K_3 \nabla_l \phi \nabla_j \phi\right)
\end{aligned}$$

- While at the same time we have,

$$\begin{aligned}
&\delta \left[\int_{\mathcal{M}} d^4x \sqrt{-g}\, V(\phi)\hat{\mathcal{G}}\right] \\
&= \int_{\mathcal{M}} d^4x \sqrt{-g}\, \delta g^{ij} \left[4(*R*)_{ikjl} \nabla^l \nabla^k V(\phi)\right] + \delta\phi[\partial_\phi V(\phi)\hat{\mathcal{G}}]
\end{aligned}$$

- Hence $\mathcal{L}_{Ringo} = V_{Ringo}(\phi)\hat{\mathcal{G}}$
- Similarly $\mathcal{L}_{John} = V_{john} G_{ij} \nabla^i \phi \nabla^j \phi$.

With a little help from my friends

- Switch on only V_{Ringo} in EoM. We find,
- $$K_1 = \frac{1}{16} V'_{ringo}, \qquad K_3 = \frac{1}{16} V''_{ringo}$$

  Note the absence of $\dot{\phi}$; Ringo cannot self-tune without a little help from his friends.
- The equation of motion reads,

$$\mathcal{E}^{ik}_{ringo} = \sqrt{-g} K_1(\phi, \rho) \delta^{aijk}_{\lambda\mu\nu\sigma} g^{\lambda b} \nabla^\mu \nabla_i \phi R_{jk}{}^{\nu\sigma} + K_3(\phi, \rho) \delta^{aijk}_{\lambda\mu\nu\sigma} g^{\lambda b} \nabla_i \phi \nabla^\mu \phi R_{jk}{}^{\nu\sigma}$$
$$= \sqrt{-g} (*R*)^{ijkl} \left(4K_1 \nabla_l \nabla_j \phi + 4K_3 \nabla_l \phi \nabla_j \phi \right)$$

- While at the same time we have,

$$\delta \left[\int_{\mathcal{M}} d^4x \sqrt{-g}\, V(\phi) \hat{\mathcal{G}} \right]$$
$$= \int_{\mathcal{M}} d^4x \sqrt{-g}\, \delta g^{ij} \left[4(*R*)_{ikjl} \nabla^l \nabla^k V(\phi) \right] + \delta\phi [\partial_\phi V(\phi) \hat{\mathcal{G}}]$$

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&= \sqrt{-g} (*R*)^{ijkl} \left(4K_1 \nabla_l \nabla_j \phi + 4K_3 \nabla_l \phi \nabla_j \phi \right) \\
&= \frac{1}{4} \sqrt{-g} (*R*)^{ijkl} \nabla_l \nabla_j V_{ringo}(\phi)
\end{aligned}$$

- While at the same time we have,

$$\delta \left[\int_{\mathcal{M}} d^4x \sqrt{-g}\, V(\phi) \hat{\mathcal{G}} \right]$$
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All three $\mathcal{L}_{George}, \mathcal{L}_{Ringo}, \mathcal{L}_{John}$ are KK Lovelock densities

Paul

- Last term is not recognisable. However, numerous Padilla tricks bring it to the form,
-

$$\mathcal{L}_{paul} = \sqrt{-g}V_{Paul}(\phi)\left[R^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi + \right.$$
$$+G^{\mu\nu}(\nabla_\mu\phi\nabla_\alpha\phi - g_{\mu\alpha}(\nabla\phi)^2)\nabla^\alpha\nabla_\nu\phi$$
$$\left.+R^{\mu\nu}(\nabla_\mu\nabla_\alpha\phi - g_{\mu\alpha}\Box\phi)\nabla^\alpha\phi\nabla_\nu\phi\right]$$

- Therefore

$$\mathcal{L}_{paul} = \sqrt{-g}V_{paul}(\phi)(*R*)^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi$$

Paul

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$$\begin{aligned} \mathcal{L}_{paul} &= \sqrt{-g} V_{Paul}(\phi) \left[R^{\mu\nu\alpha\beta} \nabla_\mu \phi \nabla_\alpha \phi \nabla_\nu \nabla_\beta \phi + \right. \\ &\quad + G^{\mu\nu} (\nabla_\mu \phi \nabla_\alpha \phi - g_{\mu\alpha} (\nabla\phi)^2) \nabla^\alpha \nabla_\nu \phi \\ &\quad \left. + R^{\mu\nu} (\nabla_\mu \nabla_\alpha \phi - g_{\mu\alpha} \Box \phi) \nabla^\alpha \phi \nabla_\nu \phi \right] \end{aligned}$$

- Therefore

$$\mathcal{L}_{paul} = \sqrt{-g} V_{paul}(\phi) (*R*)^{\mu\nu\alpha\beta} \nabla_\mu \phi \nabla_\alpha \phi \nabla_\nu \nabla_\beta \phi$$

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- ???
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- However remember,

$$(*R*)^{\mu\nu\alpha\beta} = R^{\mu\nu\alpha\beta} + 2R^{\nu[\alpha}g^{\beta]\mu} - 2R^{\mu[\alpha}g^{\beta]\nu} + Rg^{\mu[\alpha}g^{\beta]\nu} ,$$

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