Francis Bernardeau
IPhT Saclay, France

IPhT Saclay, Heart of Darkness, June 2012

Large Scale Structure Formation Beyond Linear Order

On-going collaborations with

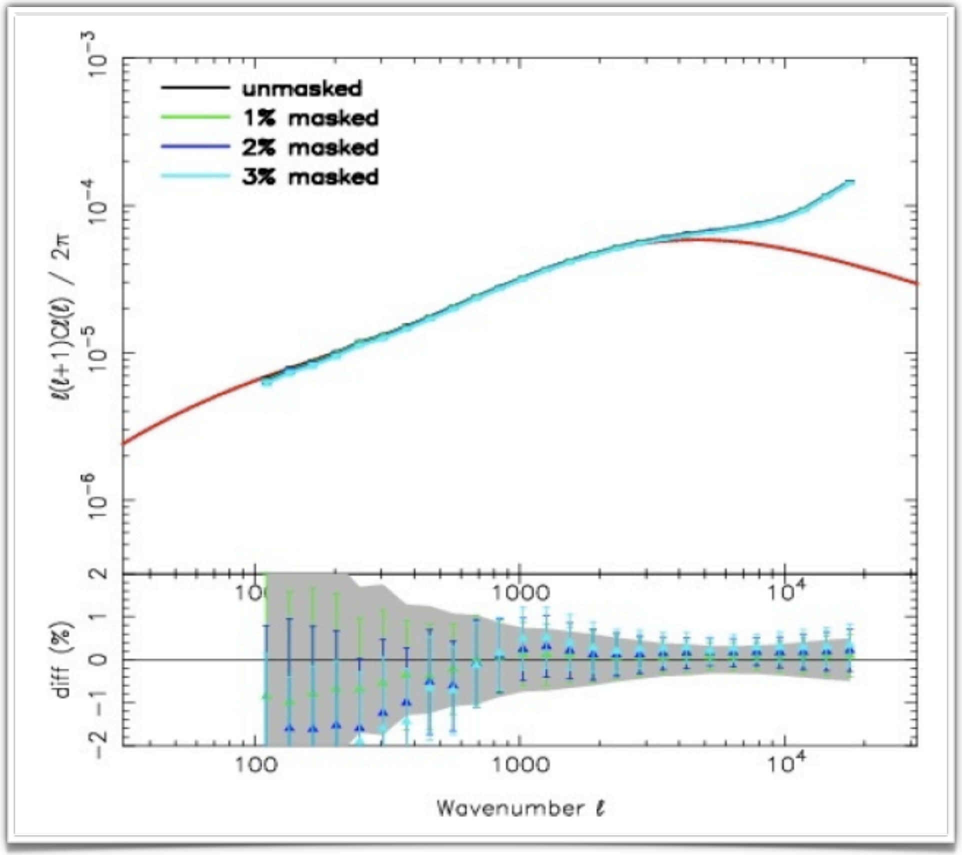
A. Taruya (RESCEU)
T. Nishimichi (IPMU)
M. Crocce (IEEC)
R. Scoccimarro (NYU)
F. Vernizzi (IPhT Saclay)
N. van de Rijt (IPhT Saclay)

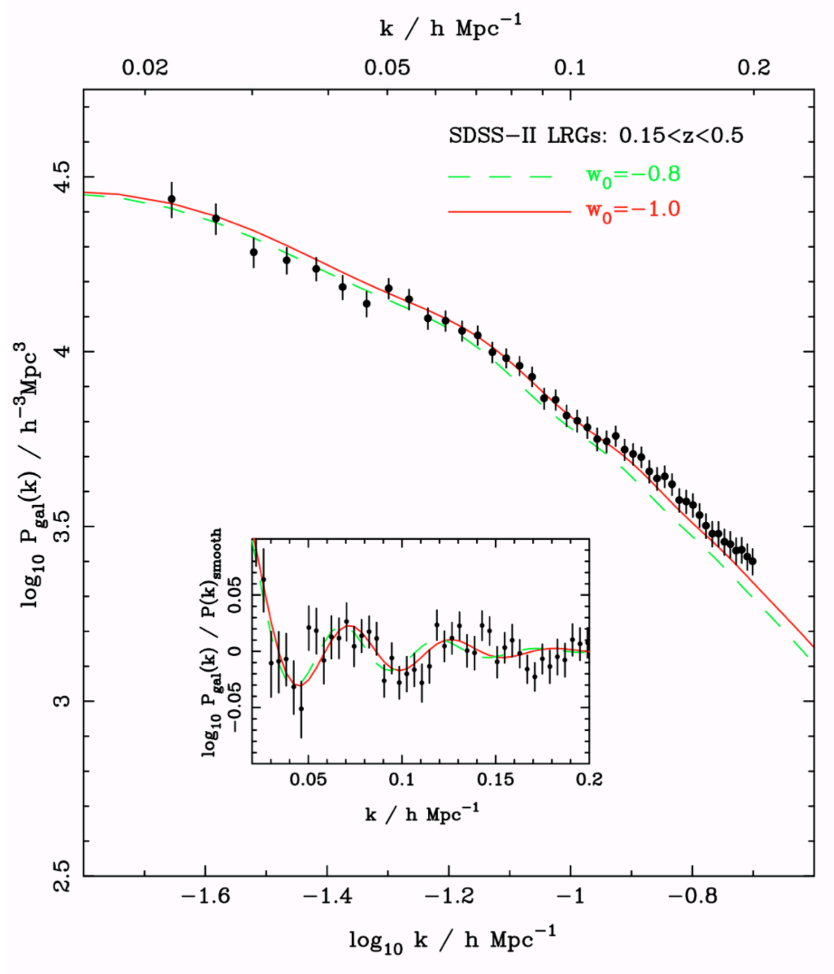
Outline

- Regime of interest
- A self-gravitating expanding dust fluid
  - *A field theory reformulation of the dynamical equations*
  - *The Gamma-expansion*
  - *The eikonal approximation*
- Into the heart of darkness
  - *kernels and possible lessons for dark energy models*

Regime of interest

- The transition from linear to quasi-linear regime





How far beyond 0.1 h Mpc^{-1} can we go?

A self-gravitating expanding dust fluid

A reformulation of the theory with a FT like approach

Scoccimarro '97

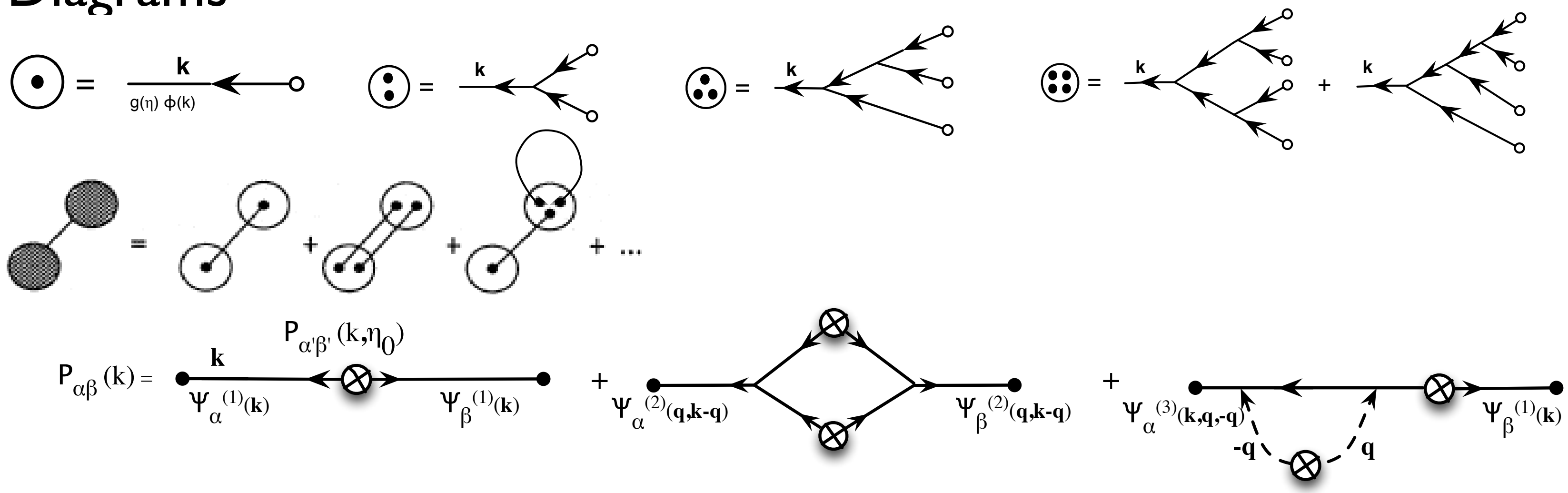
$$\frac{\partial}{\partial \eta}\Phi_a(\mathbf{k},\eta) + \Omega_a^b(\eta)\Phi_b(\mathbf{k},\eta) = \gamma_a^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)$$

$$\Phi_a(\mathbf{k},\eta) = \begin{pmatrix} \delta(\mathbf{k},\eta) \\ \theta(\mathbf{k},\eta) \end{pmatrix}$$

$$\Phi_a(\mathbf{k},\eta) = g_a^b(\eta)\Phi_b(\mathbf{k},\eta=0) + \int_0^\eta d\eta' g_a^b(\eta-\eta')\gamma_b^{cd}(\mathbf{k}_1,\mathbf{k}_2)\Phi_c(\mathbf{k}_1)\Phi_d(\mathbf{k}_2)$$

doublet *linear propagator* $g_{ab}(\eta) = \frac{\mathrm{e}^\eta}{5}\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} - \frac{\mathrm{e}^{-3\eta/2}}{5}\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$

- Diagrams



Note : detailed effects of baryons versus DM can be taken into account (Somogyi & Smith 2010; FB, Van de Rijt, Vernizzi '12) with a 4-component multiplet, for neutrinos it is more complicated...

Methods of Field Theory

Time-flow (renormalization) equations

M. Pietroni '08
Anselmi, Pietroni '12

From the field evolution equation to the spectra evolution equation

The closure theory

Taruya, Hiramatsu, ApJ 2008, 2009

Motion equations for correlators are derived using the Direct-Interaction (DI) approximation in which one separates the field expression in a DI part and a Non-DI part. At leading order in Non-DI >> DI, one gets a closed set of equations,

These equations can more rigorously be derived in a large N expansion.

Valageas P., A&A, 2007

The eikonal approximation

FB, Van de Rijt, Vernizzi 2012

The Gamma expansion

The key ingredients : the (multipoint) propagators

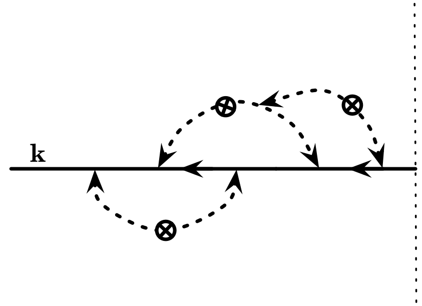
Scoccimarro and Crocce PRD, 2005

Final density / velocity div.

$$G_{ab}(k,\eta)\ \delta_{\rm D}(\mathbf{k}-\mathbf{k}') \equiv \left\langle \frac{\delta \Psi_a(\mathbf{k},\eta)}{\delta \phi_b(\mathbf{k}')} \right\rangle$$

Initial Conditions

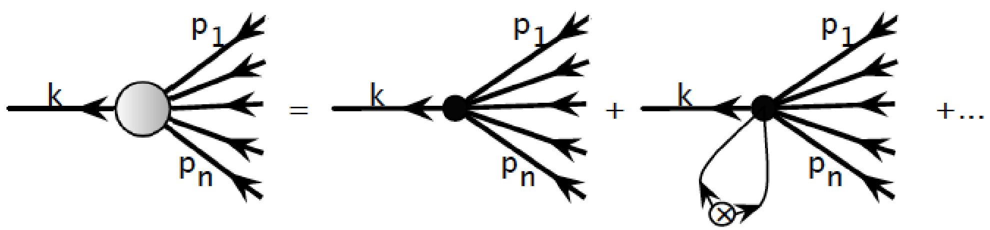
$G_{ab}(k) =$



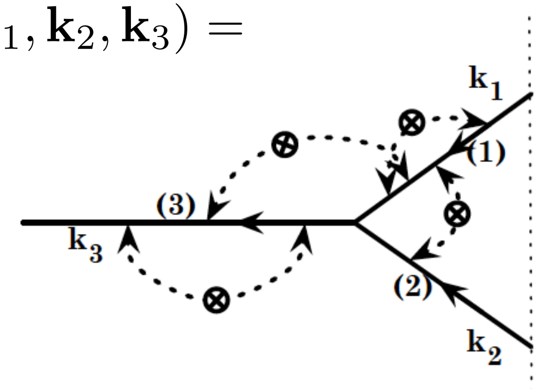
FB, Crocce, Scoccimarro, PRD, 2008

$$\Gamma^{(p)}_{ab_1\ldots b_p}(\mathbf{k}_1,\ldots,\mathbf{k}_p,\eta)\delta_{\rm D}(\mathbf{k}-\mathbf{k}_{1\ldots p}) = \frac{1}{p!}\left\langle \frac{\delta^p \Psi_a(\mathbf{k},\eta)}{\delta\phi_{b_1}(\mathbf{k}_1)\ldots\delta\phi_{b_p}(\mathbf{k}_p)} \right\rangle$$

$\Gamma^{(n)}(k,p_1,\ldots,p_n)=$



$\Gamma^{(2)}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) =$



- This suggests another scheme: to use the n-point propagators as the building blocks

FB, Crocce, Scoccimarro, PRD, 2008

  - The reconstruction of the power spectrum :

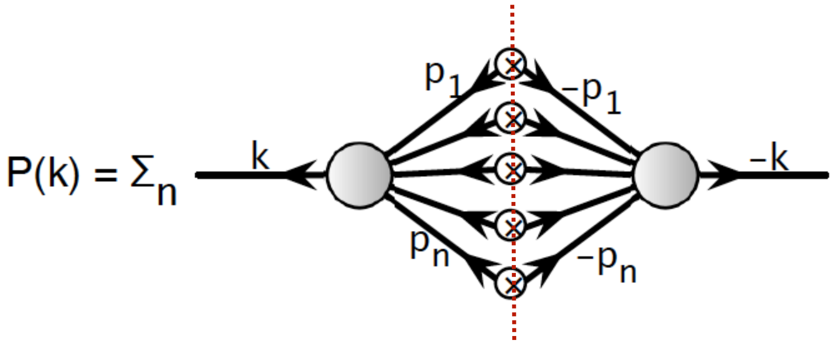
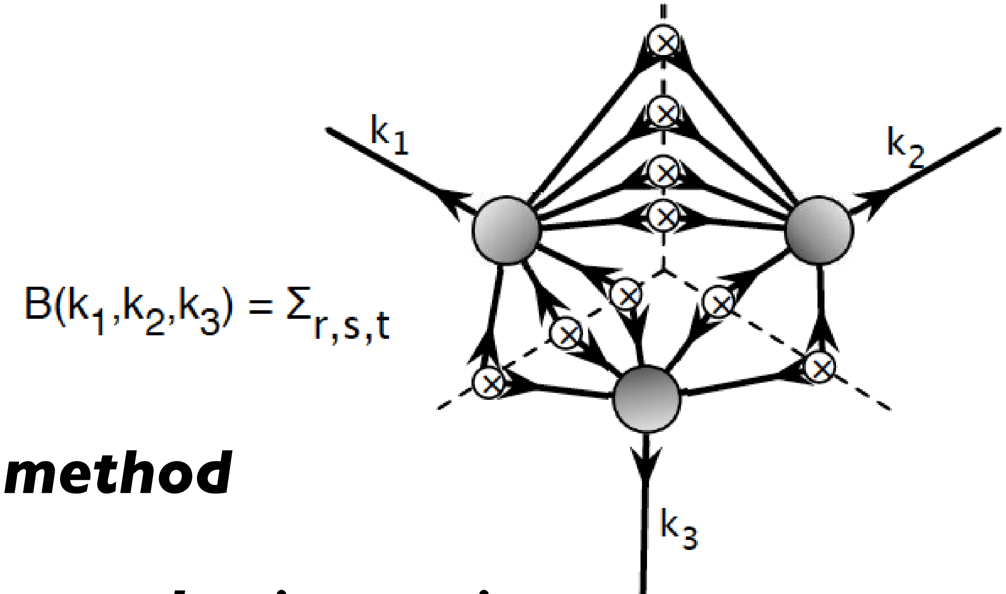


FIG. 3: Reconstruction of the power spectrum out of transfer functions. The crossed circles represent the initial spectrum. The sum runs over the number of internal co ing lines, e.g. the number of such circles. It is to be that each term of this sum is positive.

→ *Sum of positive terms*

  - Also provide the building blocks for higher order moments...



Γ-expansion method

- ***re-organisation(s) of the perturbation series***

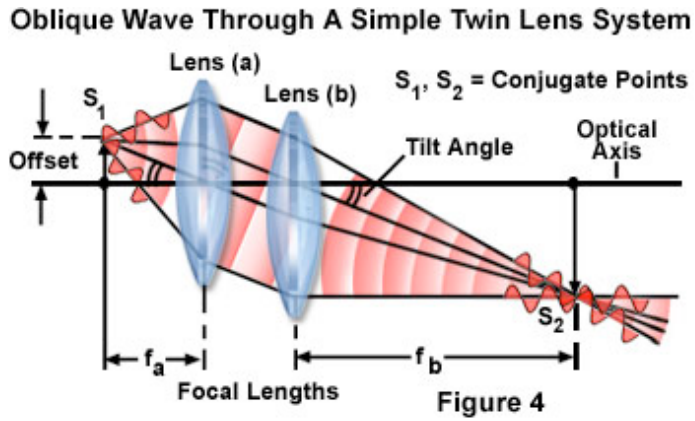
The eikonal approximation

The eikonal approximation :

FB, Van de Rijt, Vernizzi 2011

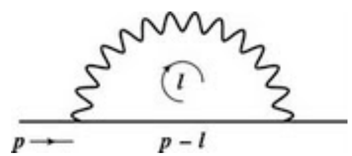
- In wave propagations: it leads to geometrical optics

wavelength is much shorter than
any other lengths $\lambda \ll \ell$



- In quantum field theory such as QED

$p \gg l \quad \text{in}$



- Now in classical random field theory

dynamics : $$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^b(\eta)\Phi_b(\mathbf{k},\eta)=\gamma_a^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)$$

Impact of the long-wave modes into the short wave modes (of interest)

1. Split the interaction term into 2 parts:

- $k_1 \ll k_2$ or $k_2 \ll k_1$ (soft domain)
- $k_1 \approx k_2$ (hard domain)

2. Compute the first part using simplified form for the vertices

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_a^b(\eta)\Phi_b(\mathbf{k},\eta)-\Xi_a^b(\mathbf{k},\eta)\Phi_b(\mathbf{k},\eta)=\gamma_a^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)|_{\text{hard domain}}$$

$$\Xi_a^b(\mathbf{k},\eta)=\int \mathrm{d}^3\mathbf{q}\left(\gamma_a^{cb}(\mathbf{q},\mathbf{k})+\gamma_a^{bc}(\mathbf{k},\mathbf{q})\right)\Phi_c(\mathbf{q},\eta)|_{\text{soft domain}}$$

It leads to a "renormalized" theory that takes into account the long wave modes in a nonlinear manner.

3. Taking ensemble average over Ξ leads to the standard results assuming linear growing modes and Gaussian initial conditions.

The "renormalized" theory at linear order

$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta) + \Omega_a^{\ b}(\eta)\Phi_b(\mathbf{k},\eta) - \Xi_a^{\ b}(\mathbf{k},\eta)\Phi_b(\mathbf{k},\eta) = 0$$

$$\Xi_a^{\ b}(\mathbf{k},\eta) = \int \mathrm{d}^3\mathbf{q}\left({}^{\mathrm{eik.}}\gamma_a^{\ cb}(\mathbf{q},\mathbf{k}) + {}^{\mathrm{eik.}}\gamma_a^{\ bc}(\mathbf{k},\mathbf{q})\right)\Phi_c(\mathbf{q},\eta)\big|_{\mathrm{soft\ domain}}$$

velocity field component only

What is in this new term ?

A **multi-component** fluid analysis with adiabatic modes and iso-curvature/density modes

$$\Xi_a^b(\mathbf{k},\eta) = \Xi^{(\mathrm{ad})}(\mathbf{k},\eta)\delta_a^b + \Xi_a^{b\,(\nabla)}(\mathbf{k},\eta)$$

diagonal term → $= \int \frac{\mathbf{k}.\mathbf{q}}{q^2}\,\delta_d(\mathbf{q})\mathrm{d}^3\mathbf{q}$

non-diagonal term → $\Xi_a^{b\,(\nabla)} = \Xi^{(\nabla)}\,h_a^b\,,\quad h_a^b \equiv \begin{pmatrix} f_2 & 0 & 0 & 0 \\ 0 & f_2 & 0 & 0 \\ 0 & 0 & -f_1 & 0 \\ 0 & 0 & 0 & -f_1 \end{pmatrix}$

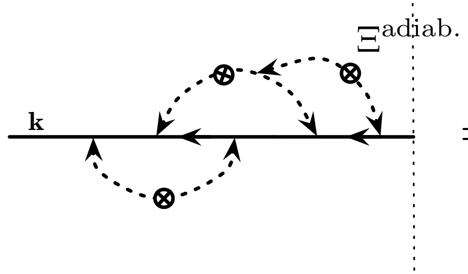
The main outcome with adiabatic modes only, is the following :

$$\xi_a{}^b(\mathbf{k},\eta,\eta_0) = g_a{}^b(\eta,\eta_0)\exp\left(\int_{\eta_0}^{\eta} \mathrm{d}\eta'\ \mathbf{k}.\mathbf{d}^{\mathrm{adiab.}}(\eta')\right)$$

(adiabatic) displacement field

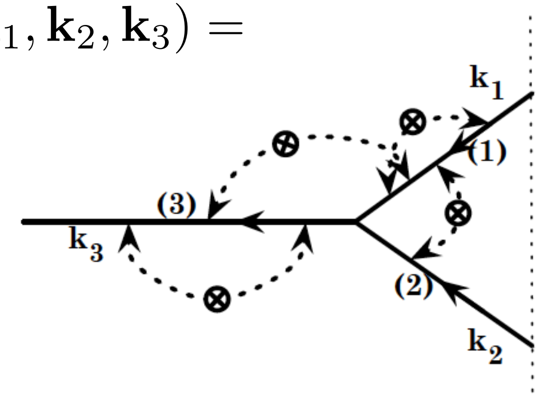
Consequences for propagators

$G_{ab}(k) =$



$$= \left\langle \xi_a{}^b(\eta) \right\rangle_{\Xi} = g_a{}^b(\eta)\exp\left(-\frac{k^2\sigma_{\mathrm{d}}^2(\eta-\eta_0)^2}{2}\right)$$

$\Gamma^{(2)}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) =$



$$= \Gamma^{\mathrm{tree}}_{abc}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\exp\left(-\frac{k_3^2\sigma_{\mathrm{d}}^2(\eta-\eta_0)^2}{2}\right)$$

The eikonal approximation is very powerful

- Re-summation can be extended to any order

$$\Gamma^{(p)} = \exp\left[-\frac{|\mathbf{k}_1 + \ldots \mathbf{k}_p|^2\sigma_v^2}{2}(e^s-1)^2\right]\Gamma^{(p)}_{\rm tree}$$

FB, Crocce, Scoccimarro, PRD, 2008

- Non-Gaussian initial conditions *Crocce, Sefusatti, FB, 2010*
  - The Gamma-expansion is still valid.
  - In the large k limit we now have : $G(k) \;\to\; \exp\left[-\sum_{p=2}^{\infty}\frac{\langle(\mathbf{v}.\mathbf{k})^p\rangle_c}{p!}(e^{\eta}-e^{\eta_0})^p\right]$

- Can be used In Lagrangian coordinates

FB, Valageas 2008, Matsubara, 08

- For any fluid content, in particular including dark matter and baryons (new modes appear)

FB, Van de Rijt, Vernizzi 2011

- The basis for the regularization schemes in which one can incorporate arbitrary order loops

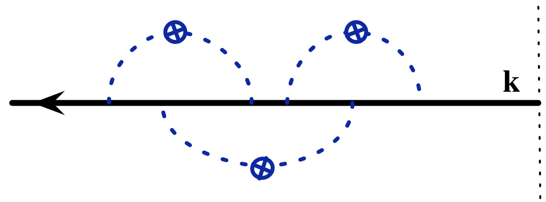
FB et al. 2011

A regularization scheme = how to interpolate between n-loop results and the large-k behavior ?

An ad-hoc solution was provided by Crocce and Scoccimarro (RPT) for the one-point propagator but it cannot be generalized all cases.

‣The proposed form is the following

$$ {}^{\mathrm{Reg}}\Gamma^{(p)}{}_{a}^{\;b_1\ldots b_p} = {}^{\mathrm{tree}}\Gamma_{a}^{\;b_1\ldots b_p}\;\exp\left(-\frac{k^2\sigma_d^2}{2}\right) $$



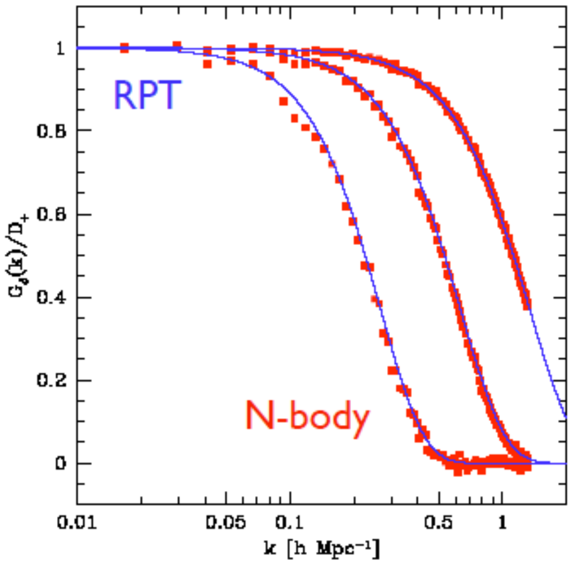
‣This is our proposition for regularized propagators: our best guess!

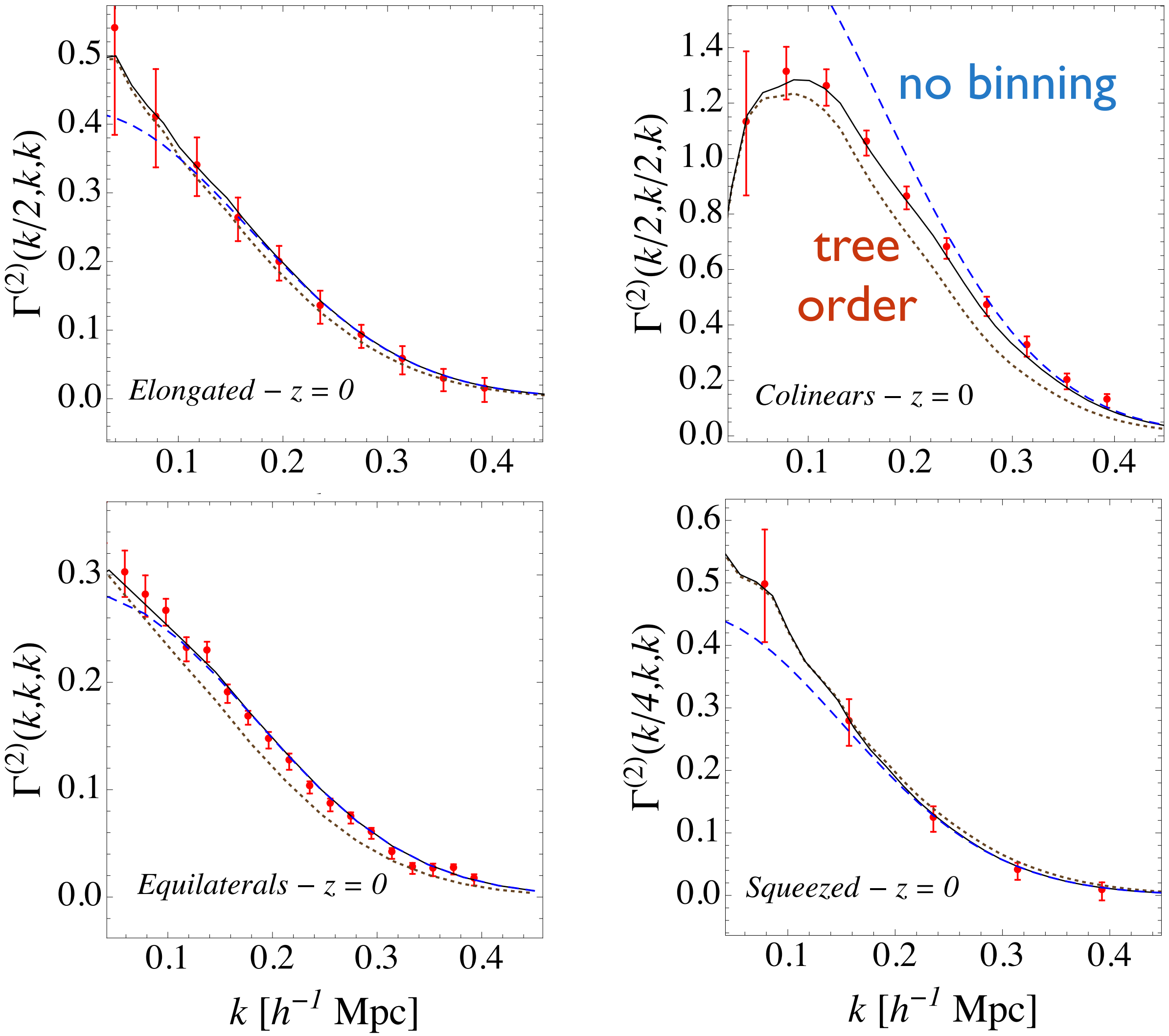
FB, Crocce, Scoccimarro '12

Comparison with numerical simulations at tree and one-loop order

three-point propagator at 0 and 1-loop order

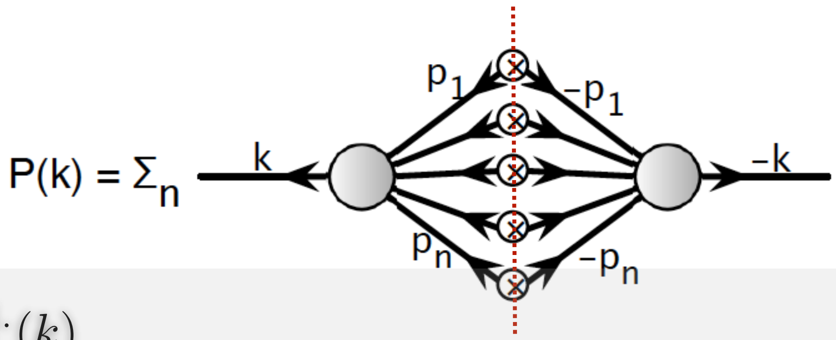
two-point propagator at 1-loop order





The RegPT proposition

Taruya , FB, Nishimichi, Codis '12 in prep.



$$P_{aa'}(k) = {}^{\rm Reg}\Gamma^{(1)}{}_a^{\,b}(k)\ {}^{\rm Reg}\Gamma^{(1)}{}_{a'}^{\,b'}(k)\ P^{\rm ini.}_{bb'}(k)$$
$$+ {}^{\rm Reg}\Gamma^{(2)}{}_a^{\,bc}(\mathbf{k}_1,\mathbf{k}_2)\ {}^{\rm Reg}\Gamma^{(2)}{}_{a'}^{\,b'c'}(\mathbf{k}_1,\mathbf{k}_2)\ P^{\rm ini.}_{bb'}(k_1)\ P^{\rm ini.}_{cc'}(k_2)$$
$$+ {}^{\rm Reg}\Gamma^{(3)}{}_a^{\,bcd}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\ {}^{\rm Reg}\Gamma^{(3)}{}_{a'}^{\,b'c'd'}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\ P^{\rm ini.}_{bb'}(k_1)\ P^{\rm ini.}_{cc'}(k_2)\ P^{\rm ini.}_{dd'}(k_2)$$

two loop order

one loop order

tree order

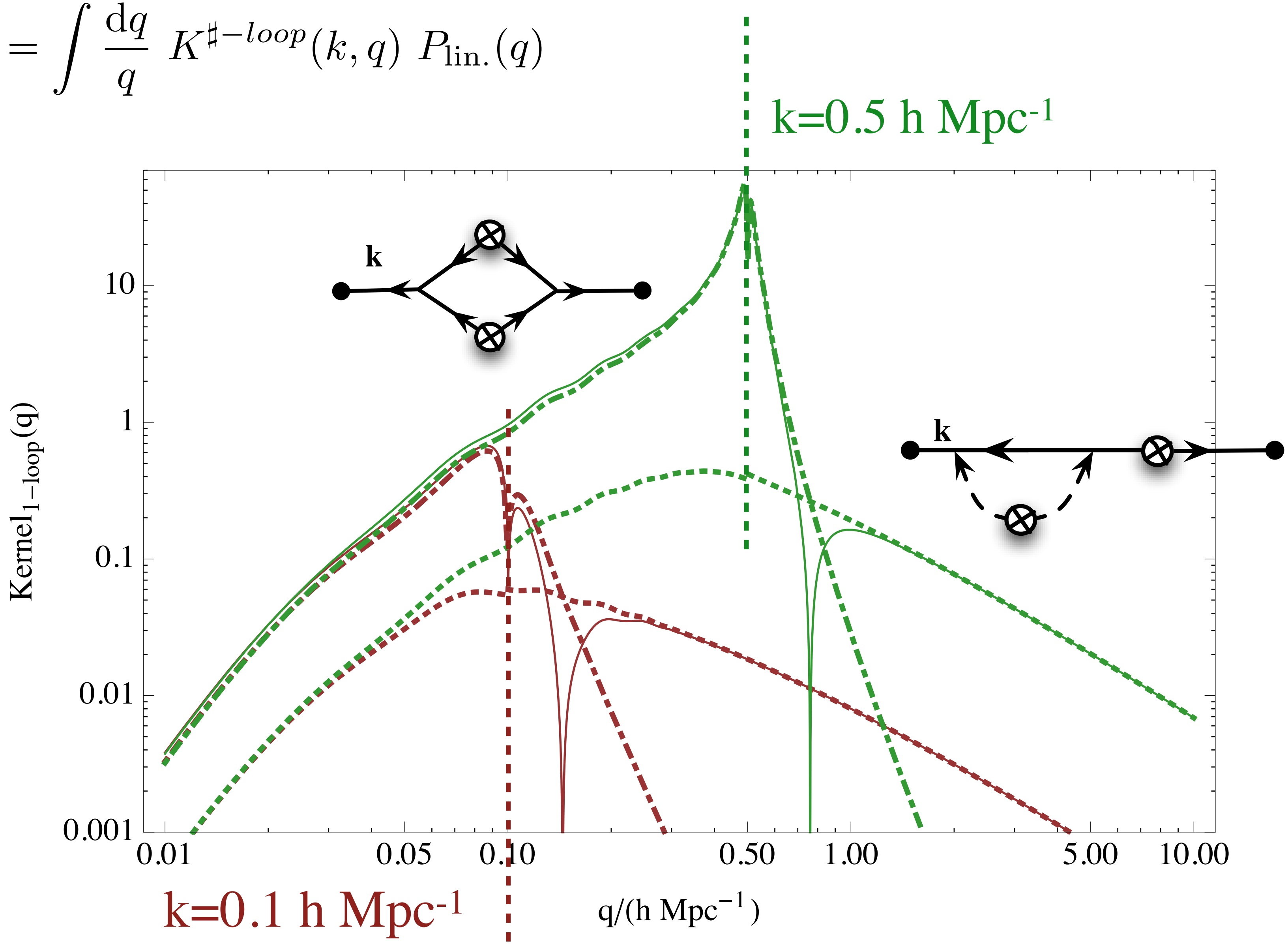
$$\begin{aligned} {}^{\rm Reg}\Gamma^{(p)}{}_a^{\,b_1\ldots b_p} = \ & {}^{\rm tree}\Gamma_a^{\,b_1\ldots b_p}\ \exp\left(-\frac{k^2\sigma_d^2(k)}{2}\right) \\ & + \left[{}^{\rm one-loop}\Gamma_a^{\,b_1\ldots b_p} + \frac{1}{2}k^2\sigma_d^2\ {}^{\rm tree}\Gamma_a^{\,b_1\ldots b_p}\right]\exp\left(-\frac{k^2\sigma_d^2(k)}{2}\right) \\ & + \left[{}^{\rm two-loop}\Gamma_a^{\,b_1\ldots b_p} + {\rm c.t.}\right]\exp\left(-\frac{k^2\sigma_d^2(k)}{2}\right) \end{aligned}$$

Into the heart of darkness
in PT calculation

Kernels in Pertubation theory calculations

FB, Taruya, Nishimichi, '12 in prep.

$$P_{\rm NL}^{\sharp-loop}(k) = \int \frac{{\rm d}q}{q}\ K^{\sharp-loop}(k,q)\ P_{\rm lin.}(q)$$

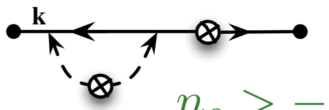


Kernels for the 2-point propagators at p-loop order

$$P_{\rm NL}^{\sharp-loop}(k) = \int \frac{{\rm d}q}{q}\; K^{\sharp-loop}(k,q)\; P_{\rm lin.}(q)$$

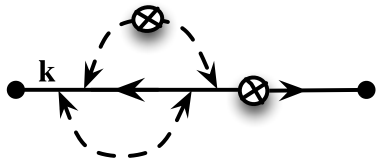
Convergence properties

1-loop



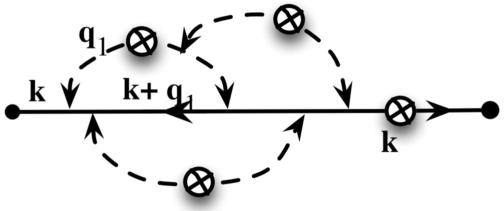
$n_s > -1$

2-loop

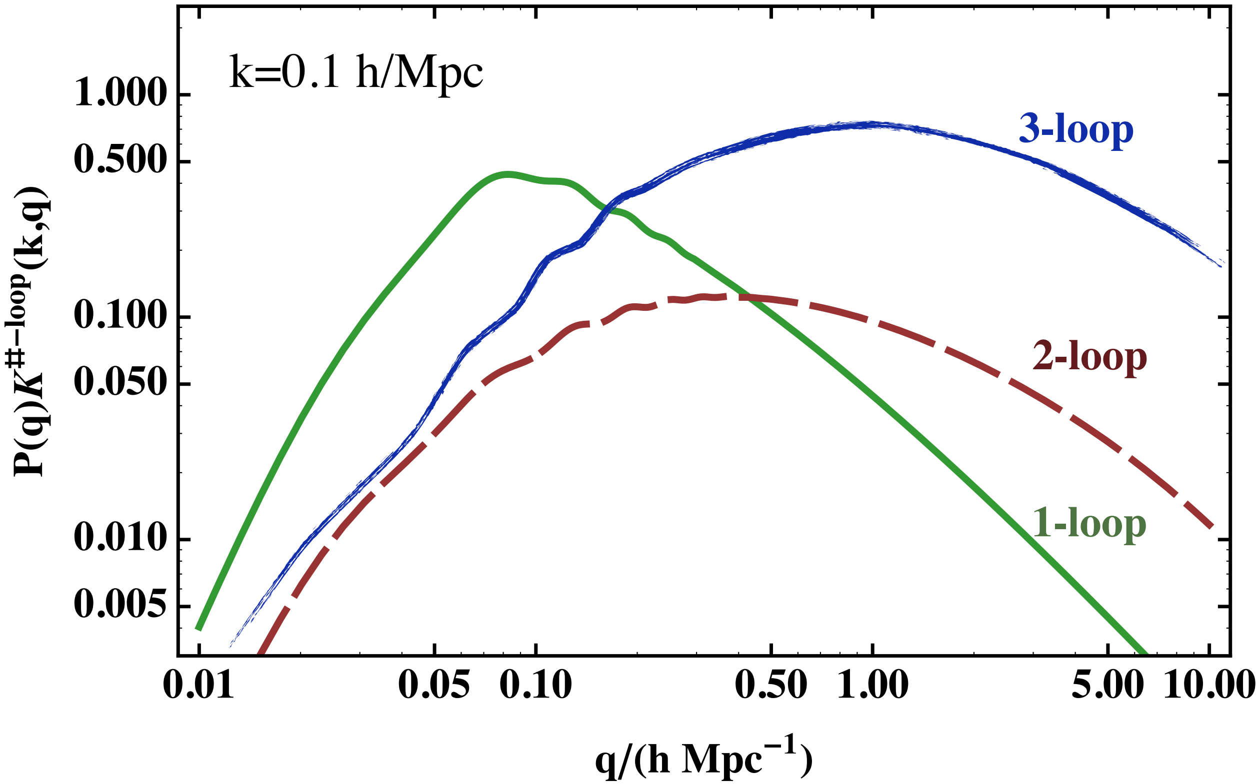


$n_s > -2$

3-loop



$n_s > -2.33$



It comes as a reminder of impact of small scale physics (e.g. shell crossings, baryon physics)

Valageas '10; Pueblas & Scoccimarro '08; Pietroni et al. '11

- Shape of kernels is key to the validity of PT calculations and comparison with numerical simulations
- It comes from the IR behavior of coupling functions

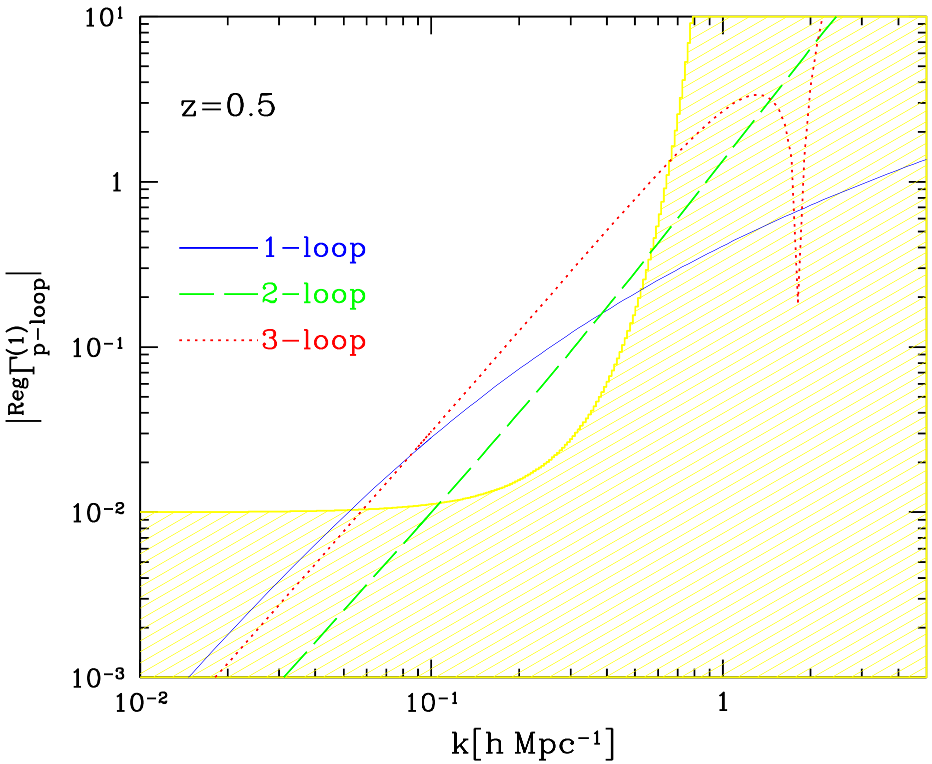
$$\frac{\partial}{\partial\eta}\Phi_a(\mathbf{k},\eta)+\Omega_{ab}(\eta)\Phi_b(\mathbf{k},\eta)=\gamma_{abc}(\mathbf{k}_1,\mathbf{k}_2)\Phi_b(\mathbf{k}_1)\Phi_c(\mathbf{k}_2)$$

$$\gamma_{abc}(k_1,k_2)=\begin{pmatrix} \left\{0,\frac{(k_1+k_2).k_2}{2k_2.k_2}\right\} & \left\{\frac{(k_1+k_2).k_1}{2k_1.k_1},0\right\} \\ \{0,0\} & \left\{0,\frac{k_1.k_2(k_1+k_2).(k_1+k_2)}{2k_1.k_1k_2.k_2}\right\} \end{pmatrix}$$

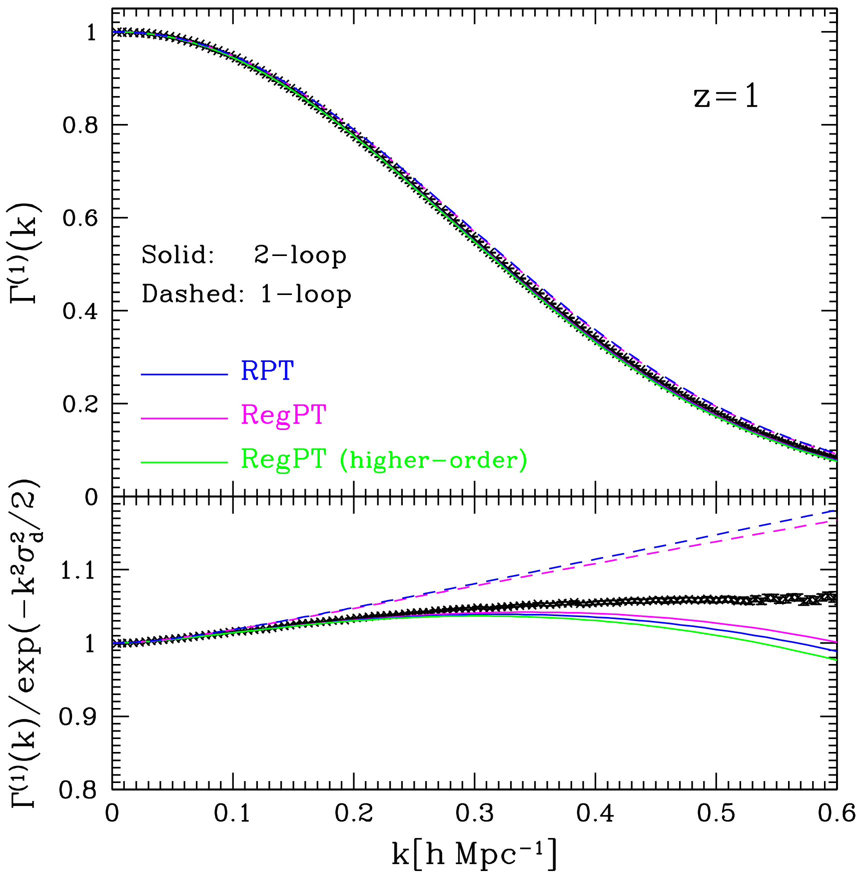
$$\gamma_{abc}(k_1,\epsilon-k_1)\ \sim\ \epsilon^2$$

- Coupling functions in dark energy models do (Sefusatti & Vernizzi '11) or do not (like DGP, see Scoccimarro, dilaton/chameleon field, see Brax & FB '11) follow this property depending on the effective sound speed.

Contribution to 2-point propagator at 1% level



contribution of loop contributions



Comparison with N-body results for the 2-point propagator

Conclusions

- The evolution of perturbations can be done in the quasi linear with a variety of methods; the best strategies (series expansion re-organization) are still under exploration

- The eikonal approximation scheme captures of lot of interesting features and it can be used in a wide variety of situations;

- First public codes implementing second-order results are to be available soon. See A. Taruya's talk.