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Solid Inflation

w/ Solomon Endlich and Junpu Wang,
to appear (soon?)

(also Gruzinov 2004)

Inflation

basic story, well known

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EFT of inflation

- The early universe: homogeneous and isotropic
- Usually modeled via $\varphi_a = \varphi_a(t)$
- Time-translations spontaneously broken

⇒ Goldstone boson = adiabatic perturbations

- Systematic effective field theory

(Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007)

Here instead:

- t-independent, x-dependent fields: $\varphi_a = \varphi_a(\vec{x})$
- time-translations unbroken
- spatial translations and rotations, broken

Apparently violates:

1. homogeneity and isotropy

2. the need for a physical "clock"

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2. the need for a physical "clock" ⇨ metric

Homogeneity and isotropy

- Ex: one scalar w/ vev $\langle\varphi\rangle = x$
- If it has a shift symmetry $\varphi \to \varphi + a$
- unbroken diagonal translation

$$\begin{cases} x \to x - a \\ \varphi \to \varphi + a \end{cases}$$

Rotations still broken

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Rotations still broken $\Longrightarrow$ need 3 fields

3 scalars: $\phi^I(\vec{x}, t)$ $I = 1, 2, 3$

vevs: $\langle \phi^I \rangle = x^I$

If internal symmetries:

$$\phi^I \to \phi^I + a^I$$
$$\phi^I \to SO(3)\,\phi^I$$

then unbroken diagonal subgroups

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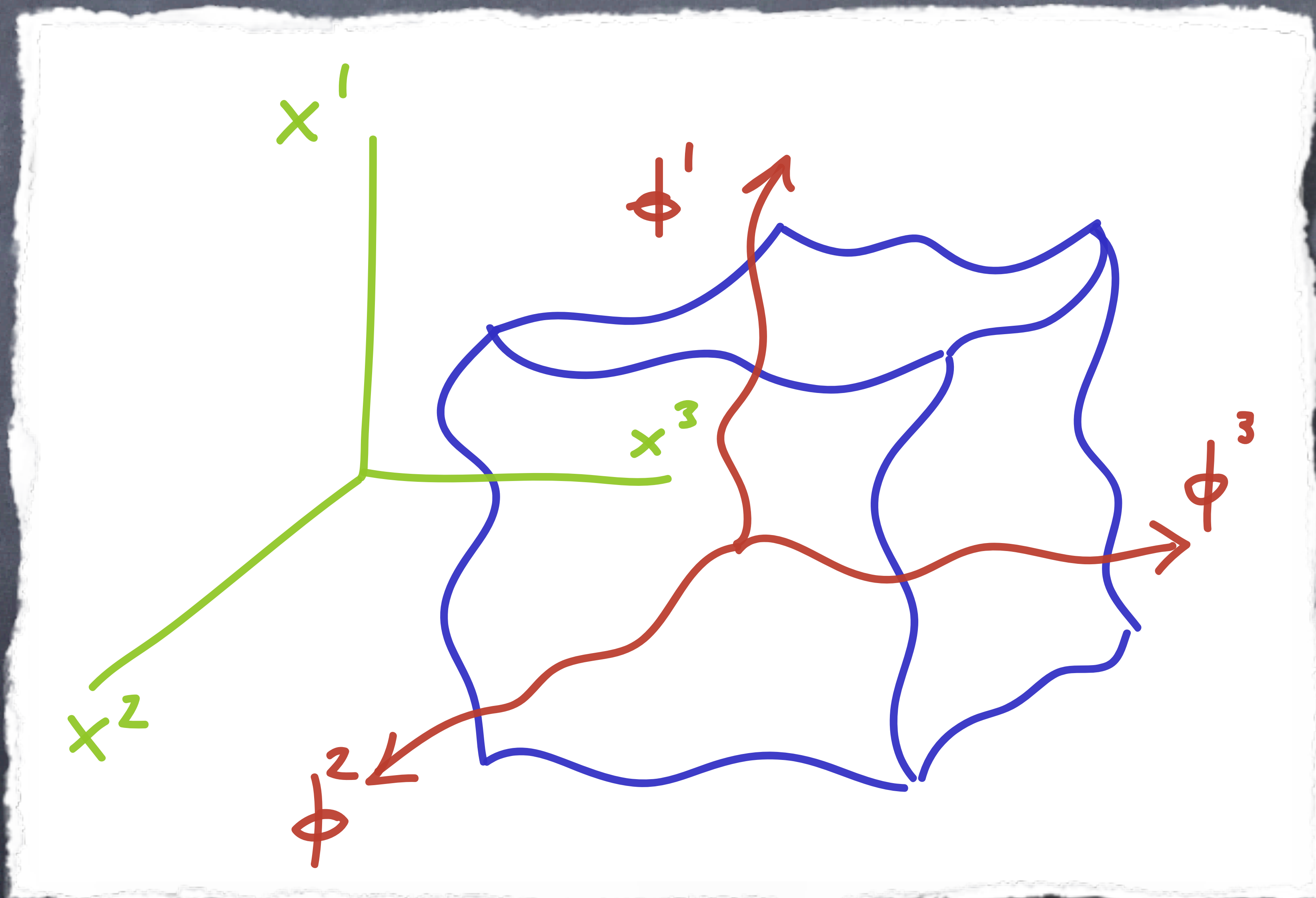
This is a solid

EFT for solids (and fluids)

Dof: volume elements' positions

$$\phi^I(\vec{x}, t) \qquad I = 1, 2, 3$$



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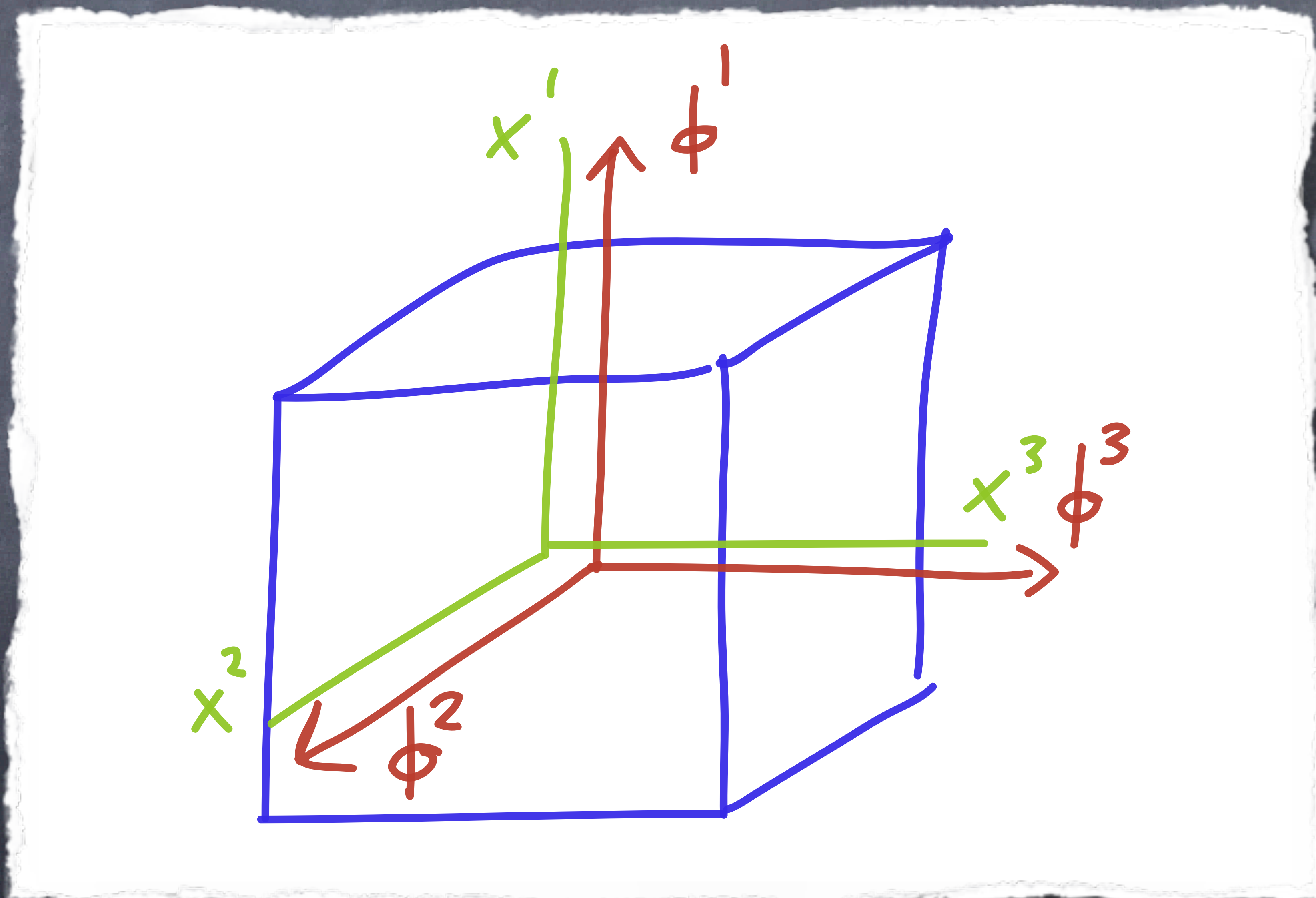

Symmetries: Poincaré + internal

$$\left.\begin{array}{l} \phi^I \to \phi^I + a^I \\ \phi^I \to SO(3)\,\phi^I \end{array}\right\}$$ recover homogeneity/isotropy

$$\phi^I \to \xi^I(\phi) \qquad \det \frac{\partial \xi^I}{\partial \phi^J} = 1$$ fluid vs solid

Action

$$B^{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J$$

$$\mathcal{L} = F\Big([B], \frac{[B^2]}{[B]^2}, \frac{[B^3]}{[B]^3}\Big) + \dots$$

↑

(X, Y, Z)

(For the fluid $\mathcal{L} = F(\det B) + \dots$)

Stress-energy tensor

$$T_{\mu\nu} \sim F, F' \times g_{\mu\nu}, \partial_\mu \phi^I \partial_\nu \phi^J \times \delta^{IJ}, B^{IJ}, B^{IK} B^{KJ}$$

On the background $B^{IJ} = \delta^{IJ}$

$$T_{\mu\nu} \to \begin{cases} \rho = -F \\ \rho + p = -2\, F_X \end{cases}$$

Slow roll ⇨ small $F_X = \mathcal{O}(\epsilon)$

⇨ Approximate internal scale invariance $\phi^I \to \lambda \phi^I$

Excitations (phonons)

$$\phi^I = x^I + \pi^I$$

$$\mathcal{L} \to F_X \left[\dot{\vec{\pi}}^2 - c_T^2(\partial_i \pi_T^j)^2 - c_L^2(\partial_i \pi_L^i)^2\right] + \text{interactions}$$

$$c_L^2 = \frac{1}{F_X} \times F_X, F_{XX}, (F_Y + F_Z)$$

$$c_T^2 \simeq 3/4 \cdot (c_L^2 + 1)$$

stability, sub-luminality ⇨ $(F_Y + F_Z) \lesssim F_X = \mathcal{O}(\epsilon)$

$\text{interactions} \sim F_Y, F_Z \times (\partial\pi)^n$ ⇨ large

Strong coupling scale

$$\mathcal{L}_2 \sim F_X \cdot (\partial\pi)^2$$

$$\mathcal{L}_{\rm int} \sim F \cdot (\partial\pi)^n$$

$$\Rightarrow \quad \Lambda_{\rm strong} \sim F^{1/4} \cdot \epsilon^{3/4} \sim (M_{\rm Pl} H)^{1/2} \cdot \epsilon^{3/4}$$

much bigger than H, for ϵ not too small

The clock

$$B^{IJ} \equiv g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J \to \frac{1}{a^2(t)}\delta^{IJ}$$

$$X \to 1/a^6$$

$$Y, Z \to 1$$

time-dependence from the metric

⇨ no associated Goldstone boson

Reheating

- Solid/fluid transition at some critical det(B)
- Similar to solid He at 0K and 25bar (30% compressible, we need e^60...)
- Fluid: same dof, more symmetries
- Sharp feature in F(X, Y, Z) -- region of enhanced symmetry in X, Y, Z space.

Cosmological perturbations

$$\phi^I = x^I + \pi^I$$

$$g_{\mu\nu} = g_{\mu\nu}^{\rm FRW} + \delta g_{\mu\nu}$$

Very roughly:

$$\mathcal{L}_2 \sim F_X \cdot (\partial\pi)^2$$

$$\mathcal{L}_3 \sim F \cdot (\partial\pi)^3$$

$$\zeta \sim \vec{\nabla} \cdot \vec{\pi}$$

$\Rightarrow$

$$\langle\zeta\zeta\rangle \sim \frac{1}{\epsilon}\frac{1}{c_L^5}\frac{H^2}{M_{\rm Pl}^2}$$ (cfr. w/ $\frac{1}{\epsilon}\frac{1}{c_L}\frac{H^2}{M_{\rm Pl}^2}$)

$$\frac{\mathcal{L}_3}{\mathcal{L}_2} \sim \frac{1}{\epsilon}\frac{1}{c_L^2}\zeta$$ (cfr. w/ $\frac{1}{c_L^2}\zeta$)

A few caveats

$$\phi^I = x^I + \pi^I$$

$$g_{\mu\nu} = g_{\mu\nu}^{\rm FRW} + \delta g_{\mu\nu}$$

for reheating

Unitary gauge: $\phi^I = x^I$

$$\det B = \det g^{IJ} = 1/a(t)^6$$

$$g_{ij} = a^2\big((1+A)\delta_{ij} + \partial_i\partial_j\chi + \partial_{(i}C_{j)} + \gamma_{ij}\big)$$

not independent

$$F(B^{IJ}) \to F(g^{IJ})$$

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not independent (A and χ)

$$F(B^{IJ}) \to F(g^{IJ})$$ ⇒ Lorentz violating massive gravity

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not independent

$F(B^{IJ}) \to F(g^{IJ})$ ⇨ Lorentz violating massive gravity $\in$ this workshop

Better: spatially flat gauge

$$\phi^I = x^I + \pi^I$$
$$g_{ij} = a^2(\delta_{ij} + \gamma_{ij})$$

- Solve the constraints for N, Ni
- Plug back into the action
- Compute $\langle \pi\pi \dots \pi \rangle$
- Translate into $\langle \zeta\zeta \dots \zeta \rangle$ or $\langle \mathcal{R}\mathcal{R} \dots \mathcal{R} \rangle$

The problem

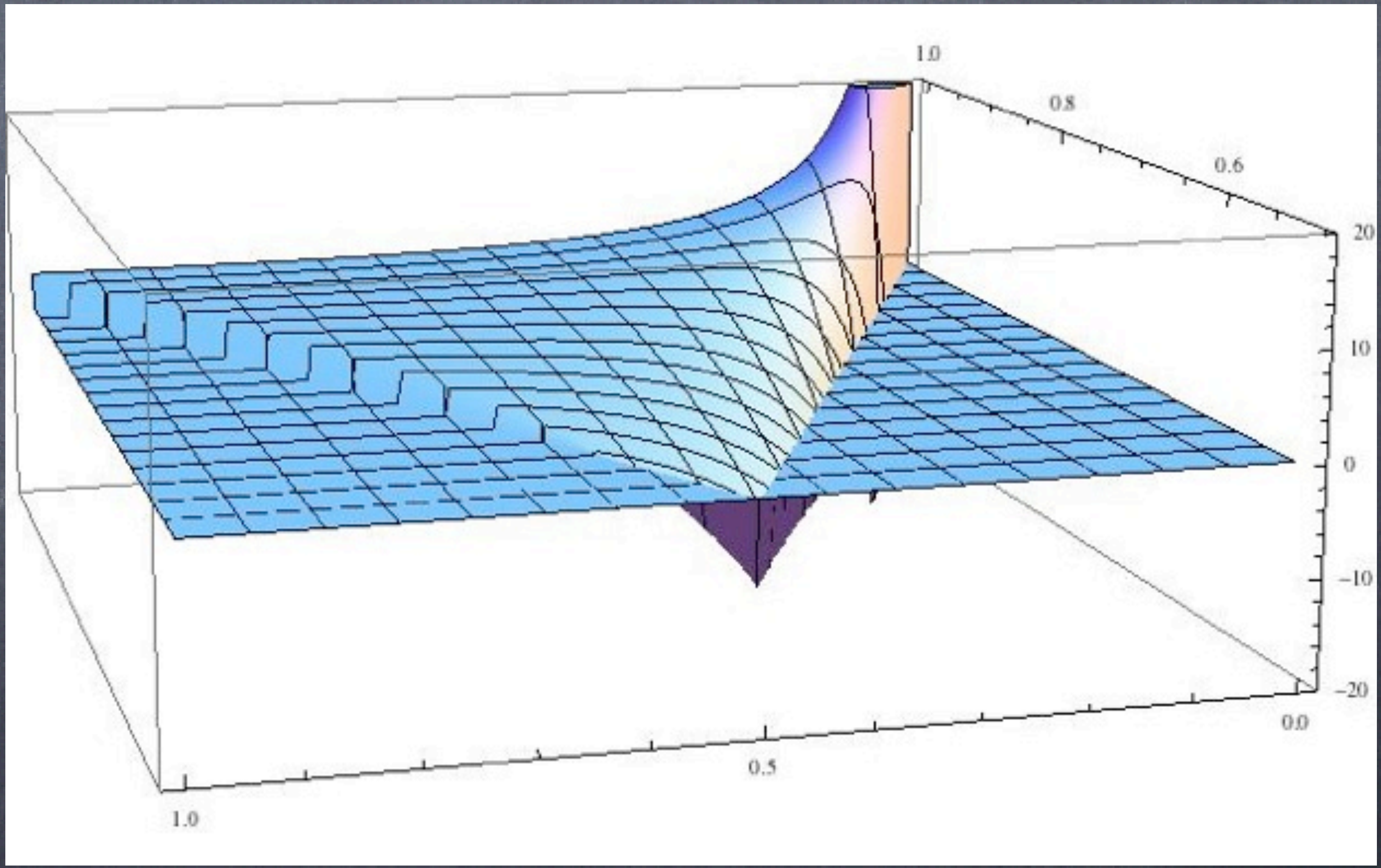
- Neither ζ nor $\mathcal{R}$ is conserved
- There are no adiabatic solutions
- For solid, long longitudinal perturbation locally distinguishable from the background (anisotropic stress).
- time-dependence is slow: $\mathcal{O}(\epsilon)$
- Model-dependent matching at reheating

For instantaneous matching

$$n_S - 1 = 2\epsilon\, c_L^2 - \eta - 5s$$

$$n_T - 1 = 2\epsilon\, c_L^2$$ (mass term $\sim c_T^2$)

$$\langle\zeta\zeta\zeta\rangle = \frac{1}{\epsilon^3}\frac{1}{c_L^{12}}\frac{H^4}{M_{\rm Pl}^4}\times$$



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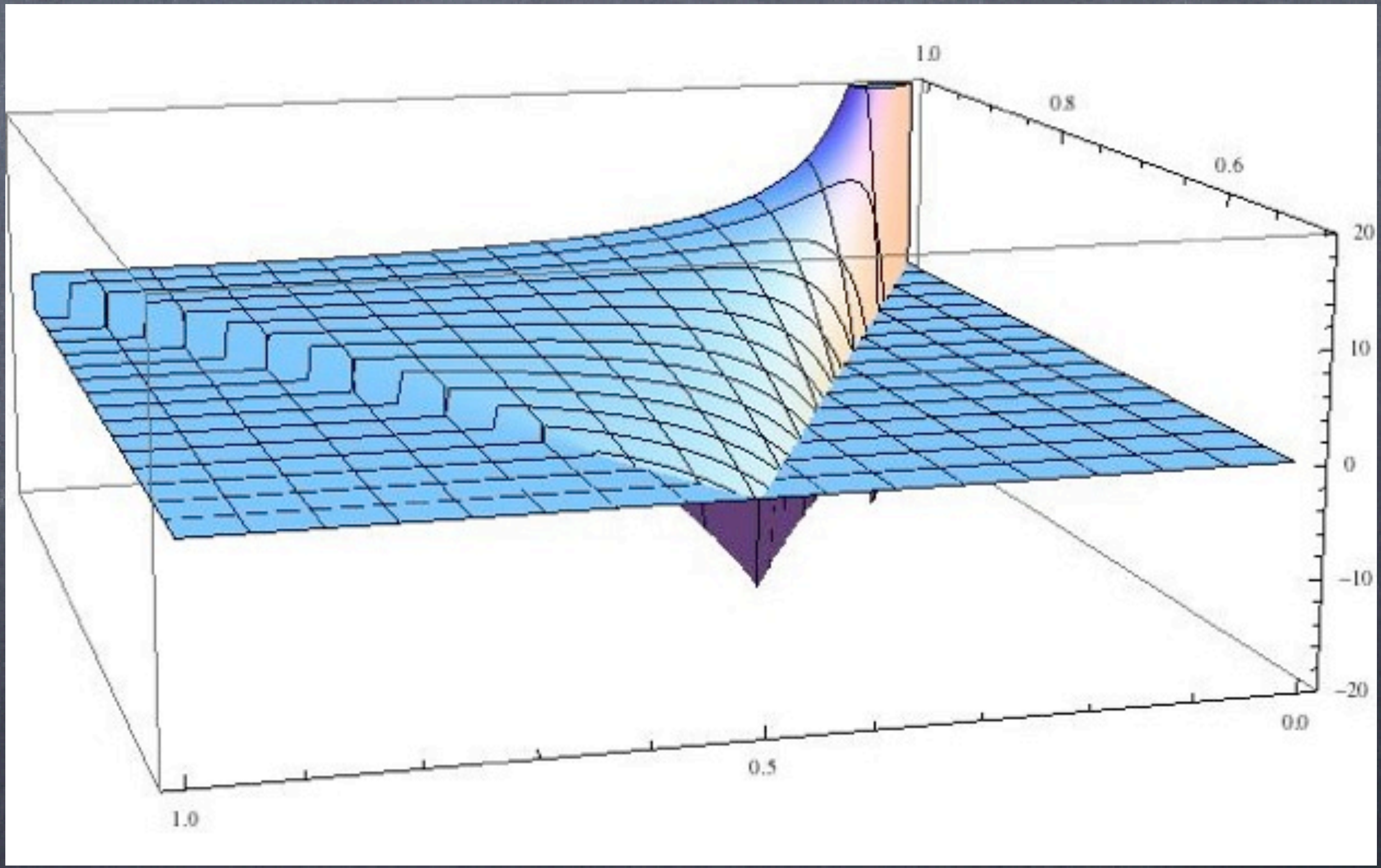
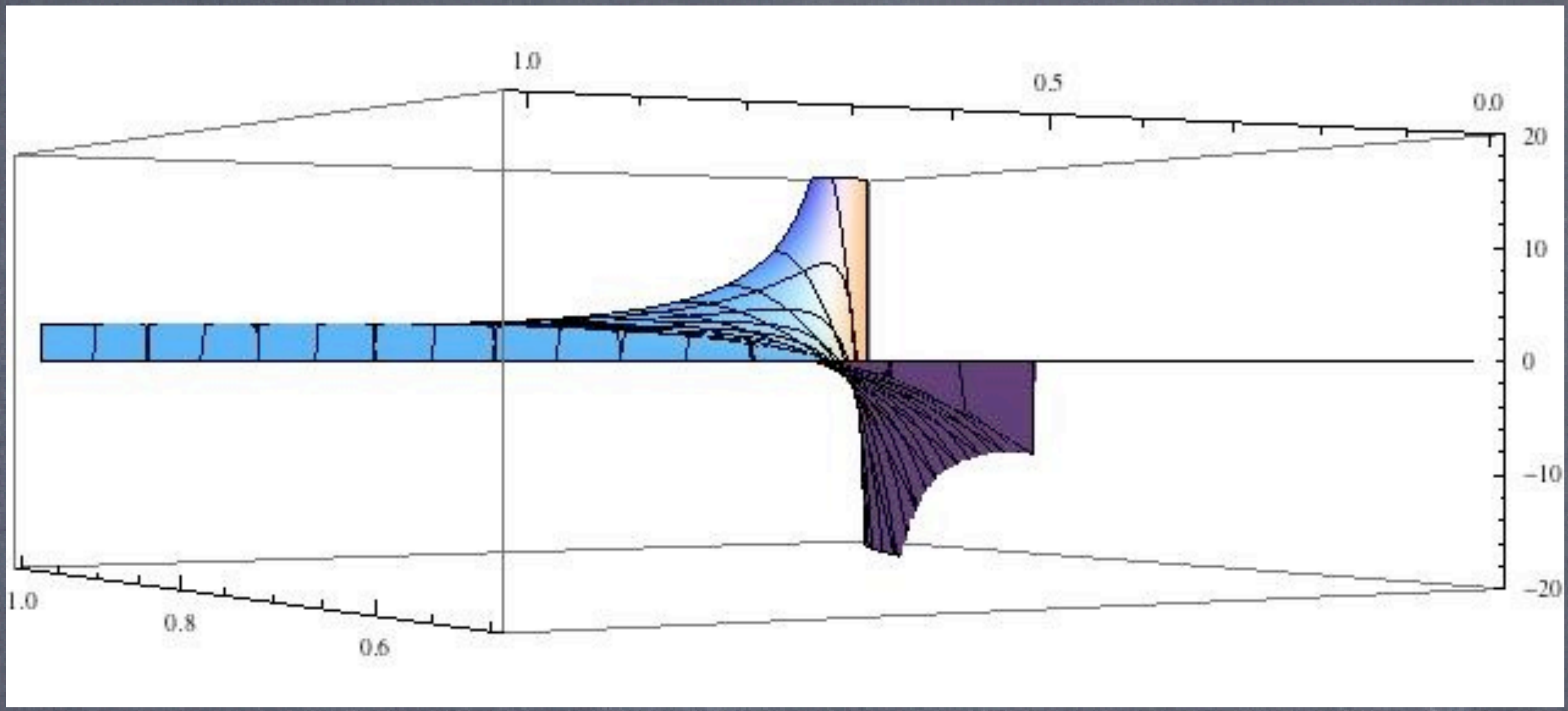


Quadrupolar squeezed limit



$$\langle\zeta\zeta\zeta\rangle \to f_{NL} \times \langle\zeta\zeta\rangle\langle\zeta\zeta\rangle \times (1 - 3\cos^2\theta)$$

$$f_{NL} \sim \frac{1}{\epsilon}\frac{1}{c_L^2}$$

2% overlap w/ "local" shape

38% w/ "equilateral"

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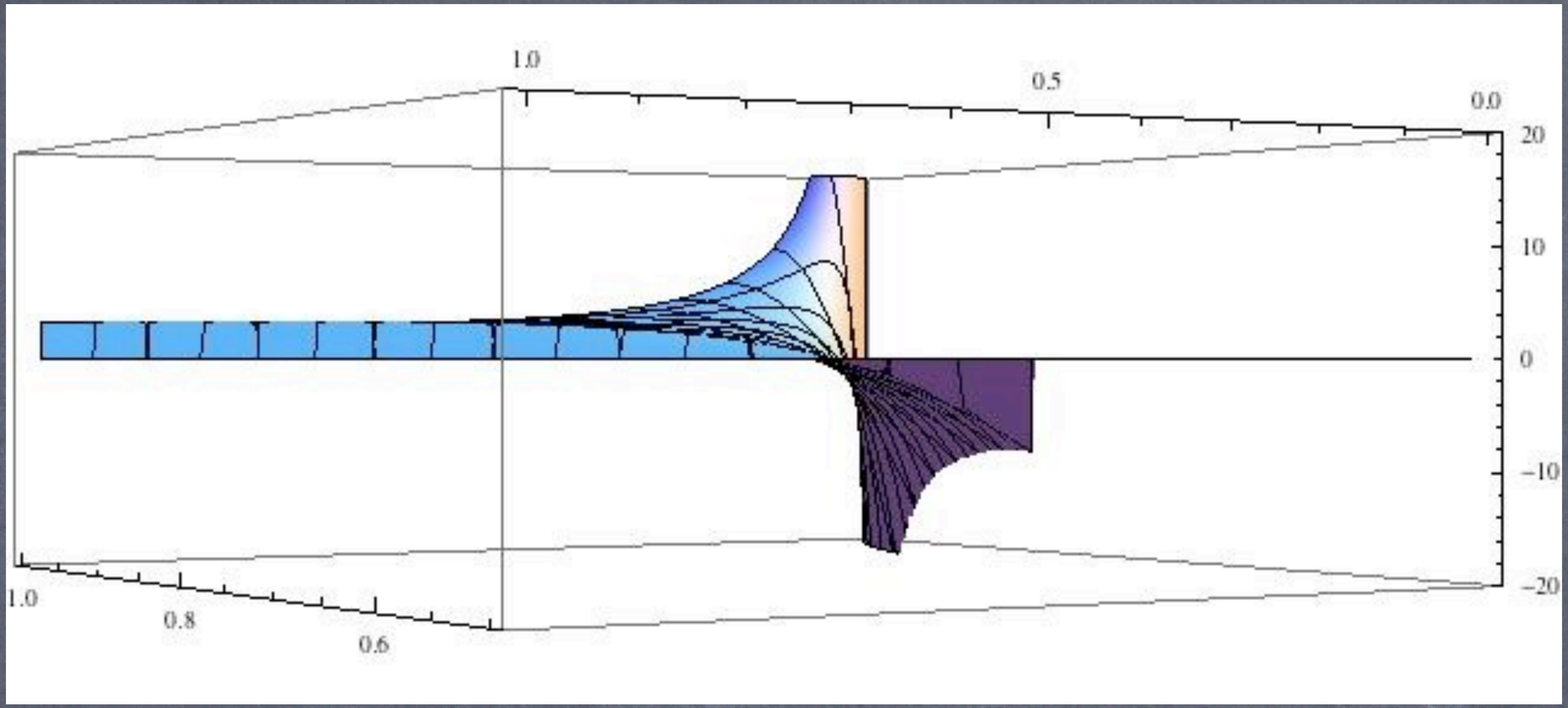

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Outlook

- Dedicated analysis for 3-pt function
- Non-adiabaticity
- Vector modes
- cubic crystal: anisotropic 3-pt function
- supersolid: add $\phi^0(t)$