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Inflation and Modified Gravity

with J. Noreña, M. Peña and M. Simonović, 1206.1083

with G. D'Amico, M. Musso, J. Noreña and E. Trincherini, 1011.3004 – JCAP

with A. Nicolis and E. Trincherini, 1007.0027 - JCAP

+ work in progress

Outline

1. Inflation is a ~ dS solution with a preferred foliation: modified gravity
2. Flow of ideas and models from late to early Universe
   - (Ghost inflation)
   - Khronon inflation - from healthy Horava gravity -
   - Galilean symmetry in inflation
3. Self acceleration in the early Universe: new way to scale invariance

Λ

Inflation IS modified gravity

Imagine this conference takes place **during primordial inflation**.

- We would measure w ~ -1 (as we do now)
- But it cannot be Λ (while it is very likely to be now)

  We know inflation ends and this does not occur in dS

  - It could be something with a sizeable $T_{\mu\nu}$. No smooth dS limit.
    Mixing relevant at Hubble scale.

    $$h_{\mu\nu}\delta T^{\mu\nu} \sim h\,\dot{\phi}^B \partial\varphi \sim M_P H h \partial\varphi$$

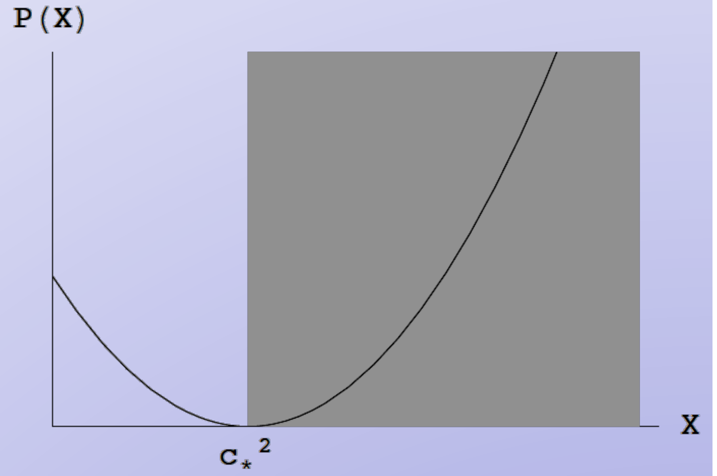
    Quintessence ~ Slow-roll inflation

  - It could have a smooth limit w → - 1
    This is what we would call a proper modification of gravity

Ghost inflation

PC, Arkani-Hamed, Mukoyama and Zaldarriaga 04

$$\mathcal{L} = \sqrt{-g}\, M^4 P(X)\ , \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$



- Spontaneous breaking of Lorentz symmetry
- Consistent derivative expansion:

$$\langle\dot\phi\rangle = M^2$$

$$S = \int d^4x\, \frac{1}{2}\dot\pi^2 - \frac{\alpha^2}{2M^2}(\nabla^2\pi)^2 - \frac{\beta}{2M^2}\dot\pi(\nabla\pi)^2 + \dots$$

$$\phi = M^2 t + \pi$$

- Non Lorentz-invariant action, standard spatial kinetic term NOT allowed

$$S = \int d^4x\sqrt{\gamma}\left[\frac{1}{8}M^4(g^{00}-1)^2 - \frac{1}{2}\tilde{M}^2K^2 - \frac{1}{2}\tilde{M}'^2K_{ij}K^{ij}\right]$$

- **EFT around the time dependent solution**

$$K_{ij} = \frac{1}{2N}\left(\dot h_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

Time translations

Inflation takes place in ~ dS $\qquad ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2$

Scale invariance of correlation function due to dilation isometry $\qquad t \to t - H^{-1}\log\lambda \quad \vec{x} \to \lambda\vec{x}$

+ invariance under time translation of inflaton dynamics: $t \to \tilde{t} = t + \mathrm{const}$

Or equivalently: $\quad \phi \to \phi + c$

$$\varphi_{\vec{k}} \to \lambda^3 \varphi_{\vec{k}/\lambda} \qquad \langle \varphi_{\vec{k}_1} \varphi_{\vec{k}_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{1}{k_1^3} F(k_1 \eta)$$

$$\langle \varphi_{\vec{k}_1} \ldots \varphi_{\vec{k}_n} \rangle = (2\pi)^3 \delta(\sum_i \vec{k}_i) F(\vec{k}_1, \ldots, \vec{k}_n)$$

Approximately valid in a given interval → Reheating

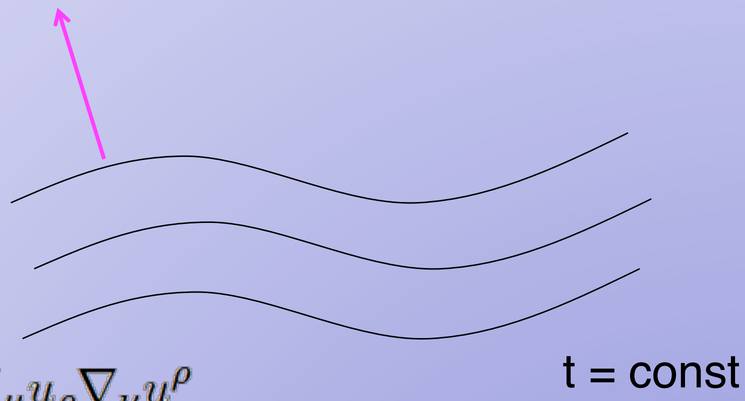
Time independence

What happens if the symmetry: $t \to \tilde{t} = t + \text{const}$ is promoted to $t \to \tilde{t}(t)$?

This is the same symmetry discussed in the healthy Horava gravity

Blas, Pujolas and Sibiryakov 10

$$u_\mu = \frac{\partial_\mu \phi}{\sqrt{-g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}}$$



$$(\nabla_\mu u^\mu)^2\,; \quad \nabla_\mu u^\nu \nabla_\nu u^\mu; \quad \nabla_\mu u^\nu \nabla^\mu u_\nu; \quad u^\mu u^\nu \nabla_\mu u_\rho \nabla_\nu u^\rho$$

By parts:

Frobenius theorem: $\nabla_\mu u^\nu \nabla_\nu u^\mu = \nabla_\mu u^\nu \nabla^\mu u_\nu + u^\mu u^\nu \nabla_\mu u_\rho \nabla_\nu u^\rho$

Only two operators + higher derivative corrections

Khronon inflation

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left(M_{Pl}^2 R - 2\Lambda - \underline{M_\lambda^2}\left(\nabla_\mu u^\mu - 3H\right)^2 + \underline{M_\alpha^2} u^\mu u^\nu \nabla_\mu u_\rho \nabla_\nu u^\rho\right)$$

Geometrically: $x_i \to \tilde{x}_i\,(\mathbf{x}, t)\,; \quad t \to \tilde{t}(t)$ $\quad\quad \cancel{g^{00}}$

$$S = \frac{M_P^2}{2}\int d^3x\, dt\sqrt{h}N\left(R^{(3)} + K_{ij}K^{ij} - \underline{\lambda}(K - 3H)^2 + \underline{\alpha} a_i a^i\right)$$

$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$a_i \equiv N^{-1}\partial_i N$$

All correlation functions depend on c_s only

Power spectrum

$$S_2 = \int d^3x\, d\eta \left(\frac{M_\alpha^2}{2} (\partial \pi')^2 - \frac{M_\lambda^2}{2} (\partial^2 \pi)^2 \right)$$

No dependence on a!

$$\pi_k(\eta) = \frac{1}{\sqrt{2k^3}} \frac{1}{\sqrt{M_\alpha M_\lambda}} e^{\pm i \frac{M_\lambda}{M_\alpha} k\eta}$$

~ Minkowski

$$c_s^2 \equiv \frac{M_\lambda^2}{M_\alpha^2} \qquad \zeta = -H\pi$$

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{1}{2k^3} \frac{H^2}{M_\alpha M_\lambda}$$

- $\omega = k/a \ll H$: the decaying mode decays, but only as 1/a (not $1/a^3$)
- ζ is conserved out of H, as its derivative is small
- Small breaking terms will become relevant

$$S = \int d^3x\, d\eta \left[\frac{M_\alpha^2}{2} (\partial \pi')^2 - \frac{M_\lambda^2}{2} (\partial^2 \pi)^2 + \beta a^2 H^2 \left(\frac{M_\alpha^2}{2} \pi'^2 - \frac{M_\lambda^2}{2} (\partial_i \pi)^2 \right) \right]$$

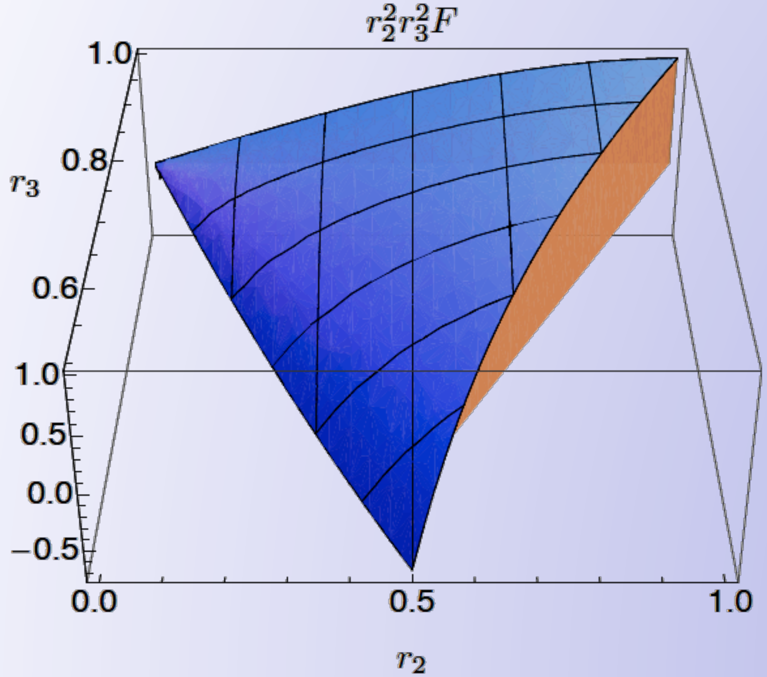
3-point function

$$S_3 = \int d^3x\, d\eta\, \frac{1}{a}\Big[M_\lambda^2\big(2\partial_i\pi'\partial_i\pi\partial^2\pi + \pi'\partial_i\partial_j\pi\partial_i\partial_j\pi\big) + M_\alpha^2\big(\pi'\partial_i\pi''\partial_i\pi - \underline{\partial_i\pi'\partial_j\pi\partial_i\partial_j\pi}\big)\Big]$$

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle \equiv (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)F_\zeta(k_1,k_2,k_3)$$

$$F_\zeta(k_1,k_2,k_3) = \frac{1}{\prod k_i^3}P_\zeta^2\Big[-\frac{k_1}{k_t^2}(k_3^2\vec{k}_1\cdot\vec{k}_2 + k_2^2\vec{k}_1\cdot\vec{k}_3) - \frac{k_1^2}{k_t}\vec{k}_2\cdot\vec{k}_3 - \underline{\frac{M_\alpha^2}{M_\lambda^2}\frac{k_1^3}{k_t^2}\vec{k}_2\cdot\vec{k}_3}\Big]$$

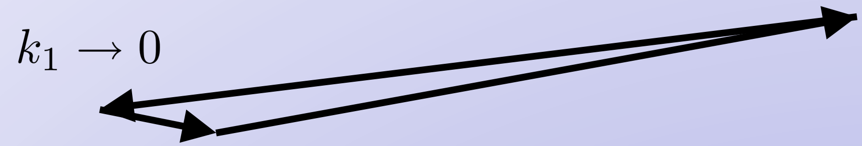
$+$ cyclic perms.



$$\propto \frac{1}{c_s^2}$$

The not-so-squeezed limit

P.C., D'Amico, Musso, Noreña 11



At lowest order in derivatives

$$S_2 + S_3 = M_{\rm Pl}^2 \int d^4x \; \epsilon a^3 \left[(1+3\zeta_B)\dot{\zeta}^2 - (1+\zeta_B)\frac{(\partial_i \zeta)^2}{a^2} \right]$$

Long mode reabsorbed by coordinate rescaling $\quad \vec{x} \to (1+\zeta_B)\vec{x}$

Corrections:

- **Time evolution of ζ is of order k²**
- Spatial derivatives will be symmetrized with the short modes, giving k²
- Constraint equations give order k² corrections

Final result: in the not-so-squeezed limit we have

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq -(2\pi)^3 \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3) P(k_1) P(k_S) \left[\frac{\mathrm{d}\ln(k_S^3 P(k_S))}{\mathrm{d}\ln k_S} + \mathcal{O}\left(\frac{k_1^2}{k_S^2}\right) \right]$$

Conformal consistency relations

(Assuming zero tilt for simplicity)

PC, Noreña, Simonović 12
Hinterbichler, Hui and Khoury 12
Kheagias, Riotto 12
in progress by Goldberger, Hinterbichler, Hui, Khoury, Nicolis

$$\langle \zeta_{\vec{q}}\, \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \dots \zeta_{\vec{k}_n} \rangle' \overset{q \to 0}{=} -\frac{1}{2} P(q) q^i D_i \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \dots \zeta_{\vec{k}_n} \rangle' + \mathcal{O}(q/k)^2$$

$$\text{with} \quad q^i D_i \equiv \sum_{a=1}^{n} \left[6 \vec{q} \cdot \vec{\partial}_{k_a} - \vec{q} \cdot \vec{k}_a \vec{\partial}^2_{k_a} + 2 \vec{k}_a \cdot \vec{\partial}_{k_a} (\vec{q} \cdot \vec{\partial}_{k_a}) \right]$$

2- and 3-pf only depends on moduli and $q^i D_i$ reduces to: $$\sum_{a=1}^{n} \vec{q} \cdot \vec{k}_a \left[\frac{4}{k_a} \frac{\partial}{\partial k_a} + \frac{\partial^2}{\partial k_a^2} \right]$$

All conformal consistency relations will be violated, due to the slow decay of decaying mode

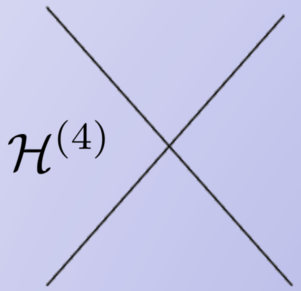
4-point function

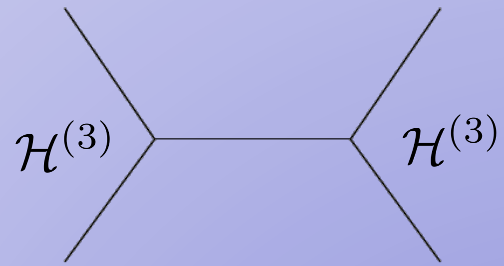
$$
\begin{aligned}
S_\alpha^{(4)} = M_\alpha^2 \int \mathrm{d}^3x\,\mathrm{d}\eta \Big(& \frac{H^2}{2}\pi'\pi'(\partial\pi)^2 - \frac{H}{a}\left((\partial\pi)^2\partial_i\pi\partial_i\pi' + \pi'\pi''(\partial\pi)^2 + \pi'\partial_i\pi\partial_j\pi\partial_i\partial_j\pi\right) \\
& -\frac{3H}{a}\pi'\pi'\partial_i\pi\partial_i\pi' + \frac{1}{2a^2}\left(\pi''\pi''(\partial\pi)^2 + 6\pi'\pi''\partial_i\pi\partial_i\pi' + 3\pi'\pi'\partial_i\pi'\partial_i\pi' + 3\partial_i\pi\partial_j\pi\partial_i\pi'\partial_j\pi'\right) \\
& +\frac{1}{a^2}\pi''\partial_i\pi\partial_j\pi\partial_i\partial_j\pi + \frac{3}{a^2}\pi'\partial_i\pi\partial_j\pi'\partial_i\partial_j\pi + \frac{1}{a^2}(\partial\pi)^2(\partial\pi')^2 + \underline{\frac{1}{2a^2}\partial_i\pi\partial_j\pi\partial_i\partial_l\pi\partial_j\partial_l\pi}\Big)\ .
\end{aligned}
$$

$$\mathcal{H}(P,\pi) = P\pi' - \mathcal{L}(P,\pi)$$

$$P = \frac{\partial\mathcal{L}}{\partial\pi'} = -M_\alpha^2\partial^2\pi' + \frac{M_\alpha^2}{2a}\partial^2(\partial_i\pi)^2$$

$$\mathcal{H}^{(4)} = \frac{1}{c_s^4}\times 0$$





$$
\begin{aligned}
\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\rangle_c = & (2\pi)^3\delta\big(\sum\vec{k}_a\big)P_\zeta^3\frac{M_\alpha^4}{M_\lambda^4}\frac{1}{\prod_a k_a^3}\frac{1}{4p^3(p+k_1+k_2)^2} \\
\times \Bigg\{ & \left(p^6(\vec{k}_1\cdot\vec{k}_2)(\vec{k}_3\cdot\vec{k}_4) - 2p^3k_1^3(\vec{p}\cdot\vec{k}_2)(\vec{k}_3\cdot\vec{k}_4)\right)\left[\frac{1}{4(p+k_3+k_4)^2} - \frac{1}{2k_t^3}(k_t + 2(p+k_1+k_2))\right] \\
& + \left(2p^3k_3^3(\vec{k}_1\cdot\vec{k}_2)(\vec{p}\cdot\vec{k}_4) - 4k_1^3k_3^3(\vec{p}\cdot\vec{k}_2)(\vec{p}\cdot\vec{k}_4)\right)\left[\frac{1}{4(p+k_3+k_4)^2} + \frac{1}{2k_t^3}(k_t + 2(p+k_1+k_2))\right]\Bigg\}
\end{aligned}
$$

Galilean symmetry

Nicolis, Rattazzi, Trincherini 08

Shift symmetry on the gradient of a scalar

$$\phi \to \phi + b_\mu x^\mu + c$$

- What happens if I impose this on the inflaton?
- In particular can I get new shapes wrt $\dot{\pi}^3$ and $\dot{\pi}(\nabla\pi)^2$?
- Neat example of the EFT of inflation

Lowest order

Lowest derivative galileons give 2nd order eom

$$\mathcal{L} \sim (\partial\phi)^2(\partial^2\phi)^n\,, \quad n \leq 3 \qquad \longrightarrow \qquad (\partial^2\phi)^{n+1}$$

Use these operators for inflaton Lagrangian

Burrage, De Rham, Seery, Tolley 10

Non-Gaussianity given by cubic operators with 4 derivatives:

$$\ddot{\pi}\dot{\pi}^2\,, \quad \dot{\pi}^2\nabla^2\pi\,, \quad \dot{\pi}\nabla\dot{\pi}\nabla\pi\,,\ \ddot{\pi}(\nabla\pi)^2\,,\ \nabla^2\pi(\nabla\pi)^2$$

.... but using

$$\ddot{\pi} + 3H\dot{\pi} - c_s^2\nabla^2\pi/a^2 = 0$$

all these operators are equivalent to the ones with 3 derivatives!

Gruzinov, Mirbabayi, Senatore, Zaldarriaga

Non renormalization

Luty, Porrati and Rattazzi 03

The leading G. operators $\mathcal{L} \sim (\partial\phi)^2(\partial^2\phi)^n\ , \quad n \leq 3$ are not renormalized

Consistent to set them to zero

We consider operators with two derivatives on each field $(\partial^2\phi)^n$

WAIT! But now the EOM are of higher order! Ghosts!??!

No, we are going to impose the symmetry on the EFT of inflation
and treat higher derivative terms perturbatively

Only an EFT of inflation

It is the theory of **small** perturbations around an inflating background

We probe $\phi_0(t+\pi(t,\vec{x}))$ $H\pi = -\zeta \simeq 10^{-5}$ and $E \sim H$

Usually the regime of validity extends to much larger values of π

We are interested in

$$M_{\rm Pl}^2 \dot{H} (\partial\pi)^2 + M(\partial^2\pi)^3 + \dots$$

Say with cubic term to be of order $\sim 10^{-3}$ wrt the kinetic one.
Higher derivative terms are small and are (as usual) treated perturbatively

The theory will break down for large $H\pi$! **In particular $\pi \sim t$ is not within the EFT**

The action

Useful to introduce a "fake" scalar which linearly realizes Lorentz symmetry

$$\psi(t,\vec{x}) \equiv t + \pi(t,\vec{x})$$

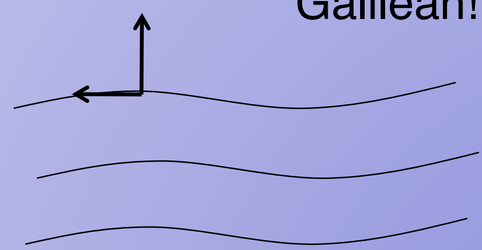
Building block: $\nabla_\mu \nabla_\nu \psi \equiv \Psi$

Build operators at a given order in π in terms of products of

$$[\Psi\Psi\ldots\Psi]\ldots[\Psi\Psi\ldots\Psi]$$

- Identify tadpoles
- Identify independent operators at each order
- Geometrical constructions does not help
- Mixing with gravity is subleading in slow-roll

Projectors are not Galilean!

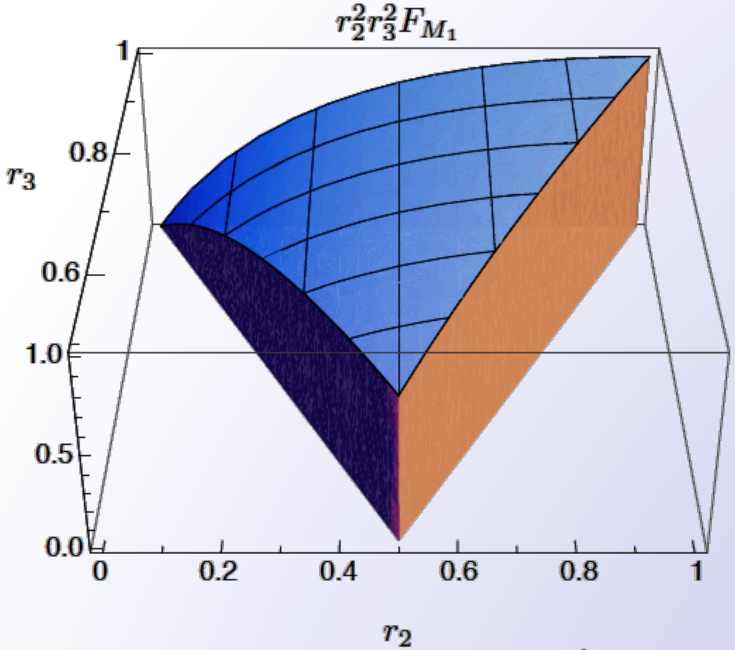


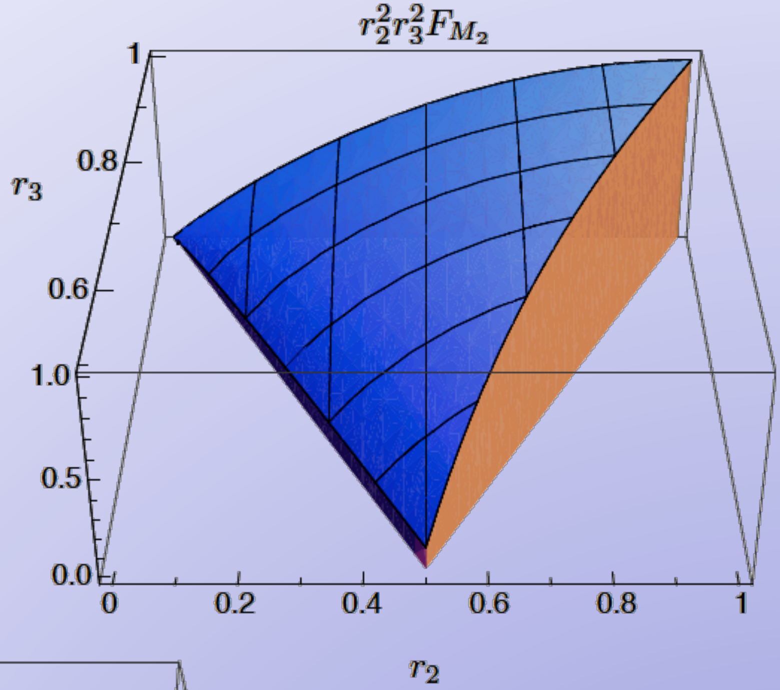
The cubic action

Final action has only 3 independent cubic operators:

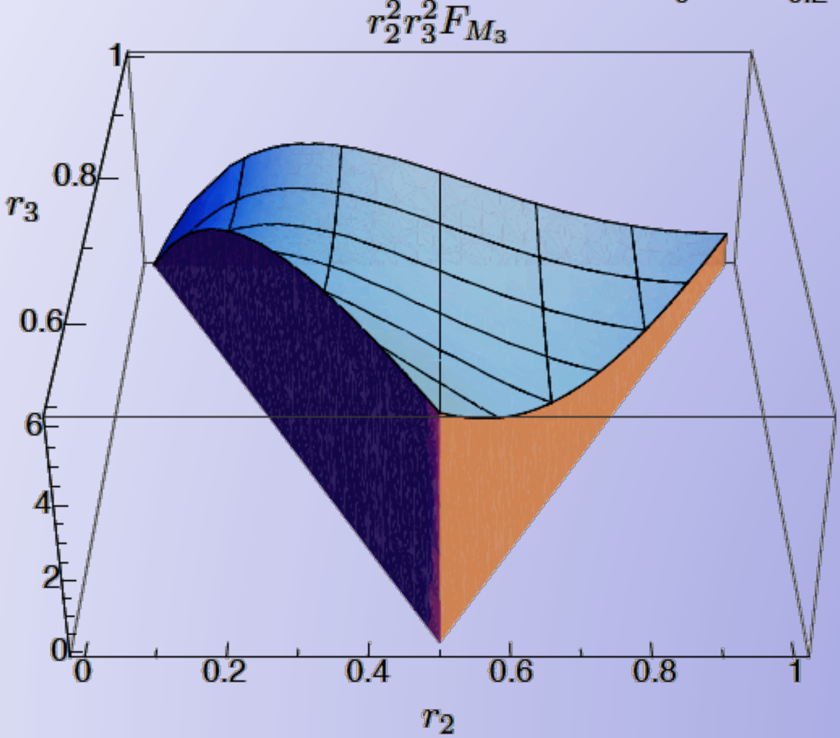
$$S = \int \mathrm{d}^4x a^3 \Big[- M_{\mathrm{Pl}}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + M_1 \ddot{\pi}^3 + M_2\, \ddot{\pi} \frac{(\partial_i \dot{\pi} - H \partial_i \pi)^2}{a^2}$$
$$+ M_3 \left(\ddot{\pi} \frac{(\partial_i \partial_j \pi)^2}{a^4} - 2H \dot{\pi} \ddot{\pi}^2 + 3 H^3 \dot{\pi}^3 \right) \Big]$$

Shapes





Surfing NG!



Similar to Bartolo etal 10

4-point function

Standard EFT: $\mathcal{L}_{1-\partial} = (\partial\pi_c)^2 + \frac{1}{\Lambda^2}(\partial\pi_c)^3 + \frac{1}{\Lambda^4}(\partial\pi_c)^4 + \ldots$

$$\mathrm{NG}_3 \equiv \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^{3/2}} \simeq \frac{\mathcal{L}_3}{\mathcal{L}_2}\bigg|_{E\sim H} \simeq \left(\frac{H}{\Lambda}\right)^2 \qquad \mathrm{NG}_4 \equiv \frac{\langle\zeta^4\rangle}{\langle\zeta^2\rangle^2} \simeq \frac{\mathcal{L}_4}{\mathcal{L}_2}\bigg|_{E\sim H} \simeq \left(\frac{H}{\Lambda}\right)^4$$

$$\Longrightarrow \quad \mathrm{NG}_4 \sim \mathrm{NG}_3^2$$

Non-minimal galilean action: $\mathcal{L} = (\partial\pi_c)^2 + \frac{1}{\Lambda^2}(\partial^2\pi_c)^2 + \frac{1}{\Lambda^5}(\partial^2\pi_c)^3 + \frac{1}{\Lambda^8}(\partial^2\pi_c)^4 + \ldots$

$$\mathrm{NG}_3 \simeq \left(\frac{H}{\Lambda}\right)^5 \qquad \mathrm{NG}_4 \simeq \left(\frac{H}{\Lambda}\right)^8$$

$$\Longrightarrow \quad \mathrm{NG}_4 \sim \mathrm{NG}_3^{8/5}$$

For a given cubic NG our model predicts a larger 4 pt function

$f_{\mathrm{NL}} = 100$ implies $\tau_{\mathrm{NL}} \sim 10^4$ vs. $\tau_{\mathrm{NL}} \sim 10^5$

Possible to impose an approximate π -> -π symmetry to make 4pf dominant

Self acceleration

Deffayet 00
Nicolis, Rattazzi, Trincherini 09

The present acceleration might be explained as **self-acceleration**, as in DGP

Matter is coupled with a scalar that describes an effective accelerating metric

$$\mathcal{S}_\pi = \int d^4x\ f^2 \left[e^{2\pi}(\partial\pi)^2 + \frac{2}{3}\frac{1}{H_0^2}(\partial\pi)^2\Box\pi + \frac{1}{3}\frac{1}{H_0^2}(\partial\pi)^4 \right]$$

$$e^{\pi_{\rm dS}} = -\frac{1}{H_0 t} \qquad -\infty < t < 0 \qquad g_{\mu\nu}^{(\pi)} \equiv e^{2\pi(x)}\eta_{\mu\nu}$$ is "de Sitter"

Self-inflation: fake de Sitter

Rubakov 09
Nicolis, Rattazzi, Trincherini 09
PC, Nicolis, Trincherini 10
Hinterbichler, Khoury 11
Hinterbichler, Joyce, Khoury 12
PC, Joyce, Khoury and Simonović in progress

This might happen also in primordial cosmology.

We "observe" de Sitter as a scale-invariant spectrum of perturbations

If a test scalar σ couples to the π "metric": **correlation functions are the same as in dS**

If the theory is conformal invariant, SO(4,2), quite natural to find SO(4,1) invariant solutions

SO(4,2) → SO(4,1)

Symmetry pattern is different from inflation and gives some distinct predictions in higher order correlators

Conclusions

1. For sure we have an accelerating phase which is not Λ: **inflation**
2. Models of MG → models of inflation
3. Self-inflation? Perturbations as in de Sitter space
4. Probe: **higher order correlation functions**