

International Spring School of the GDR PH-QCD

QCD prospects for future ep and eA colliders

LECTURERS:

Alfred Mueller High energy ep and eA scattering

Piet Mulders TMDs: theory and phenomenology

George Sterman Factorization of hard processes

Marc Vanderhaeghen GPDs and spatial structure of hadrons



ORSAY 4-8 June 2012

Amphi I, Laboratoire de Physique Théorique,
bâtiment 210, Université d'Orsay

<http://indico.in2p3.fr//event/QCD-ep-eA-colliders>

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Sponsors:



GPDs & spatial structure of hadrons

Part 2

Marc Vanderhaeghen
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Outline

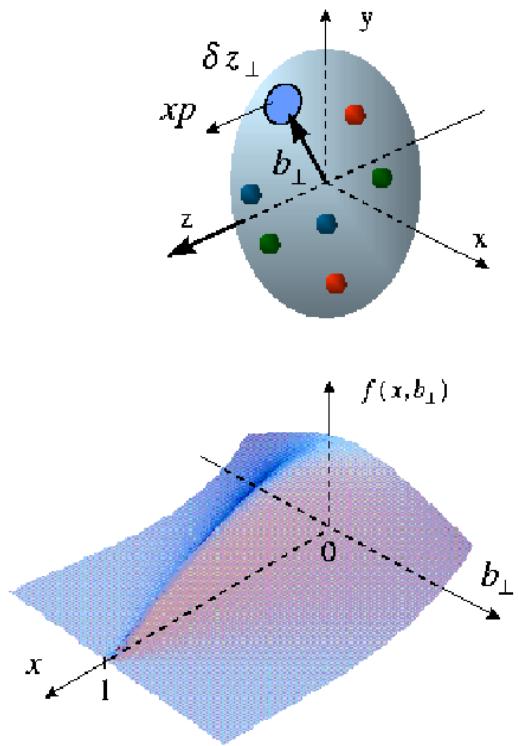
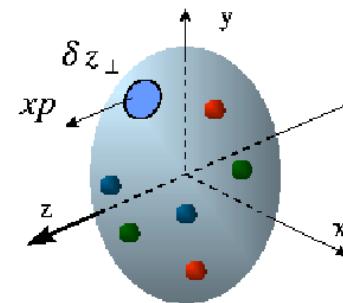
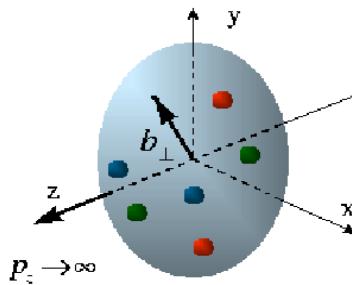
→ What is the physics contained in GPDs ?

- GPDs: basic definitions and properties
- 3D imaging of the nucleon: link between elastic nucleon Form Factors and GPDs, connection between longitudinal momentum and transverse position
- Generalizations: Wigner distributions
- GPDs and nucleon spin
- Hard exclusive processes : DVCS, hard meson production, $N \rightarrow \Delta$ DVCS, ...

Reviews on GPDs

- > Goeke, Polyakov, Vdh : *Prog.Part.Nucl.Phys.* 47, 401 (2001)
- > Diehl : *Phys.Rept.* 388, 41 (2003)
- > Ji : *Ann.Rev.Nucl.Part.Sci* 54, 413 (2004)
- > Belitsky, Radyushkin : *Phys.Rept.* 418, 1 (2005)
- > Boffi, Pasquini : *Riv.Nuovo.Cim.* 30, 387(2007)

Generalized Parton Distributions (GPDs): 3D picture of nucleon

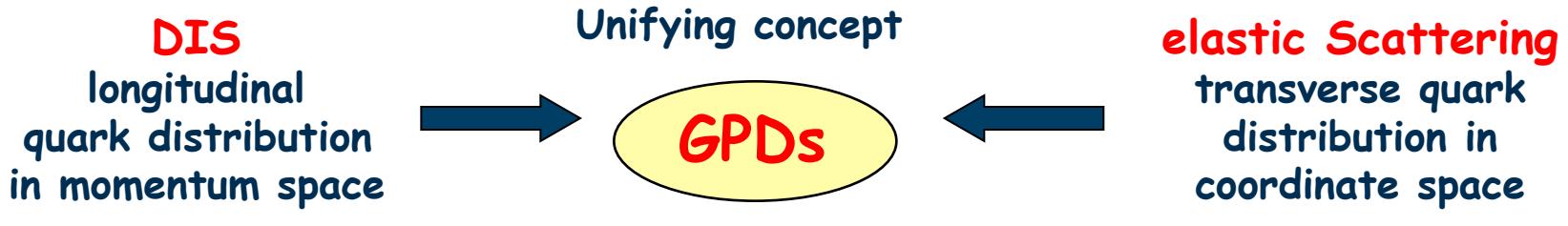


Elastic Scattering
transverse quark
distribution in
coordinate space

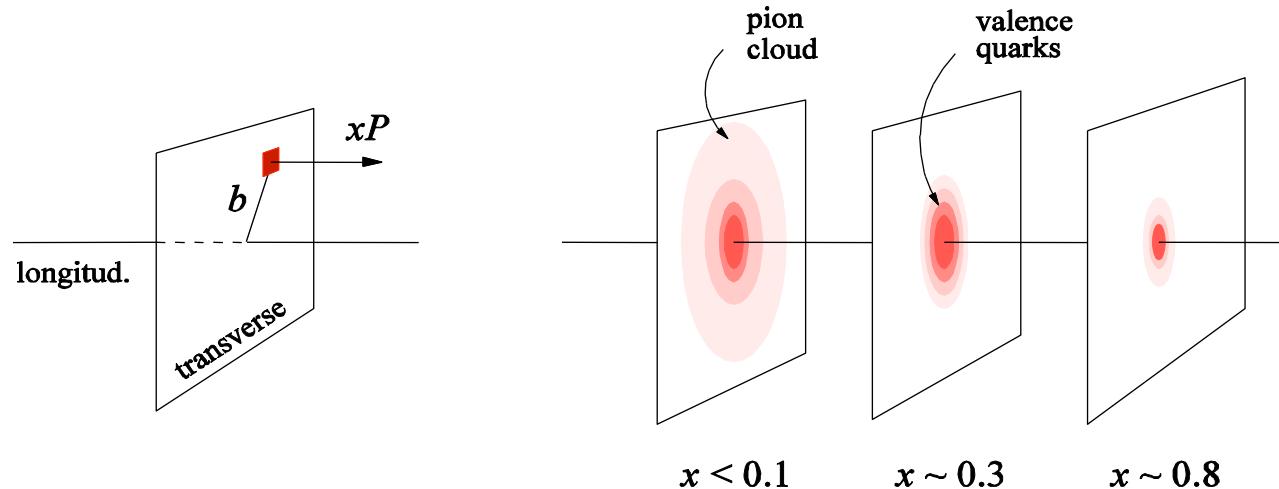
DIS
longitudinal
quark distribution
in momentum space

DES (GPDs)
fully-correlated
quark distribution in
both coordinate and
momentum space

Generalized Parton Distributions (GPDs): 3D picture of nucleon



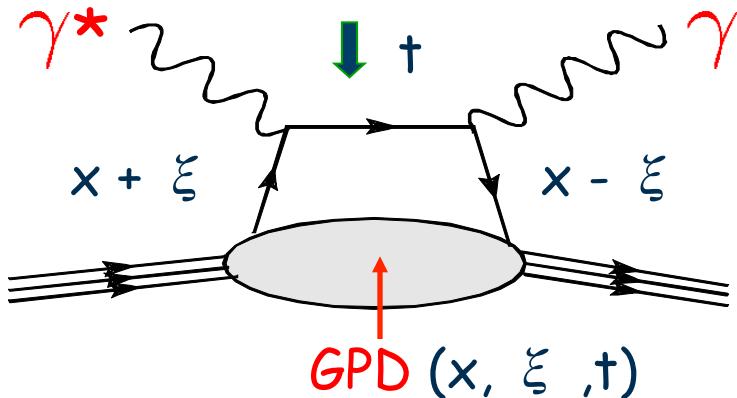
fully-correlated
quark distributions in **both**
coordinate and momentum space



Burkardt (2000, 2003),
Belitsky, Ji, Yuan (2004)

QCD factorization: tool to access GPDs

$Q^2 \gg 1 \text{ GeV}^2$

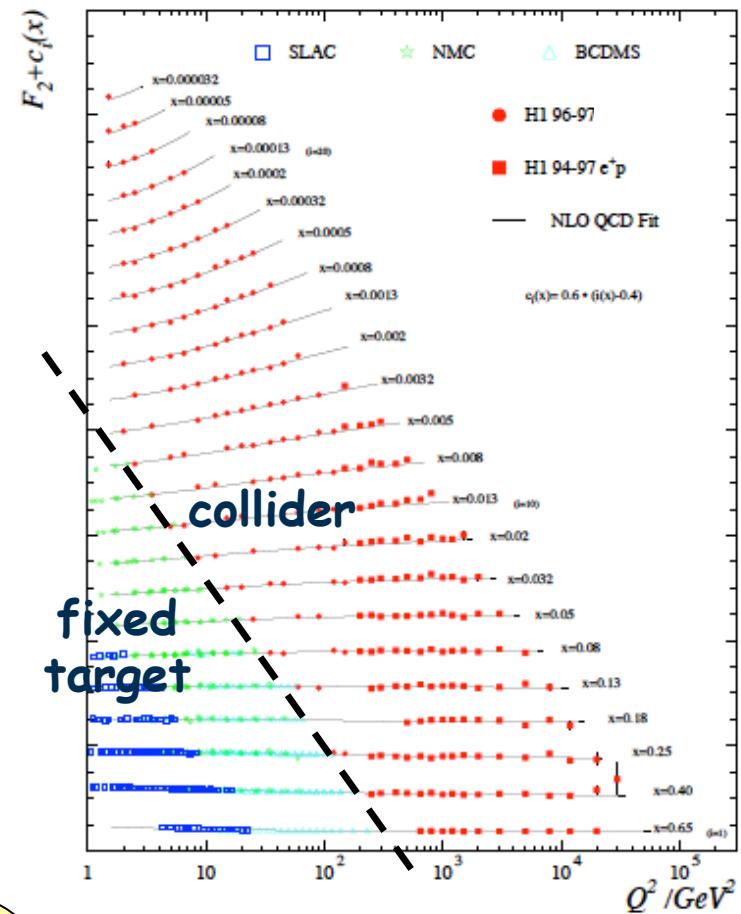


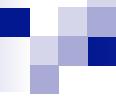
→ at large Q^2 : **QCD factorization theorem** :
hard exclusive process described by **GPDs**
model independent !

Müller et al. (1994),
Ji (1995), Radyushkin (1995),
Collins, Frankfurt, Strikman (1996)

→ **KEY** Q^2 leverage required to test
QCD scaling → **e N collider**

world data on proton F_2

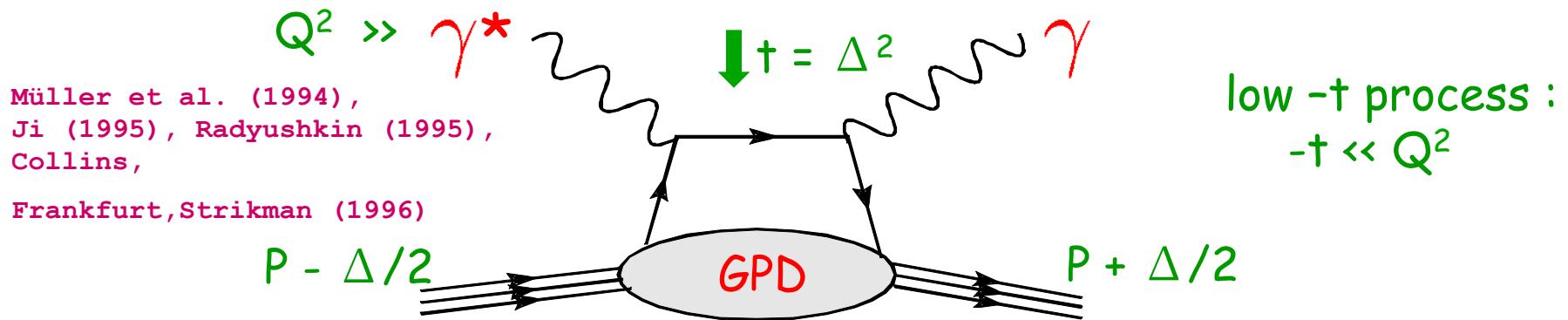




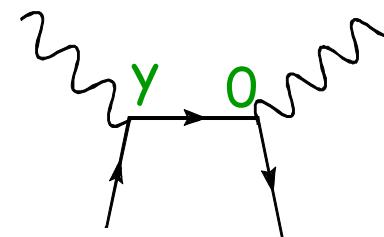
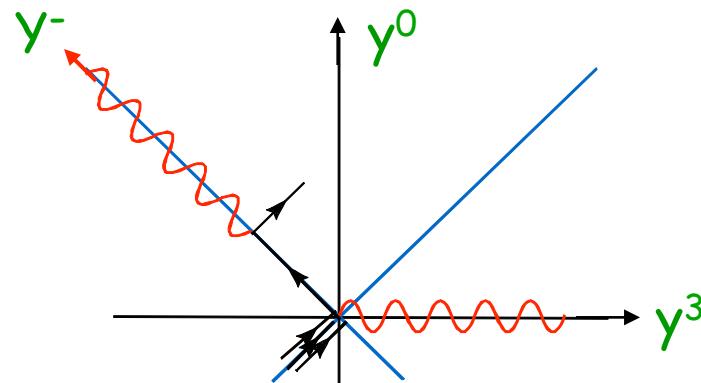
GPDs: definitions

hard exclusive processes: factorization

→ Deeply virtual Compton scattering (DVCS)

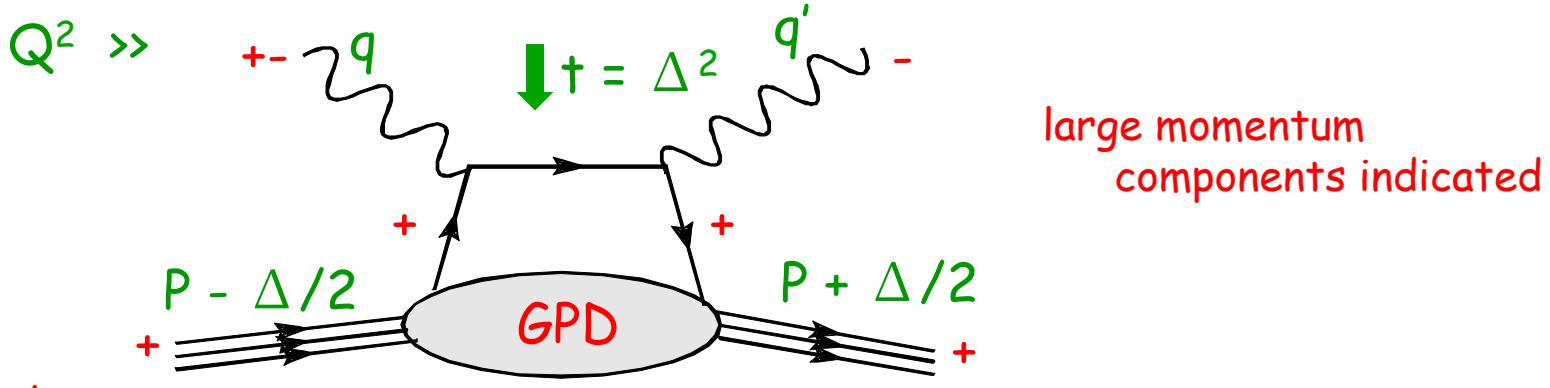


→ large $Q^2 \rightarrow$ light-cone dominated process with $y^+ = 0, y_\perp = 0$



generalized probe

DVCS: kinematics



light-cone vectors:

$$\tilde{p}^\mu = P^+ (1, 0, 0, +1) \quad \& \quad n^\mu = (1, 0, 0, -1) / (2 P^+)$$

$$\text{with } \tilde{p} \cdot n = 1$$

in Bjorken limit:

$$Q^2 \gg m^2, -t$$

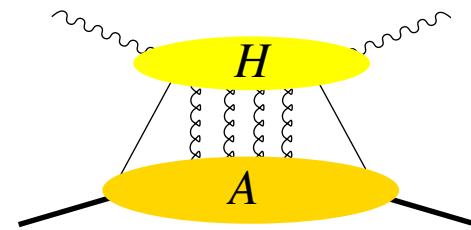
$P^\mu \equiv \frac{1}{2}(p^\mu + p'^\mu) = \tilde{p}^\mu + \frac{\bar{m}^2}{2} n^\mu$	$\rightarrow \tilde{p}^\mu$
$q^\mu = -(2\xi') \tilde{p}^\mu + \left(\frac{Q^2}{4\xi'}\right) n^\mu$	$\rightarrow -(2\xi) \tilde{p}^\mu + \left(\frac{Q^2}{4\xi}\right) n^\mu$
$\Delta^\mu \equiv p'^\mu - p^\mu = -(2\xi) \tilde{p}^\mu + (\xi \bar{m}^2) n^\mu + \Delta_\perp^\mu$	$\rightarrow -(2\xi) \tilde{p}^\mu + \Delta_\perp^\mu$
$q'^\mu \equiv q^\mu - \Delta^\mu = -2(\xi' - \xi) \tilde{p}^\mu + \left(\frac{Q^2}{4\xi'} - \xi \bar{m}^2\right) n^\mu - \Delta_\perp^\mu$	$\rightarrow \left(\frac{Q^2}{4\xi}\right) n^\mu - \Delta_\perp^\mu$

$$\bar{m}^2 \equiv m_N^2 - \Delta^2/4$$

$$\xi, \xi' \rightarrow \frac{x_B/2}{1 - x_B/2}$$

Wilson lines

- exchange of more than 2 partons between hard scattering process (H) and soft amplitude (A) is suppressed **except** for gluons with polarization A^+



- resummation of gluon exchange to all orders gives rise to **Wilson line** between $\bar{q}(a)$ and $q(b)$

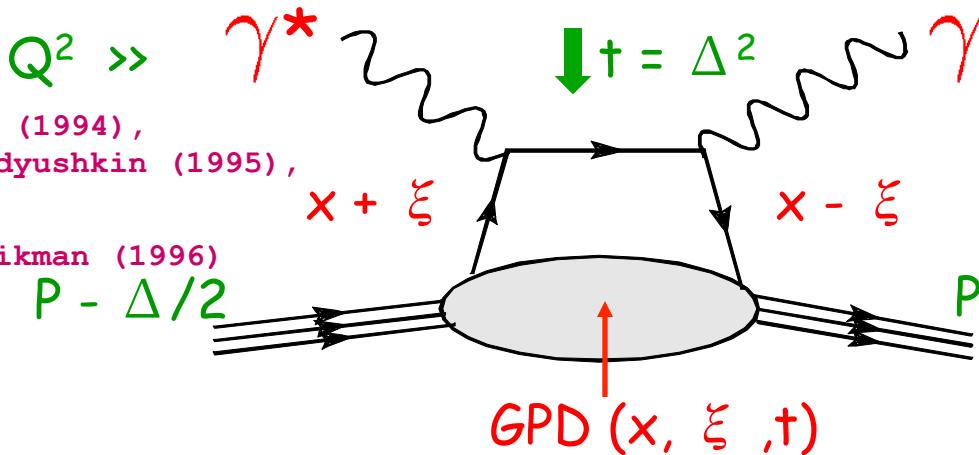
$$W[a, b] = P \exp \left[ig \int_a^b dy^- A^+(y) \right]_{y^+ = 0, y_\perp = 0}$$

- **gauge invariant bilocal operator:**

$$\bar{q}(a) \Gamma W[a, b] q(b) \quad \text{with} \quad \Gamma = \gamma^+ \text{ or } \gamma^+ \gamma_5$$

- for convenience, choose **light-cone gauge**: $A^+ = 0$ in which $W[a,b] = 1$

GPDs: definitions



low -t process :
 $-t \ll Q^2$

$$\Delta^+ = -(2 \xi) P^+$$

light-cone dominance: $n^\mu (1, 0, 0, -1) / (2 P^+)$

$$\begin{aligned} & \frac{P^+}{2\pi} \int dy^- e^{ixP^+y^-} \langle P + \frac{\Delta}{2} | \bar{q}\left(-\frac{y}{2}\right) \gamma \cdot n q\left(\frac{y}{2}\right) | P - \frac{\Delta}{2} \rangle_{y^+=0, y_\perp=0} \\ &= \bar{N} \left\{ H(x, \xi, t) \gamma \cdot n + E(x, \xi, t) i\sigma^{\mu\nu} \frac{\Delta_\nu}{2M} n_\mu \right\} N \end{aligned}$$

$$\begin{aligned} & \frac{P^+}{2\pi} \int dy^- e^{ixP^+y^-} \langle P + \frac{\Delta}{2} | \bar{q}\left(-\frac{y}{2}\right) \gamma \cdot n \gamma_5 q\left(\frac{y}{2}\right) | P - \frac{\Delta}{2} \rangle_{y^+=0, y_\perp=0} \\ &= \bar{N} \left\{ \tilde{H}(x, \xi, t) \gamma \cdot n \gamma_5 + \tilde{E}(x, \xi, t) \gamma_5 \frac{\Delta^\mu}{2M} n_\mu \right\} N \end{aligned}$$

properties of GPDs (I)

- forward limit : ordinary parton distributions

$$H^q(x, \xi = 0, t = 0) = q(x) \quad \text{unpolarized quark distr}$$

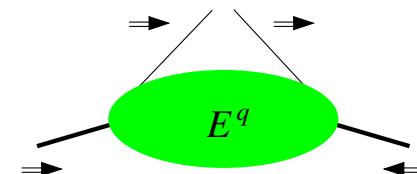
$$\tilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x) \quad \text{polarized quark distr}$$

$x > 0$: quarks, $x < 0$: anti-quarks

- analogous relation for gluons: $H^g(x, \xi = 0, t = 0) = x g(x)$

- E^q, \tilde{E}^q : do NOT appear in DIS \rightarrow new information

- $E^q \neq 0$ requires orbital angular momentum between partons ($\Delta L^z = \pm 1$)

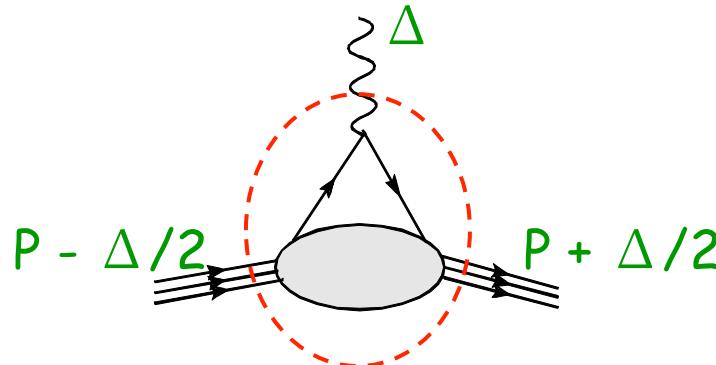


- Time-reversal invariance: GPDs are even functions in ξ

$$H^q(x, \xi, t) = H^q(x, -\xi, t) \quad \text{same for other distr}$$

properties of GPDs (II)

- first moments : nucleon **electroweak form factors**



ξ -independence :
Lorentz invariance

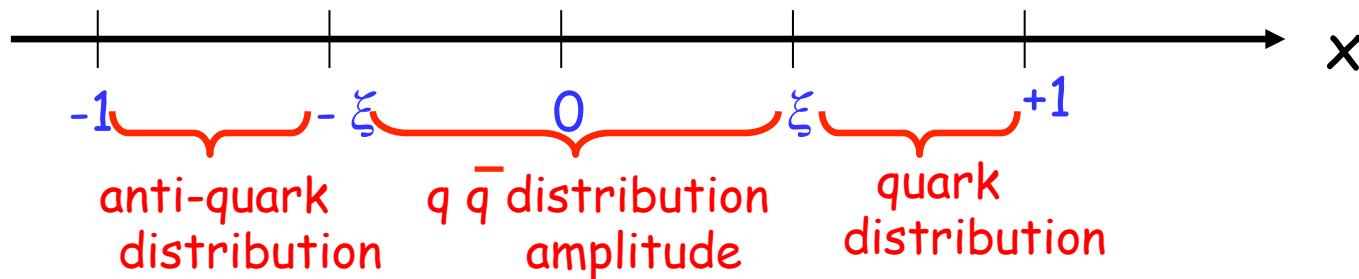
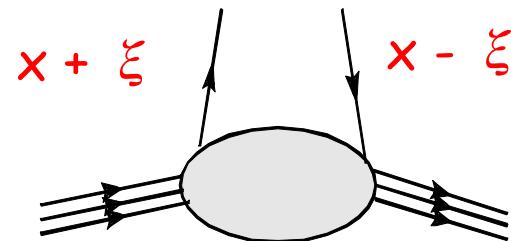
$\int_{-1}^1 dx H^q(x, \xi, t) = F_1^q(t)$ $\int_{-1}^1 dx E^q(x, \xi, t) = F_2^q(t)$ $\int_{-1}^1 dx \tilde{H}^q(x, \xi, t) = G_A^q(t)$ $\int_{-1}^1 dx \tilde{E}^q(x, \xi, t) = G_P^q(t)$	Dirac Pauli axial pseudo-scalar
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- x^n Mellin moments of GPDs correspond to local operators
→ can be calculated in lattice QCD
- Lorentz invariance: polynomiality condition in ξ (series terminates)

$$\int_{-1}^1 dx x^{n-1} H^q(x, \xi, t) = \sum_{k=0}^n (2\xi)^k A_{n,k}^q(t)$$

n even → terms up to n
n odd → terms up to n - 1

GPDs: x and ξ -dependencies

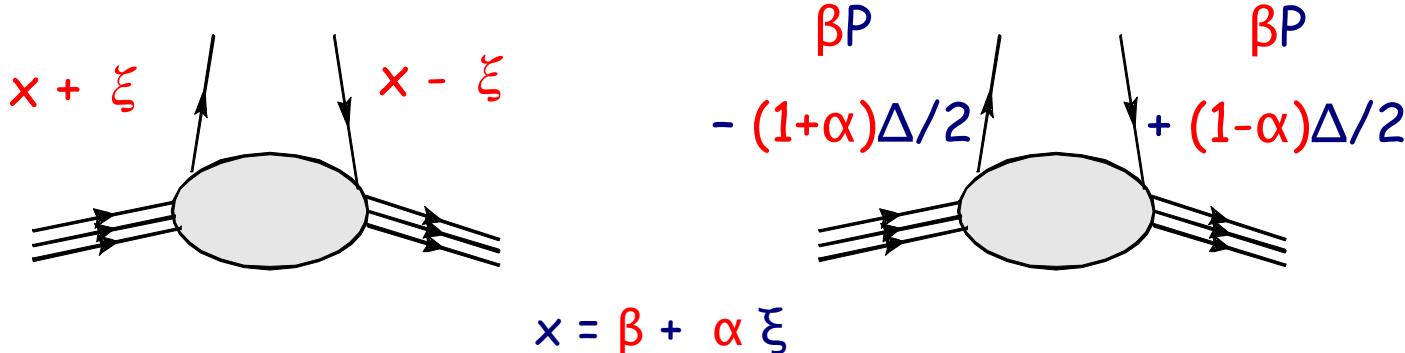


ERBL evolution as DGLAP evolution as
for meson DAs for PDFs

GPDs: x and ξ -dependencies

- double distributions

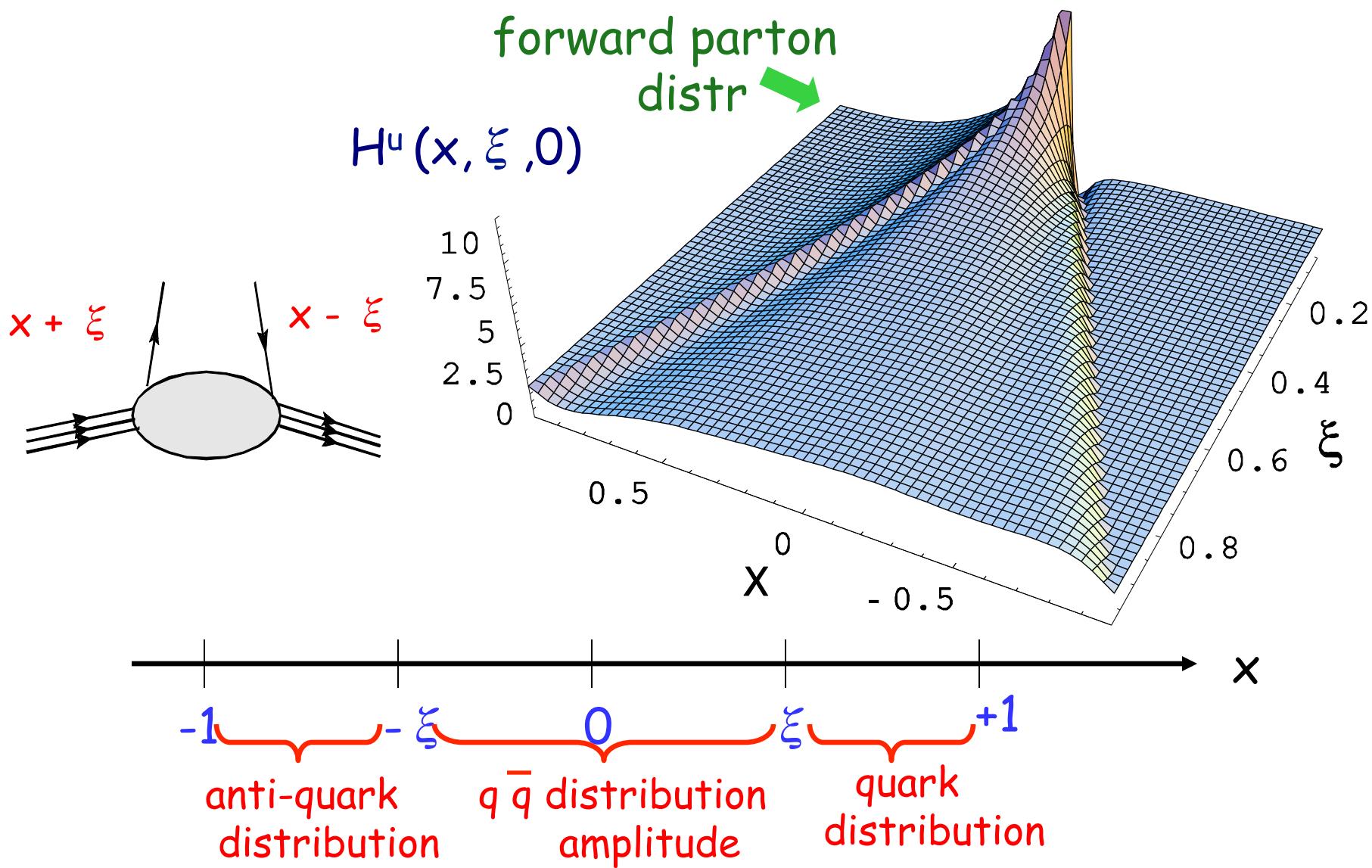
Müller et al. (1994)
Radyushkin (1996, 1997)

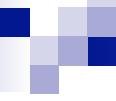


$$H(x, \xi, t) = \int d\alpha \int d\beta \delta(x - \beta - \alpha\xi) f(\beta, \alpha, t)$$

- forward limit $\int d\alpha f(\beta, \alpha, 0) = q(\beta)$
- physics in β : like a parton distribution function
- physics in α : like a meson distribution amplitude
- to satisfy polynomiality (highest power in ξ) \rightarrow need to add "D-term"

GPDs: x and ξ -dependencies





GPDs and nucleon imaging

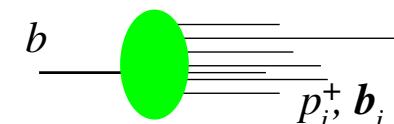
GPDs: t -dependence \rightarrow impact parameter

→ State with definite light-cone momentum p^+ & transverse position \vec{b}_\perp

$$|p^+, \vec{b}_\perp\rangle = \int \frac{d^2 \vec{p}_\perp}{(2\pi)^2} e^{-i \vec{b}_\perp \cdot \vec{p}_\perp} |p^+, \vec{p}_\perp\rangle$$

such a state localizes proton in 2dim

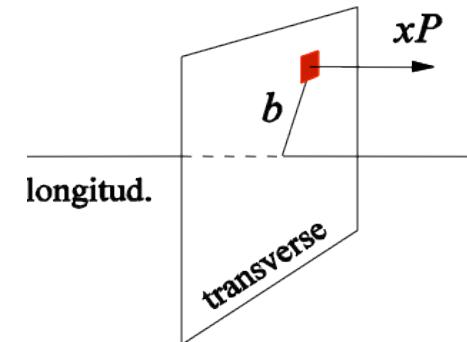
in frame where proton moves fast \rightarrow parton interpretation



→ for $\xi = 0 \longrightarrow t = -\Delta_\perp^2$

$$H^q(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-i \vec{b}_\perp \cdot \vec{\Delta}_\perp} H^q(x, \xi = 0, -\vec{\Delta}_\perp^2)$$

Burkardt (2000, 2003)



2dim Fourier transform of GPDs :

simultaneous distributions of quarks w.r.t. longitudinal momentum $x P$ and transverse position b

GPDs: t -dependence

modified Regge parametrization :

$$H^q(x, 0, t) = q_v(x) x^{-\alpha'_1 (1-x) t}$$

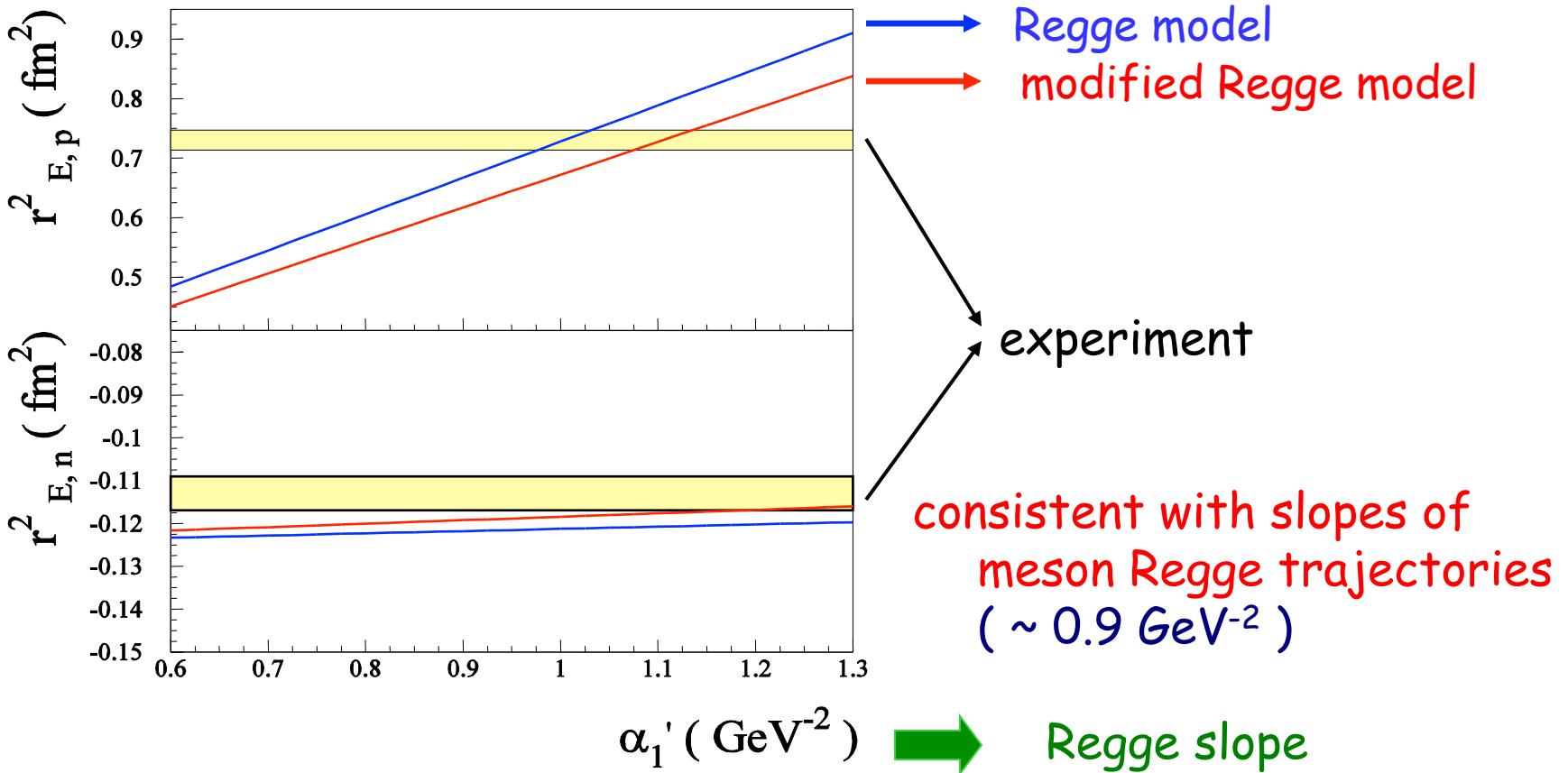
$$E^q(x, 0, t) = \frac{\kappa_q}{N_q} (1 - x)^{\eta_q} q_v(x) x^{-\alpha'_2 (1-x) t}$$

- Input : forward parton distributions at $\mu^2 = 1 \text{ GeV}^2$ (MRST2002 NNLO)
- Drell-Yan-West relation : $\exp(-\alpha' t) \rightarrow \exp(-\alpha' (1-x)t)$: Burkardt (2001)
- parameters :
 - regge slopes : $\alpha'_1 = \alpha'_2$ determined from rms radii
 - η_u, η_d determined from F_2 / F_1 at large $-t$
- future constraints : moments from lattice QCD

proton and neutron charge radii

$$r_{E,p}^2 = r_{1,p}^2 + \frac{3}{2} \frac{\kappa_p}{M_N^2} \quad r_{1,p}^2 = -6 \alpha_1' \int_0^1 dx \left\{ e_u u_v(x) + e_d d_v(x) \right\} \ln x$$

$$r_{E,n}^2 = r_{1,n}^2 + \frac{3}{2} \frac{\kappa_n}{M_N^2} \quad r_{1,n}^2 = -6 \alpha_1' \int_0^1 dx \left\{ e_u d_v(x) + e_d u_v(x) \right\} \ln x$$



connection FF@large Q^2 \leftrightarrow GPD@large x

$$\begin{aligned} I &= \int_0^1 dx (1-x)^\nu e^{\alpha' Q^2 (1-x) \ln x} = \int_0^1 dx e^{\nu \ln(1-x) + \alpha' Q^2 (1-x) \ln x} \\ &= \int_0^1 dx e^{f(x, Q^2)} \end{aligned}$$

at large Q^2 : integral dominated by maximum of $f(x, Q^2)$, remainder region is exp. suppressed (method of steepest descent)

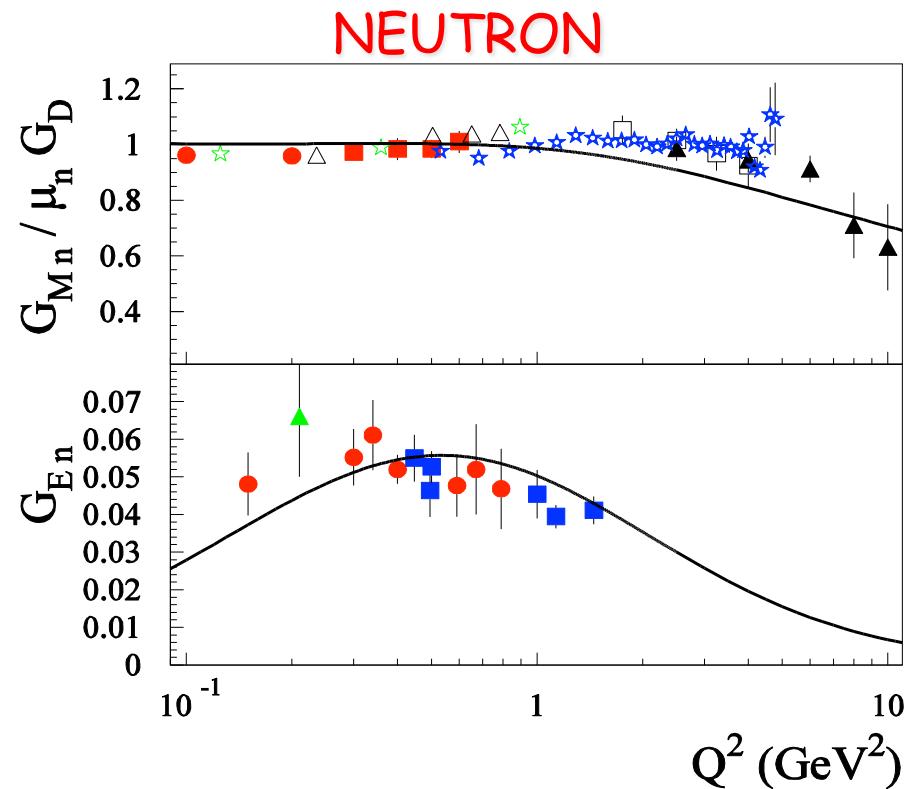
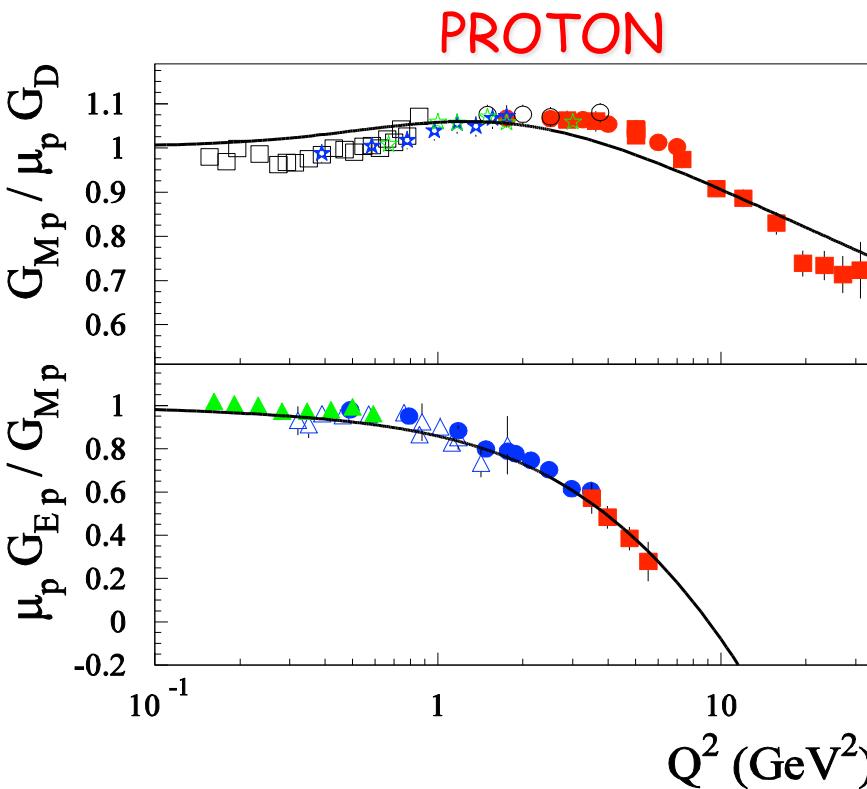
$$f(x, Q^2) \text{ reaches maximum for : } x = x_0 \simeq 1 - \frac{\nu}{\alpha' Q^2}$$

“Drell-Yan-West” relation for PDF/GPD :

at large Q^2 : I is dominated by its behavior around $x \rightarrow 1$

$$I \simeq e^{f(x_0, Q^2)} \left(\frac{2}{f''(x_0, Q^2)} \right)^{1/2} \frac{\sqrt{\pi}}{2} \sim \left(\frac{1}{\alpha' Q^2} \right)^{(\nu+1)/2}$$

electromagnetic form factors



→ modified Regge GPD parameterization

3-parameter fit $\begin{cases} 1 : \text{Regge slope} \rightarrow \text{proton Dirac (Pauli) radius} \\ 2, 3 : \text{large } x \text{ behavior of GPD } E^u, E^d \rightarrow \text{large } Q^2 \text{ behavior of } F_{2p}, F_{2n} \end{cases}$

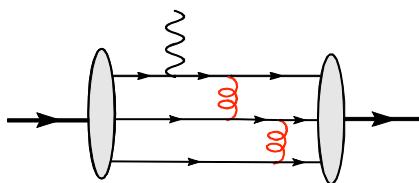
Guidal, Polyakov, Radyushkin, Vdh (2005)

also Diehl, Feldmann, Jakob, Kroll (2005)

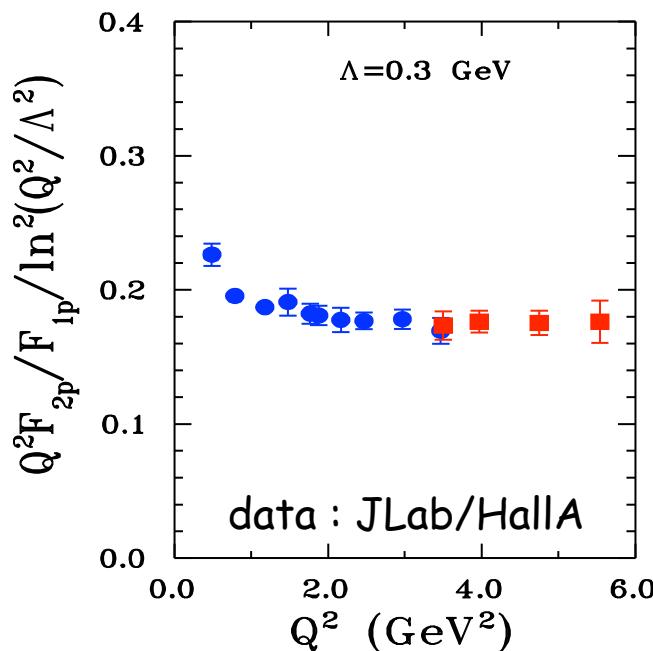
world data (2006)

proton e.m. FFs

PQCD



$$\frac{F_{2p}}{F_{1p}} \sim \frac{\ln^2(Q^2/\Lambda^2)}{Q^2}$$

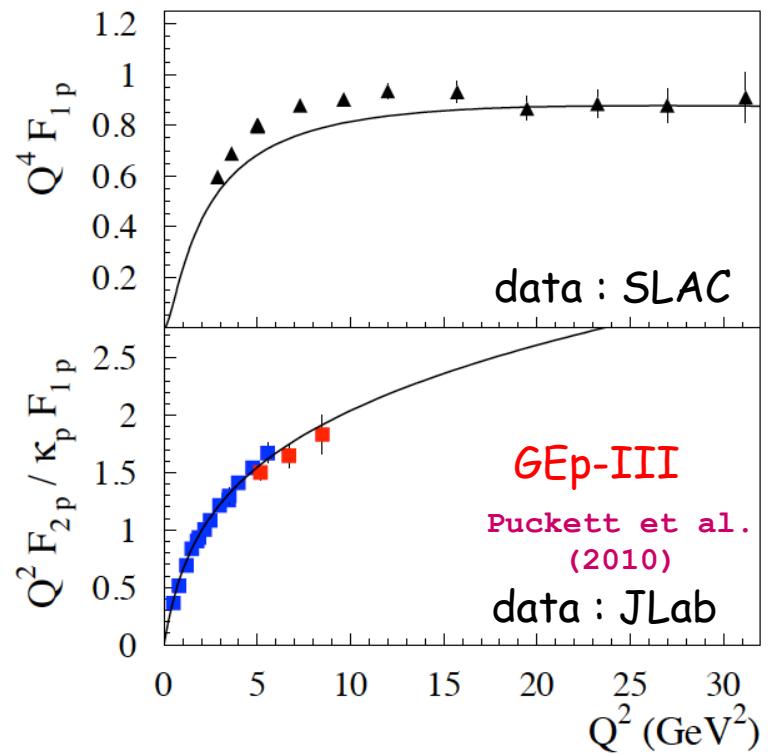


Belticky, Ji, Yuan (2003)

$$\begin{cases} G_M &= F_1 + F_2 \\ G_E &= F_1 - \left(\frac{Q^2}{4M_N^2} \right) F_2 \end{cases}$$

GPD framework

modified Regge GPD model

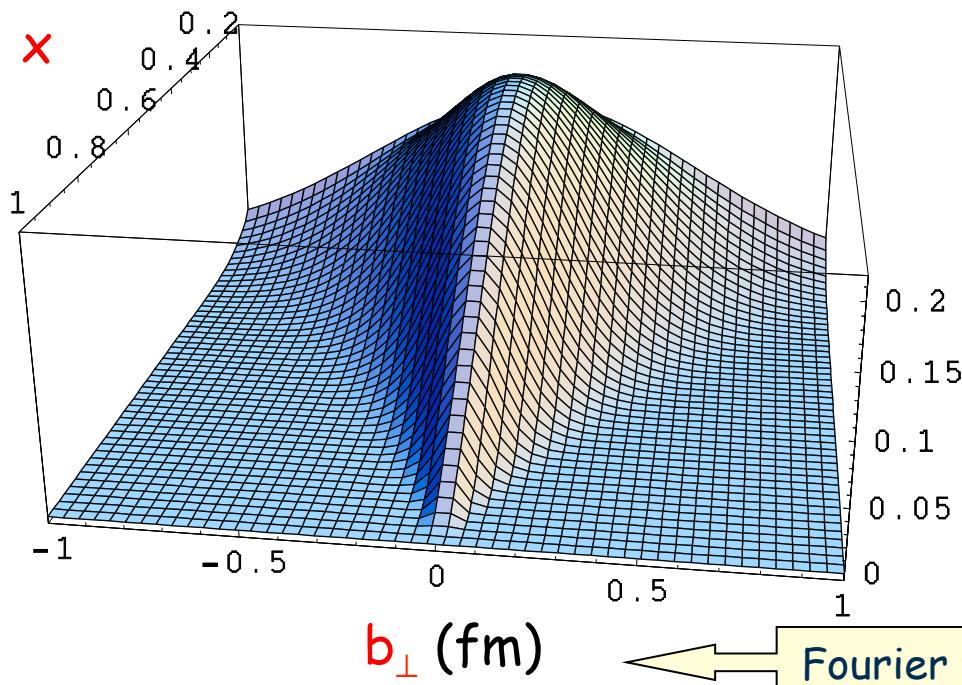


Guidal, Polyakov, Radyushkin, Vdh (2005)

GPDs: transverse image of nucleon

GPDs : quark distributions w.r.t.
longitudinal momentum x and
transverse position b_\perp

$$H^u(x, b_\perp)$$

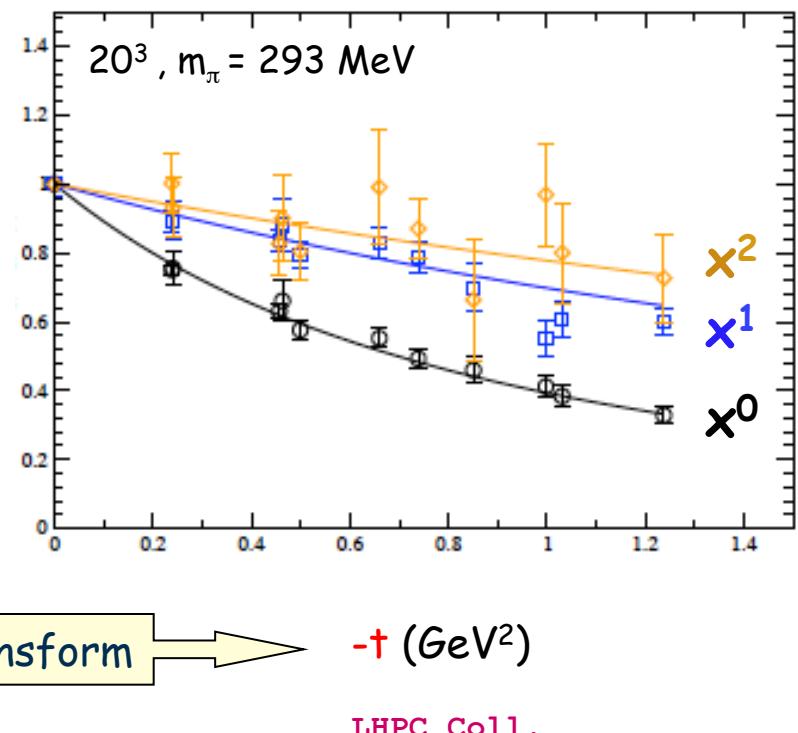


Guidal, Polyakov, Radyushkin, Vdh (2005),

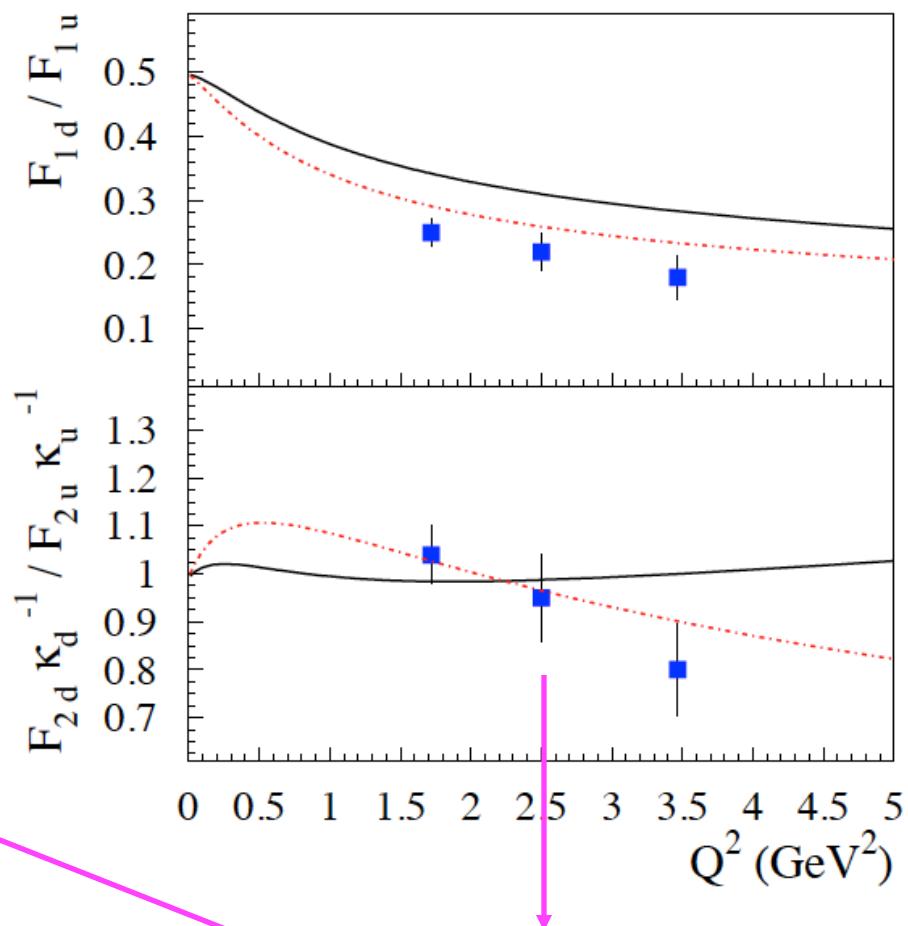
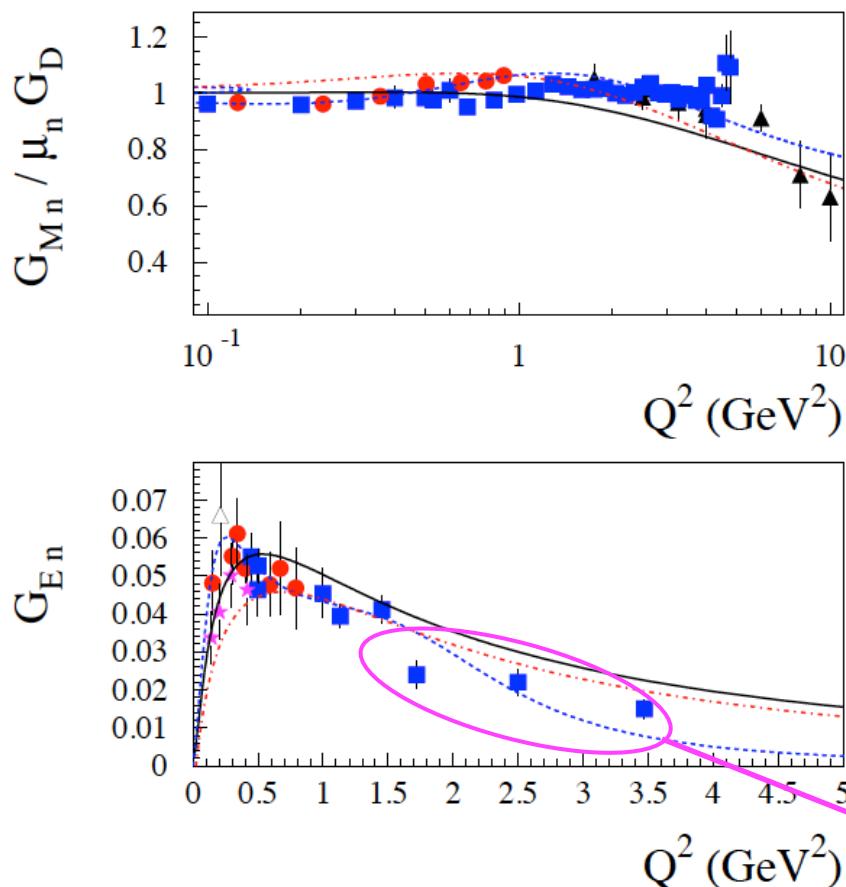
Diehl, Feldmann, Jakob, Kroll (2005)

lattice QCD : moments of GPDs

$$x^n \text{ moment of } H^{u-d}$$



neutron e.m. form factors



— · — · — Phenomenological fit : Bradford et al.

— — — modified Regge GPD parameterization (3 parameters)

— · — · — modified Regge GPD parameterization (6 parameters)

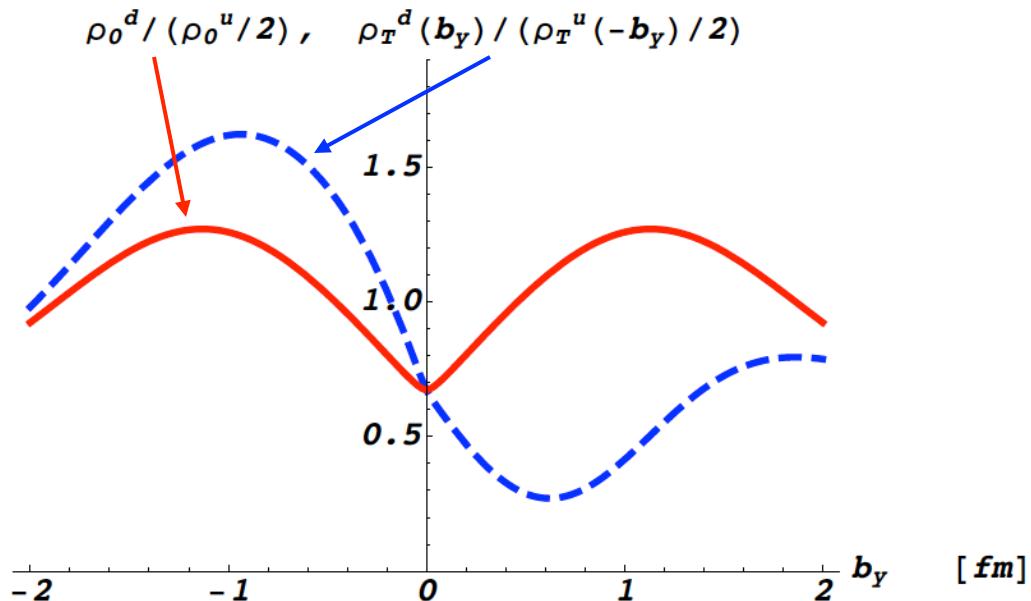
Jlab/HallA E02-013

d/u quark spatial distributions

2D spatial distr. :

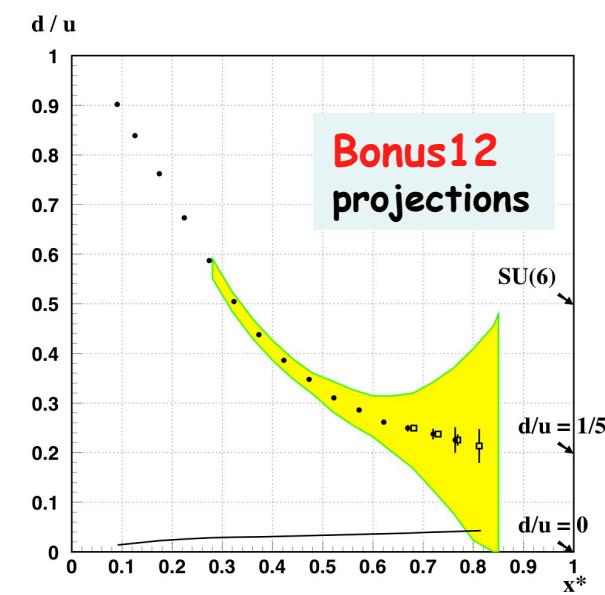
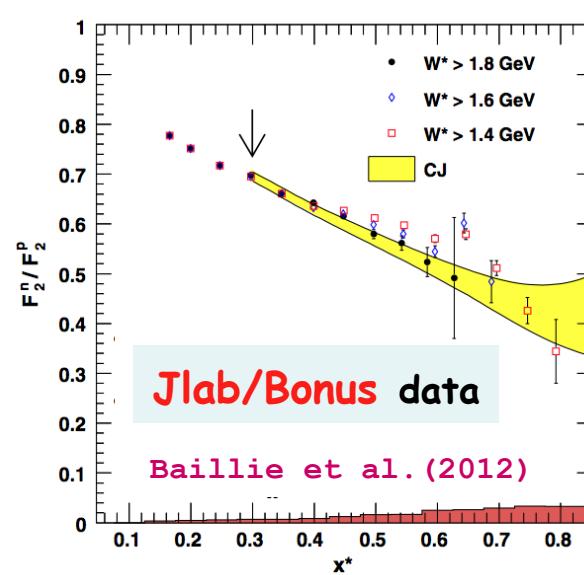
d-quark distr. spread out further in proton compared to u-quark distr.

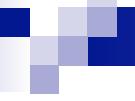
Opposite behavior for neutron



d/u momentum distributions

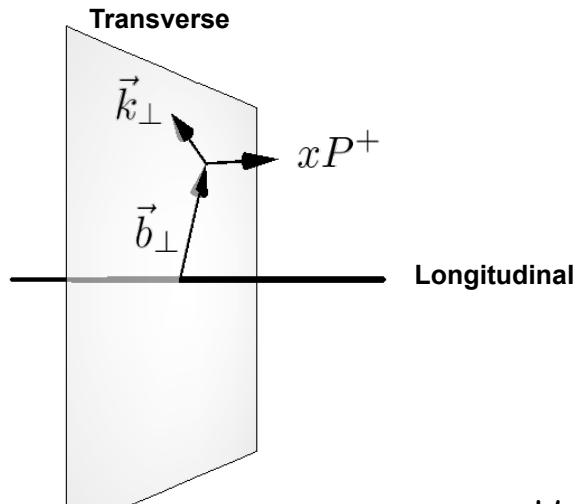
longitudinal momentum distr. correlated with transverse spatial distr. through a GPD





Generalizations: Wigner distributions

Wigner distributions



Wigner (1932)
 Belitsky, Ji, Yuan (2004)
 Lorce', Pasquini (2011)

QM
 QFT (Breit frame)
 QFT (light cone)

$$\begin{array}{ccc}
 \vec{b}_\perp = \frac{\vec{r}_{f\perp} + \vec{r}_{i\perp}}{2} & \xleftrightarrow{\text{Fourier conjugate}} & \vec{\Delta}_\perp = \vec{k}_{f\perp} - \vec{k}_{i\perp} \\
 \vec{z}_\perp = \vec{r}_{i\perp} - \vec{r}_{f\perp} & \xleftrightarrow{\text{Fourier conjugate}} & \vec{k}_\perp = \frac{\vec{k}_{f\perp} + \vec{k}_{i\perp}}{2}
 \end{array}$$

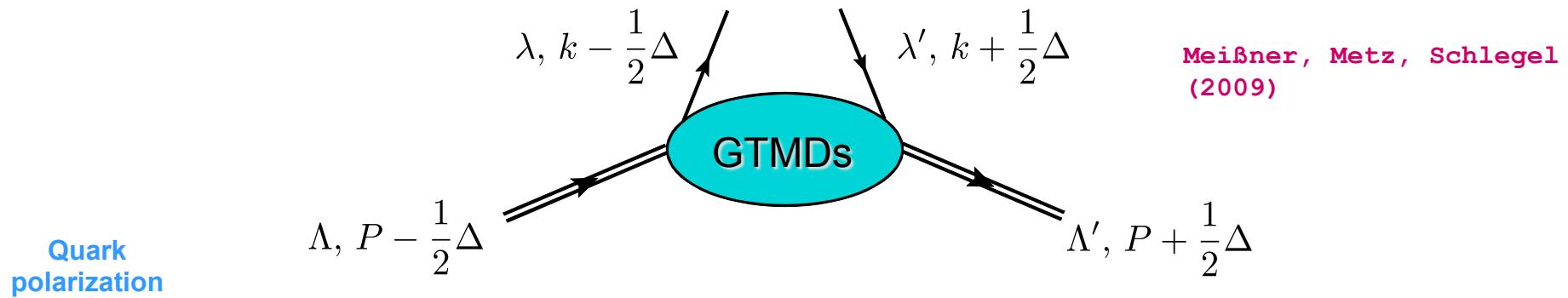
$[\vec{b}_\perp, \vec{k}_\perp] \neq 0$ Heisenberg's uncertainty relations



quasi-probabilistic

- ❖ real functions, but in general not-positive definite
 - ➡ correlations of quark momentum and position in the transverse plane as function of quark and nucleon polarizations
- ❖ quantum-mechanical analogous of classical density on the phase space
 - ➡ one-body density matrix in phase-space in terms of overlap of light-cone wf (LCWF)
- ❖ not directly measurable in experiments
 - ➡ needs phenomenological models with input from experiments on GPDs and TMDs

generalized TMDs & Wigner distributions



$$W_{\Lambda'\Lambda}^{[\Gamma]} = \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}_{\lambda'}^q(-\frac{z}{2}) \Gamma \mathcal{W}(-\frac{z}{2}, \frac{z}{2}|n) \psi_\lambda^q(\frac{z}{2}) | P, \Lambda \rangle e^{i(xP^+ z^- \vec{k}_\perp \cdot \vec{z}_\perp)}$$

4 X 4 = 16 polarizations \longleftrightarrow 16 complex GTMDs (at twist-2)

$$W_{\Lambda,\Lambda'}^\Gamma(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$$

x: average fraction of quark longitudinal momentum

Fourier transform \downarrow

ξ : fraction of longitudinal momentum transfer

$$\tilde{W}_{\Lambda,\Lambda'}^\Gamma(x, \xi, \vec{k}_\perp, \vec{b}_\perp)$$

16 real Wigner distributions

\vec{k}_\perp : average quark transverse momentum

$\vec{\Delta}$: nucleon momentum transfer

Meißner, Metz, Schlegel
(2009)

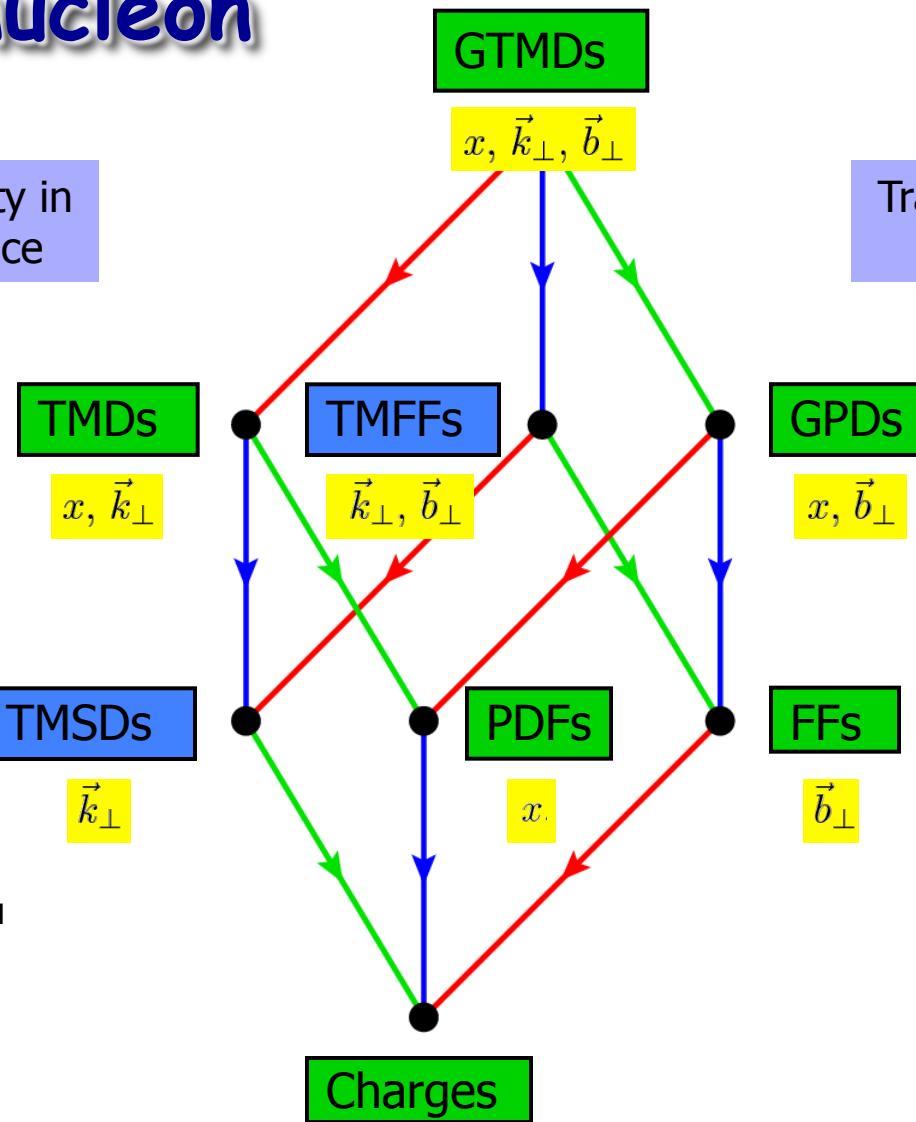
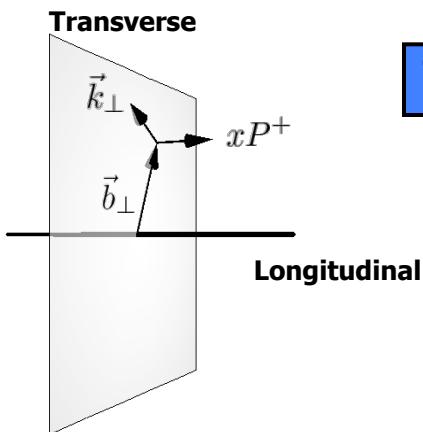
“complete” picture of nucleon

$$\xi = 0$$

Momentum space	$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$	Position space
	$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$	

Transverse density in momentum space

Lorcé (2011)

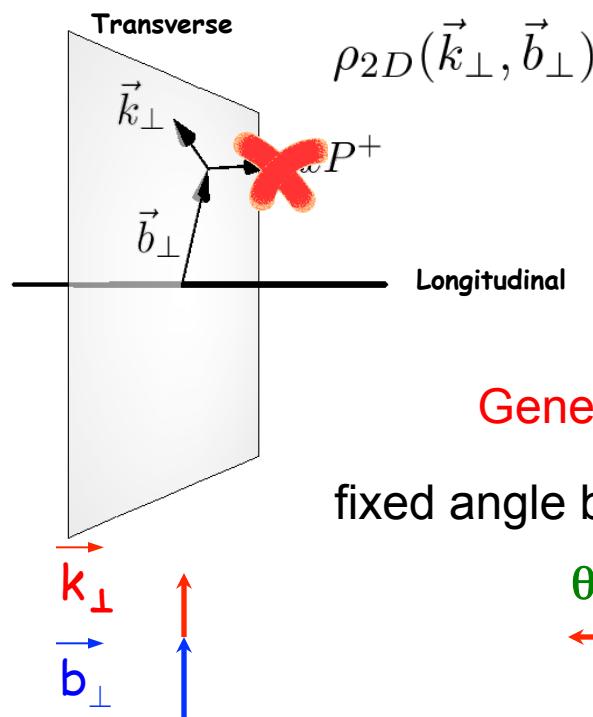


Transverse density in position space

$$\begin{aligned}
 \text{Red arrow: } & \int d^2 b_\perp \\
 \text{Blue arrow: } & \int dx \\
 \text{Green arrow: } & \int d^2 k_\perp
 \end{aligned}$$

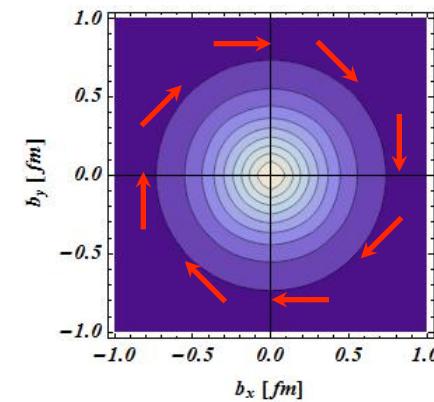
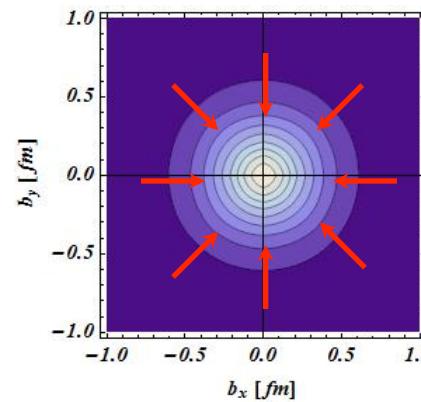
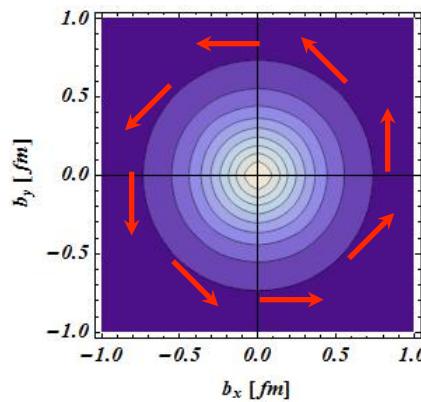
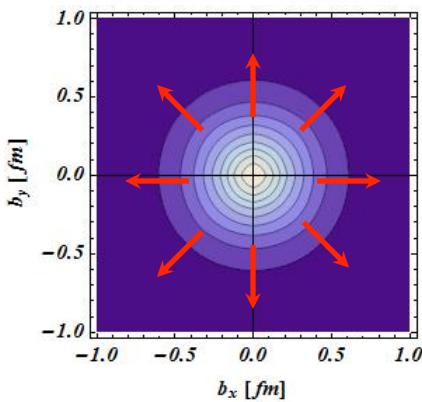
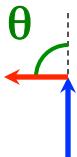
Unpol. up Quark in Unpol. Proton

Lorce', Pasquini (2011)

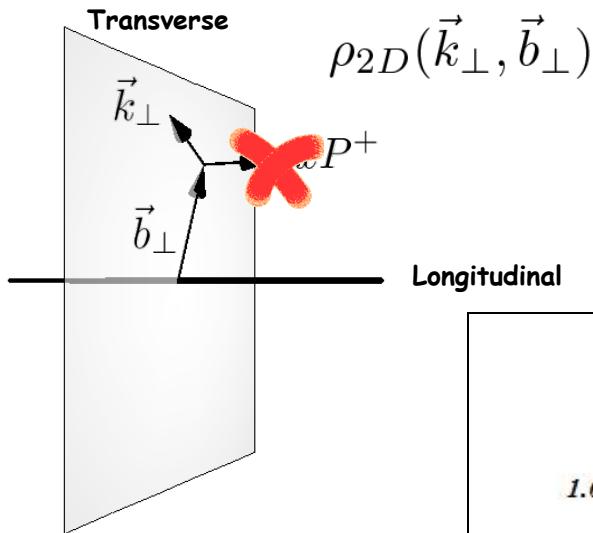


Generalized Transverse Charge Density

fixed angle between \vec{k}_\perp and \vec{b}_\perp and fixed value of $|\vec{k}_\perp|$

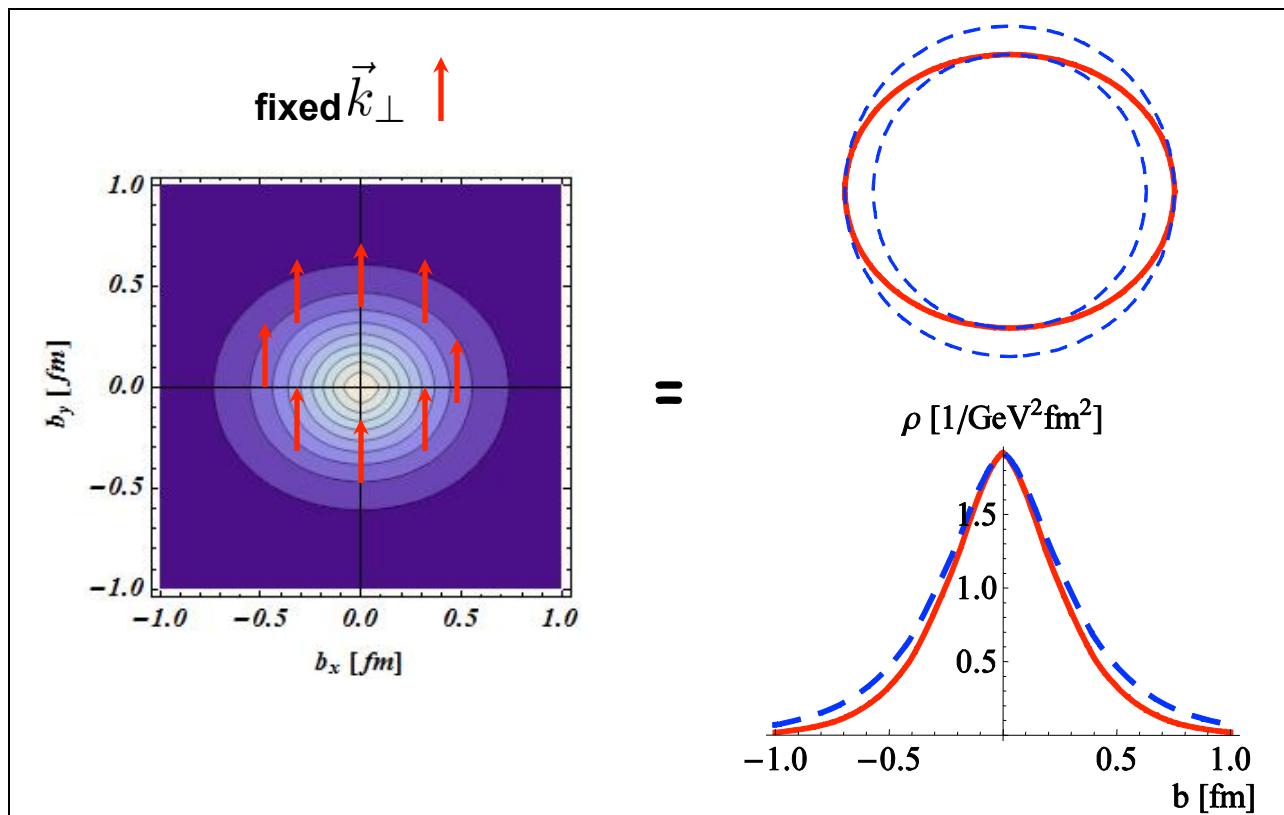


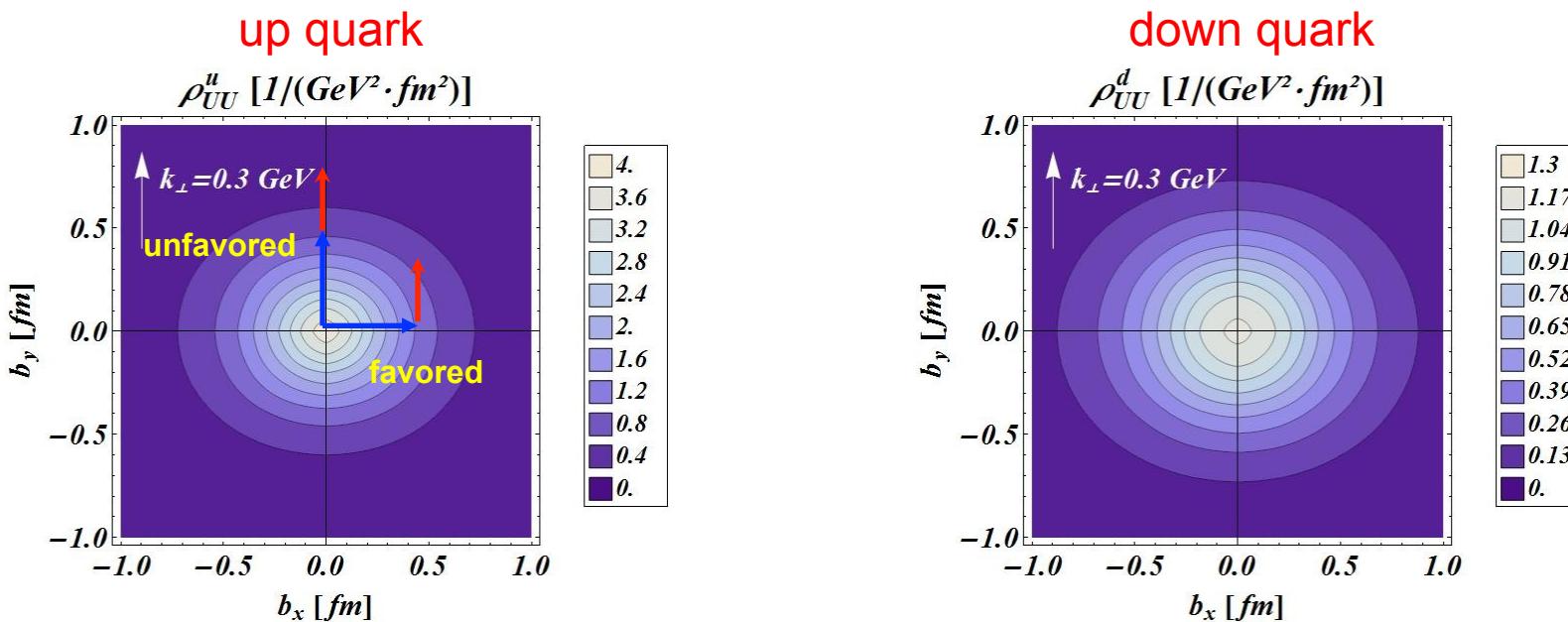
Unpol. up Quark in Unpol. Proton



3Q light-cone model

Lorce', Pasquini (2011)





- ❖ left-right symmetry of distributions
→ quarks are as likely to rotate clockwise as to rotate anticlockwise
- ❖ up quarks are more concentrated at the center of the proton than down quark
- ❖ integrating over \vec{b}_\perp → transverse-momentum **density**

$$f_1^{(1)}(k_\perp^2) = \int dx f_1(x, k_\perp^2)$$
- ❖ integrating over \vec{k}_\perp → charge **density** in the transverse plane \vec{b}_\perp

$$\rho^q(b_\perp^2) = e^q \int d^2 \Delta_\perp e^{-i \vec{b}_\perp \cdot \vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$