

International Spring School of the GDR PH-QCD

QCD prospects for future ep and eA colliders

LECTURERS:

Alfred Mueller High energy ep and eA scattering

Piet Mulders TMDs: theory and phenomenology

George Sterman Factorization of hard processes

Marc Vanderhaeghen GPDs and spatial structure of hadrons



ORSAY 4-8 June 2012

Amphi I, Laboratoire de Physique Théorique,
bâtiment 210, Université d'Orsay

<http://indico.in2p3.fr//event/QCD-ep-eA-colliders>

Organisation:

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Samuel Wallon (LPT Orsay) Chair

Sponsors:



GPDs & spatial structure of hadrons

Part 3

Marc Vanderhaeghen
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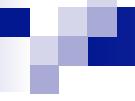
Outline

→ What is the physics contained in GPDs ?

- GPDs: basic definitions and properties
- 3D imaging of the nucleon: link between elastic nucleon Form Factors and GPDs, connection between longitudinal momentum and transverse position
- Generalizations: Wigner distributions
- GPDs and nucleon spin
- Hard exclusive processes : DVCS, hard meson production, $N \rightarrow \Delta$ DVCS, ...

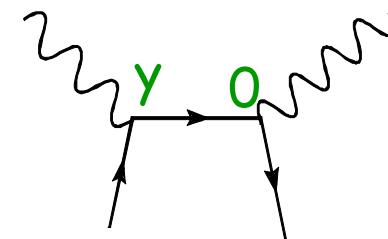
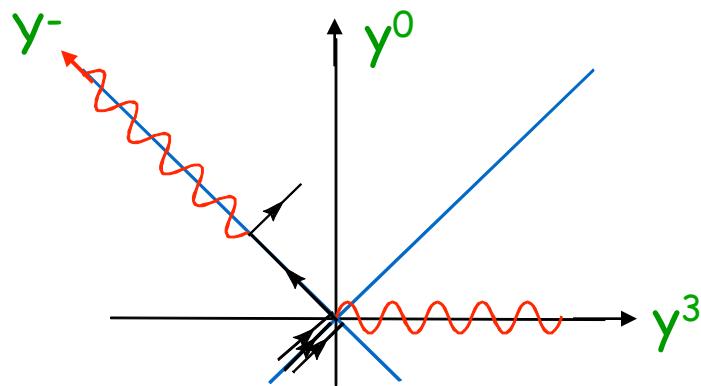
Reviews on GPDs

- > Goeke, Polyakov, Vdh : Prog.Part.Nucl.Phys. 47, 401 (2001)
- > Diehl : Phys.Rept. 388, 41 (2003)
- > Ji : Ann.Rev.Nucl.Part.Sci 54, 413 (2004)
- > Belitsky, Radyushkin : Phys.Rept. 418, 1 (2005)
- > Boffi, Pasquini : Riv.Nuovo.Cim. 30, 387(2007)



GPDs and nucleon spin

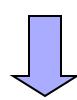
handbag (bilocal) operator: generalized probe



generalized probe

$$\bar{q}(0) \gamma^\mu q(y) = \bar{q}(0) \gamma^\mu q(0) + y^- \bar{q}(0) \gamma^\mu \partial^+ q(0) + \dots$$

($y \approx 0$)



γ (W^\pm, Z^0) probe



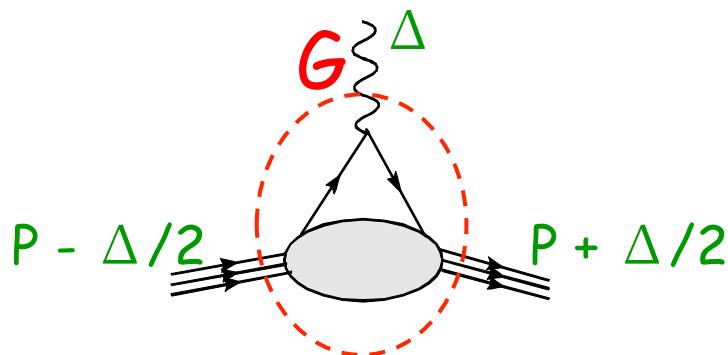
$\langle N | \dots | N \rangle$ electroweak
form factors

spin 2 (graviton) probe



energy-momentum
form factors

energy-momentum form factors



nucleon in external classical gravitational field

\rightarrow G couples to energy-momentum tensor

symmetric (Belinfante)

$$\langle P + \frac{\Delta}{2} | T^{\mu\nu}(0) | P - \frac{\Delta}{2} \rangle \quad (\mu, \nu) \equiv \frac{1}{2}(\mu\nu + \nu\mu)$$

$$= \bar{N} \left(P + \frac{\Delta}{2} \right) \left\{ A(t) \gamma^{(\mu} P^{\nu)} + B(t) P^{(\mu} i\sigma^{\nu)\alpha} \frac{\Delta_\alpha}{2M} \right.$$

$$\left. + C(t) (\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) \frac{1}{M} \right\} N \left(P - \frac{\Delta}{2} \right)$$

Gordon identity

$$= \bar{N} \left(P + \frac{\Delta}{2} \right) \left\{ A(t) P^\mu P^\nu / M + (A(t) + B(t)) P^{(\mu} i\sigma^{\nu)\alpha} \frac{\Delta_\alpha}{2M} \right.$$

$$\left. + C(t) (\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) \frac{1}{M} \right\} N \left(P - \frac{\Delta}{2} \right)$$

momentum sum rule

$$\begin{aligned}\langle P | \hat{P}^\nu | P \rangle &= \langle P | \int d^3\vec{x} T^{0\nu}(x) | P \rangle \\ &= \lim_{\Delta \rightarrow 0} \langle P + \frac{\Delta}{2} | \int d^3\vec{x} T^{0\nu}(x) | P - \frac{\Delta}{2} \rangle \\ &= \lim_{\Delta \rightarrow 0} \int d^3\vec{x} e^{-i\vec{x} \cdot \vec{\Delta}} \langle P + \frac{\Delta}{2} | T^{0\nu}(0) | P - \frac{\Delta}{2} \rangle \\ &= \lim_{\Delta \rightarrow 0} (2\pi)^3 \delta^3(\vec{\Delta}) \langle P + \frac{\Delta}{2} | T^{0\nu}(0) | P - \frac{\Delta}{2} \rangle \\ &= A(0) P^\nu (2P^0) (2\pi)^3 \delta^3(0) \\ &= A(0) \underbrace{P^\nu}_{\langle P | P \rangle} \langle P | P \rangle\end{aligned}$$

momentum sum rule (cont.)

$$\langle P | \hat{P}^\nu | P \rangle = A(0) P^\nu \langle P | P \rangle$$

→ Total system : energy-momentum conservation



$$A(0) = 1$$

→ Physical interpretation in terms of :

quarks : $A_q(0)$
& gluons : $A_g(0)$

$$A_q(0) + A_g(0) = 1$$

angular momentum sum rule

consider N in rest frame : $P^\mu (M, 0, 0, 0)$ $S^\mu (0, 0, 0, 1)$

$$\begin{aligned} \langle P, +\frac{1}{2} | \hat{J}^{12} | P, +\frac{1}{2} \rangle &= J \langle P, +\frac{1}{2} | P, +\frac{1}{2} \rangle \\ &= \langle P, +\frac{1}{2} | \int d^3 \vec{x} \left\{ x^1 T^{02}(x) - x^2 T^{01}(x) \right\} | P, +\frac{1}{2} \rangle \\ &= \varepsilon_{ij3} \lim_{\Delta \rightarrow 0} \langle P + \frac{\Delta}{2}, +\frac{1}{2} | \int d^3 \vec{x} x^i T^{0j}(x) | P - \frac{\Delta}{2}, +\frac{1}{2} \rangle \\ &= \varepsilon_{ij3} \lim_{\Delta \rightarrow 0} \int d^3 \vec{x} x^i e^{-i\vec{x} \cdot \vec{\Delta}} \langle P + \frac{\Delta}{2}, +\frac{1}{2} | T^{0j}(0) | P - \frac{\Delta}{2}, +\frac{1}{2} \rangle \\ &= \varepsilon_{ij3} \lim_{\Delta \rightarrow 0} \left[i \frac{\partial}{\partial \Delta^i} (2\pi)^3 \delta^3(\vec{\Delta}) \right] \\ &\quad \times \langle P + \frac{\Delta}{2}, +\frac{1}{2} | T^{0j}(0) | P - \frac{\Delta}{2}, +\frac{1}{2} \rangle \end{aligned}$$

angular momentum sum rule (cont.)

$$= \varepsilon_{ij3} \lim_{\Delta \rightarrow 0} (2\pi)^3 \delta^3(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right) \left\{ [A(t) + B(t)] \bar{N} P^{(0)} i\sigma^{j)\alpha} \frac{\Delta_\alpha}{2M} N \right.$$

+ terms independent of Δ
+ terms quadratic in $\Delta \}$

$$= \varepsilon_{ij3} (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M} \\ \times \bar{N}(P, +\frac{1}{2}) \left(-\frac{1}{2} \right) \left\{ P^0 \sigma^{ji} + P^j \sigma^{0i} \right\} N(P, +\frac{1}{2})$$

\downarrow \downarrow
 M 0 in rest frame

$$= (2\pi)^3 \delta^3(0) [A(0) + B(0)] \frac{1}{2M} M \underbrace{\bar{N}(P, +\frac{1}{2}) \sigma^{12} N(P, +\frac{1}{2})}_{2M}$$

$$= \frac{1}{2} [A(0) + B(0)] \langle P | P \rangle$$

angular momentum sum rule (cont.)

$$J = \frac{1}{2} [A(0) + B(0)]$$

→ Total system : angular momentum conservation



$$A(0) + B(0) = 1 \quad \rightarrow \quad B(0) = 0$$

→ Physical interpretation in terms of quarks & gluons

$$J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad \frac{1}{2} = J_q + J_g = \frac{1}{2} \Delta \Sigma + L_q + J_g$$

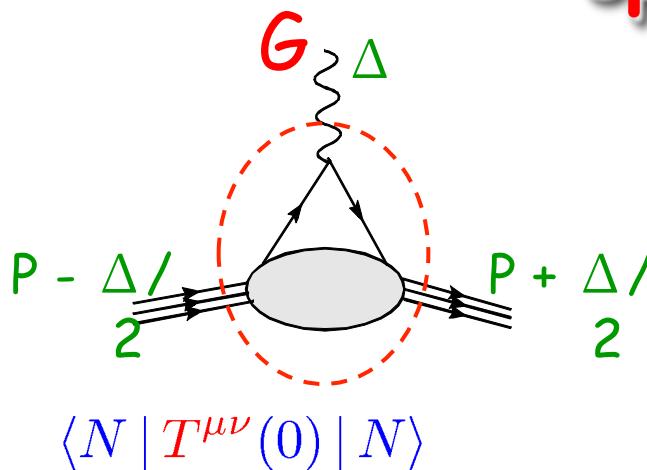
$$\vec{J}_q = \int d^3\vec{x} \left[\Psi^\dagger \frac{\vec{\Sigma}}{2} \Psi + \underbrace{\Psi^\dagger \vec{x} \times (-i \vec{D}) \Psi}_{L_q} \right] \quad \vec{J}_g = \int d^3\vec{x} \vec{x} \times (\vec{E} \times \vec{B})$$

$\frac{1}{2} \Delta \Sigma$

$T^{\mu\nu}$ form factors in terms of GPDs

$$\begin{aligned}
 \rightarrow & \langle P + \frac{\Delta}{2} | T_q^{\mu\nu}(0) | P - \frac{\Delta}{2} \rangle n_\mu n_\nu \\
 &= \langle P + \frac{\Delta}{2} | \bar{q} i \gamma^{(\mu} \overset{\leftrightarrow}{D}{}^{\nu)} q(0) | P - \frac{\Delta}{2} \rangle n_\mu n_\nu \\
 &= \bar{N} \left\{ \frac{1}{M} [A(t) + 4\xi^2 C(t)] + [A(t) + B(t)] i \sigma^{\nu\alpha} \frac{\Delta_\alpha}{2M} n_\nu \right\} N \\
 \rightarrow & \frac{P^+}{2\pi} \int dy^- e^{ixP^+y^-} \langle P + \frac{\Delta}{2} | \bar{q}\left(-\frac{y}{2}\right) i\gamma \cdot n n \cdot \overset{\leftrightarrow}{D}{} q\left(\frac{y}{2}\right) | P - \frac{\Delta}{2} \rangle_{y^+=0, y_\perp=0} \\
 &\quad \text{---} i n \cdot \overset{\leftrightarrow}{D}{} = x P \cdot n = x \\
 &= x \frac{P^+}{2\pi} \int dy^- e^{ixP^+y^-} \langle P + \frac{\Delta}{2} | \bar{q}\left(-\frac{y}{2}\right) \gamma \cdot n q\left(\frac{y}{2}\right) | P - \frac{\Delta}{2} \rangle_{y^+=0, y_\perp=0} \\
 &= \bar{N} \left\{ \frac{1}{M} x H(x, \xi, t) + x [H(x, \xi, t) + E(x, \xi, t)] i \sigma^{\mu\nu} \frac{\Delta_\nu}{2M} n_\mu \right\} N \\
 \rightarrow & \int_{-1}^1 dx \quad \text{of both lhs and rhs}
 \end{aligned}$$

energy-momentum form factors & spin of nucleon



nucleon in external classical gravitational field

$\rightarrow G$ couples to energy-momentum tensor

$$(\mu, \nu) \equiv \frac{1}{2}(\mu\nu + \nu\mu)$$

$$= \bar{N} \left\{ A(t) \gamma^{(\mu} P^{\nu)} + B(t) P^{(\mu} i\sigma^{\nu)\alpha} \frac{\Delta_\alpha}{2M} + C(t) (\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) \frac{1}{M} \right\} N$$

\rightarrow link to GPDs :

x. Ji (1995)

$$\int_{-1}^1 dx x H(x, \xi, t) = A(t) + 4\xi^2 C(t)$$

$$\int_{-1}^1 dx x E(x, \xi, t) = B(t) - 4\xi^2 C(t)$$

\rightarrow SPIN
sum rule

$$\int_{-1}^1 dx x \left\{ H^q(x, \xi, 0) + E^q(x, \xi, 0) \right\} = A(0) + B(0) = 2 J^q$$

GPDs: angular momentum sum rule



total angular momentum $J^q = \frac{1}{2} \Delta q + L^q$

quark orbital angular momentum

x. Ji
(1997)

$$2 J^q = M_2^q + \int_{-1}^1 dx x E^q(x, 0, 0)$$

with known $M_2^q = \int_0^1 dx x [q(x) + \bar{q}(x)]$

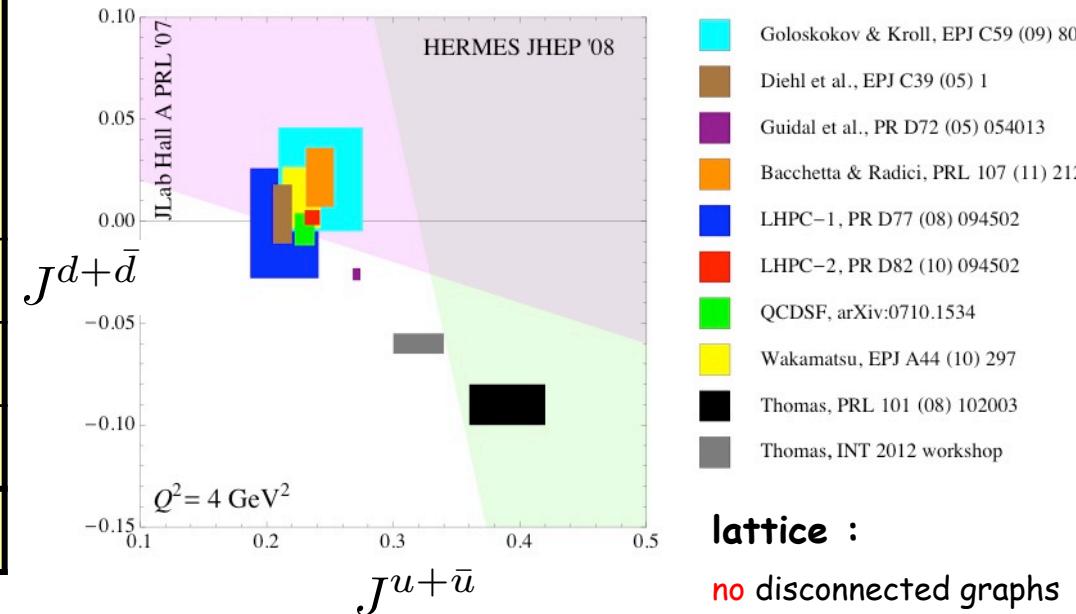


Valence parametrization for GPD E^q : $E^q(x, 0, 0) = \kappa_q / N_q (1 - x)^{\eta_q} q_v(x)$

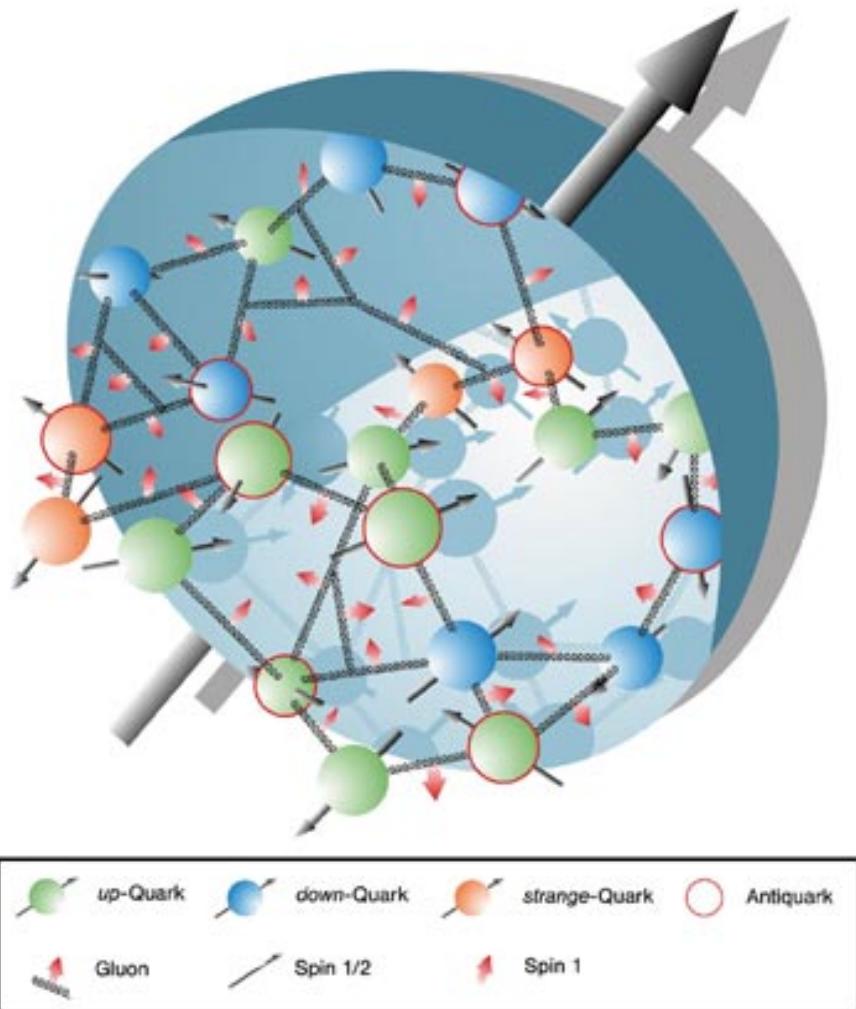
Goeke, Polyakov, vdh (2001)

Proton	M_2^q	$2 J^q$ GPD model ($\mu^2 = 2 \text{ GeV}^2$)	$2 J^q$ Lattice (LHPC) (4 GeV^2)
u	0.37	0.58	≈ 0.47
d	0.20	-0.06	≈ 0.00
s	0.04	0.04	
u+d+s	0.61	0.56	

lattice : no disconnected diagrams

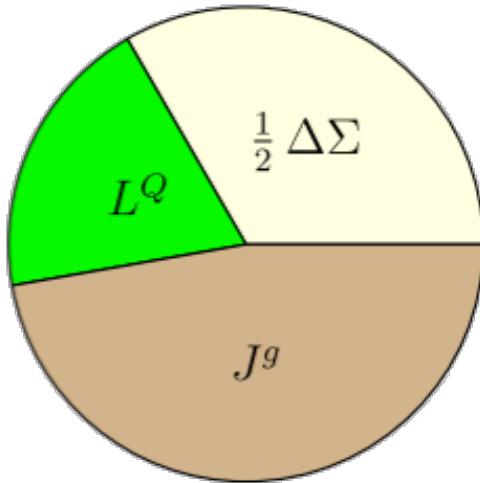


nucleon spin content



Ji proton spin decomposition

Ji 1997



$$\vec{J}_{QCD} = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi + \int d^3r \psi^\dagger \vec{r} \times (-i\vec{D}) \psi + \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a)$$

Kinetic

potential terms attributed to gluons

PROs: - gauge-invariant decomposition

- accessible in DIS and hard exclusive processes (DVCS)

CONs: - kinetic momentum $\vec{\pi} = m\vec{v}$ and angular momentum do not satisfy canonical commutation relations, no simple partonic interpretation
 - incomplete decomposition: only gluon total J^g

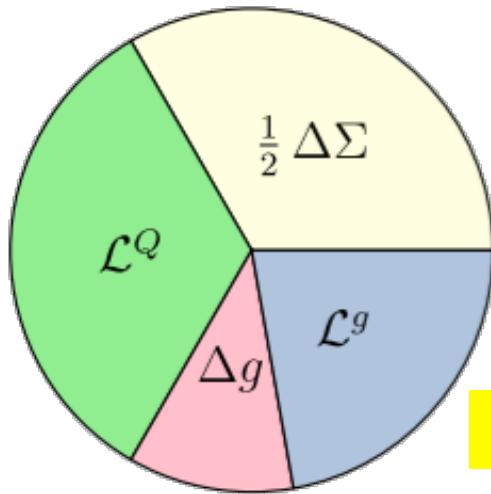
NEWS: - light-front gauge invariant extension (GIE)

$J^g = \Delta g + L^g$ (price to pay: preferred direction)

Wakamatsu (2009, 2010)

Jaffe-Manohar proton spin decomposition

Jaffe-Manohar 1990



Canonical

$$\begin{aligned}
 \vec{J}_{\text{QCD}} &= \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi \\
 &= \int d^3r \psi^\dagger \vec{r} \times (-i \vec{\nabla}) \psi \\
 &= \int d^3r \vec{E}^a \times \vec{A}^a \\
 &= \int d^3r E^{ai} \vec{r} \times \vec{\nabla} A^{ai}
 \end{aligned}$$

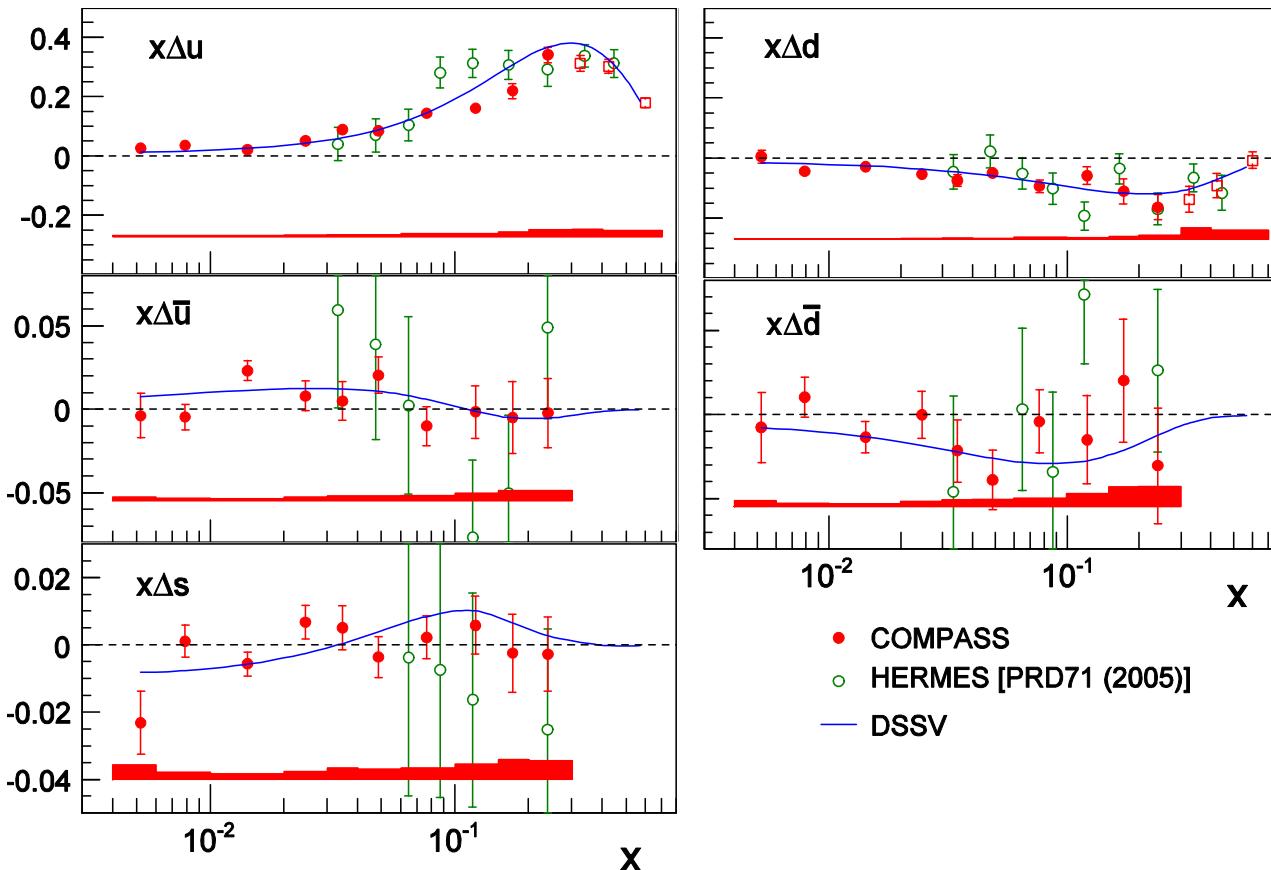
- PROs:**
- canonical momentum $\vec{p} = \partial L / \partial \vec{v}$ and angular momentum do satisfy **canonical commutation relations**, partonic interpretation
 - **complete** decomposition (quarks and gluons)

- CONs:**
- gauge-variant decomposition (light-cone framework & gauge $A^+ = 0$)
 - no observables for orbital angular momenta

- NEWS:**
- gauge invariant extensions
 - relating OAM to Wigner distributions

Chen et al. (2008)
Lorce, Pasquini (2011)
Lorce: arXiv 1205.6483 [hep-ph]

quark helicity distributions from SIDIS



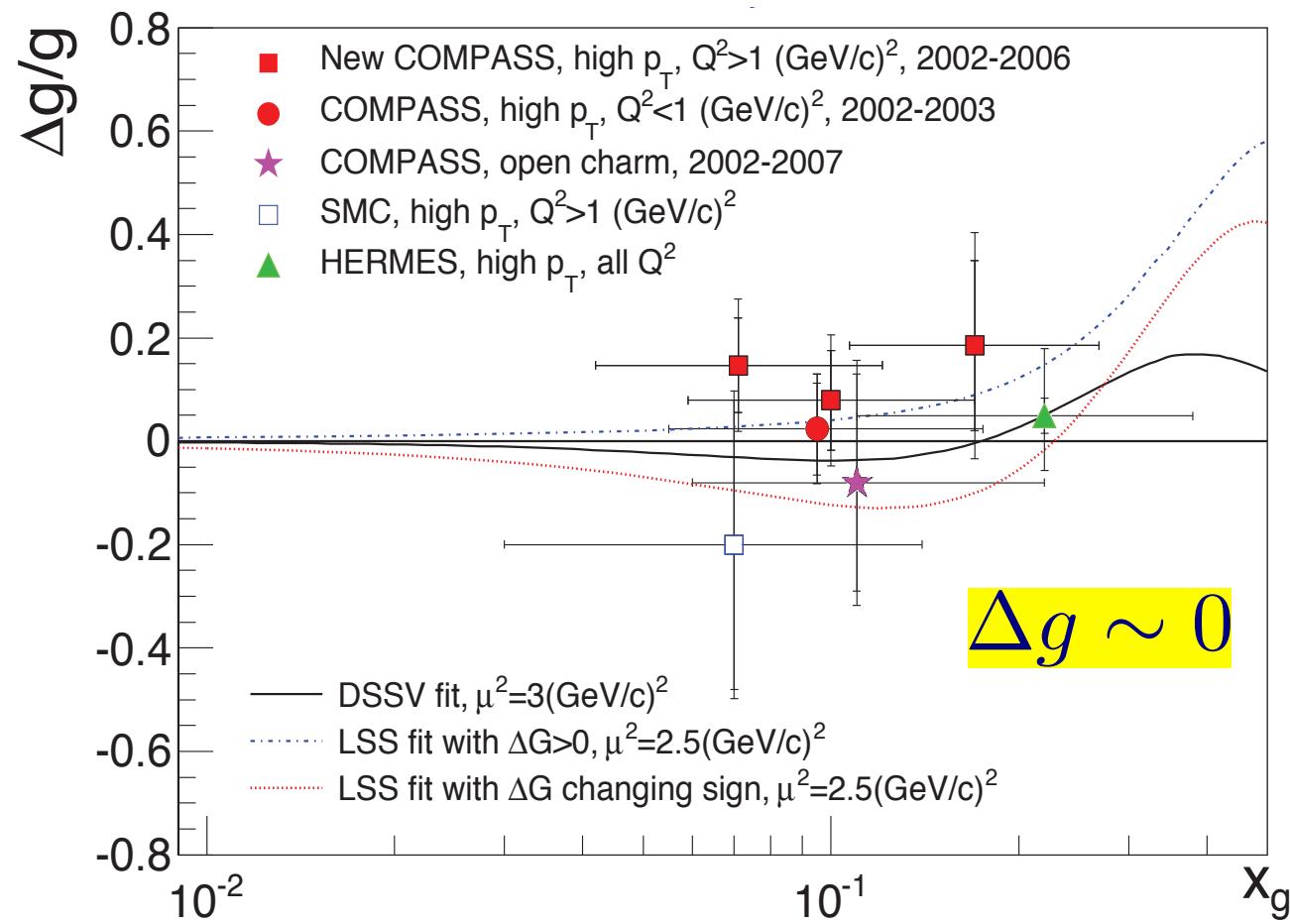
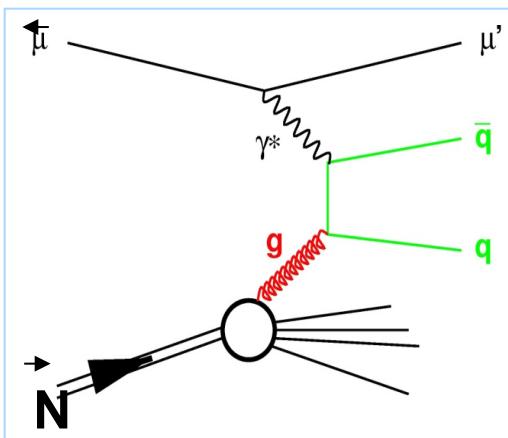
Results from inclusive and semi-inclusive experiments
(COMPASS, HERMES, Jlab) are consistent

$$\Delta\Sigma \sim 0.3$$

HERMES (2007) : $\Delta\Sigma = 0,330 \pm 0,025 \pm 0,011 \pm 0,028$

COMPASS (2007) : $\Delta\Sigma = 0,33 \pm 0,03 \pm 0,05$

gluon helicity



what have we learned from polarized DIS so far ?

→
$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \mathcal{L}^Q + \Delta g + \mathcal{L}^g$$

quark spin ↓
 $\frac{1}{2} \Delta \Sigma$ ~ 0.15

gluon spin ↓
 Δg ≈ 0

Jaffe-Manohar

→ remaining ~ 70% must come from orbital angular momenta

→
$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L^Q + J^g$$

$$L^Q \neq \mathcal{L}^Q$$

but difference may be interpreted as change in OAM as quark leaves the nucleon

Burkardt (2012)

quantifying the orbital angular momentum

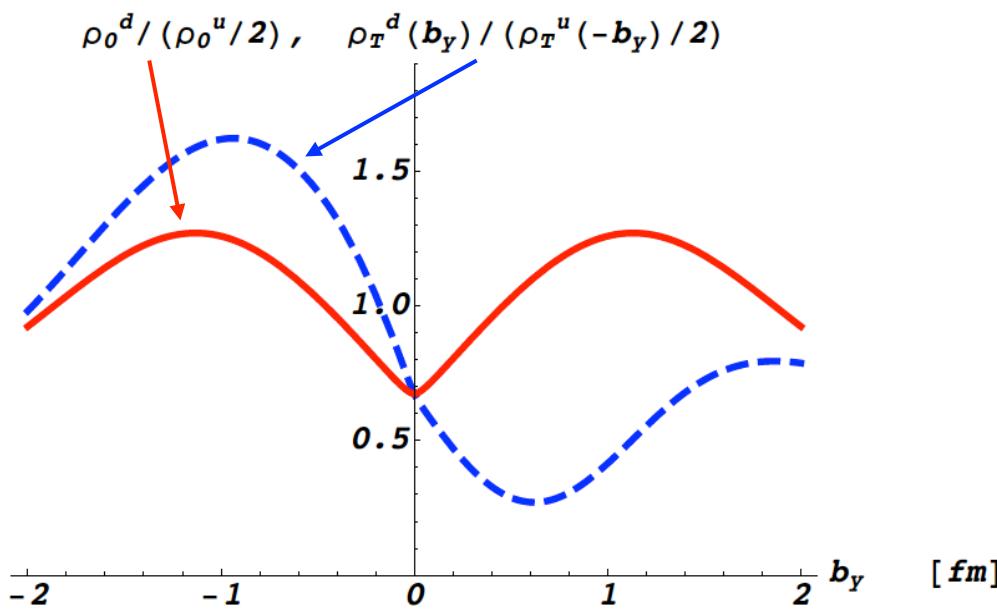
- $$2 J^q = M_2^q + \int_{-1}^1 dx x E^q(x, 0, 0)$$
 with $M_2^q = \int_0^1 dx x [q(x) + \bar{q}(x)]$
- $$J^q = \frac{1}{2} \Delta q + L^q$$
 x. Ji (1997)

using a valence parametrization for GPD E^q : Goeke, Polyakov, Vdh (2001)

Proton	$2 J^q$ GPD model	Δq HERMES	$2 L^q$	evaluated at $\mu^2 = 2.5 \text{ GeV}^2$
u	0.61	0.57 ± 0.04	0.04 ∓ 0.04	
d	-0.05	-0.25 ± 0.08	0.20 ∓ 0.08	
s	0.04	-0.01 ± 0.05	0.05 ∓ 0.05	
u + d + s	0.60	0.30 ± 0.10	0.30 ∓ 0.10	

- remaining part $\sim 40\%$ originates from gluon angular momentum J^g

connection with d/u quark spatial distribution



2D spatial distr. :

d-quark distr. spread out further in proton compared to u-quark distr.

Opposite behavior for neutron

