Factorization of Hard Processes

Orsay, June 2012

- **III.-IV A few applications**

  A. Infrared safety of jets

  B. Factorization in DIS

  C. Factorization of 1PI cross sections

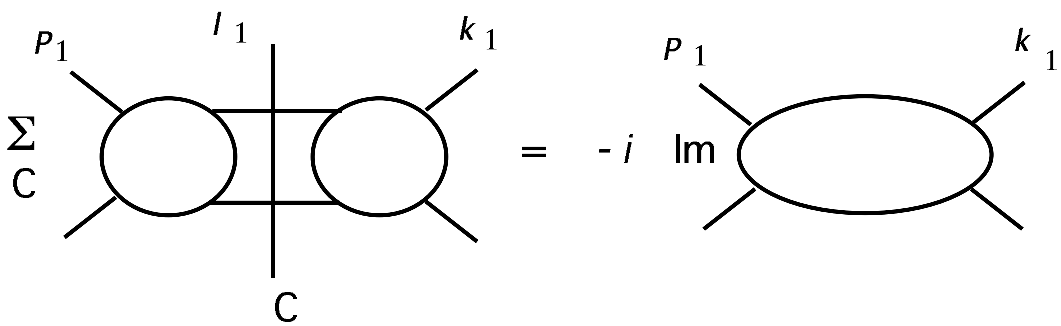
  D. Drell-Yan applications

  E. An outline of DVCS factorization

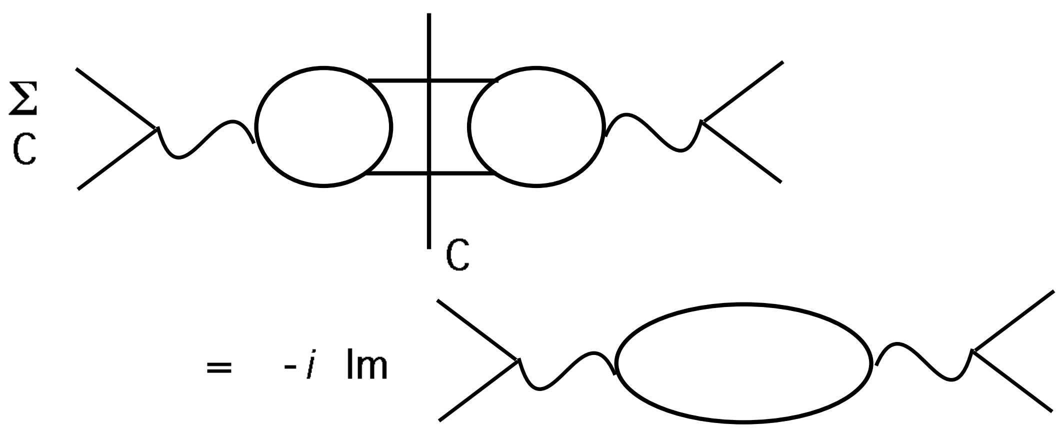
III-IVA. IR Safety for inclusive cross sections and jets

Cross sections, cut diagrams and generalized unitarity: an application and ingredient in factorization for cross sections

- **The optical theorem (total cross section $\propto$ imaginary part of forward scattering)**



- **Or for $\mathrm{e}^+\mathrm{e}^-$,**



- **We can use this to show the Infrared safety of inclusive annihilation and decay**

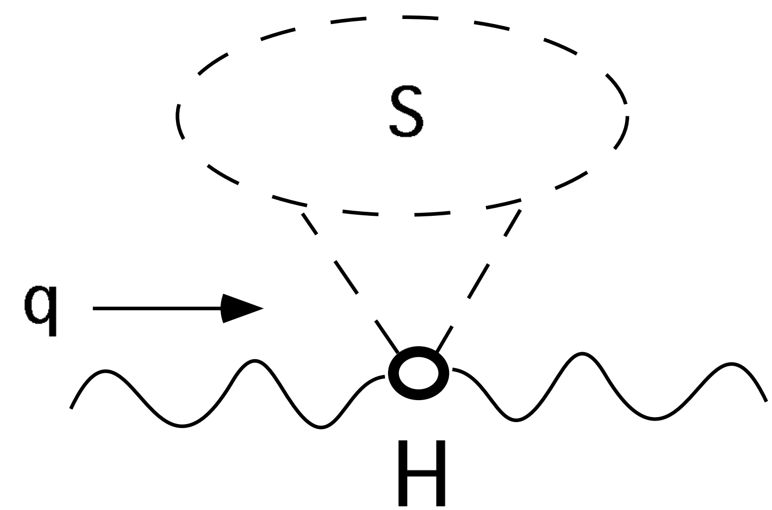
$$\sigma^{(\mathrm{tot})}_{e^+e^-}(q^2) = \frac{e^2}{q^2}\mathrm{Im}\,\pi(q^2)\,,$$

where the function π is defined in terms of the two-point correlation function of the relevant electroweak currents J_μ (with their couplings included) as

$$\pi(q^2)(q_\mu q_\nu - q^2 g_{\mu\nu}) = i\int d^4x\, e^{iqx} < 0|\, T\ J_\mu(x) J_\nu(0)\ |0 >$$

- **We only have to look for pinch surfaces in the forward scattering amplitude!**

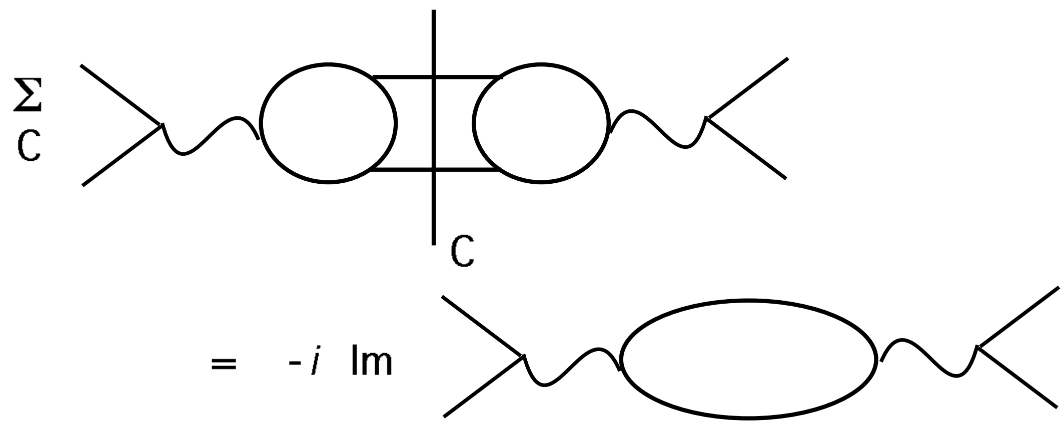
- **The only physical pictures for $\langle JJ \rangle$ and hence for π:**



- **No on-shell finite energy lines**

- **Power counting confirms finiteness.**

- **And the method is much more general – unitarity holds point-by-point in spatial loop momenta $\vec{l}$ inside the diagrams**



$$\sum_{\text{all } C} G_C(p_i, k_j, l) = 2\ \mathrm{Im}\ \left(-iG(p_i, k_j, l)\right)\ .$$

- This only requires doing energy integrals.

- Unitarity at fixed loop momenta then follows from the resulting time-ordered perturbation theory relations

$$\begin{aligned}\sum_m G_m^* G_m &= \sum_{m=1}^{A} \prod_{j=m+1}^{A} \frac{1}{E_j - S_j - i\epsilon}\,(2\pi)\delta(E_m - S_m)\,\prod_{i=1}^{m-1} \frac{1}{E_i - S_i + i\epsilon} \\ &= -i\left[-\prod_{j=1}^{A} \frac{1}{E_j - S_j + i\epsilon} + \prod_{j=1}^{A} \frac{1}{E_j - S_j - i\epsilon}\right]\end{aligned}$$

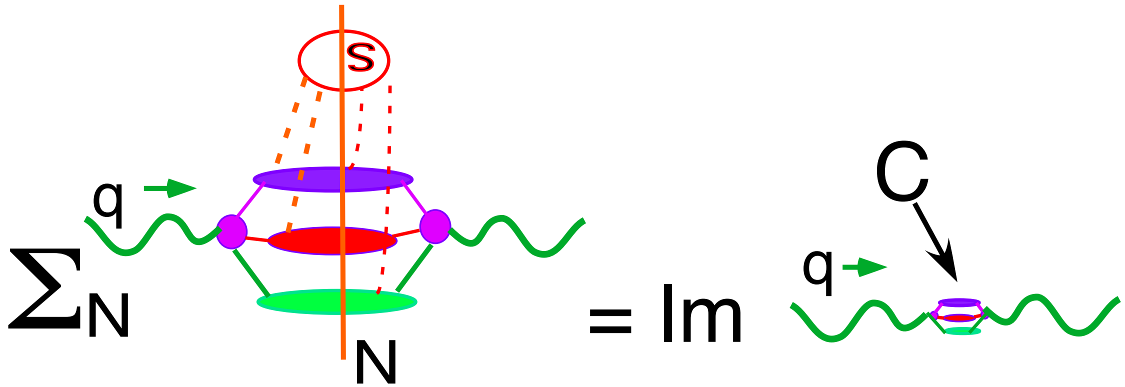
- Exercise: derive this result using only:

$$i\left(\frac{1}{x + i\epsilon} - \frac{1}{x - i\epsilon}\right) = 2\pi\delta(x)$$

- **Unitarity (and therefore cancellation) works at the level of the integrands of TOPT.**

  ⇒ the sum of cut diagrams has the same set of Landau equations as total cross section. Hence the sum at fixed loop momenta is infrared safe. For jet cross sections in e^+e^- annihilations, all states pinched on-shell are automatically included.

- **Apply this is physical pictures for jet production.**



  ⇒ When we sum over states that differ by collinear rearrangements and emission/absorption of soft partons, pinches disappear and jet cross sections are IR safe.

- **And by power counting soft gluons are suppressed.**

III-IVB. Factorization in DIS

- Finiteness of $\mathrm{e}^+\mathrm{e}^-$ jet cross sections is fine, but the challenge remains to use AF in observables σ (cross sections, also some amplitudes) that are not infrared safe because their physical pictures have to include one or more incoming hadrons.

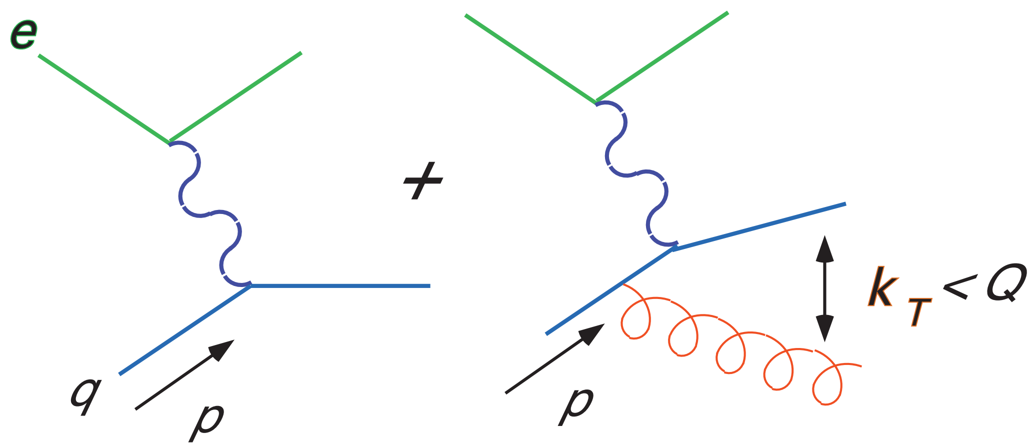
- We can use IR safety if: σ has a short-distance subprocess.
  We then separate *IR Safe* from IR: this is factorization

- IR Safe part (short-distance) is calculable in pQCD

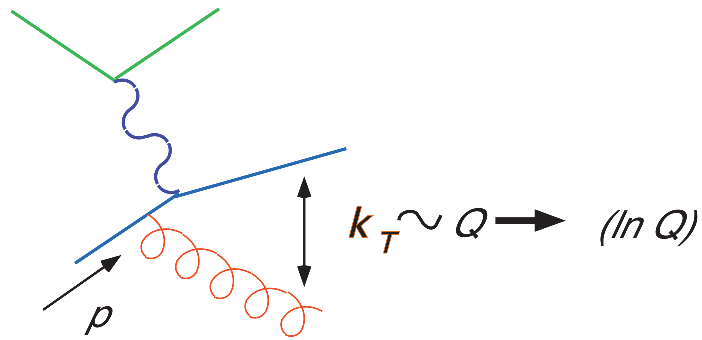
- Infrared part – example: parton distribution – measureable and universal

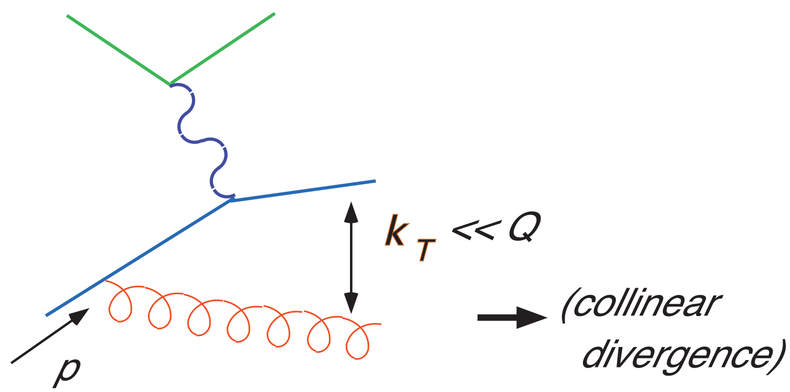
- Infrared safe part – insensitive to soft gluon emission and collinear rearrangements

- For DIS, will find a result . . .
- Just like Parton Model except in Parton Model the infrared safe part is $\sigma_{\mathrm{LO}} \Rightarrow \phi(x)$ normalized uniquely
- In pQCD must define parton distributions more carefully: the factorization scheme
- Basic observation: virtual states are not truly frozen. Some states fluctuate on scale $1/Q$ . . .



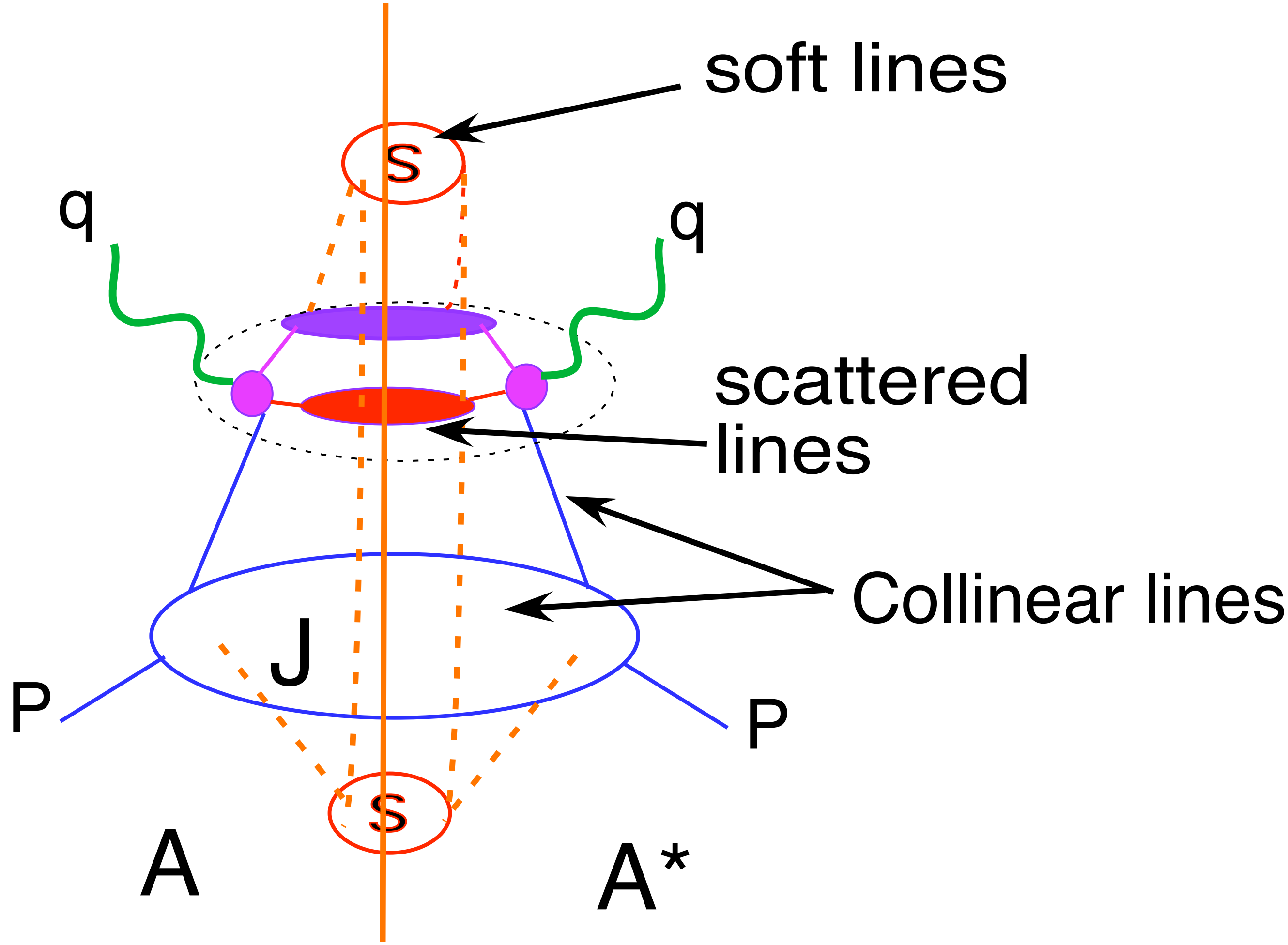
Short-lived states $\Rightarrow \ln(Q)$





- **Longer-lived states $\Rightarrow$ Collinear Singularity (IR)**
- **How we systematize to all orders in perturbation theory . . . a taste of "all-orders" proofs in pQCD.**

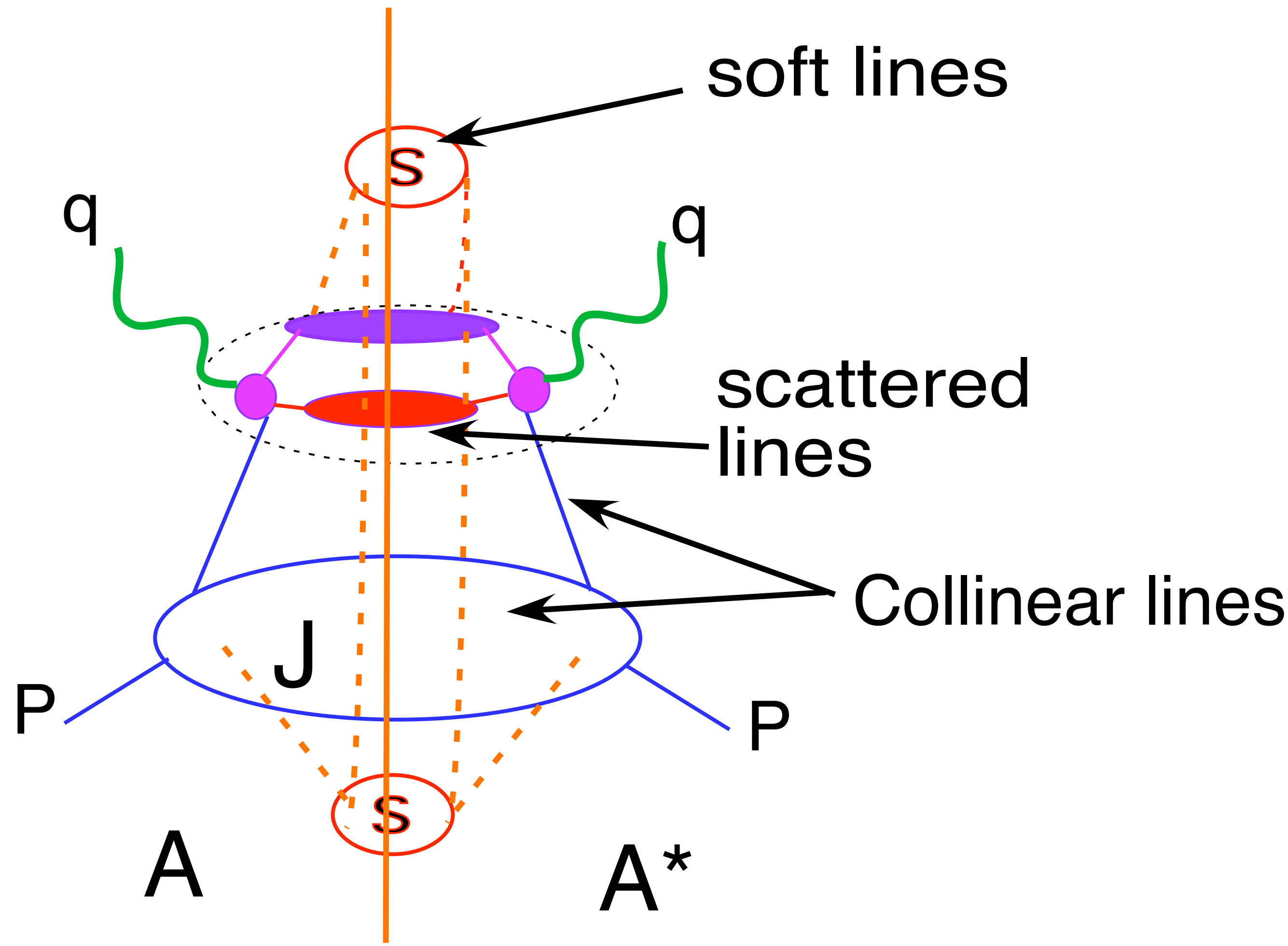
- **We can generalize to <u>all sources of mass dependence</u>. Always from classical processes with on-shell particles.**



- **This is the same "cut diagram notation", representing the amplitude and complex conjugate. Adding up all cut diagrams is the same as summing diagrams of A and then taking $|A|^2$.**

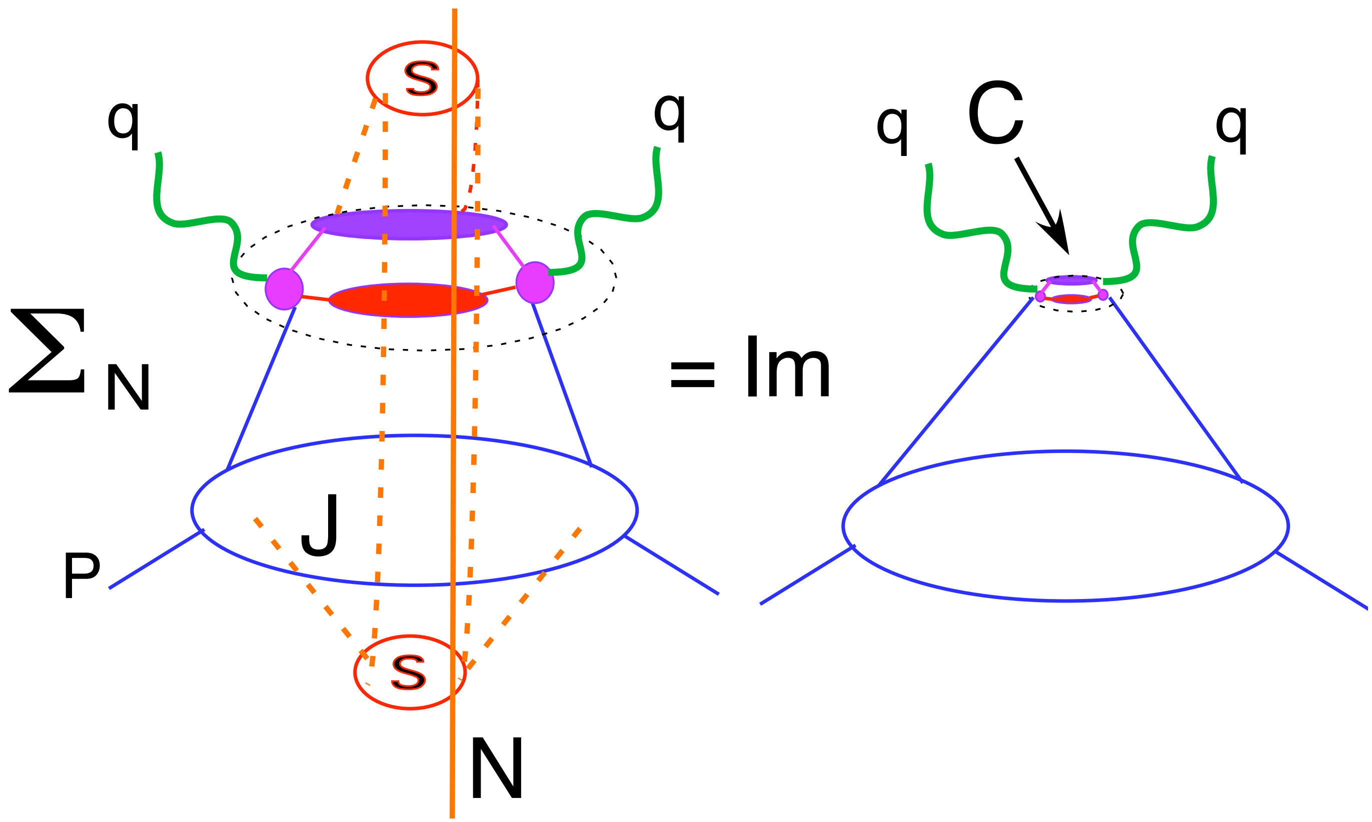
- **The scattered parton line is accompanied by arbitrary numbers of longitudinally-polarized gluons, just as in elastic scattering.**

- **Again: the structure of on-shell lines in an arbitrary cut diagram.**



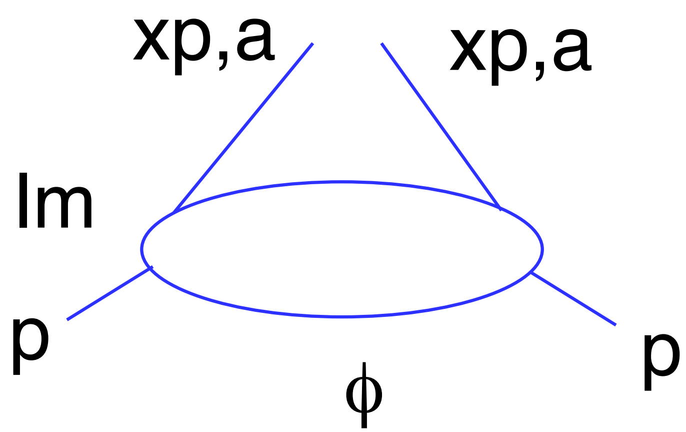
- **The story: h splits into collinear partons, then one of them scatters, producing jets that recede at speed of light, connected only by "infinite wavelength soft" quanta.**

- **Use of the optical theorem – relate the cut diagram to forward scattering. No classical processes are possible, because the scattered quarks must rescatter, and all interactions after the hard scattering collapse to a "short-distance" function C, that depends only on xp and q:**



- **All long-distance logs cancels because of the inclusive sum over states.**

- The partons on each side of the short distance function $C(p,q)$ must have the same flavor and momentum fraction.



- Definition of parton distribution generates all the same long-distance behavior left in in the original diagrams (quark case) after the sum over hadronic final states:

$$\phi_{a/h}(x,\mu_F) = \sum_{\text{spins }\sigma} \int \frac{dy^-}{2\pi} e^{-ixp^+y^-} \langle p,\sigma|\bar{q}(y^-)\gamma^+ P \exp[-ig\int_0^{y^-} A^+(\lambda)]\, q(0)|p,\sigma\rangle$$

- This matrix element requires renormalization: thus the 'μ_F'.

- **The result: factorized DIS**

$$F_2^{\gamma q}(x,Q^2) = \int_x^1 d\xi\; C_2^{\gamma q}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right) \times \phi_{q/q}(\xi,\mu_F,\alpha_s(\mu))$$

$$\equiv C_2^{\gamma q}\left(\frac{x}{\xi},\frac{Q}{\mu},\frac{\mu_F}{\mu},\alpha_s(\mu)\right)\otimes \phi_{q/q}(\xi,\mu_F,\alpha_s(\mu))$$

- $\phi_{q/q}$ **has** $\ln(\mu_F/\Lambda_{\mathrm{QCD}})$ ... **with** μ_F **its independent renormalization scale.**

- C **has** $\ln(Q/\mu),\ \ln(\mu_F/\mu)$

- **Often pick** $\mu=\mu_F$ **and often pick** $\mu_F=Q$**. So often see:**

$$F_2^{\gamma q}(x,Q^2) = C_2^{\gamma q}\left(\frac{x}{\xi},\alpha_s(Q)\right) \otimes \phi_{q/q}(\xi,Q^2)$$

Demand $\mu d/d\mu F = 0$, The two functions in convolution have only $\alpha_s(\mu)$ in common $\Rightarrow$

$$\mu\frac{d}{d\mu}\,\phi_{qq}(x,\mu^2) \;=\; \int_x^1 \frac{d\xi}{\xi}\,P(x/\xi,\alpha_s(\mu))\;\phi(\xi,\mu^2)$$

$$\mu\frac{d}{d\mu}\,C_2^{\gamma q}\left(\frac{x}{\xi},\alpha_s(Q)\right) \;=\; \int_x^1 \frac{d\xi}{\xi}\,C_2^{\gamma q}(x/\xi,\alpha_s(\mu))\;\;P(\xi,\mu^2)$$

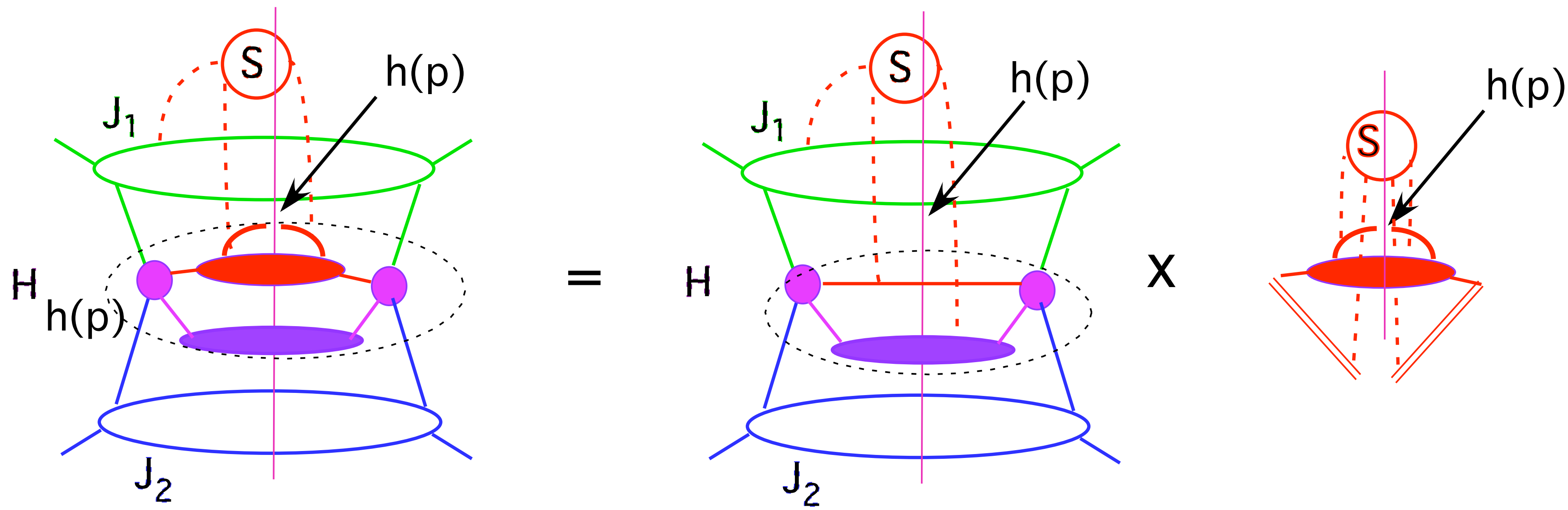
Here we've suppressed sum of parton types, but emphasized the analogy to the renormalization group.

- **P is the DGLAP splitting function. Note it can be calculated from C directly, and is hence IR safe. Some details in appendix.**

III-IVC. Factorization for 1PI cross sections in hadron-hadron scattering

- **How it works, with pictures as in DIS:**

- **Separation of soft quanta from fragmenting partons because soft radiation cannot resolve collinear-moving particles.**



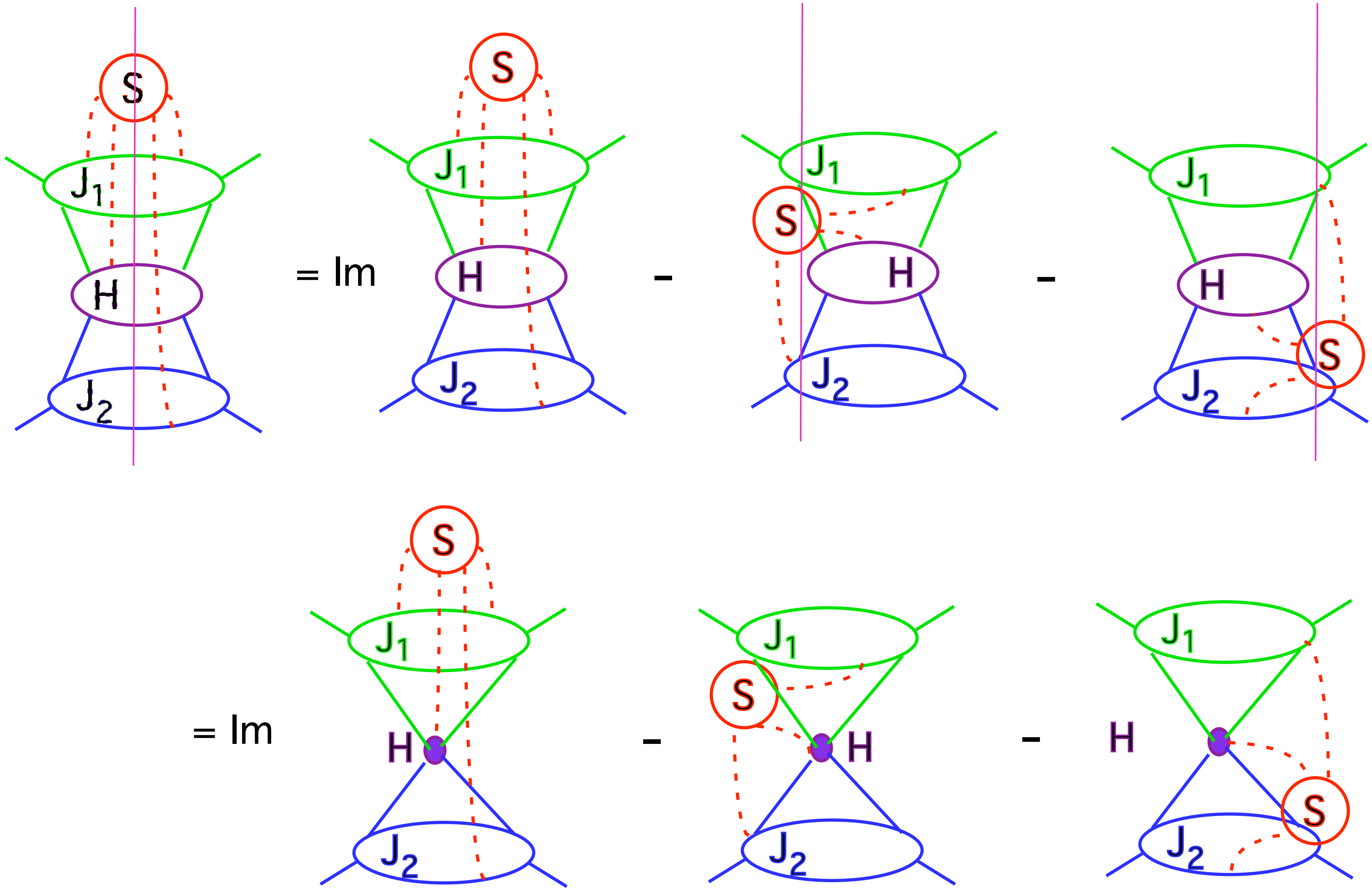
- **Can now sum over all final states in the first factor on the RHS ⇒ the all-orders cancellation of soft singularities that connect initial and final states for single-particle inclusive and other short-distance cross sections in hadron-hadron scattering:**

- **The fragmentation function as an operator**

$$D_{H/q}(z,\zeta,\zeta') \;=\; \sum_X \int \frac{p^+ dy^-}{2\pi}\, \mathrm{e}^{-i(p^+/z)y^-} \langle 0|\, q(0) \frac{\gamma^+}{2p^+}\, |H(p), X\rangle \langle H(p), X|\bar{q}(y^-)\,|0\rangle$$

- **with a gauge link $W(y^-, 0)$ between the fields.**

- **And then in the rest of the diagram . . .**



- **all terms on RHS are power-suppressed by power counting unless soft gluons don't attach to the final state at all. Collinear factorization arguments then have to with soft gluons that only attach to initial-state jets. This leads us to . . .**

- **III-IVD. Drell-Yan applications**

- **Here we can only remind ourselves of the results:**
- **Collinear factorization for the total cross section at pair mass Q**

$$d\sigma_{\mathrm{H_1H_2}}(p_1, p_2, M) = \sum_{a,b} \int_0^1 d\xi_a \, d\xi_b d\hat{\sigma}_{ab\to F+X} \left(\xi_a p_1, \xi_b p_2, Q, \mu\right) \times \phi_{a/H_1}(\xi_a, \mu) \, \phi_{b/H_2}(\xi_b, \mu),$$

- **Transverse momentum-dependent factorization at measured $Q_T \ll Q$**

$$\frac{d\sigma_{NN\to QX}}{dQ d^2Q_T} = \int d\xi_1 d\xi_2 \; d^2\mathrm{k}_{1T} d^2\mathrm{k}_{2T} d^2\mathrm{k}_{sT} \times \; H(\xi_1 p_1, \xi_2 p_2, Q, n)_{a\bar{a}\to Q+X} \times \Phi_{a/N}(\xi_1, p_1\cdot n, k_{1T}) \, \Phi_{\bar{a}/N}(\xi_2, p_2\cdot n, k_{2T}) \times \; \delta\left(Q_T - k_{1T} - k_{2T} - k_{sT}\right)$$

- **The $\Phi's$: new Transverse momentum-dependent PDFs**

$$\Phi_{a/N}(\xi_1, p_1 \cdot n, k_{1T}) = \sum_{\text{spins } \sigma} \int \frac{dy^-}{2\pi} e^{-ixp^+y^-} \frac{d^2k_T}{(2\pi)^2} e^{i\mathrm{k}_T\cdot \mathrm{x}} \times \langle p,\sigma|\bar{q}(y^-)\gamma^+ W_n^\dagger(-\infty, y^-) W_n(-\infty, 0) q(0)|p,\sigma\rangle$$

where W is the gauge link starting at the field in the n^μ direction:

$$W_n(y^-) = P \exp[-ig \int_{y^-}^{-\infty} n \cdot A(\lambda n)]$$

- **At $\mathrm{b} = 0$, formally reduces to our operator definition of $\phi(\xi, \mu^2)$.**

- **With proper choice of gauge link direction, no separate function for soft gluons is required (Collins)**

- **III-IVE. Quote as Epilogue and Summary: From Collins, Frankfurt and Strikman (1997)**

- **who set out to prove factorization for exclusive electroproduction of mesons ...**

$$\mathcal{M} = \sum_{i,j} \int_0^1 dz \int dx_1 f_{i/p}(x_1, x_1 - x, t, \mu)\, H_{ij}(Q^2 x_1/x, Q^2, z, \mu)\, \phi_j^V(z, \mu)$$
$$+\text{power-suppressed corrections.} \qquad (3)$$

- **with generalized parton distribution**

$$f_{i/p}(x_1, x_2, t, \mu) = \int_{-\infty}^{\infty} \frac{dy^-}{4\pi}\, e^{-ix_2 p^+ y^-} \langle p'|\, T\bar{\psi}(0, y^-, \mathbf{0}_T)\gamma^+ \mathcal{P}\psi(0)\; |p\rangle, \qquad (4)$$

where $\mathcal{P}$ is a path-ordered exponential of the gluon field along the light-like line joining the two operators for a quark of flavor i. We have defined x_1 to be the fractional momentum

...

1. Scale all momenta by a factor Q/m, so that we are in effect attempting to take a massless on-shell limit of the amplitude.

2. Use the Coleman-Norton theorem to locate all pinch-singular surfaces in the space of loop integration momenta, in the zero-mass limit.

3. Identify the relevant regions of integration as neighborhoods of these pinch singular surfaces.

4. The scattering amplitude is a sum of contributions, one for each pinch singular surface, plus a term where all lines have virtuality of at least of order Q^2. Appropriate subtractions are made to prevent double counting.

5. Perform power counting to determine which regions give the largest power of Q.

---

6. Finally, show that the contributions for the leading power of Q give the factorization formula Eq. (3).

That about says it all.