Spring School on QCD prospects for future ep and eA colliders, Orsay, June 4-8 2012

TMDs: Theory and Phenomenology I

Piet Mulders

mulders@few.vu.nl

Abstract

Transverse momentum dependent (TMD) distribution and fragmentation functions are described as Fourier transforms of matrix elements containing non-local combinations of quark and gluon fields. While the collinear functions are light-cone correlators in which the non-locality is restricted along the light-cone, the transverse momentum dependent functions are light-front correlators including a transverse (space-like) separation away from the light-cone. In the matrix elements the time-ordering is superfluous and they are parts of the full (squared) amplitudes that account for the connections to the hadrons (soft parts).

The collinear (x-dependent) parton (quark or gluon) distribution functions (PDF's) that appear in the parameterization of collinear leading-twist correlators are interpreted as momentum densities including polarized parton densities in polarized hadrons. They involve only spin-spin densities and they do not allow for a description of single-spin asymmetries in high-energy scattering processes at leading 1/Q order in the hard scale Q.

TMD (x and p_T-dependent) PDF's that appear in the parameterization of TMD correlators include spin-spin as well as momentum-spin correlations and they are able to describe single-spin and azimuthal asymmetries, such as Sivers and Collins effects in semi-inclusive deep inelastic scattering (SIDIS), but there are many open issues on p_T-factorization. Upon taking moments in p_T (or taking Bessel weights) the correlators involve higher-twist operators, but evaluated at zero-momentum (gluonic pole matrix elements). They can be incorporated in a 'generalized' factorization scheme with specific gluonic pole factors such as the sign in SIDIS versus Drell-Yan, which can be traced back to having TMD's with non-trivial process-dependent past- or future-pointing gauge links appearing in the light-front separated, non-local operator combinations.

P.J. Mulders

Introduction

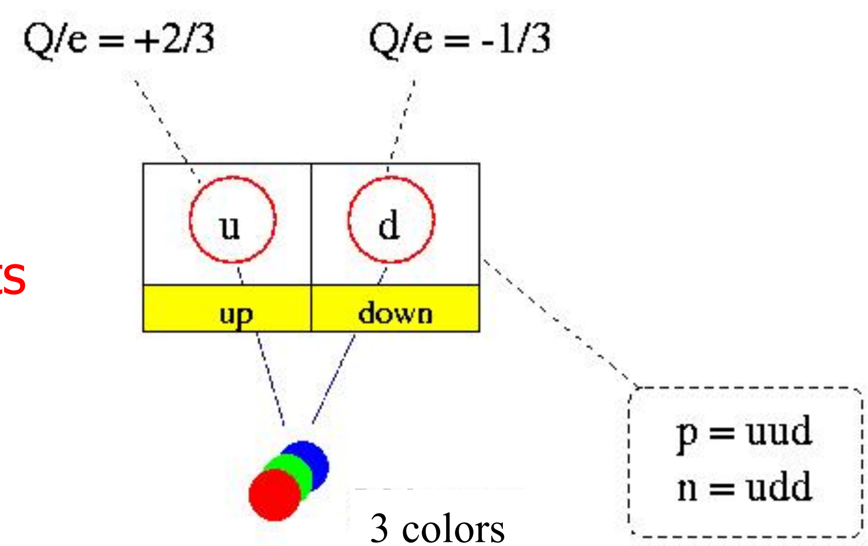
- What are we after?
  - Structure of proton (quarks and gluons)
  - Use of proton as a tool (spin, flavor, …)
- What are our means?
  - QCD as part of the Standard Model
  - Features of QCD: asymptotic freedom
  - Confinement scale ~ GeV

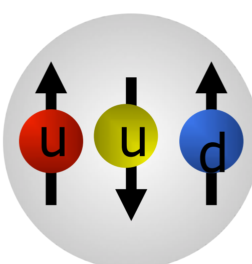
Valence structure of hadrons: global properties of nucleons

- mass
- charge
- spin
- magnetic moment
- isospin, strangeness
- baryon number

- $M_p \approx M_n \approx 940$ MeV
- $Q_p = 1$, $Q_n = 0$
- $s = ½$
- $g_p \approx 5.59$, $g_n \approx -3.83$
- $I = ½$: (p,n) $S = 0$
- $B = 1$

Quarks as constituents

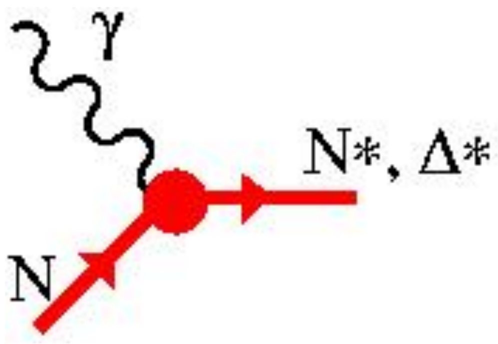


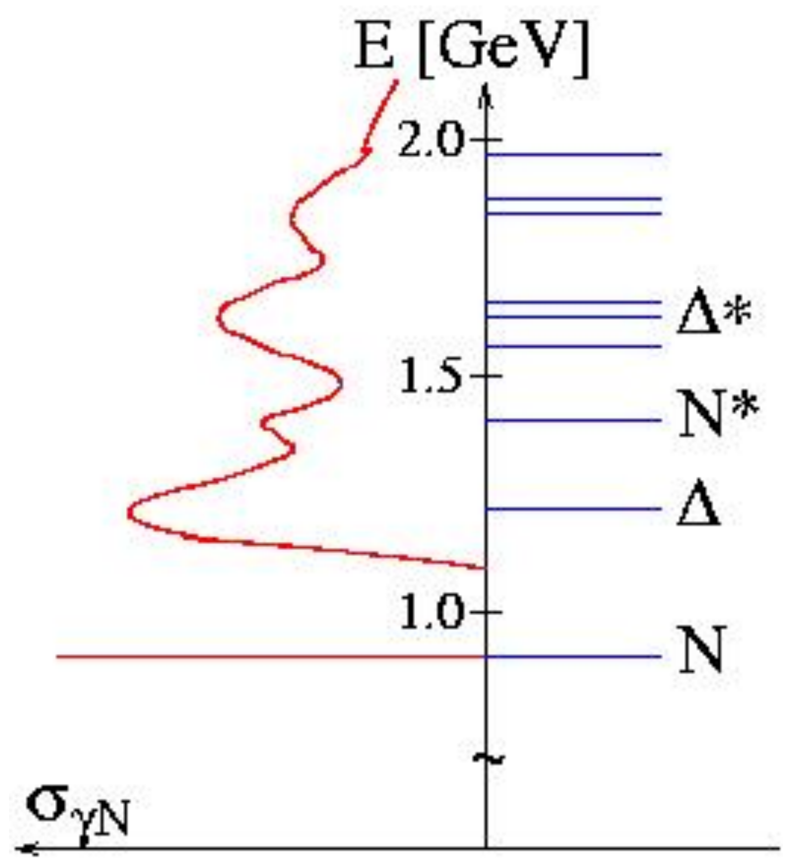


proton

A real look at the proton

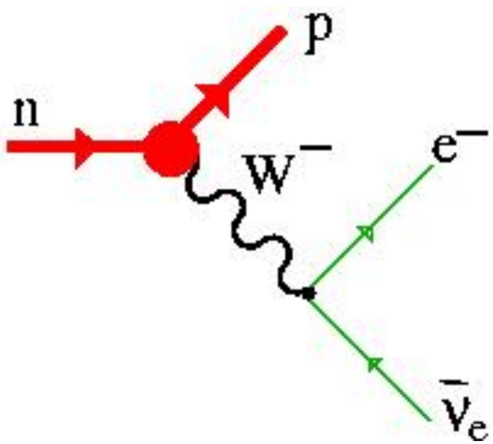
$\gamma + N \rightarrow ....$





Nucleon excitation spectrum
E ~ 1/R ~ 200 MeV
R ~ 1 fm

A (weak) look at the nucleon

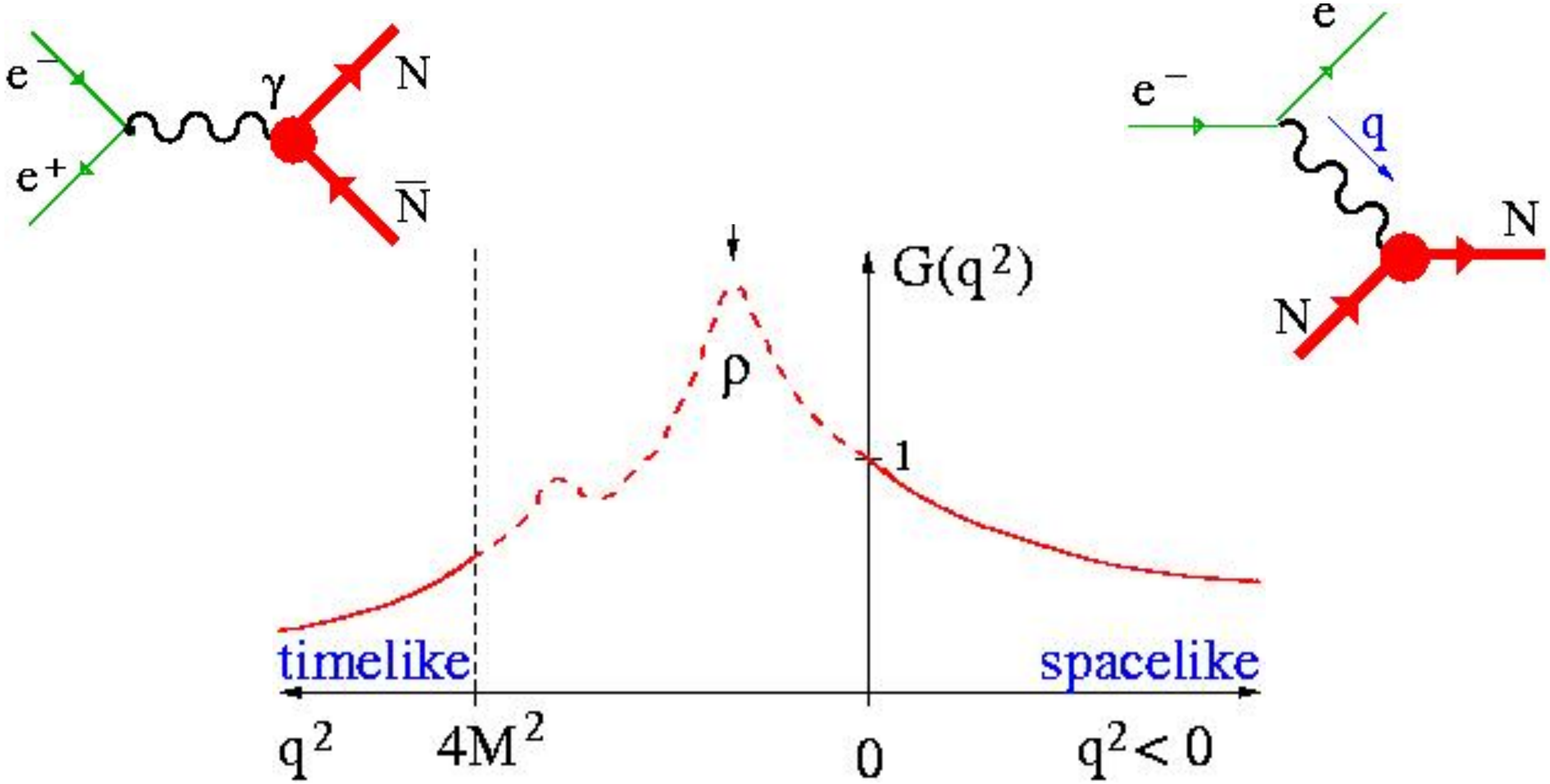


$n \rightarrow p + e^- + n$

τ = 900 s
→ Axial charge
$G_A(0) = 1.26$

A virtual look at the proton

$\gamma^* \rightarrow N\,\bar{N}$ $\qquad\qquad$ $\gamma^* + N \rightarrow N$



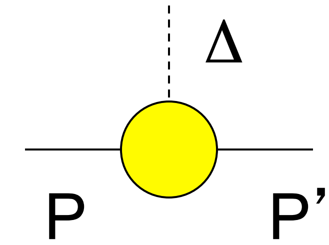
Local – forward and off-forward m.e.

Local operators (coordinate space densities):

$$< P'|O(x)|P > \; = e^{i\Delta . x}\left[G_1(t) - i\Delta_\mu G_2^\mu(t)\right]$$

$t = \Delta^2$

Form factors



Static properties:

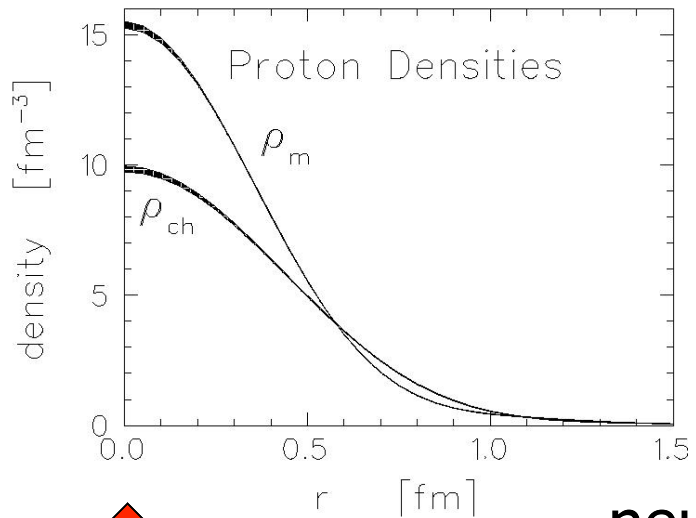
$$G_1(0) = \; < P|O(x)|P >$$

$$G_2^\mu(0) = \; < P|x^\mu O(x)|P >$$

Examples:
(axial) charge
mass
spin
magnetic moment
angular momentum

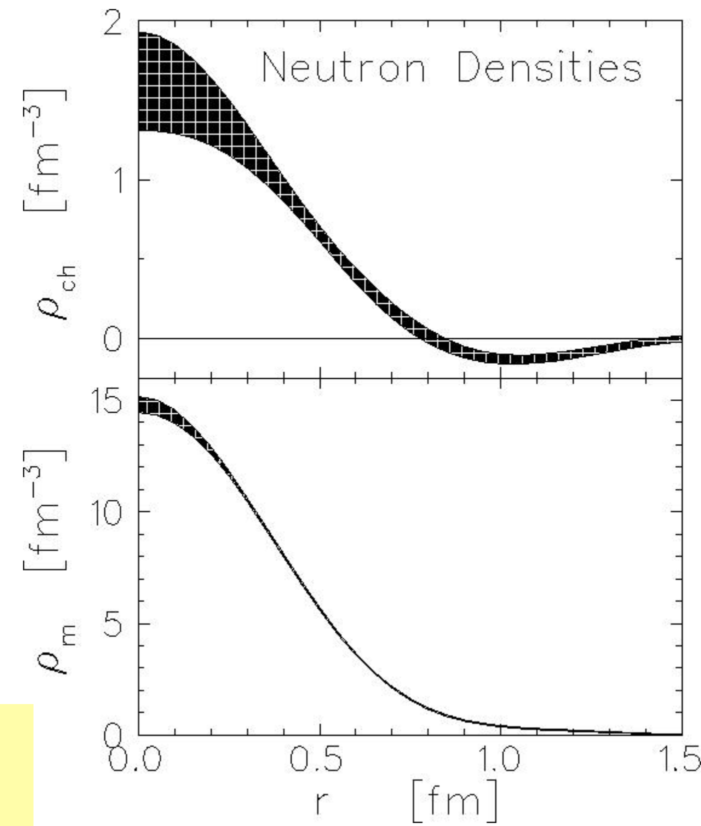
Nucleon densities from virtual look

$$G_i(t) \rightarrow \rho_i(x)$$



proton

neutron



- charge density ≠ 0
- u more central than d?
- role of antiquarks?
- $n = n_0 + p\pi^- + \ldots$?

Quark and gluon operators

Given the QCD framework, the operators are known quarkic or gluonic currents such as

(axial) vector currents

$$V_\mu^q(x) = \bar{q}(x)\gamma_\mu q(x)$$

$$A_\mu^{q'q}(x) = \bar{q}(x)\gamma_\mu\gamma_5 q'(x)$$

probed in specific combinations by photons, Z- or W-bosons

$$J_\mu^{(\gamma)} = \tfrac{2}{3}V_\mu^u - \tfrac{1}{3}V_\mu^d - \tfrac{1}{3}V_\mu^s + \ldots$$

$$J_\mu^{(Z)} = \tfrac{1}{2}\left(V_\mu^u - A_\mu^u\right) - \tfrac{4}{3}\sin^2\theta_W V_\mu^u + \ldots$$

$$J_\mu^{(W)} = V_\mu^{ud'} - A_\mu^{ud'} + \ldots$$

energy-momentum currents

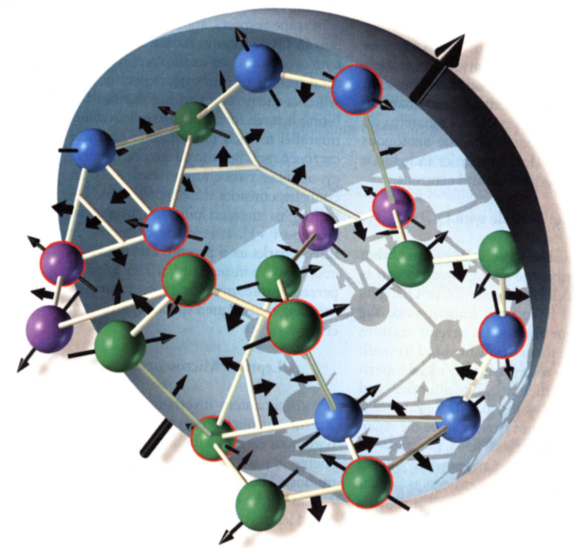
$$T_{\mu\nu}^q(x) \sim \bar{q}(x)\gamma_{\{\mu}D_{\nu\}}q(x)$$

$$T_{\mu\nu}^G(x) \sim G_{\mu\alpha}(x)G^\alpha{}_\nu(x)$$

probed by gravitons

Towards the quarks themselves

- The current provides the densities but only in specific combinations, e.g. *quarks minus antiquarks* and only flavor weighted
- No information about their correlations, (effectively) pions, or …
- Can we go beyond these global observables (which correspond to local operators)?
- Yes, in high energy (semi-)inclusive measurements we will have access to non-local operators!

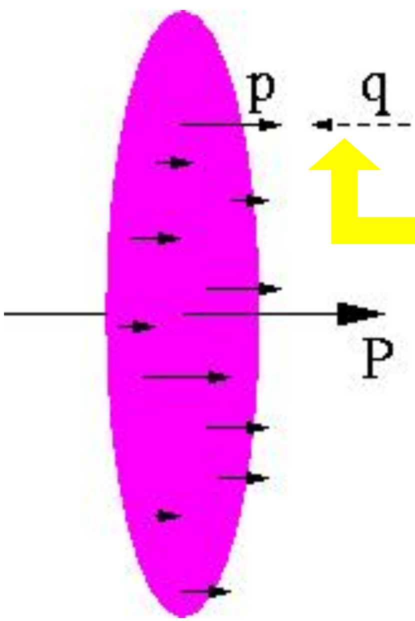


- L_{QCD} (quarks, gluons) known!

Non-local probing

Nonlocal forward operators (correlators):

$$< P \,|\, O\left(x - \tfrac{y}{2}, x + \tfrac{y}{2}\right) |\, P > \; = < P \,|\, O\left(-\tfrac{y}{2}, +\tfrac{y}{2}\right) |\, P >$$



Selectivity at high energies: q = p

Specifically useful: ‘squares’

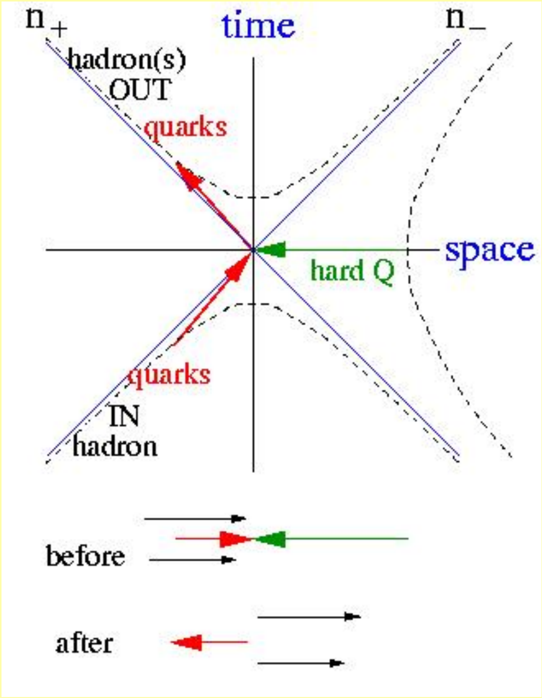
$$O\left(x - \tfrac{y}{2}, x + \tfrac{y}{2}\right) = \Phi^{\dagger}\left(x - \tfrac{y}{2}\right) \ldots \Phi\left(x + \tfrac{y}{2}\right)$$

Momentum space densities of Φ-ons:

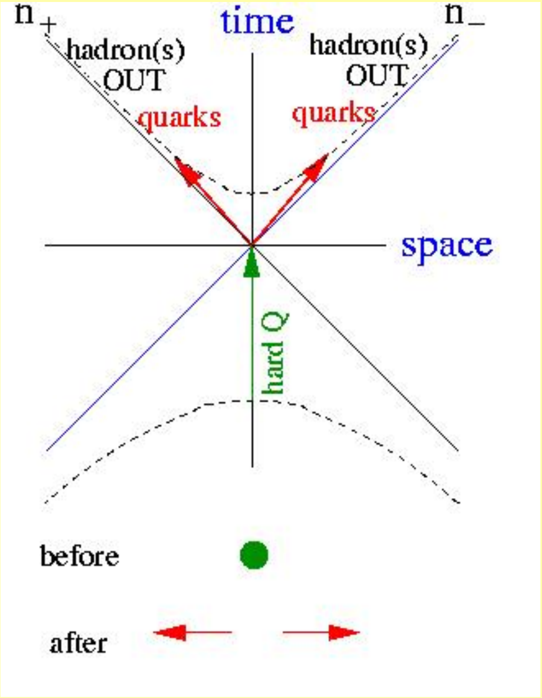
$$\int dy\; e^{ip\cdot y} < P \,|\, \Phi^{\dagger}\left(-\tfrac{y}{2}\right)\Phi\left(+\tfrac{y}{2}\right) |\, P > \; = \; \left| < P - p \,|\, \Phi(0) \,|\, P > \right|^{2} = f(p)$$

A hard look at the proton

- Hard virtual momenta ($\pm\, q^2 = Q^2 \sim$ many GeV^2) can couple to (two) soft momenta

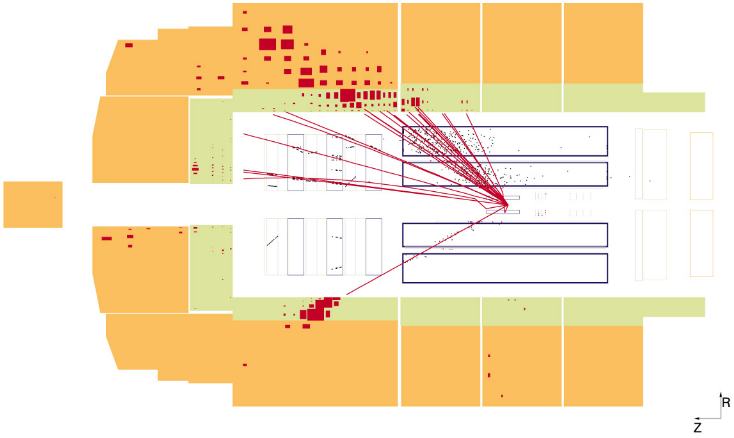


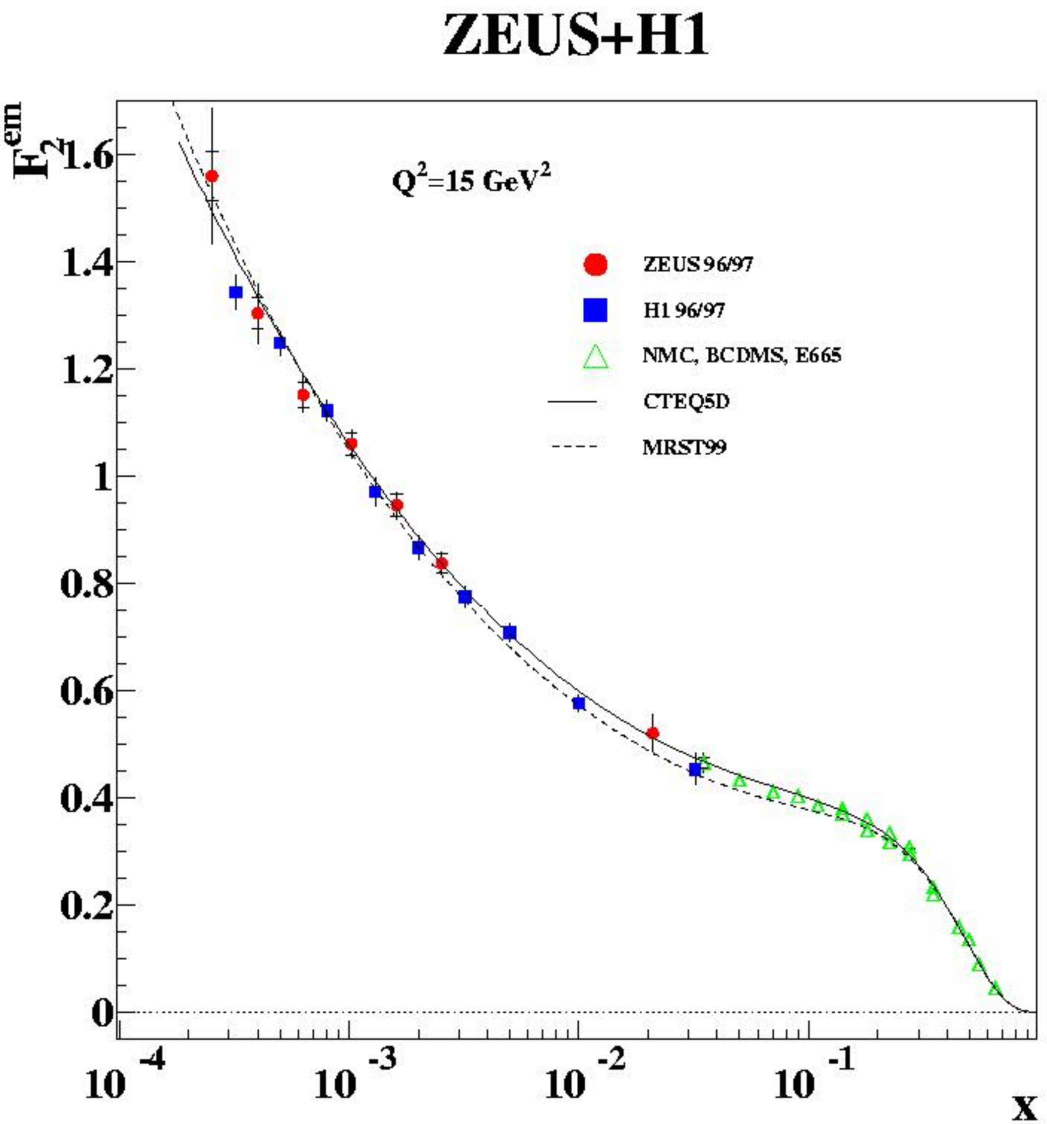
$\gamma^* + N \rightarrow$ jet



$\gamma^* \rightarrow$ jet + jet

Experiments!

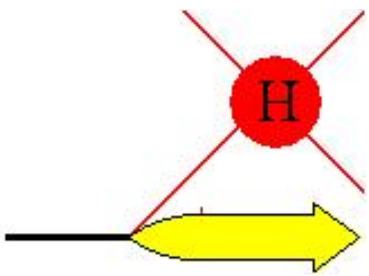




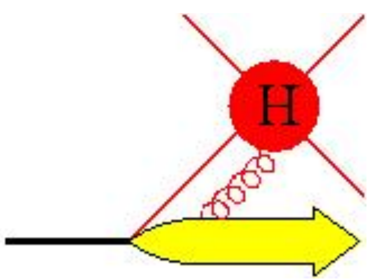
Hadron correlators

- Hadronic correlators establish the diagrammatic link between hadrons and partonic hard scattering amplitude
- Quark, quark + gluon, gluon, …

$$\langle 0|\psi_i(\xi)|p,s\rangle = u_i(p,s)\, e^{-ip.\xi}$$

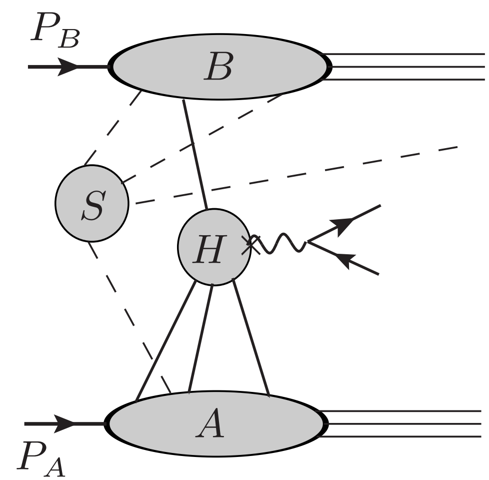


$$\langle X|\psi_i(\xi)|P\rangle\, e^{+ip.\xi}$$



$$\langle X|\psi_i(\xi)A^{\mu}(\eta)|P\rangle\, e^{+i(p-p_1).\xi+ip_1.\eta}$$

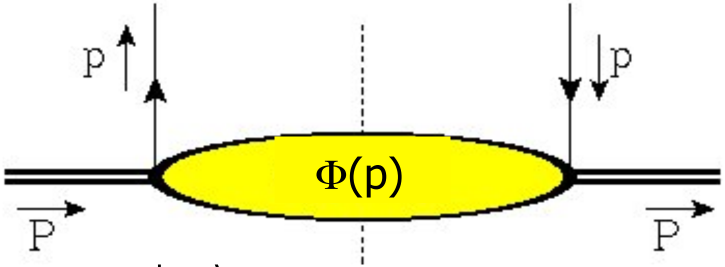
- Disentangling a hard process into parts involving hadrons, hard scattering amplitude and soft part is non-trivial



J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011

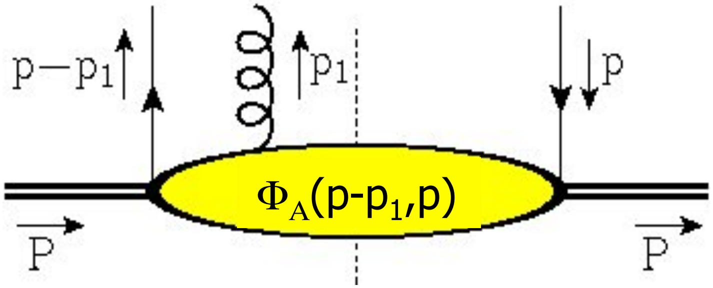
Hadron correlators

- Basically at high energies soft parts are combined into forward matrix elements of parton fields to account for distributions and fragmentation



$$\Phi_{ij}(p;P) = \Phi_{ij}(p\,|\,p) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}_j(0)\,\psi_i(\xi) \middle| P \right\rangle$$

- Also needed are multi-parton correlators



$$\Phi^{\alpha}_{A;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi}{(2\pi)^4} e^{i(p-p_1).\xi+ip_1.\eta} \left\langle P \middle| \bar{\psi}_j(0)A^{\alpha}(\eta)\,\psi_i(\xi) \middle| P \right\rangle$$

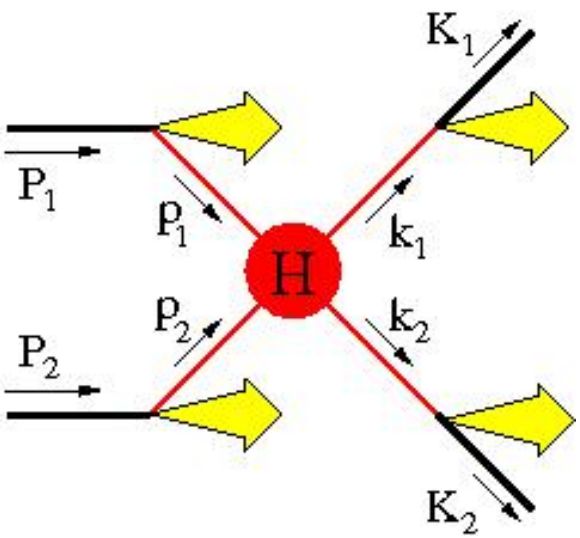
- Correlators usually just will be parametrized (nonperturbative physics)

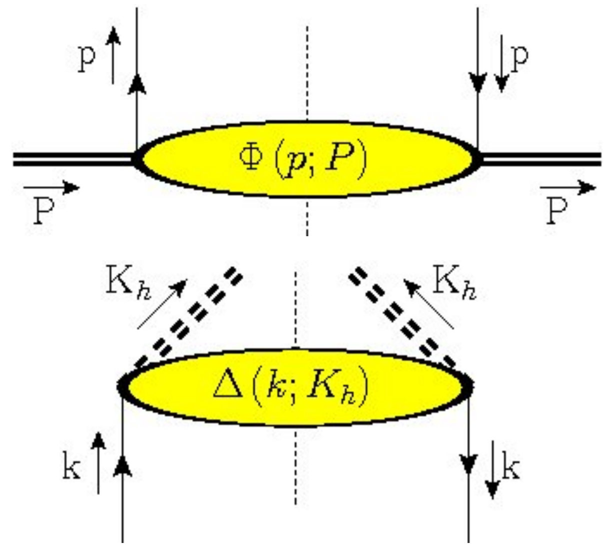
PDFs and PFFs

Basic idea of PDFs is to get a full factorized description of high energy scattering processes

$$\hat{\sigma} = |H(p_1, p_2, \ldots)|^2$$

calculable





defined (!)
&
portable

$$\sigma(P_1, P_2, \ldots) = \iiint \ldots dp_1 \ldots \Phi_a(p_1, P_1; \mu) \otimes \Phi_b(p_2, P_2; \mu)$$

$$\otimes\, \hat{\sigma}_{ab,c\ldots}(p_1, p_2, \ldots; \mu) \otimes \Delta_c(k_1, K_1; \mu) \ldots.$$

Give a meaning to integration variables!

Hard scale

- In high-energy processes other momenta available, such that P.P′ ~ s with a hard scale $s = Q^2 >> M^2$

- Employ light-like vectors P and n, such that P.n = 1 (e.g. n = P′/P.P′) to make a Sudakov expansion of parton momentum

$$p = \underbrace{xP^\mu}_{\sim Q} + \underbrace{p_T^\mu}_{\sim M} + \underbrace{\sigma n^\mu}_{\sim M^2/Q}$$

$$x = p^+ = p.n \sim 1$$

$$\sigma = p.P - xM^2 \sim M^2$$

- Enables importance sampling (twist analysis) for integrated correlators,

$$\Phi(p) = \Phi(x, p_T, p.P) \;\Rightarrow\; \Phi(x, p_T) \;\Rightarrow\; \Phi(x) \;\Rightarrow\; \Phi$$

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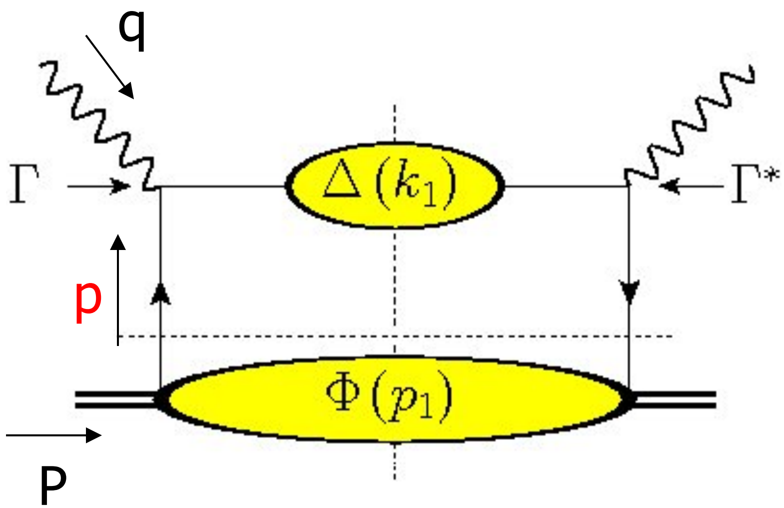
TMDs: Theory and Phenomenology II

Piet Mulders

mulders@few.vu.nl

Principle for DIS



$$\sum_s u(p,s)\,\bar{u}(p,s)$$

$$\Rightarrow \Phi(p,P) \sim (\not{p} + m) f(p)$$

- Instead of partons use correlators
- Expand parton momenta (for SIDIS take e.g. n= $P_h/P_h.P$)

$$p = \underbrace{xP^\mu}_{\sim Q} + \underbrace{p_T^\mu}_{\sim M} + \underbrace{\sigma n^\mu}_{\sim M^2/Q}$$

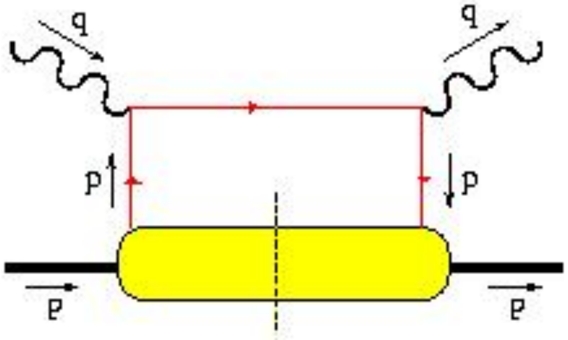
$$x = p^+ = p.n \sim 1$$

$$\sigma = p.P - xM^2 \sim M^2$$

Light-cone dominance in DIS

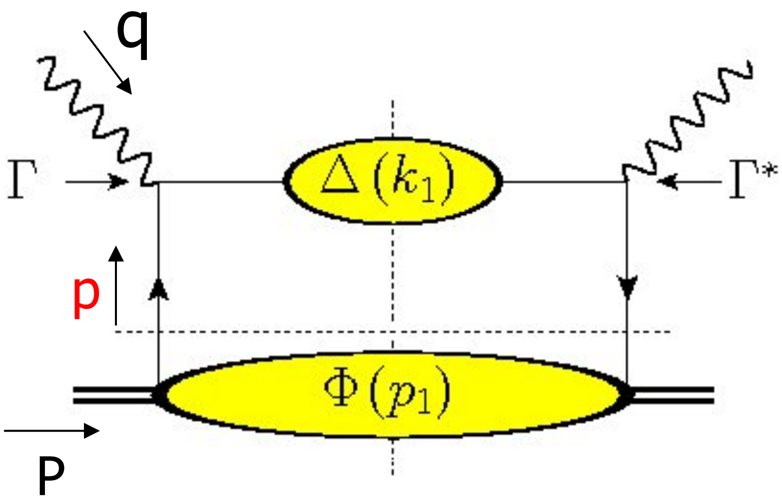
Large scale Q leads in a natural way to the use of lightlike vectors:
$n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ 2\,P\cdot q = \frac{Q^2}{x_B} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P = \frac{x_B M^2}{Q\sqrt{2}}\,n_- + \frac{Q}{x_B\sqrt{2}}\,n_+ \\ \\ q = \frac{Q}{\sqrt{2}}\,n_- - \frac{Q}{\sqrt{2}}\,n_+ \end{array}\right.$$



part	'components' $-$	$+$	
HARD	$\sim Q$	$\sim Q$	
$H \to q$	$\sim 1/Q$	$\sim Q$	$\to \int dp^- d^2p_T \ldots$

Result for DIS



$$2MW^{\mu\nu}(P,q) = -\tfrac{1}{2}\, g_T^{\mu\nu} \int dx\, dp.P\, d^2 p_T\, Tr[\Phi(p,P)\gamma^+]\,\delta(x - x_B)$$

$$= -\tfrac{1}{2}\, g_T^{\mu\nu}\, Tr[\Phi(x_B)\gamma^+]$$

Twist analysis (1)

- Dimensional analysis to determine importance in an expansion in inverse hard scale
- Maximize contractions with n

$$\dim[\overline{\psi}(0)\not{n}\,\psi(\xi)] = 2$$

$$\dim[F^{n\alpha}(0)F^{n\beta}(\xi)] = 2$$

$$\dim[\overline{\psi}(0)\not{n}\,A_T^{\alpha}(\eta)\,\psi(\xi)] = 3$$

- ... or maximize # of P's in parametrization of Φ

$$\boxed{\Phi^q(x) = f_1^q(x)\frac{\not{P}}{2} \;\Leftrightarrow\; f_1^q(x) = \int \frac{d\lambda}{(2\pi)} e^{ix\lambda}\langle P|\overline{\psi}(0)\not{n}\,\psi(\lambda n)|P\rangle}$$

Parametrization of lightcone correlator

DISTRIBUTION FUNCTIONS

Parameterization of p_T-integrated soft part including subleading order and including T-odd parts for a spin 1/2 hadron:

leading part

$$\begin{aligned}\Phi(x) = {} & \boxed{\frac{1}{2}\left\{ f_1(x)\,\not{n}_+ + S_L\, g_1(x)\gamma_5\,\not{n}_+ + h_1(x)\,\frac{[\not{S}_T,\,\not{n}_+]\gamma_5}{2}\right\}} \\ & + \frac{M}{2P^+}\left\{ e(x) + g_T(x)\,\gamma_5\,\not{S}_T + S_L\, h_L(x)\,\frac{[\not{n}_+,\,\not{n}_-]\gamma_5}{2}\right\} \\ & + \frac{M}{2P^+}\left\{ \cancel{f_T(x)}\,\epsilon_T^{\rho\sigma} S_{T\rho}\gamma_\sigma - i\, S_L\, \cancel{e_L(x)}\,\gamma_5 + \cancel{i\,h(x)}\,\frac{[\not{n}_+,\,\not{n}_-]}{2}\right\}\end{aligned}$$

T-odd

- M/P^+ parts appear as M/Q terms in cross section
- T-reversal applies to $\Phi(x)$ → no T-odd functions

Jaffe & Ji NP B 375 (1992) 527
Jaffe & Ji PRL 71 (1993) 2547

Basis of partons

- 'Good part' of Dirac space is 2-dimensional
- Interpretation of DF's

TWO 'SPIN' STATES FOR (GOOD) QUARK FIELDS

chiral eigenstates:

$$\psi_{R/L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi : \qquad |R\rangle \quad \text{and} \quad |L\rangle$$

or

transverse spin eigenstates:

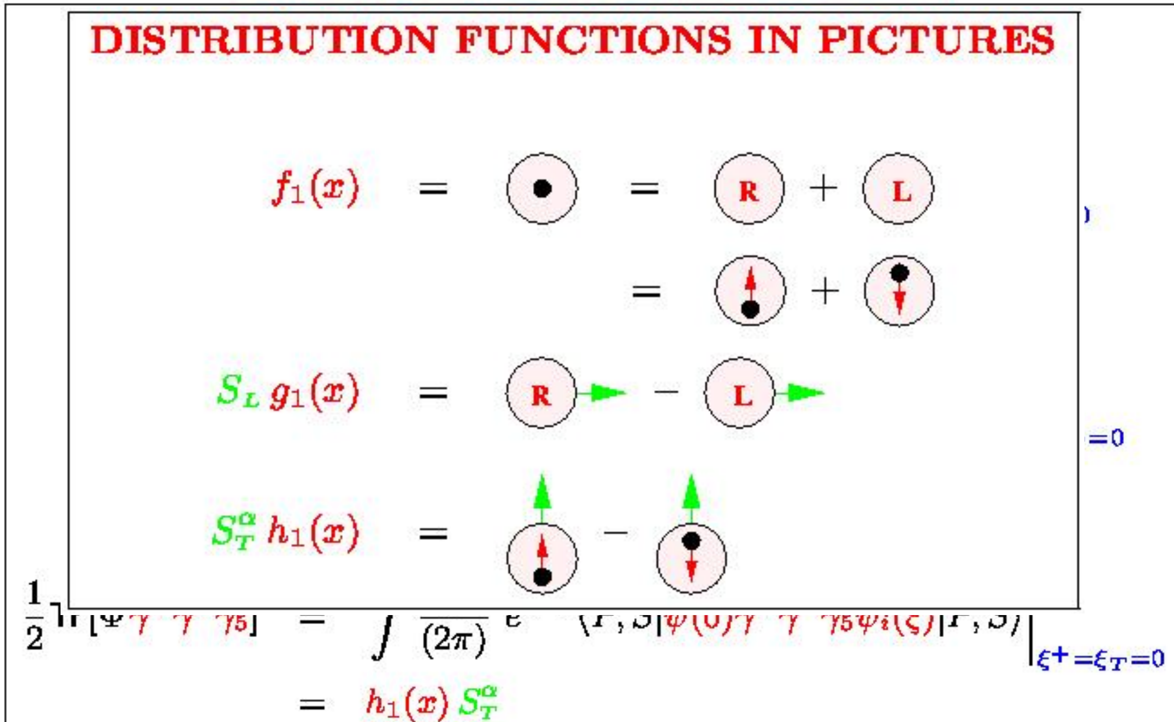
$$\psi_{\uparrow/\downarrow} \equiv \frac{1}{2}(1 \pm \gamma^\alpha \gamma_5)\psi : \qquad |\uparrow\rangle \quad \text{and} \quad |\downarrow\rangle$$

Note: $[\mathcal{P}_{R/L}, \mathcal{P}_+] = [\mathcal{P}_{\uparrow/\downarrow}, \mathcal{P}_+] = 0$

unpolarized quark distribution

helicity or chirality distribution

transverse spin distr. or transversity



Matrix representation for M = $[\Phi(x)\gamma^+]^T$

Bacchetta, Boglione, Henneman & Mulders
PRL 85 (2000) 712

Quark production matrix, directly related to the helicity formalism

Anselmino et al.

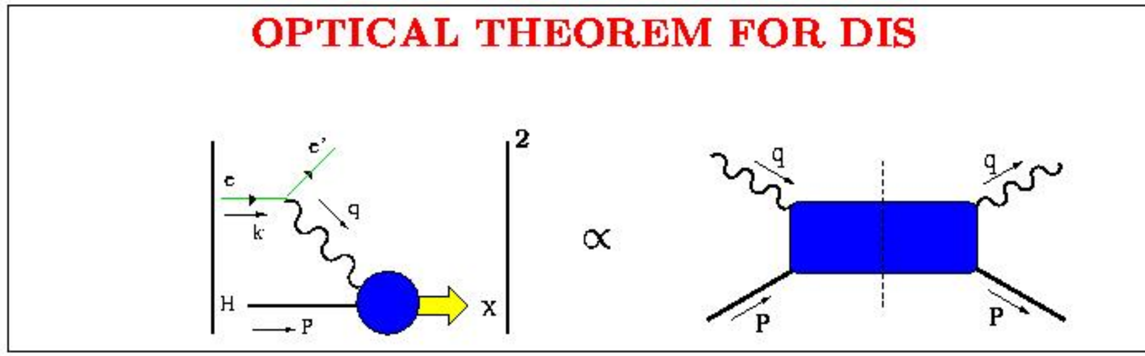
MATRIX REPRESENTATION FOR SPIN 1/2

p_T-integrated distribution functions:
For a spin 1/2 hadron (e.g. nucleon) the quark production matrix in quark⊗nucleon spin space is given by

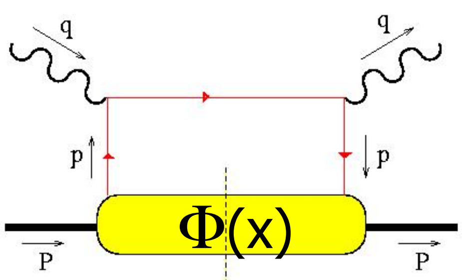
$$M^{(\text{prod})} = \begin{array}{cc} \begin{pmatrix} f_1 + g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1 - g_1 & 0 & 0 \\ 0 & 0 & f_1 - g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1 + g_1 \end{pmatrix} & \begin{array}{c} \text{R}\rightarrow \\ \leftarrow\text{R} \\ \text{L}\rightarrow \\ \leftarrow\text{L} \end{array} \\ \begin{array}{cccc} \text{R}\rightarrow & \leftarrow\text{R} & \text{L}\rightarrow & \leftarrow\text{L} \end{array} & \end{array}$$

- Off-diagonal elements (RL or LR) are chiral-odd functions
- Chiral-odd soft parts must appear with partner in e.g. SIDIS, DY

(calculation of) cross section in DIS



Full calculation



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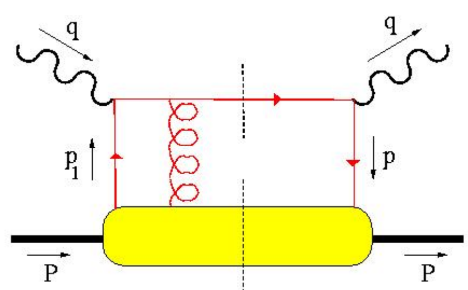

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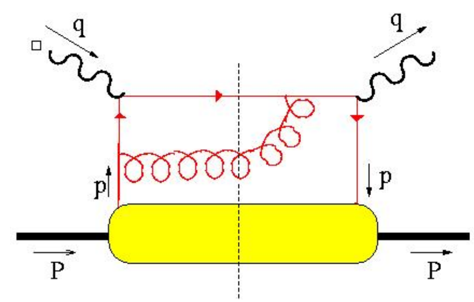
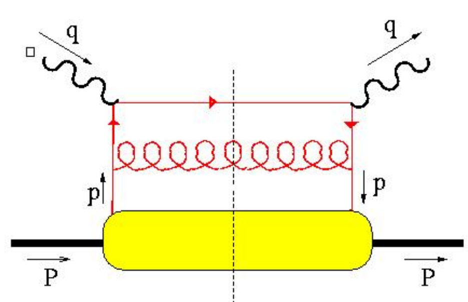


LEADING (in 1/Q)

$x = x_B = -q^2/P.q$

+



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TMDs: Theory and Phenomenology III

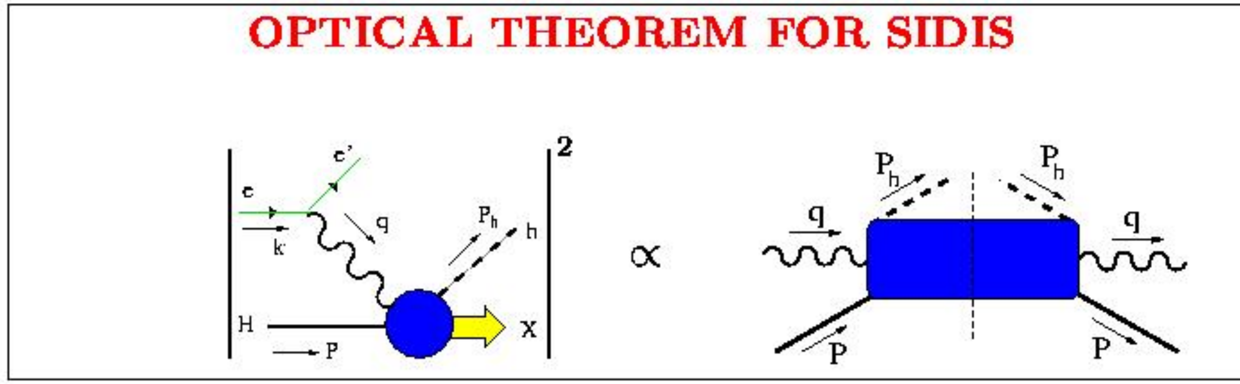
Piet Mulders

VU UNIVERSITY AMSTERDAM

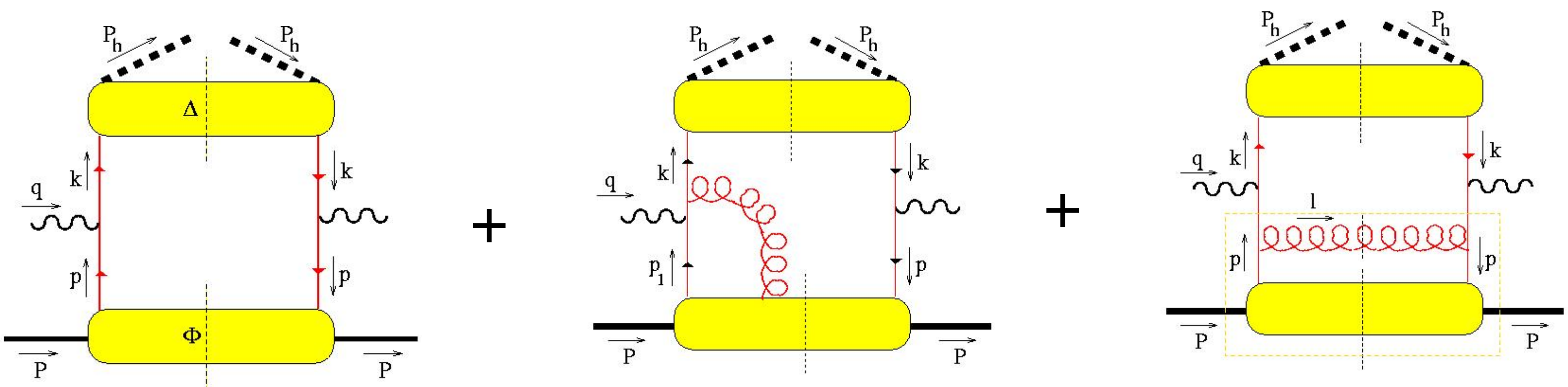
NIKHEF

mulders@few.vu.nl

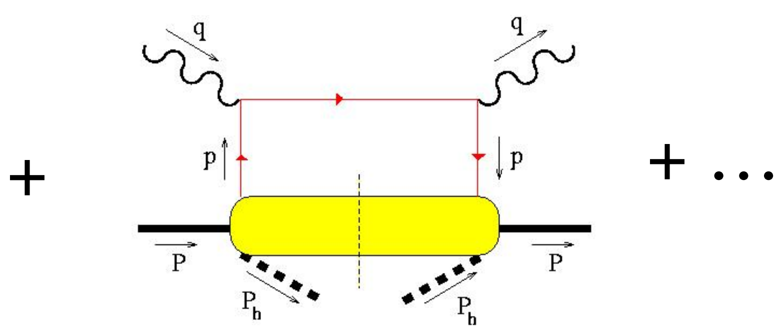
(calculation of) cross section in SIDIS



Full calculation



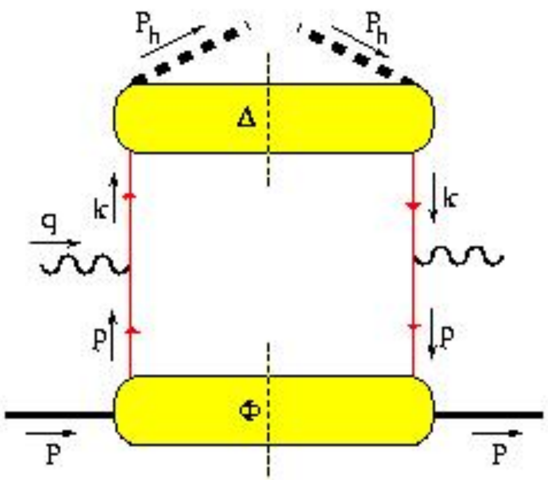
LEADING
(in 1/Q)



Light-front dominance in SIDIS

Large scale Q leads in a natural way to the use of lightlike vectors:
$n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$

$$\left.\begin{array}{l} q^2 = -Q^2 \\ P^2 = M^2 \\ P_h^2 = M_h^2 \\ 2\,P\cdot q = \dfrac{Q^2}{x_B} \\ 2\,P_h\cdot q = -z_h\,Q^2 \end{array}\right\} \longleftrightarrow \left\{\begin{array}{l} P_h = \dfrac{z_h\,Q}{\sqrt{2}}\,n_- + \dfrac{M_h^2}{z_h\,Q\sqrt{2}}\,n_+ \\[2ex] q = \dfrac{Q}{\sqrt{2}}\,n_- - \dfrac{Q}{\sqrt{2}}\,n_+ + q_T \\[2ex] P = \dfrac{x_B\,M^2}{Q\sqrt{2}}\,n_- + \dfrac{Q}{x_B\,\sqrt{2}}\,n_+ \end{array}\right.$$



Three external momenta

$P \quad P_h \quad q$

transverse directions relevant

$q_T = q + x_B\,P - P_h/z_h$

or

$q_T = -P_{h\perp}/z_h$

Relevance of transverse momenta in hadron-hadron scattering

$$p_1 \approx x_1 P_1 + p_{1T}$$
$$p_2 \approx x_2 P_2 + p_{2T}$$

- At high energies fractional parton momenta fixed by kinematics (external momenta) up to M²/Q² !!

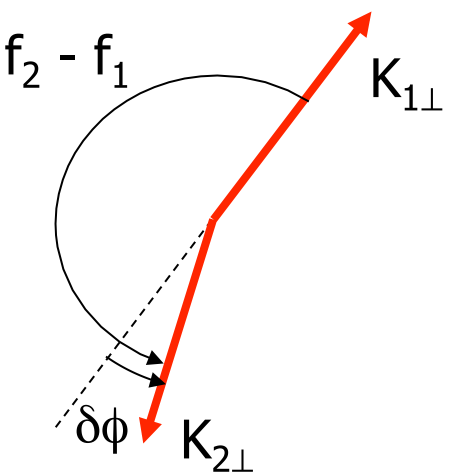
DY $\quad x_1 = p_1.n = \dfrac{p_1.P_2}{P_1.P_2} = \dfrac{q.P_2}{P_1.P_2}$

- Also possible for transverse momenta of partons

DY $\quad q_T = q - x_1 P_1 - x_2 P_2 = p_{1T} + p_{2T}$

2-particle inclusive hadron-hadron scattering

$$q_T = z_1^{-1} K_1 + z_2^{-1} K_2 - x_1 P_1 - x_2 P_2$$
$$= p_{1T} + p_{2T} - k_{1T} - k_{2T}$$

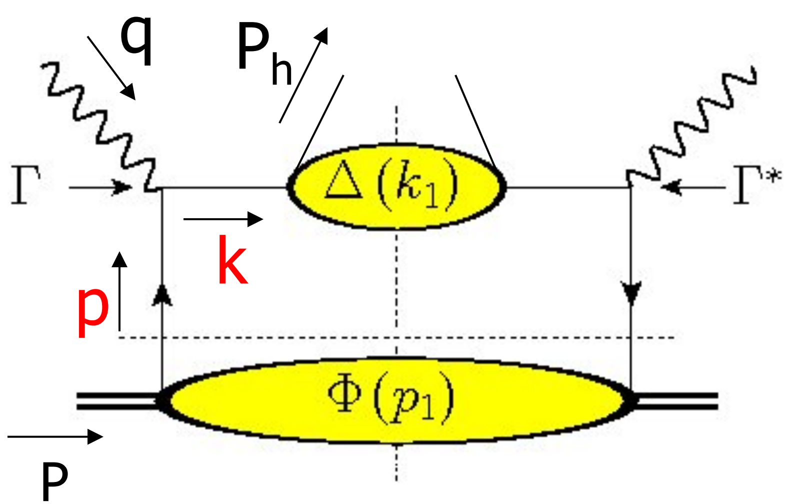


pp-scattering

Second scale!

Care is needed: we need more than one hadron and knowledge of hard process(es)!

Result for SIDIS



$$q_T = q + x_B P - \frac{P_h}{z_h}$$

$$2MW^{\mu\nu}(P, P_h, q) = \int d^2 p_T \int d^2 k_T$$

$$\times\, Tr[\Phi(x_B, p_T)\gamma^\mu \Delta(z_h, k_T)\gamma^\mu]\,\delta^2(p_T + q_T - k_T)$$

$$= -\tfrac{1}{2} g_T^{\mu\nu} \int d^2 p_T \int d^2 k_T$$

$$\times\, Tr[\Phi(x_B, p_T)\gamma^+]\, Tr[\Delta(z_h, k_T)\gamma^-]\,\delta^2(p_T + q_T - k_T)$$

Twist analysis (2)

- Dimensional analysis to determine importance in an expansion in inverse hard scale
- Maximize contractions with n

$$\dim[\overline{\psi}(0)\not{n}\,\psi(\xi)] = 2$$

$$\dim[F^{n\alpha}(0)F^{n\beta}(\xi)] = 2$$

$$\dim[\overline{\psi}(0)\not{n}\,A_T^{\alpha}(\eta)\,\psi(\xi)] = 3$$

- ... or maximize # of P's in parametrization of Φ

$$\boxed{\Phi^q(x) = f_1^q(x)\frac{\not{P}}{2} \Leftrightarrow f_1^q(x) = \int \frac{d\lambda}{(2\pi)} e^{ix\lambda} \left\langle P \middle| \overline{\psi}(0)\not{n}\,\psi(\lambda n) \middle| P \right\rangle}$$

- Next for $\Phi(x,p_T)$: availability of p_T

Symmetry constraints

$$\Phi^{T*}(p;P,S) = \gamma_0 \Phi(p;P,S)\gamma_0$$

- Hermiticity

$$\Phi(p;P,S) = \gamma_0 \Phi(\bar{p};\bar{P},-\bar{S})\gamma_0$$

- Parity

$$\Phi^{[U]}(p;P,S) = (-i\gamma_5 C)\Phi^{[-U]}(\bar{p};\bar{P},\bar{S})(-i\gamma_5 C)$$

- Time reversal

$$\Phi^{c}(p;P,S) = C\,\Phi^{T}(-p;P,S)C$$

- Charge conjugation (giving antiquark corr)

Parametrization of TMD correlator for unpolarized hadron:

$$\boxed{\Phi^{[\pm]q}(x,p_T) = \left(f_1^q(x,p_T^2) \pm i h_1^{\perp q}(x,p_T^2)\frac{\not{p}_T}{M}\right)\frac{\not{P}}{2}}$$

(unpolarized and transversely polarized quarks)

T-even T-odd

Mulders, Tangerman, Boer; Bacchetta, Diehl, Goeke, Metz, M, Schlegel, JHEP02 (2007) 093

Parametrization of $\Phi(x,p_T)$

- Also T-odd functions are allowed
- Functions $h_1^{\perp}$ (BM) and $f_{1T}^{\perp}$ (Sivers) nonzero!
- Similar functions (of course) exist as fragmentation functions (no T-constraints) $H_1^{\perp}$ (Collins) and $D_{1T}^{\perp}$

DISTRIBUTION FUNCTIONS

Parameterization of p_T-dependent soft part at leading order and including T-odd parts for polarized hadrons:

$$\Phi_0(x,p_T) = \left\{ f_1(x,p_T^2) + i\, h_1^{\perp}(x,p_T^2)\, \frac{\not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_L(x,p_T) = \left\{ S_L\, g_{1L}(x,p_T^2)\, \gamma_5 + S_L\, h_{1L}^{\perp}(x,p_T^2) \gamma_5\, \frac{\not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_T(x,p_T) = \left\{ g_{1T}(x,p_T^2)\, \frac{p_T \cdot S_T}{M}\, \gamma_5 + f_{1T}^{\perp}(x,p_T^2)\, \frac{\epsilon_{T\,\rho\sigma} p_T^{\rho} S_T^{\sigma}}{M} \right.$$

$$\left. + h_{1T}(x,p_T^2)\, \gamma_5\, \not{S}_T + h_{1T}^{\perp}(x,p_T^2)\, \frac{p_T \cdot S_T}{M}\, \frac{\gamma_5\, \not{p}_T}{M} \right\} \not{n}_+$$

$$\Phi_{LL}(x,p_T) = \ldots$$

Fermionic structure of TMDs

unpolarized quark distribution

with p_T

T-odd

helicity or chirality distribution

with p_T

$f_1^q(x) = q(x)$

$g_1^q(x) = \Delta q(x)$

$h_1^q(x) = \delta q(x)$

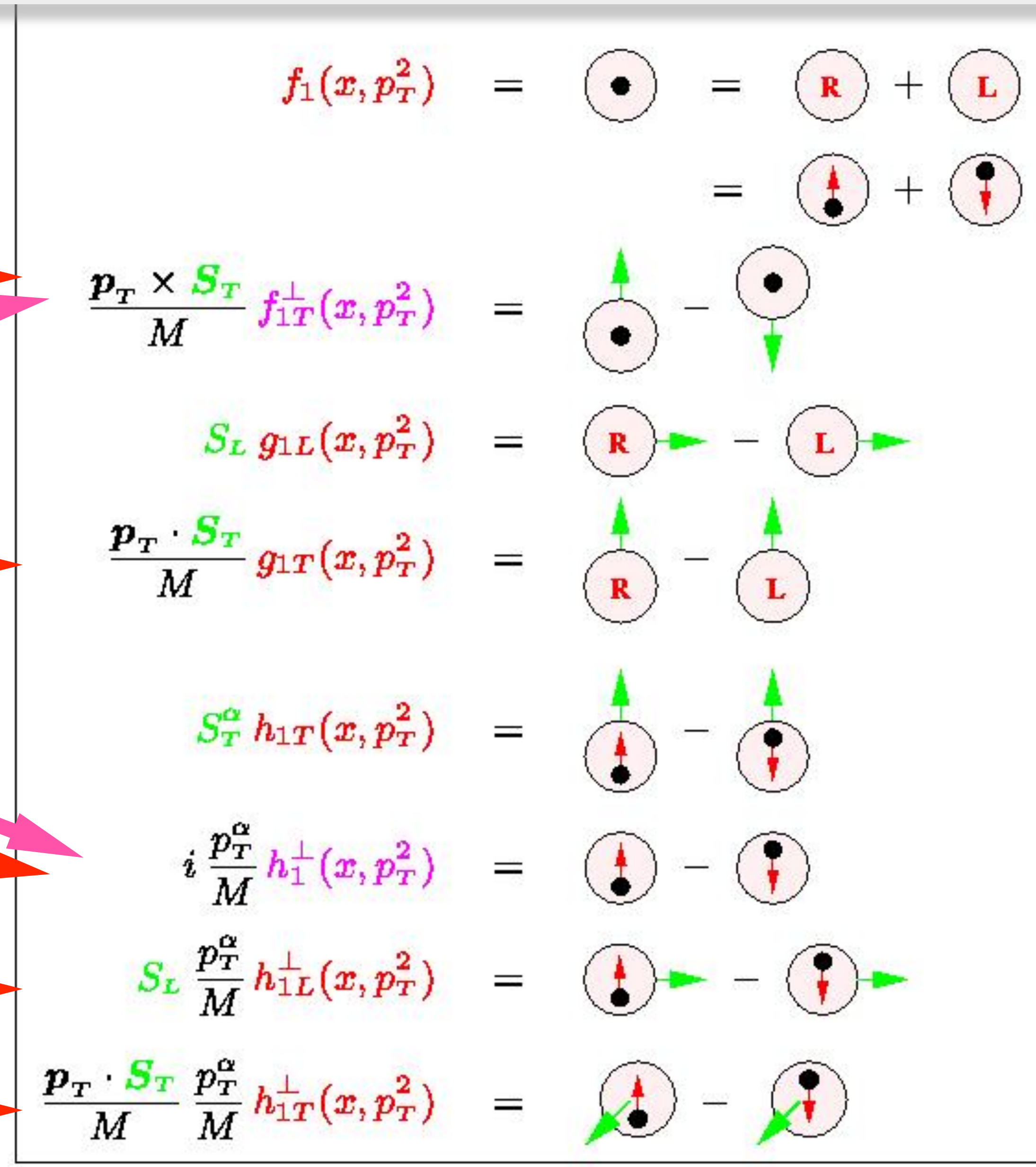
T-odd

with p_T

transverse spin distr. or transversity

with p_T

with p_T



Matrix representation for M = $[\Phi^{[\pm]}(x,p_T)\gamma^+]^T$

- p_T-dependent functions

MATRIX REPRESENTATION FOR SPIN 1/2

p_T-dependent quark distributions:

$$
\begin{array}{cccc}
\text{R} \rightarrow & \leftarrow \text{R} & \text{L} \rightarrow & \leftarrow \text{L}
\end{array}
$$

$$
\begin{pmatrix}
f_1 + g_{1L} & \frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} & \frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} & 2\, h_1 \\
\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 - g_{1L} & \frac{|p_T|^2}{M^2}\, e^{-2i\phi}\, h_{1T}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, h_{1L}^{\perp} \\
\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & \frac{|p_T|^2}{M^2}\, e^{2i\phi}\, h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M}\, e^{i\phi}\, g_{1T} \\
2\, h_1 & -\frac{|p_T|}{M}\, e^{i\phi}\, h_{1L}^{\perp} & -\frac{|p_T|}{M}\, e^{-i\phi}\, g_{1T} & f_1 + g_{1L}
\end{pmatrix}
$$

T-odd: $g_{1T} \rightarrow g_{1T} - i\, f_{1T}^{\perp}$ and $h_{1L}^{\perp} \rightarrow h_{1L}^{\perp} + i\, h_1^{\perp}$ (imaginary parts)

Bacchetta, Boglione, Henneman & Mulders
PRL 85 (2000) 712

(Un)integrated forward correlators

$$\Phi(x,p_T,p.P) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P \left| \bar{\psi}(0)\,\psi(\xi) \right| P \right\rangle$$

- unintegrated

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \left| \bar{\psi}(0)\,\psi(\xi) \right| P \right\rangle_{\boxed{\xi.n=0}}$$

- TMD (light-front)
  - Time-ordering automatic, allowing interpretation as forward anti-parton – target scattering amplitude
  - Involves operators of twists starting at a lowest value (which is usually called the 'twist' of a TMD)

$$\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P \left| \bar{\psi}(0)\,\psi(\xi) \right| P \right\rangle_{\boxed{\xi.n=\xi_T=0}\ or\ \boxed{\xi^2=0}}$$

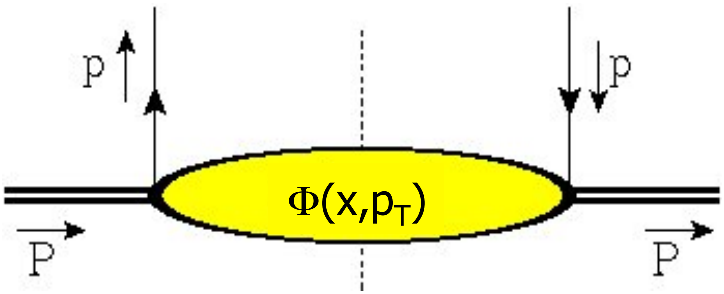
- collinear (light-cone)
  - Involves operators of a definite twist. Evolution via splitting functions (moments are anomalous dimensions)

$$\Phi = \left\langle P \left| \bar{\psi}(0)\,\psi(\xi) \right| P \right\rangle_{\boxed{\xi=0}}$$

- local
  - Local operators with calculable anomalous dimension

Large p_T

- p_T-dependence of TMDs



$$\int^{\mu} d^2p_T\, \Phi(x,p_T) = \Phi(x;\mu^2)$$

Fictitious measurement

Large μ^2 dependence governed by anomalous dim (i.e. splitting functions)

- $$\Phi(x,p_T) \underset{p_T^2 > \mu^2}{\longrightarrow} \frac{1}{\pi p_T^2}\frac{\alpha_s(p_T^2)}{2\pi}\int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right)\Phi(y;p_T^2)$$

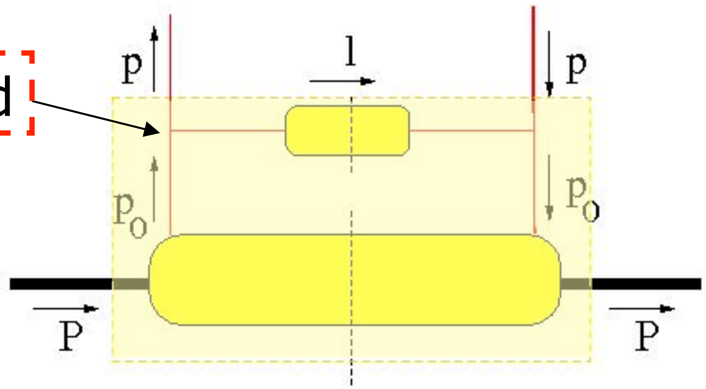
- Consistent matching to collinear situation: CSS formalism

JC Collins, DE Soper and GF Sterman, NP B 250 (1985) 199

Large values of momenta

- Calculable!

hard



$$p^2 \approx \frac{p_T^2 - x_p M_1^2}{1 - x_p} < 0$$

$$p.P \approx \frac{x_p (p_T^2 - M_1^2)}{2x(1-x_p)} < 0$$

$$M_R^2 \approx \frac{(x - x_p) p_T^2 + x_p (1-x) M_1^2}{x(1-x_p)} > 0$$

$$p_0 \approx \frac{x}{x_p} P + p_{0T} \qquad (x \le x_p \le 1)$$

$$l_T \approx -p_T$$

$p_{0T} \sim M$

$M << p_T < Q$

$$\Phi(p,P) \rightarrow \frac{\alpha_s}{p_T^2} \ldots$$ etc.

Bacchetta, Boer, Diehl, M
JHEP 0808:023, 2008 (arXiv:0803.0227)

Twist analysis (3)

- Dimensional analysis to determine importance in an expansion in inverse hard scale
- Maximize contractions with n

$$\dim[\overline{\psi}(0)\not{n}\,\psi(\xi)] = 2$$

$$\dim[F^{n\alpha}(0)F^{n\beta}(\xi)] = 2$$

$$\dim[\overline{\psi}(0)\not{n}\,A_T^{\alpha}(\eta)\,\psi(\xi)] = 3$$

- … or maximize # of P's in parametrization of Φ

$$\boxed{\Phi^q(x) = f_1^q(x)\frac{\not{P}}{2} \quad \Leftrightarrow \quad f_1^q(x) = \int \frac{d\lambda}{(2\pi)} e^{ix\lambda} \left\langle P \middle| \overline{\psi}(0)\not{n}\,\psi(\lambda n) \middle| P \right\rangle}$$

- In addition any number of collinear n.A(ξ) = $A^n(x)$ fields (dimension zero!), but of course in color gauge invariant combinations

dim 0: $\quad i\partial^n \longrightarrow iD^n = i\partial^n + gA^n$

dim 1: $\quad i\partial_T^\alpha \longrightarrow iD_T^\alpha = i\partial_T^\alpha + gA_T^\alpha$

Color gauge invariance

- Gauge invariance in a nonlocal situation requires a gauge link U(0,ξ)

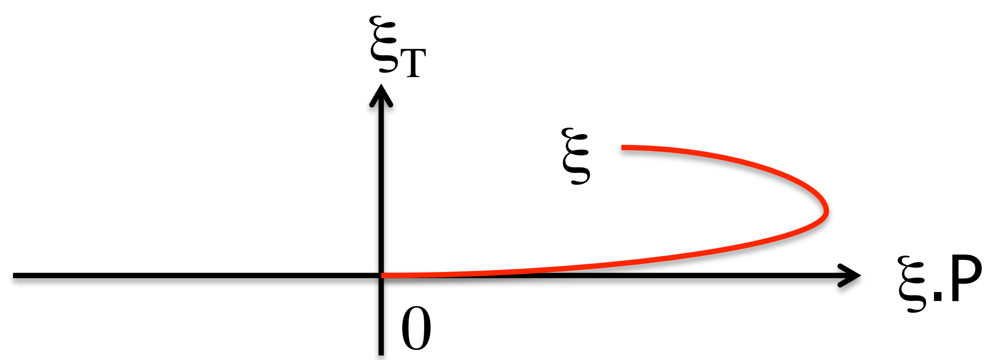
$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

$$U(0,\xi) = \mathcal{P}\exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

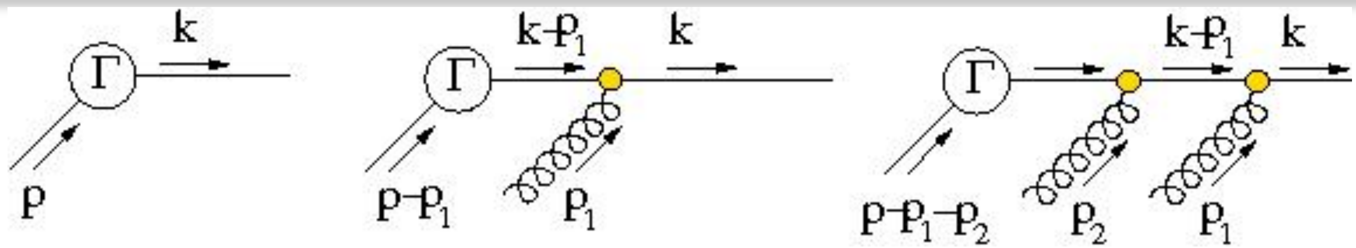
- Introduces path dependence for $\Phi(x,p_T)$

$$\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$$



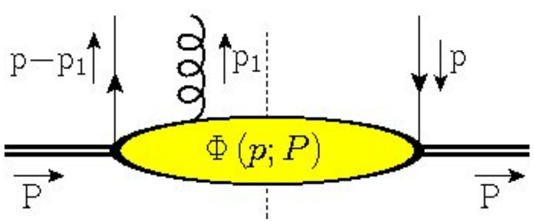
A.V. Belitsky, X.Ji, F. Yuan, NPB 656 (2003) 165
D. Boer, PJM, F. Pijlman, NPB 667 (2003) 201

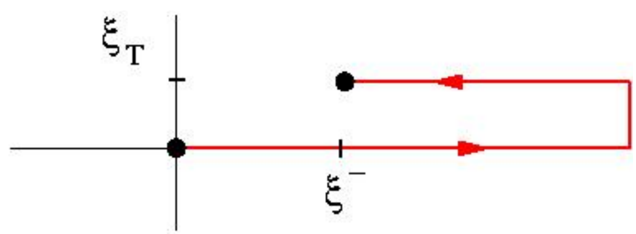
Gauge link results from leading gluons



Expand gluon fields and reshuffle a bit:

$$A^{\mu}(p_1) = n.A(p_1)\,\frac{P^{\mu}}{n.P} + i\,A_T^{\mu}(p_1) + \ldots \;=\; \frac{1}{p_1.n}\left[A^{n}(p_1)\,p_1^{\mu} + iG_T^{n\mu}(p_1) + \ldots\right]$$





Coupling only to final state partons, the collinear gluons add up to a U_+ gauge link, (with transverse connection from $A_T^a \rightarrow G^{na}$ reshuffling)

Which gauge links?

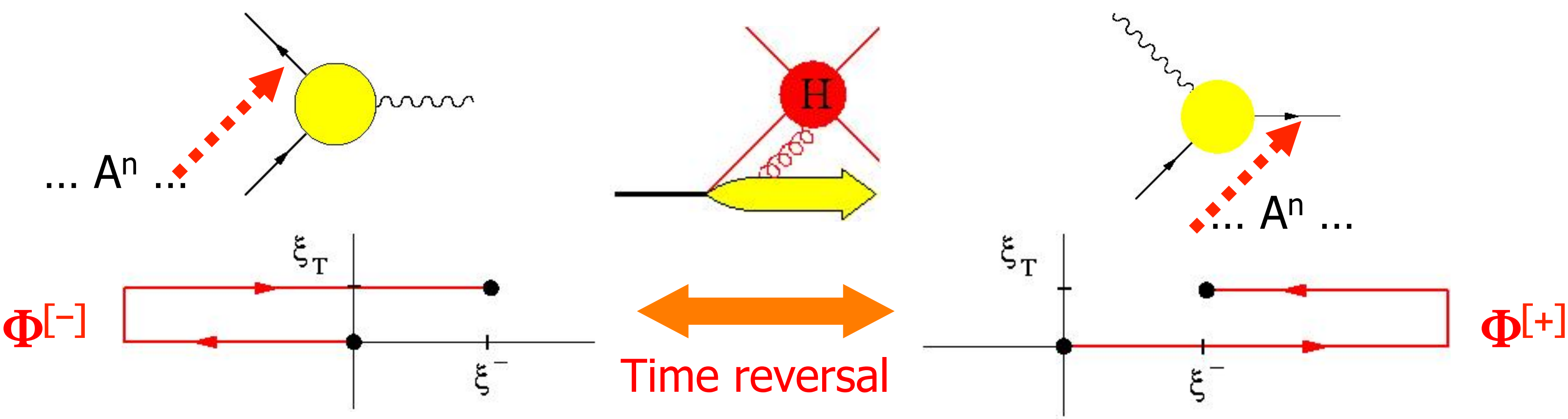
$$\Phi_{ij}^{q[C]}(x,p_T;n)=\int\frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\,e^{i\,p.\xi}\left\langle P\middle|\bar{\psi}_j(0)\,U_{[0,\xi]}^{[C]}\,\psi_i(\xi)\middle|P\right\rangle_{\xi.n=0}$$

TMD

$$\Phi_{ij}^{q}(x;n)=\int\frac{d(\xi.P)}{(2\pi)}\,e^{i\,p.\xi}\left\langle P\middle|\bar{\psi}_j(0)\,U_{[0,\xi]}^{[n]}\,\psi_i(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$

collinear

- Gauge links for TMD correlators process-dependent with simplest cases



AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJ Mulders and F Pijlman, NP B 667 (2003) 201

Which gauge links?

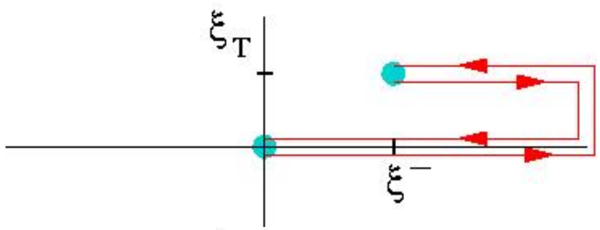
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\middle|U_{[\xi,0]}^{[C]}F^{n\alpha}(0)U_{[0,\xi]}^{[C']}F^{n\beta}(\xi)\middle|P\right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'

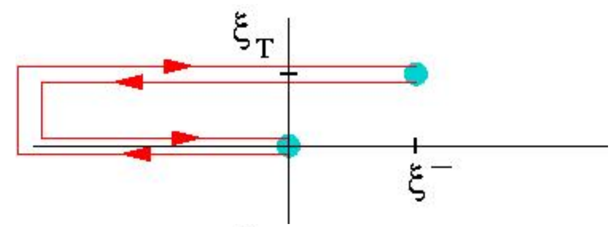
$$F^{\alpha\beta}(\xi)\rightarrow U_{[\eta,\xi]}^{[C]}F^{\alpha\beta}(\xi)U_{[\xi,\eta]}^{[C]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

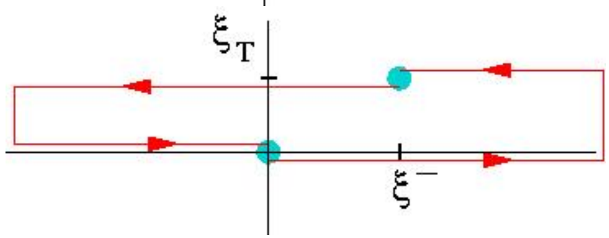
$\Phi_g^{[+,+]}$



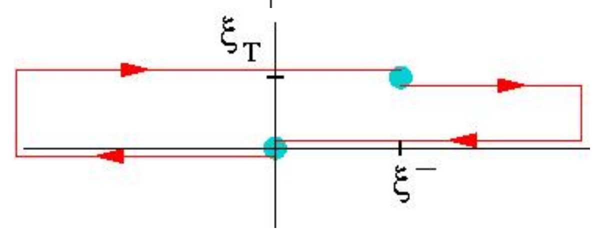
$\Phi_g^{[-,-]}$



$\Phi_g^{[+,-]}$



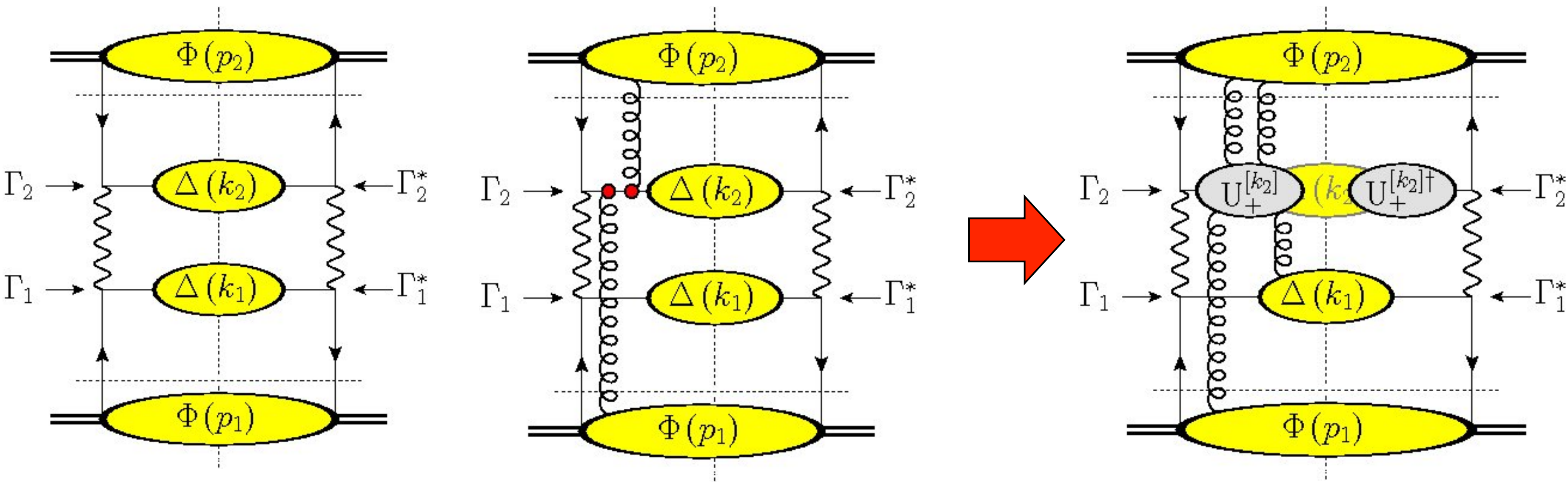
$\Phi_g^{[-,+]}$

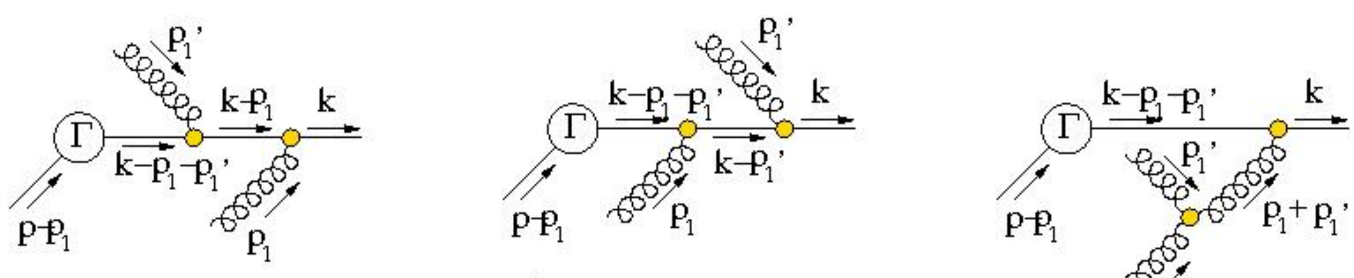


C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

T.C. Rogers, PJM, PR D81 (2010) 094006

Complications (example: qq → qq)



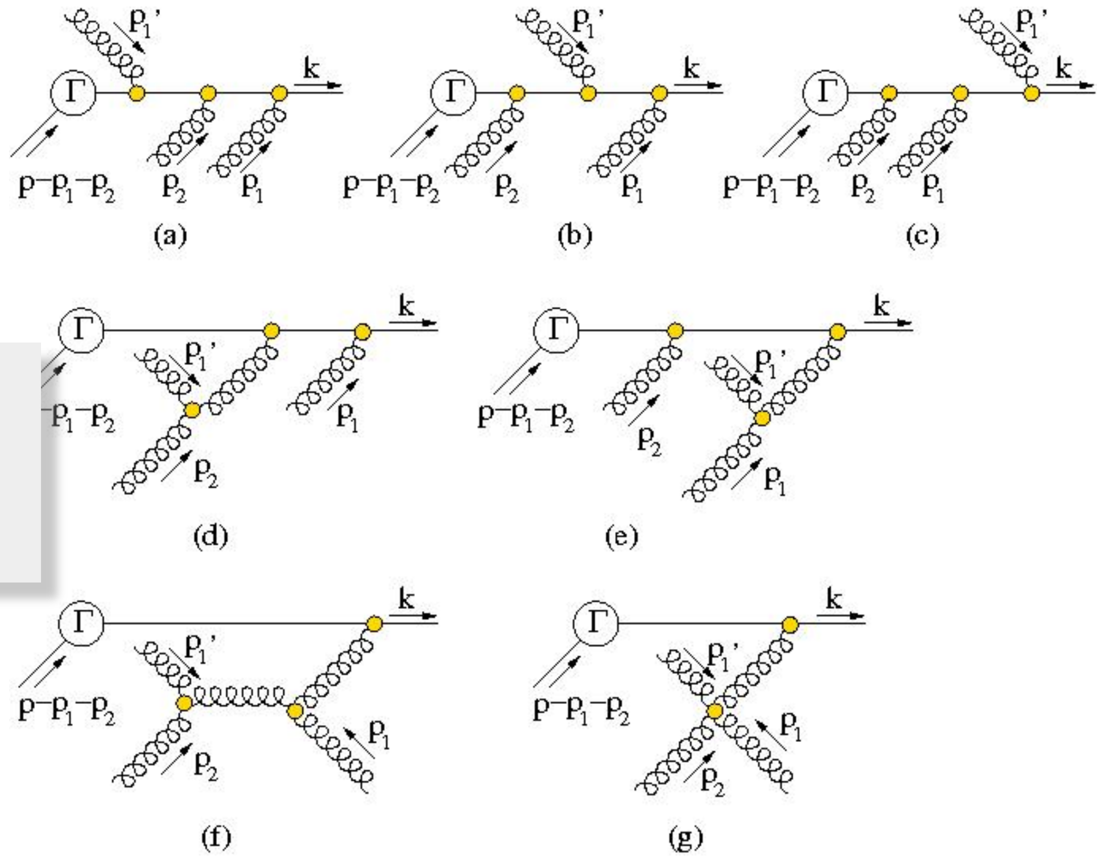


(a) (b) (c)

$U_+^{[n]}[p_1,p_2,k_1]$ modifies color flow, spoiling universality (and factorization)

$$U_{+\infty}^{[k](11)}(p,p')\ldots\Gamma\ldots\psi(p)\ldots\psi(p') = \frac{1}{2}\left\{U_{+\infty}^{[k](1)}(p), U_{+\infty}^{[k](1)}(p')\right\}\ldots\Gamma\ldots\psi(p)\ldots\psi(p')$$

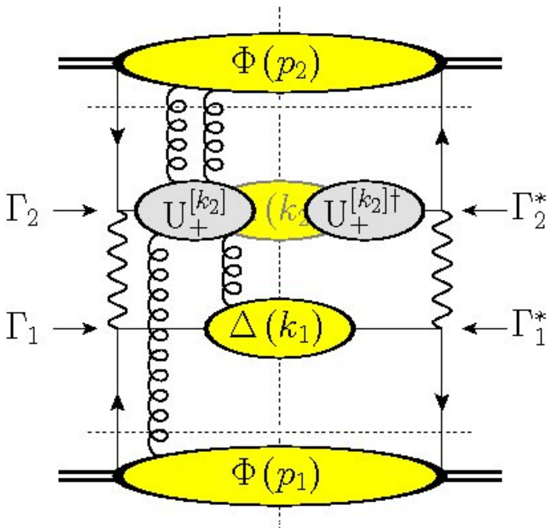
Color entanglement

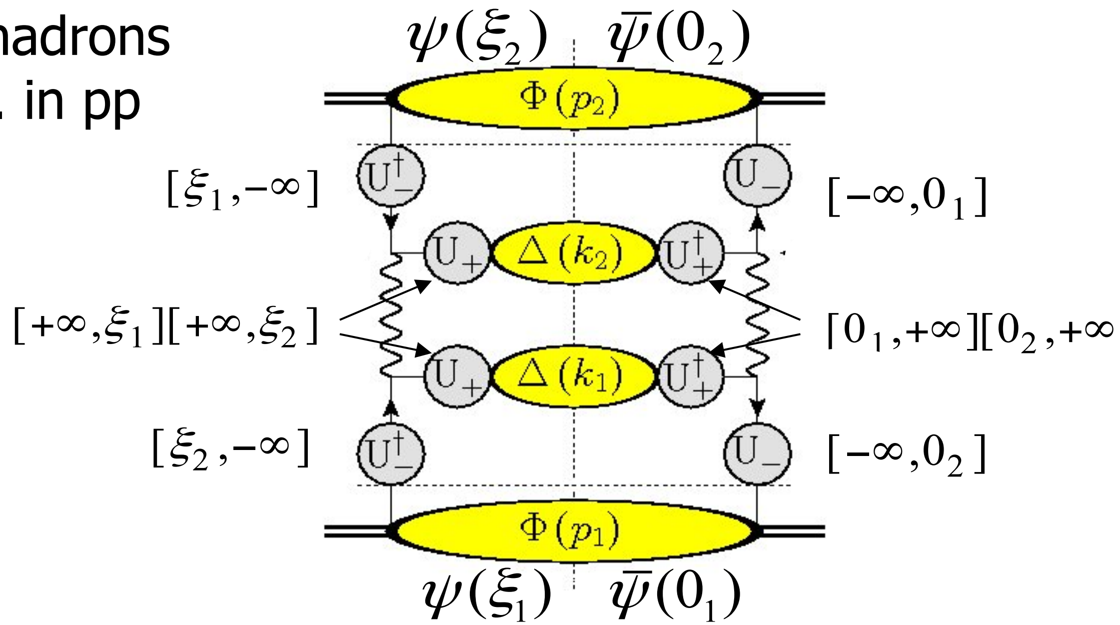


$$U_-^{[k](21)}(p,p') = \frac{1}{4}U_-^{[k](2)}(p)U_-^{[k](1)}(p')$$
$$+\frac{1}{4}U_-^{[k](1)}(p)U_-^{[k](1)}(p')U_-^{[k](1)}(p)$$
$$+\frac{1}{4}U_-^{[k](1)}(p')U_-^{[k](2)}(p)$$

Which gauge links?

- With more (initial state) hadrons color gets entangled, e.g. in pp





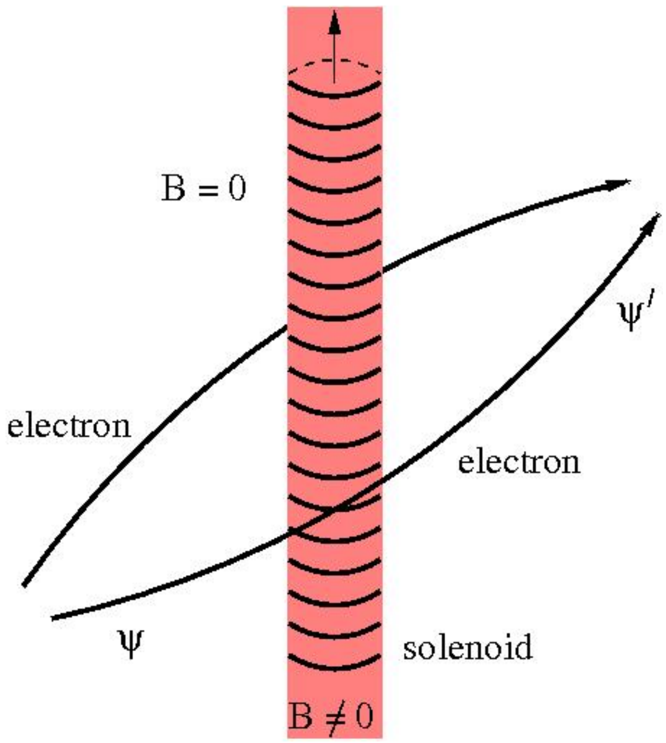
- Outgoing color contributes future pointing gauge link to $\Phi(p_2)$ and future pointing part of a loop in the gauge link for $\Phi(p_1)$

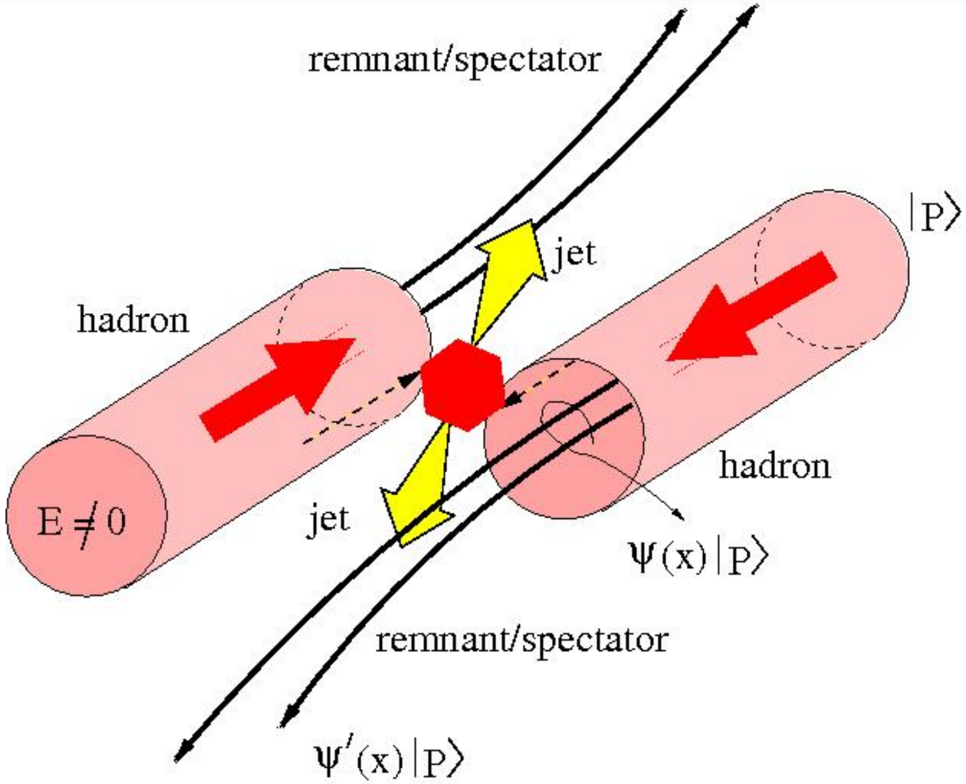
- Can be color-detangled if only p_T of one correlator is relevant (using polarization, …) but include Wilson loops in final U

T.C. Rogers, PJM, PR D81 (2010) 094006

M.G.A. Buffing, PJM, JHEP 07 (2011) 065

Featuring: phases in gauge theories





$$\psi' = P e^{ie\int ds.A}\psi$$

$$\psi_i(x)\left|P\right\rangle = P e^{-ig\int_x^{x'} ds_\mu A^\mu}\psi_i(x')\left|P\right\rangle$$

Spring School on QCD prospects for future ep and eA colliders, Orsay, June 4-8 2012

TMDs: Theory and Phenomenology IV

Piet Mulders

UNIVERSITY AMSTERDAM

NIKHEF

mulders@few.vu.nl

Experimental consequences

- Even if (elaborated on below) transverse moments involve twist-3, they may show up at leading order in azimuthal asymmetries, cf DY

$$\sigma(q_T) = \iint d^2p_T\, d^2k_T\, \delta^2(p_T + k_T - q_T)\, \Phi_1(p_T)\, \Phi_2(k_T) \ldots$$

- Integrated: $\int d^2q_T\ \sigma(q_T) = \int d^2p_T\, \Phi_1(p_T) \int d^2k_T\, \Phi_2(k_T) \ldots$
- Weighted: $\int d^2q_T\ q_T^\alpha\, \sigma(q_T) = \int d^2p_T\, \Phi_1(p_T) \boxed{\int d^2k_T\ k_T^\alpha\, \Phi_2(k_T)} \ldots + \boxed{\int d^2p_T\ p_T^\alpha\, \Phi_1(p_T)} \int d^2k_T\, \Phi_2(k_T) \ldots$

- Examples in DY: Ralston & Soper; Kotzinian; M & Tangerman
- Examples in SIDIS: Collins & Sivers asymmetries
- Examples in pp-scattering: deviations from back-to-back situation in 2-jet production: Boer, Vogelsang
- In particular single-spin asymmetries: T-odd observables (!) requiring T-odd correlators (hard T-odd effects are higher order or mass effects)

T-odd ↔ single spin asymmetry

- $W_{\mu\nu}(q;P,S;P_h,S_h) = -W_{\nu\mu}(-q;P,S;P_h,S_h)$ — symmetry structure
- $W^*_{\mu\nu}(q;P,S;P_h,S_h) = W_{\nu\mu}(q;P,S;P_h,S_h)$ — hermiticity
- $W_{\mu\nu}(q;P,S;P_h,S_h) = \overline{W}_{\mu\nu}(\bar{q};\bar{P}, -\bar{S};\bar{P}_h, -\bar{S}_h)$ — parity
- $W^*_{\mu\nu}(q;P,S;P_h,S_h) = \overline{W}_{\mu\nu}(\bar{q};\bar{P},\bar{S};\bar{P}_h,\bar{S}_h)$ — time reversal

- with time reversal constraint only even-spin asymmetries
- the time reversal constraint cannot be applied in DY or in ≥ 1-particle inclusive DIS or e^+e^-
- In those cases single spin asymmetries can be used to measure T-odd quantities (such as T-odd distribution or fragmentation functions)

Lepto-production of pions

MATRIX REPRESENTATION FOR SPIN 0

p_T-dependent quark fragmentation functions:

$$M^{(\mathrm{dec})} = \begin{array}{cc} \begin{pmatrix} D_1 & i\,\frac{|k_T|\,e^{-i\phi}}{M_h}\,H_1^{\perp} \\ -i\,\frac{|k_T|\,e^{+i\phi}}{M_h}\,H_1^{\perp} & D_1 \end{pmatrix} & \begin{array}{c} \mathrm{R} \\ \mathrm{L} \end{array} \\ \begin{array}{cc} \mathrm{R} & \qquad\qquad \mathrm{L} \end{array} & \end{array}$$

SIDIS: $\ell + H^{\uparrow} \to \ell + h + X$

$$\left\langle \frac{Q_T}{M}\sin(\phi_h^\ell - \phi_S^\ell)\right\rangle_{OTO} = \frac{2\pi\alpha^2 s}{Q^4}\,|S_T|\left(1 - y + \frac{1}{2}y^2\right)\sum_{a,\bar a} e_a^2\, x_B\, f_{1T}^{\perp(1)a}(x_B)\,D_1^a(z_h)$$

$$\left\langle \frac{Q_T}{M_h}\sin(\phi_h^\ell + \phi_S^\ell)\right\rangle_{OTO} = \frac{2\pi\alpha^2 s}{Q^4}\,|S_T|\,2(1-y)\sum_{a,\bar a} e_a^2\, x_B\, h_1^a(x_B)\,H_1^{\perp(1)a}(z_h)$$

$H_1^{\perp}$ is T-odd and chiral-odd

Operator structure in collinear case

- Collinear functions

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\xi]}^{[n]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \left\langle P \middle| \bar{\psi}(0) (\partial^n)^{N-1} U_{[0,\xi]}^{[n]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{ip.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\xi]}^{[n]} (D^n)^{N-1} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements with calculable anomalous dimensions, that can be Mellin transformed to splitting functions

$$\Phi^{(N)} = \left\langle P \middle| \bar{\psi}(0) (D^n)^{N-1} \psi(0) \middle| P \right\rangle$$

- All operators have same twist since dim(D^n) = 0

Operator structure in TMD case

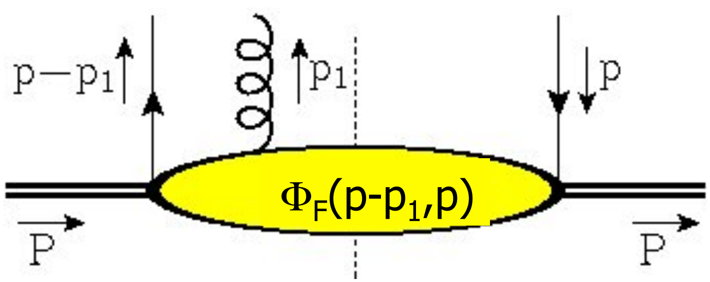
- For TMD functions one can consider transverse moments

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \left| \bar{\psi}(0)\, \psi(\xi) \right| P \right\rangle_{\xi.n=0}$$

$$p_T^\alpha \Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \left| \bar{\psi}(0) U D_T^\alpha(\pm\infty) U \psi(\xi) \right| P \right\rangle_{\xi.n=0}$$

- Transverse moments involve collinear twist-3 multi-parton correlators Φ_D and Φ_F built from non-local combination of three parton fields

$$\Phi_F^\alpha(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P}{(2\pi)} e^{i(p-p_1).\xi+ip_1.\eta} \left\langle P \left| \bar{\psi}(0) F^{n\alpha}(\eta)\, \psi(\xi) \right| P \right\rangle_{\xi.n=\xi_T=0}$$



$$\Phi_D^\alpha(x) = \int dx_1\, \Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\Phi_A^\alpha(x) = PV\int dx_1 \frac{1}{x_1} \Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

T-invariant definition

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators
- $$\int d^2p_T\; p_T^\alpha \Phi^{[\pm]}(x, p_T; n) = \tilde{\Phi}_\partial^\alpha(x) \pm \pi \Phi_G^\alpha(x)$$

T-even T-odd (gluonic pole or ETQS m.e.)

$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha} \qquad \boxed{\Phi_G^\alpha(x) = \Phi_F^{n\alpha}(x, 0 \,|\, x)}$$

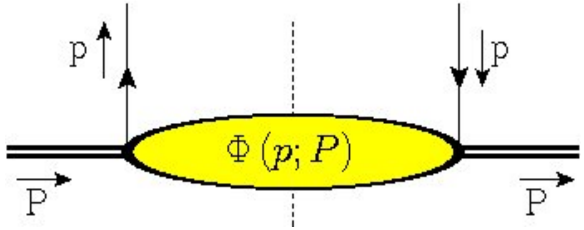
- Transverse moments connect to local operators; it is sometimes nicer to work in b_T-space with Bessel-weighted asymmetries

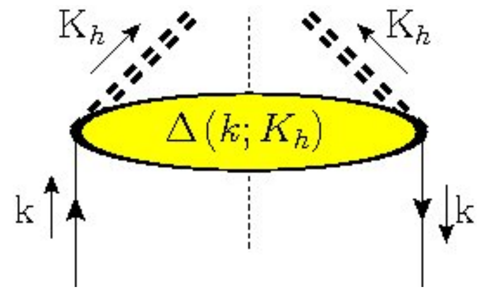
$$e^{iM\varphi_b} F(b_T) = \int d^2p_T\; e^{ip_T.b_T} e^{iM\varphi_p} \tilde{F}(p_T) \sim e^{iM\varphi_b} \int p_T\, dp_T J_M(p_T b_T) \tilde{F}(p_T)$$

  - Operators remain nonlocal because of gauge link
  - ✓ Accounts in natural way for asymptotic behavior
  - ✓ Advantages when considering evolution (cancellation of soft factors)

D Boer, L Gamberg, B Musch, A Prokudin, JHEP 1110 (2011) 021

Distributions versus fragmentation





- Operators:

$$\Phi^{[\pm]}(p\,|\,p) \sim \langle P\,|\,\overline{\psi}(0)U_{\pm}\psi(\xi)\,|\,P\rangle$$

$$\Phi_{\partial}^{\alpha}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) \pm \pi\,\Phi_{G}^{\alpha}(x)$$

T-even T-odd (gluonic pole)

$$\boxed{\Phi_{G}^{\alpha}(x) = \Phi_{F}^{n\alpha}(x,0\,|\,x) \neq 0}$$

- Operators:

out state

$$\Delta(k\,|\,k) \sim \sum_X \langle 0\,|\,\psi(\xi)\,|\,K_h X\rangle\langle K_h X\,|\,\overline{\psi}(0)\,|\,0\rangle$$

$$\boxed{\Delta_{G}^{\alpha}(x) = \Delta_{F}^{n\alpha}(\tfrac{1}{z},0\,|\,\tfrac{1}{z}) = 0}$$

$$\Delta_{\partial}^{\alpha}(x) = \tilde{\Delta}_{\partial}^{\alpha}(x)$$

T-even operator combination, but no T-constraints!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503

Higher azimuthal asymmetries

- Transverse moments can be extended to higher moments, involving twist-4 correlators Φ_{FF} etc., where each of the gluon fields can be a gluonic pole. This is relevant for cos(2ϕ) and sin(2ϕ) asymmetries. Relevant e.g. in the study of transversely polarized quarks in a proton

$$\Phi_T^q(x,p_T) = \ldots + \left(h_{1T}^q(x,p_T^2)\gamma_5 \not{S} - h_{1T}^{\perp q}(x,p_T^2)\frac{p_T.S_T}{M}\frac{\gamma_5 \not{p}_T}{M} \right)\frac{\not{P}}{2}$$

- For gluons one needs operators <F F>, <F [F,F]>, <F, {F,F}>, <[F,F] [F,F]>, <{F,F} {F,F}> etc. again with increasing twist and several gluonic poles. Relevant in study of linearly polarized gluons in proton

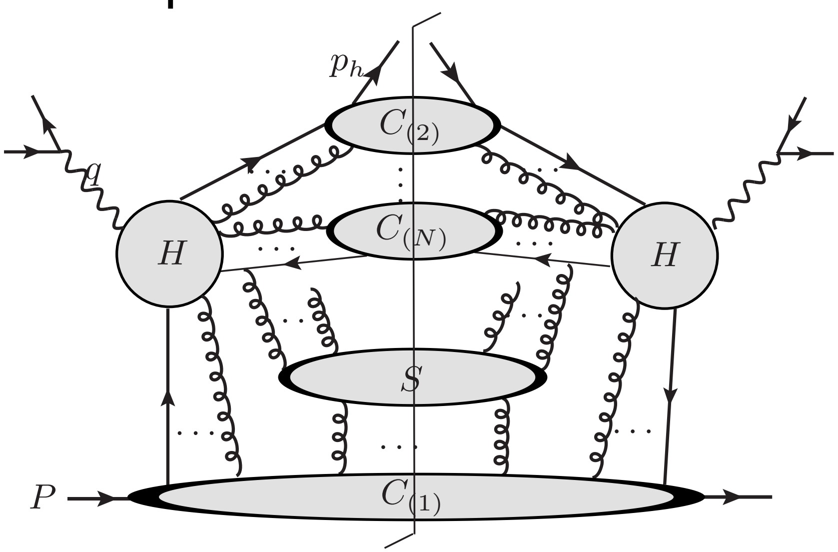
$$\Phi_g^{\mu\nu}(x,p_T) = \frac{1}{2x}\left(-g_T^{\mu\nu} f_1^g(x,p_T^2) + \left(\frac{p_T^\mu p_T^\nu + \frac{1}{2} g_T^{\mu\nu}}{M^2} \right) h_1^{\perp g}(x,p_T^2) \right)$$

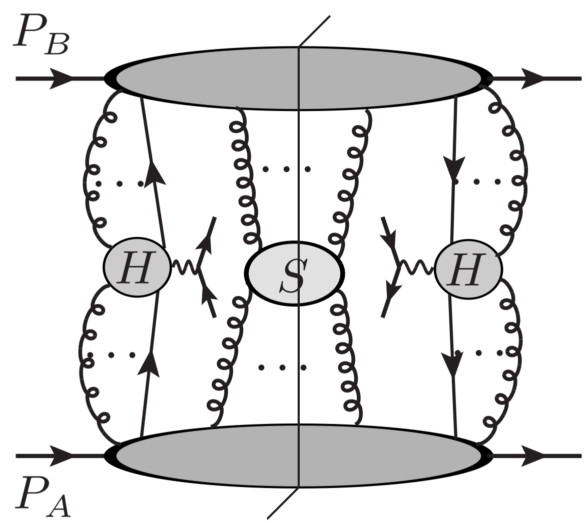
- Both $\Phi_g^{[+,+]} + \Phi_g^{[-,-]}$ and $\Phi_g^{[+,-]} + \Phi_g^{[-,+]}$ are T-even with 2nd moments containing <[F,G] [G,F]> and <{F,G} {G,F}> operator terms respectively

C Bomhof and PJ Mulders, NPB 795 (2008) 409
F Dominguez, J-W Qiu, B-W Xiao and F Yuan, PR D85 (2012) 045003
M.G.A. Buffing, A. Mukherjee, PM, in preparation

TMD-factorization

- TMD moments involve higher twist operators, with many possibilities, distinguishing T-even/odd, chiral-even/odd, …
- To study scale dependence one needs to have a full definition that accounts for (transitions to) all regions, requiring renormalization scale, regularization of rapidity divergences, …
- In a process one needs to consider collinear and soft gluons, …





J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011

http://projects.hepforge.org/tmd/

Aybat, Prokudin, Rogers, Qiu, Collins,

Conclusions

- TMDs enter in processes with more than one hadron involved (e.g. SIDIS and DY)
- Rich phenomenology and experiments!
- Relevance for JLab, Compass, RHIC, JParc, GSI, LHC, EIC and LHeC
- Role for models using light-cone wf (Barbara Pasquini) and lattice gauge theories (Philipp Haegler)
- Link of TMD (non-collinear) and GPDs (off-forward)
- Link to small x (k_T-factorization, Emil Avsar)
- Easy cases are collinear and 1-parton un-integrated (1PU) processes, with in the latter case for the TMD a (complex) gauge link, depending on the color flow in the tree-level hard process
- Finding gauge links is only first step, (dis)proving QCD factorization is next (recent work of Ted Rogers and Mert Aybat).