

# EARTHQUAKE TOMOGRAPHY: PROBLEM SETTING

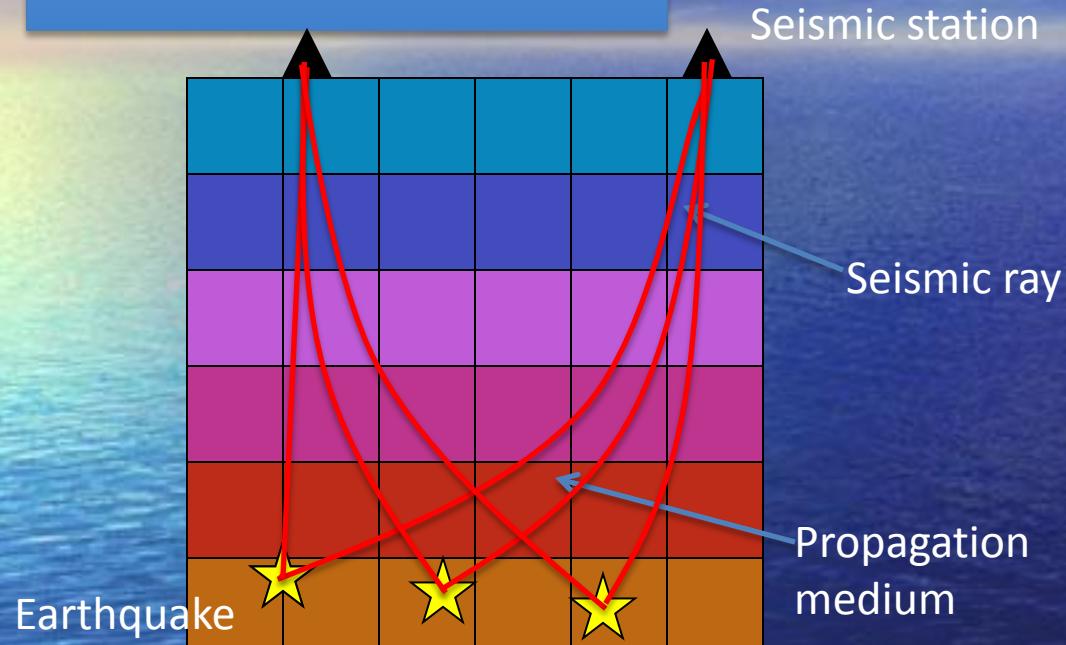
Data: EQ arrival times

Parameters: EQ hypoc. param.

Velocity param.

1. Model discretization:

Cells/nodes → Velocity parameters



4. Uncertainty and non-linearity:

- Data :  $d$
- Theoretical travel times :  $g(m)$
- $g$  is non-linear in heterogeneous media

RAYS:

2. EQ and stations: discrete distrib.

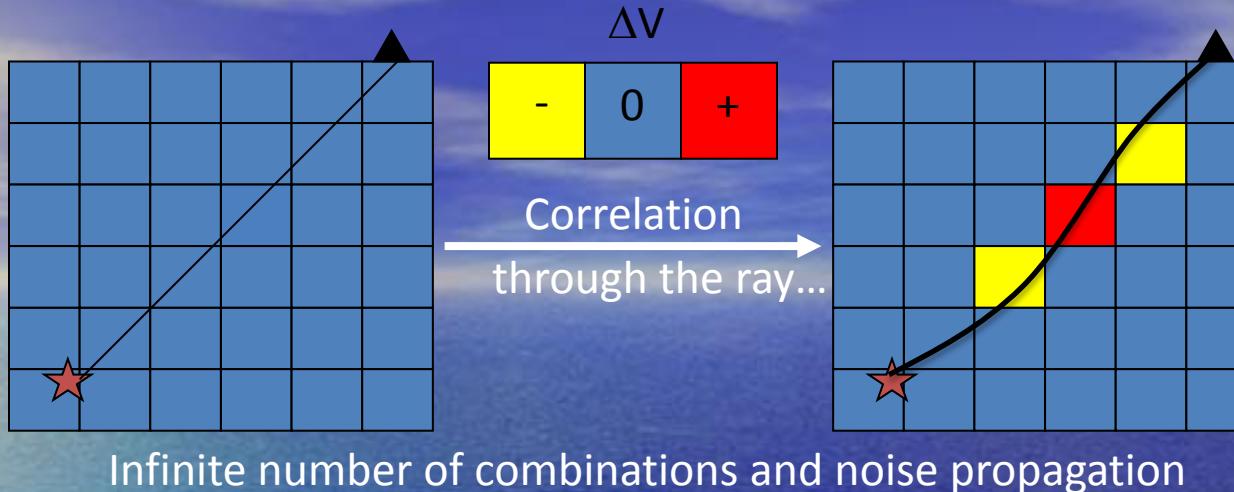


Incomplete sampling by seismic rays

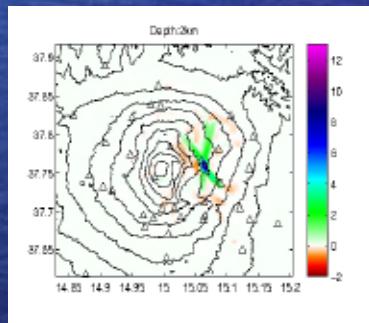
3. Travel times:  $\Delta t = \int_0^l \frac{ds}{V}$  « Direct problem »

Tomography:  
Non-linear parametric estimation  
→ inversion  
(velocity, EQ hyp.)

## Understanding correlation length (2) Ray propagation through model cells



Resolution



For each cell:

Parameter value = true value + noise (neighbouring cells)

Optimal filtering : bias-variance OPTIMAL compromise

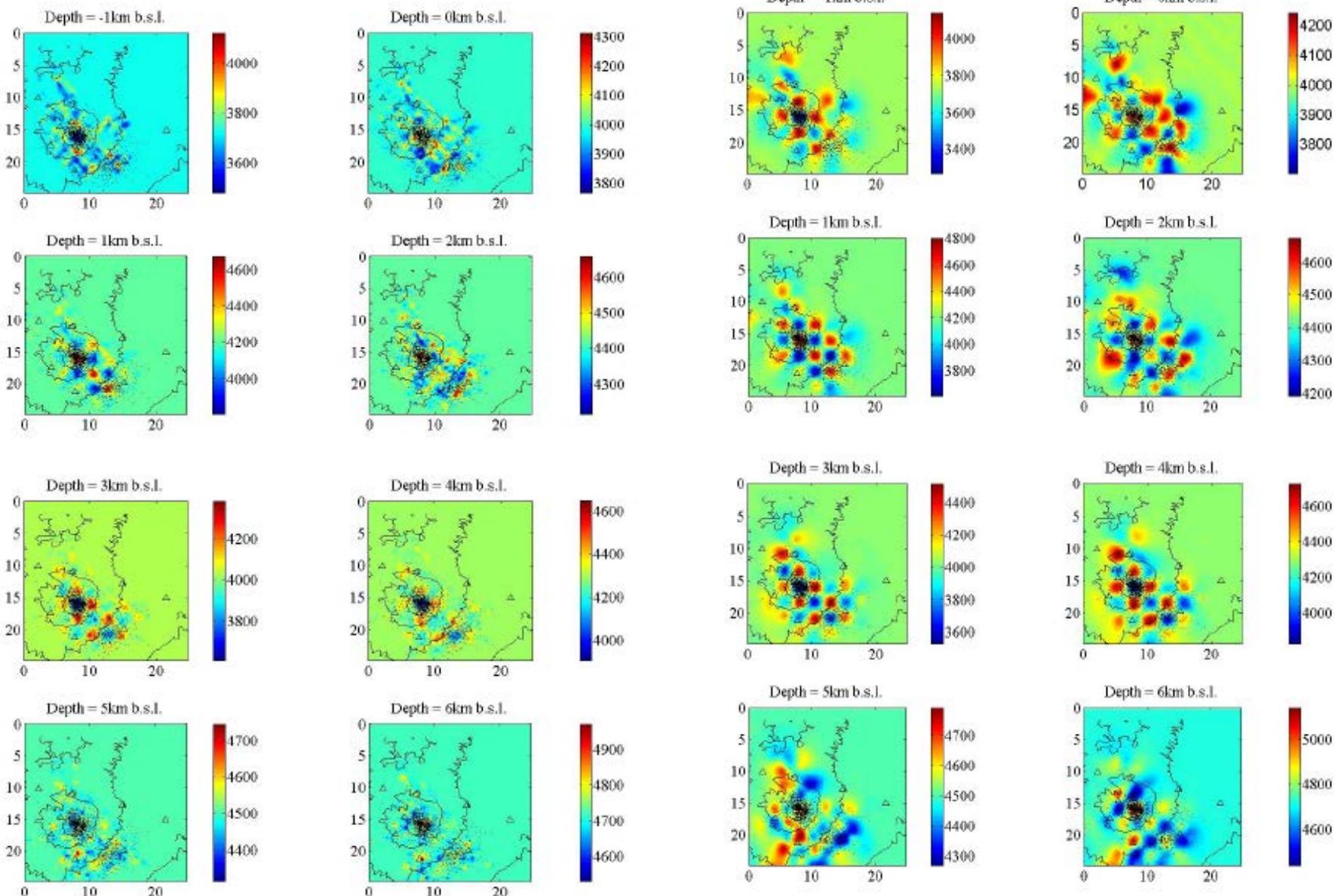
Correlating parameters with a length :

- not too large to bias physical values
- but sufficiently large to filter out the noise and reduce variance

LOW-PASS FILTERING OF HIGH-FREQUENCY NOISE

# Understanding correlation length (1)

Berger, Got, Valdes, Monteiller, 2011

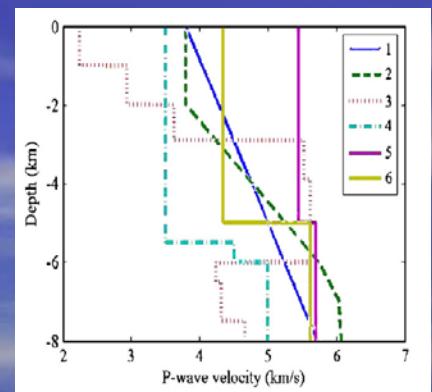


Checkerboard test: (1) under-regularization ( $\lambda = 0.5$  km)  
High-frequency noise

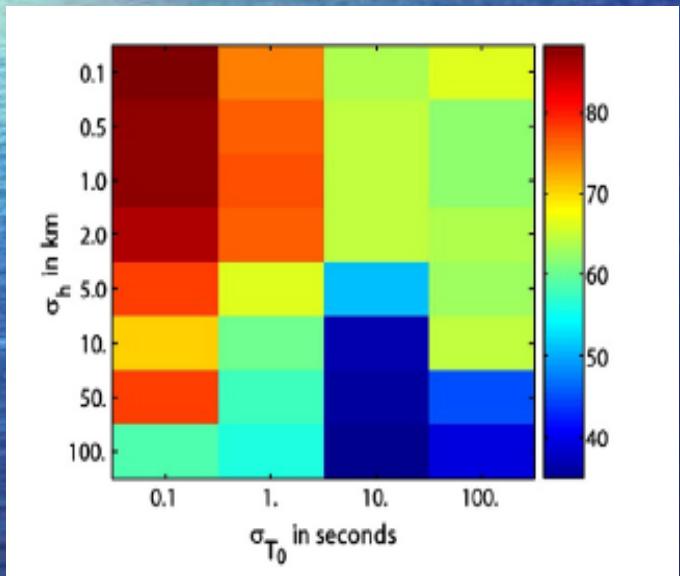
(2) Optimal regularization ( $\lambda = 3$  km)  
→ Low-pass filter

## Choosing the regularization:

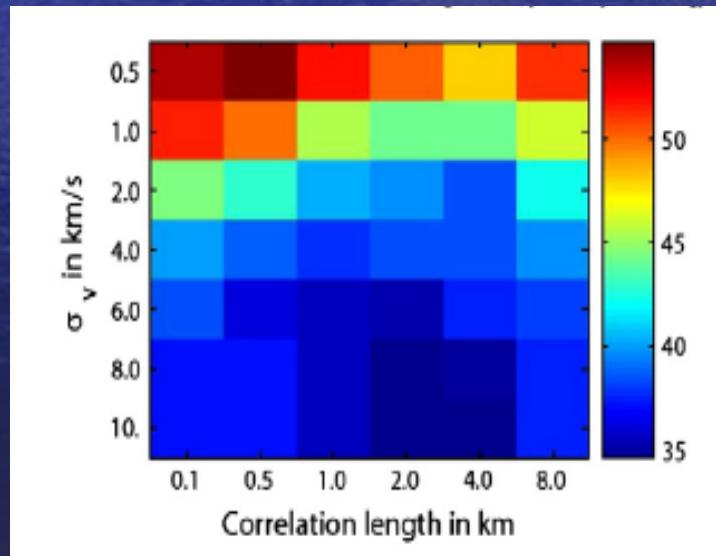
1. A priori average model ( $m_0$ ): constant gradient



2. Cost function as a function  
of regularization hyper-parameters

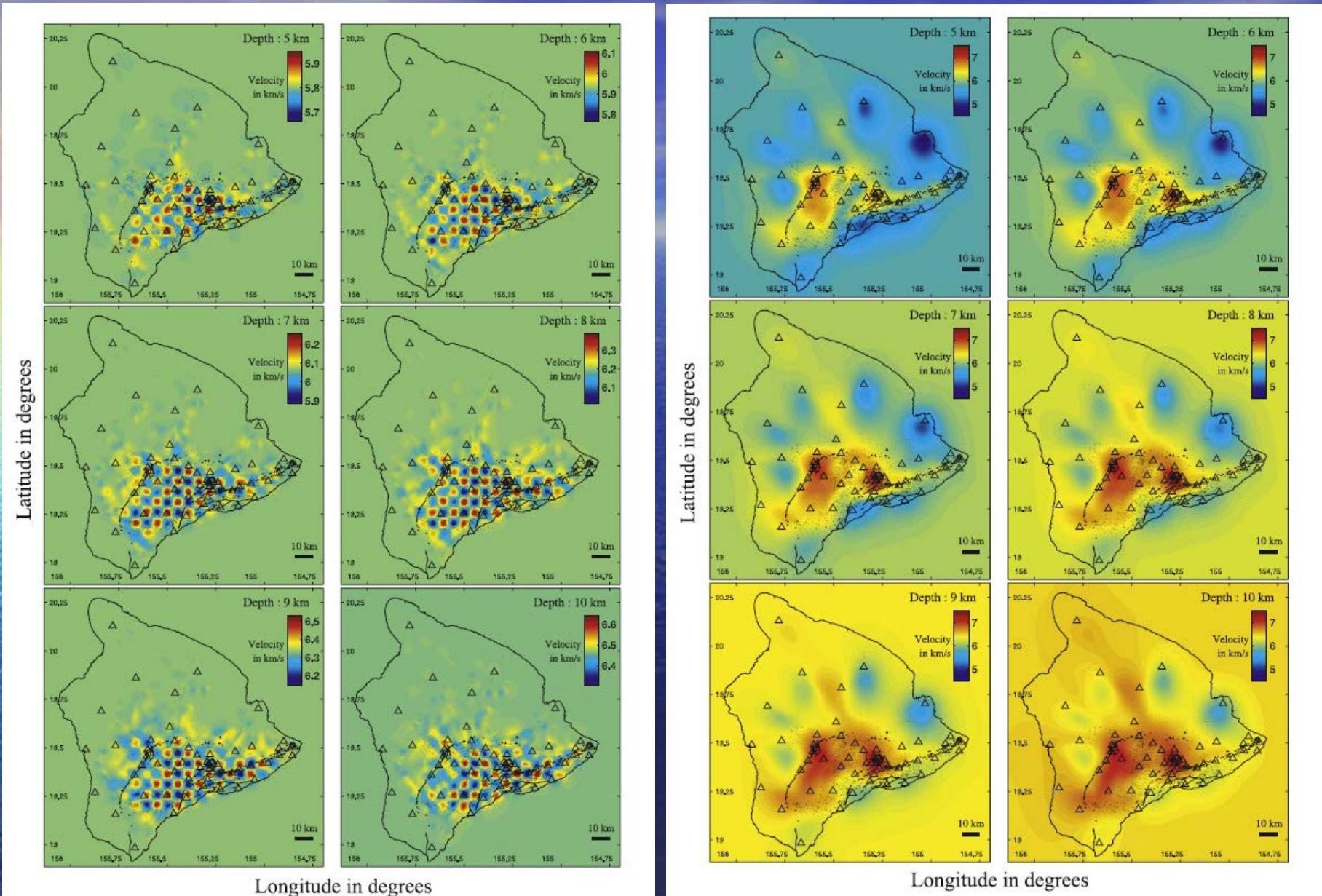


Hypocentral hyper-parameters



Velocity hyper-parameters

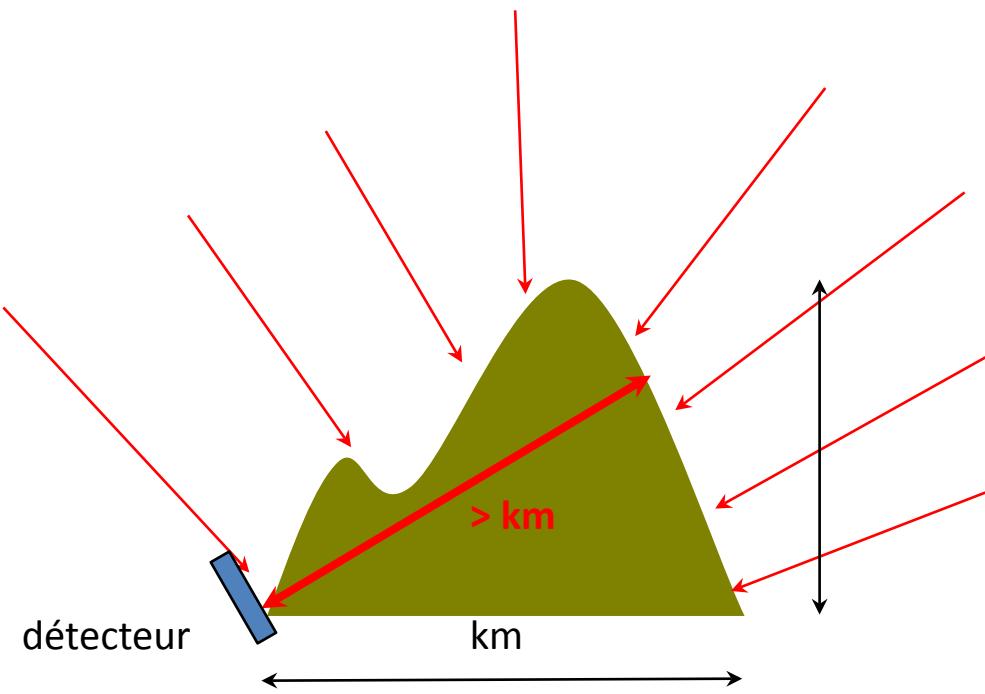
# Travel-time earthquake tomography results, Hawaii Monteiller, Got, Virieux, Okubo, JGR, 2005



41886 P travel-time, 1358 evts, 31 arr. times/evt (av.) , 959077 nodes

$$\lambda = 5 \text{ km}, \sigma_v = 1 \text{ km/s}$$

# Some Mathematics for the Tomography of Large Structures with Atmospheric Muons



## Aim

3D density imaging of km scale targets with metric spatial resolution ( $10^{-3}$ ) and few % contrast

## Methodology

counting detectors

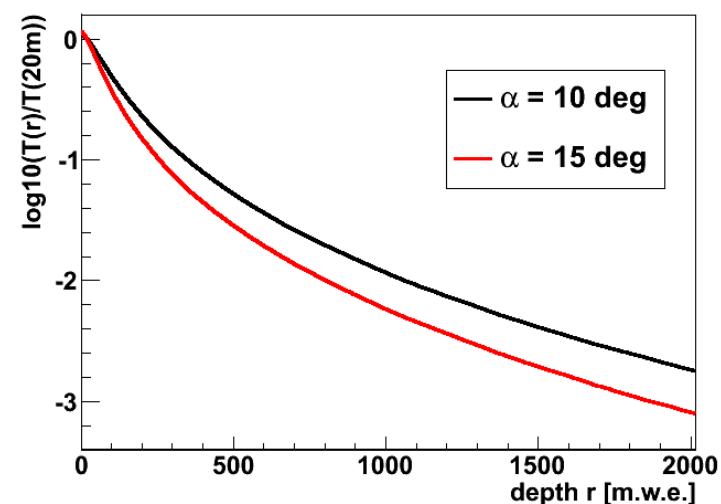
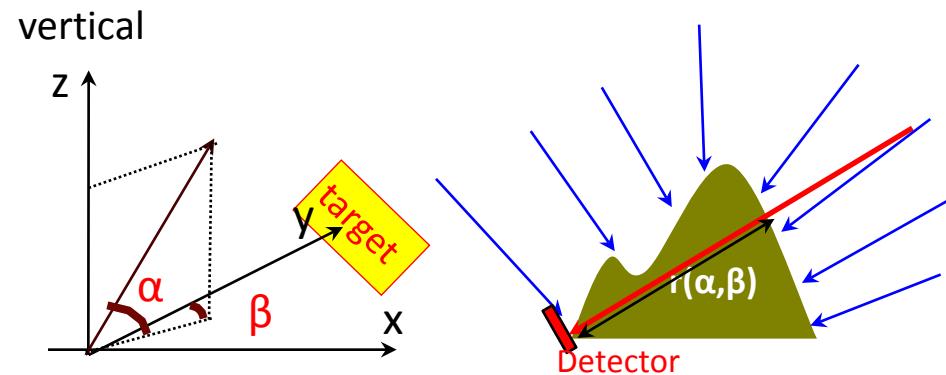
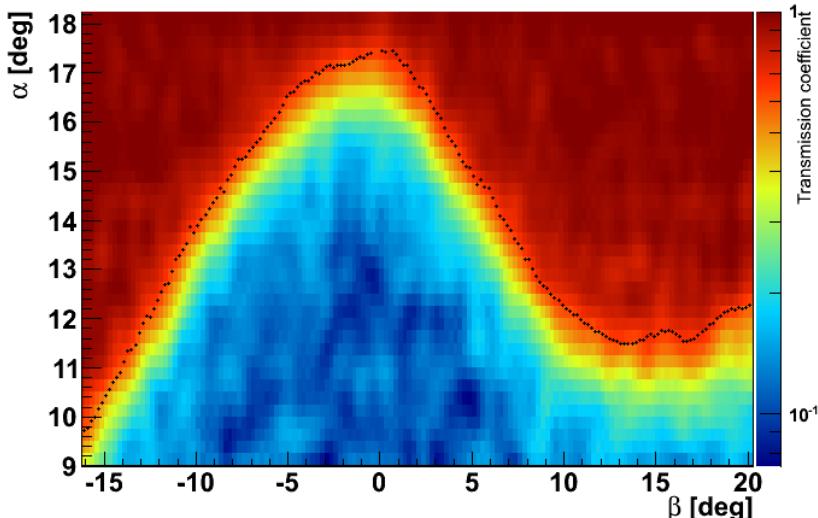
$$\varphi(\alpha, r(\alpha, \beta)) = \int_{E_{th}(\alpha, r)} \varphi(\alpha, r(\alpha, \beta), E) dE$$

$$T_p(\alpha, r(\alpha, \beta)) = \frac{\Phi(\alpha, r(\alpha, \beta))}{\Phi_0(\alpha)}$$

measured flux through volcano  
open sky flux

unknown density!

$$-\log(T_p(\alpha, r(\alpha, \beta))) \sim \int \rho(\alpha, \beta, r) dr$$

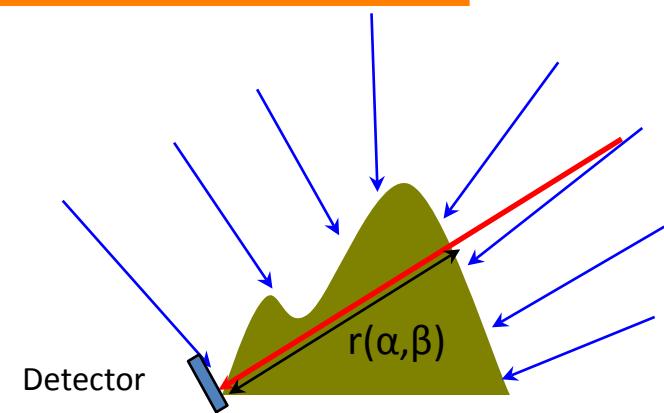


## Radiation: non-monochromatic radiation

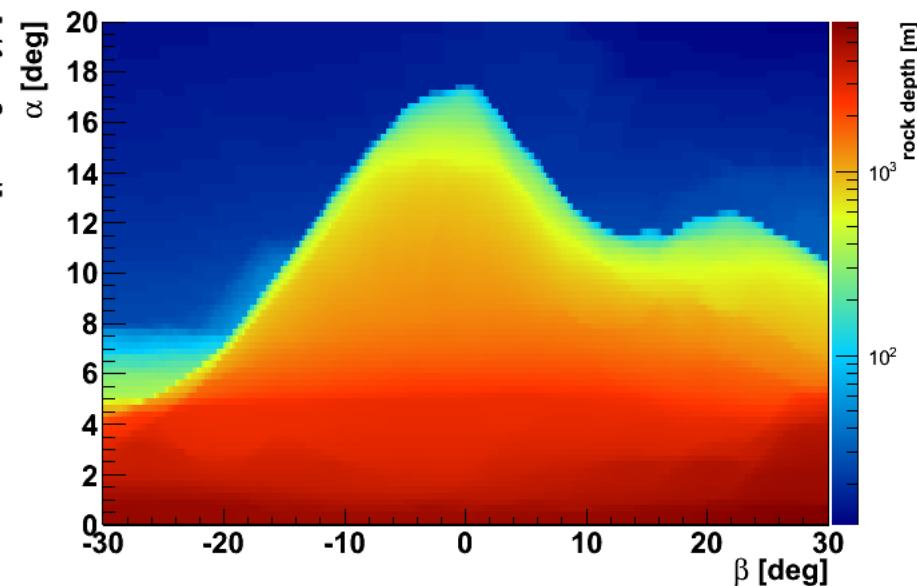
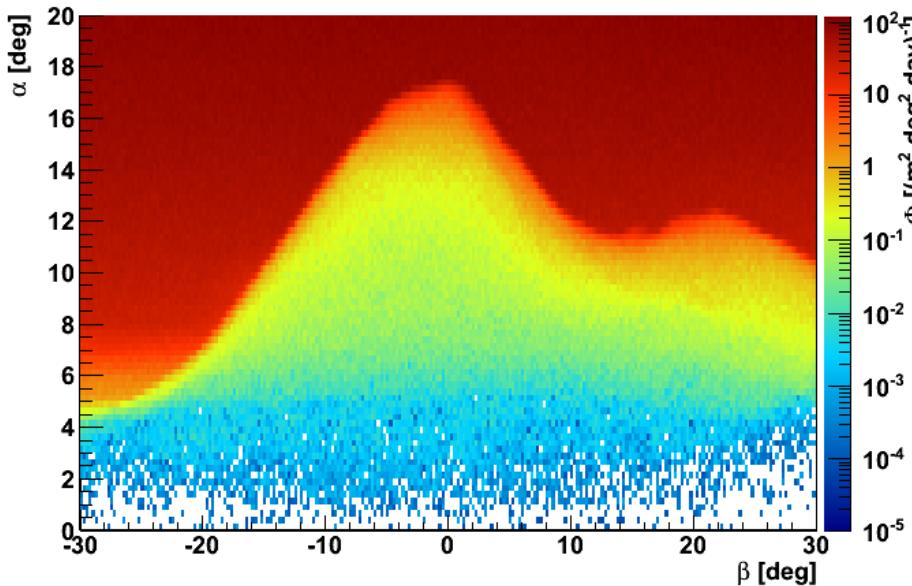
$$\frac{dN}{dE} = \frac{0.14}{\text{cm}^2 \text{ sr s GeV}} \cdot A \cdot \left( \frac{E_\mu}{\text{GeV}} \right)^{-\gamma} \cdot \left( \frac{1}{1 + \frac{1.1E_\mu \cos(\theta)}{115 \text{ GeV}}} + \frac{0.054}{1 + \frac{1.1E_\mu \cos(\theta)}{850 \text{ GeV}}} \right)$$

$A=0.701$   $\gamma=2.715$

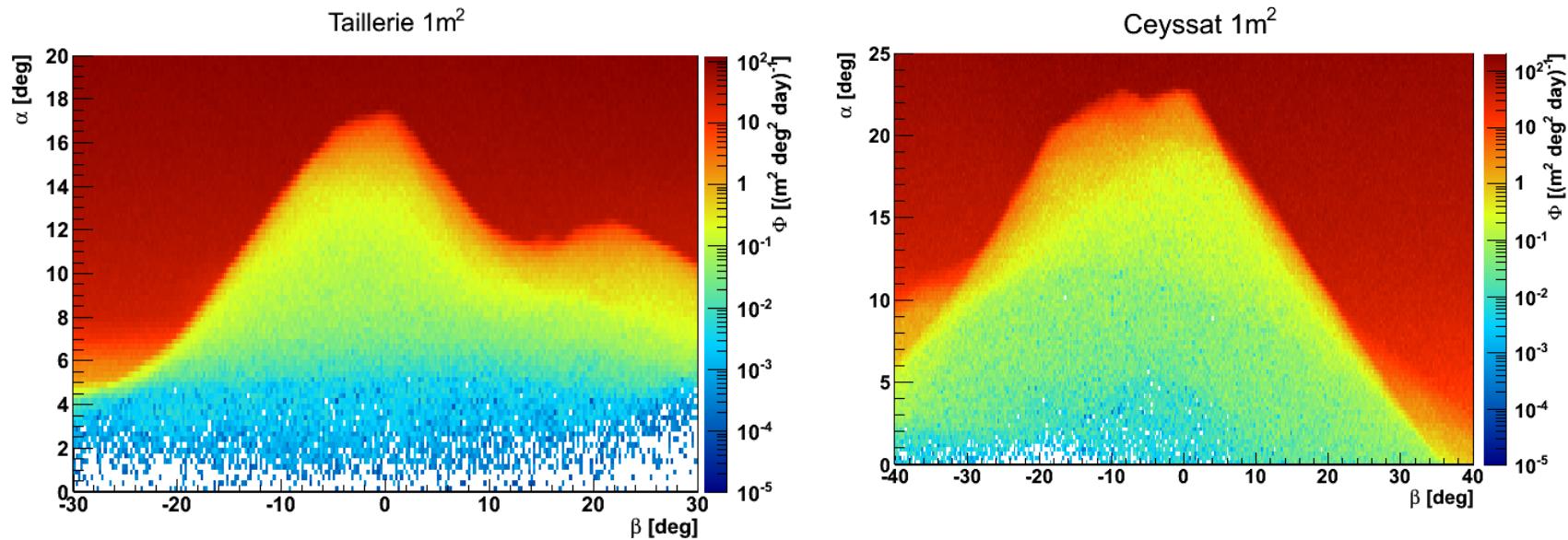
$$\varphi(\alpha, r(\alpha, \beta)) = \int_{E_{\text{th}}(\alpha, r)} \varphi(\alpha, r(\alpha, \beta), E) dE$$



Taillerie 1m<sup>2</sup>



highly variable number of events along each direction



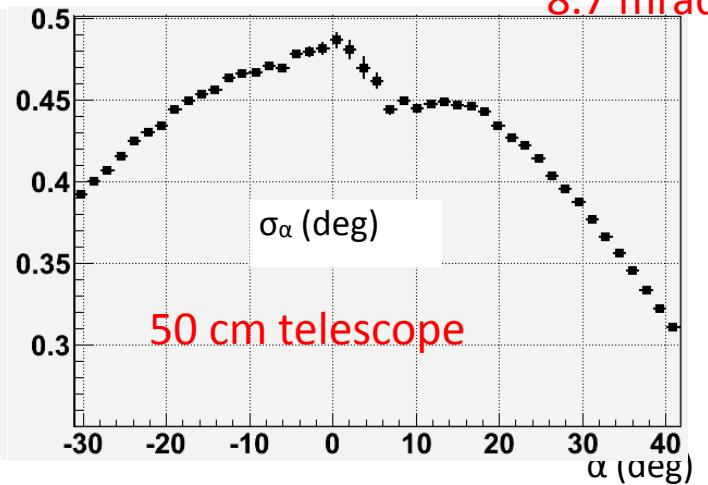
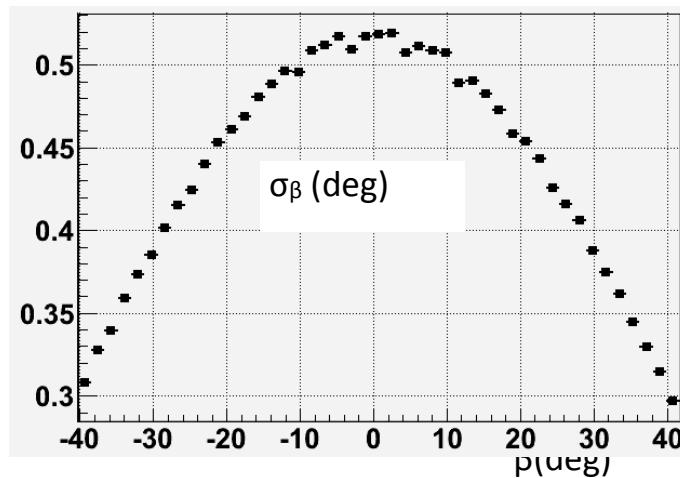
$$\sigma_{\alpha}^2 = \sigma_{\text{stat}}^2(\alpha, \beta) + \sigma_{\text{det}}^2(\alpha, \beta) + \sigma_{\text{rad}}^2(\alpha, \beta)$$

$\sigma_{\text{stat}}$  -> given by the statistical fluctuations of the expected nb of events along each binned direction

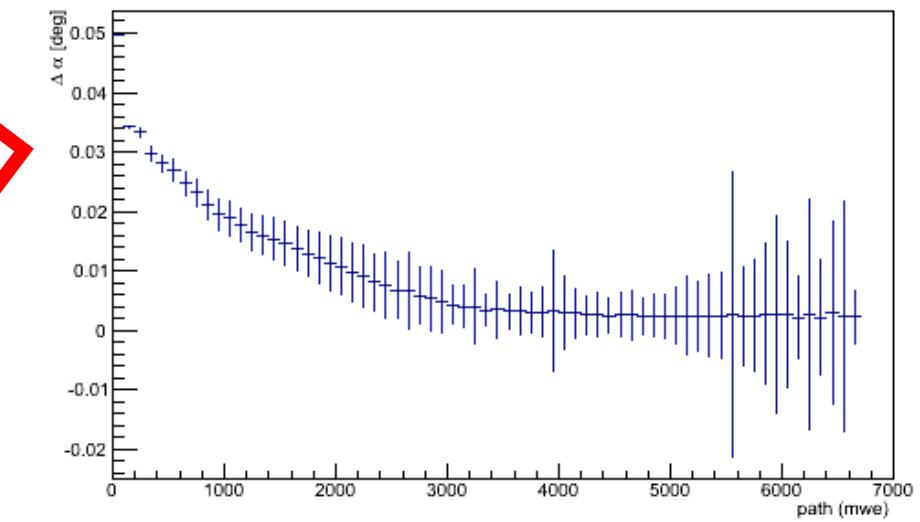
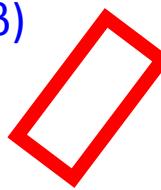
$\sigma_{\text{det}}$  -> reconstruction (detection) uncertainty

$\sigma_{\text{rad}}$  -> muon scattering in the target and outside it in the Earth magnetic field

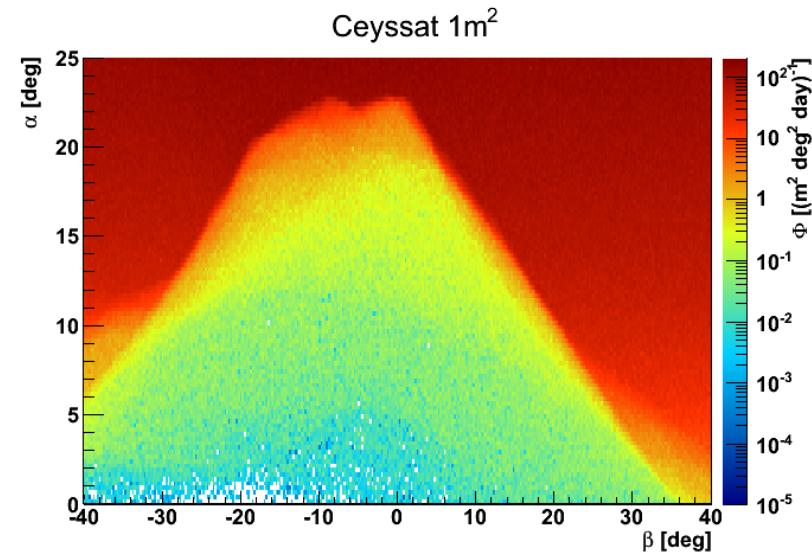
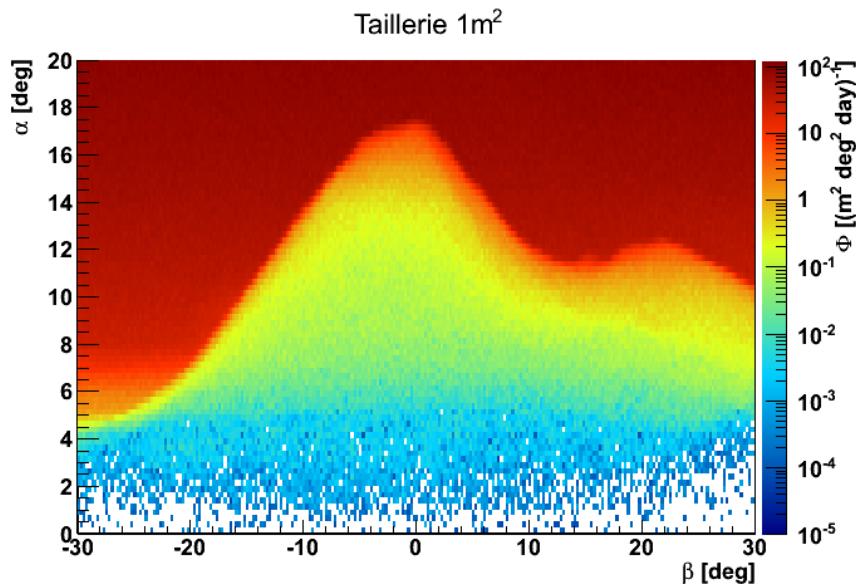
Not necessarily the same from each detection site ... challenge for CT...



$$\sigma^2_\alpha = \sigma^2_{\text{stat}}(\alpha, \beta) + \sigma^2_{\text{det}}(\alpha, \beta) \cdot \sigma^2_{\text{rad}}(\alpha, \beta)$$



highly variable number of events along each direction



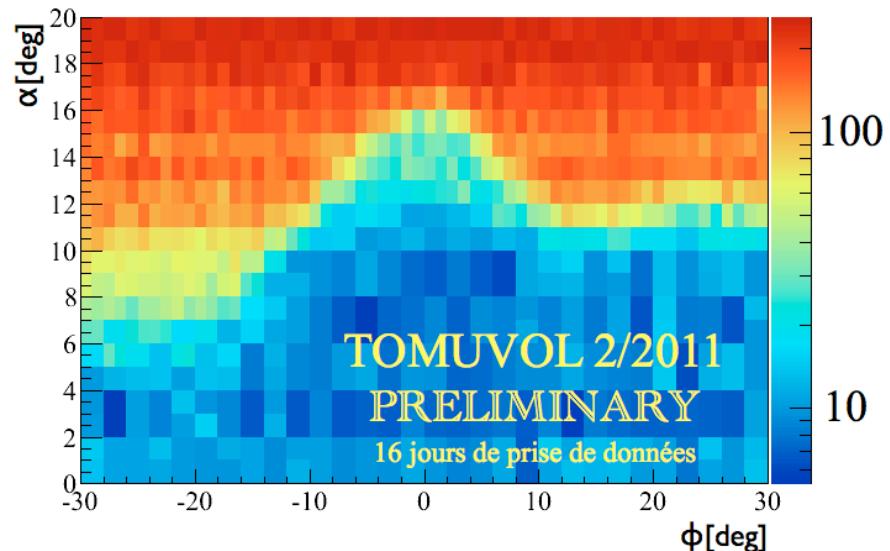
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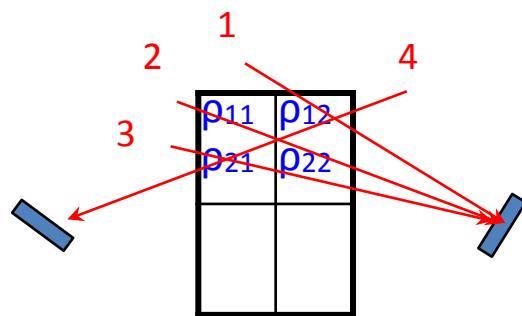
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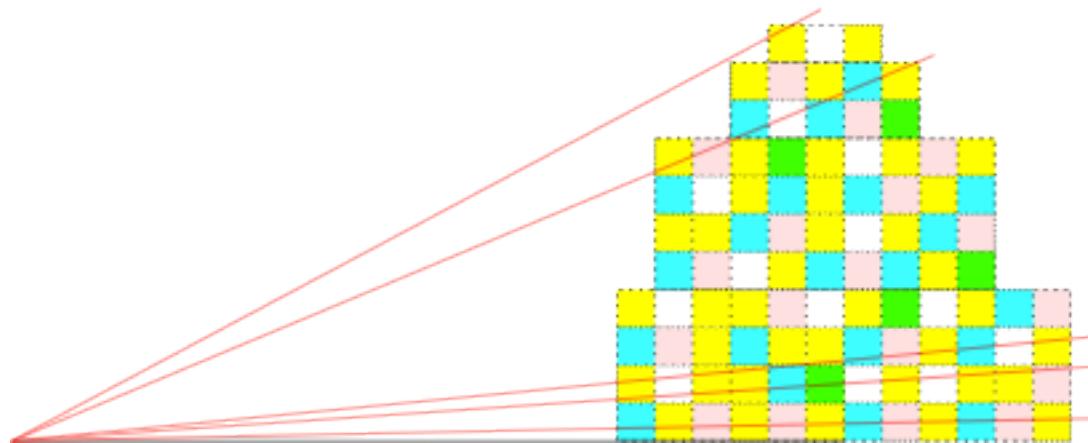
Not necessarily the same from each detection site ... challenge for CT...





$$\begin{aligned}\rho_1 &= 0 * \rho_{11} + 1 * \rho_{12} + 0 * \rho_{21} + 0 * \rho_{22} \\ \rho_2 &= 1 * \rho_{11} + 0 * \rho_{12} + 0 * \rho_{21} + 1 * \rho_{22} \\ \rho_3 &= 0 * \rho_{11} + 0 * \rho_{12} + 1 * \rho_{21} + 1 * \rho_{22} \\ \rho_4 &= 0 * \rho_{11} + 1 * \rho_{12} + 1 * \rho_{21} + 0 * \rho_{22}\end{aligned}$$

3D density imaging of large structures (>km) with metric spatial resolution -> HUGE nb of pixels



# Seismic vs Muon Tomography

Which similarities? in both cases

- « Rays »:
  - Incomplete sampling, correlation between parameters, noise propagation;
  - Variable resolution;
- Same possible artefacts

Which differences?

- Particle trajectories are almost straight lines in muon tomography, not in seismic...
- Far more data (rays) in Muon Tomography than in seismic tomography (potentially!...)
- Input Muon fluxes are not time-dependent (→ not waves)
  - no Fresnel volume → resolution is not limited from this aspect
- Muon tomography limited at the « surface » → increasing time and sensor resolution

Conclusion:

- the inverse problem is probably less unstable in Muon Tomo than in seismic tomo
- Good ray coverage in surface in Muon Tomo → good resolution in surface → natural risk
- Which actual resolution can be expected from Muon Tomography? (pot. powerful surf)

Which a priori knowledge in Muon Tomography?

- Density increases with depth → have an estimation of the average density  
and constant gradient  
physical standard deviation
- Variable standard deviation
- Variable correlation length

# Bayesian approach

(Tarantola and Valette, 1982;  
Tarantola, 1987, 2005)

(A posteriori) knowledge on the model =  
a priori knowledge on the model AND knowledge from the data

A posteriori pdf:

$$\sigma_m(m) = \rho_m(m) f(d - g(m))$$

↑                      ↑  
A priori pdf      Data+modelling error pdf

Case of gaussian pdf for data and model parameters

$$\rho_m(m) = (2\pi \det C_m)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(m - m_0)^T C_m^{-1} (m - m_0)\right)$$
$$\rho_\varepsilon(d - d) = (2\pi \det C_d)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(d - g(m))^T C_d^{-1} (d - g(m))\right)$$

Maximize the a posteriori pdf (« MAP estimation »)

$$\sigma_m(m) \propto \exp\left(-\frac{1}{2}(m - m_0)^T C_m^{-1} (m - m_0) - \frac{1}{2}(d - g(m))^T C_d^{-1} (d - g(m))\right)$$

Minimize

$$(\mathbf{d} - g(\mathbf{m}))^T \mathbf{C}_{\mathbf{d}}^{-1} (\mathbf{d} - g(\mathbf{m})) + (\mathbf{m} - \mathbf{m}_0)^T \mathbf{C}_{\mathbf{m}}^{-1} (\mathbf{m} - \mathbf{m}_0)$$

Gauss-Newton

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \left( \mathbf{G}_k^T \mathbf{C}_{\mathbf{d}}^{-1} \mathbf{G}_k + \mathbf{C}_{\mathbf{m}}^{-1} \right)^{-1} \left( \mathbf{G}_k^T \mathbf{C}_{\mathbf{d}}^{-1} (\mathbf{d} - g(\mathbf{m}_k)) + \mathbf{C}_{\mathbf{m}}^{-1} (\mathbf{m}_k - \mathbf{m}_0) \right)$$

Implementation

$$\begin{pmatrix} \mathbf{C}_{\mathbf{d}}^{-1/2} \mathbf{G} \\ \mathbf{C}_{\mathbf{m}}^{-1/2} \end{pmatrix} \Delta \mathbf{m}_{k+1} = \begin{pmatrix} \mathbf{C}_{\mathbf{d}}^{-1/2} (\mathbf{d} - g(\mathbf{m}_k)) \\ \mathbf{C}_{\mathbf{m}}^{-1/2} (\mathbf{m}_k - \mathbf{m}_0) \end{pmatrix}$$

(Tarantola and Valette, 1982; Tarantola, 1987, 2005)

Physical interpretation:  
Fitting the data with the most simple (smooth) model