Integral equation methods for Electrical Impedance Tomography

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OUTLINE

- **EIT and the inverse conductivity problem**
- **Integral equation formulation**
- **A Tikhonov regularization method**
- **A mollifier method**
- **Numerical examples**
- **Conclusions**

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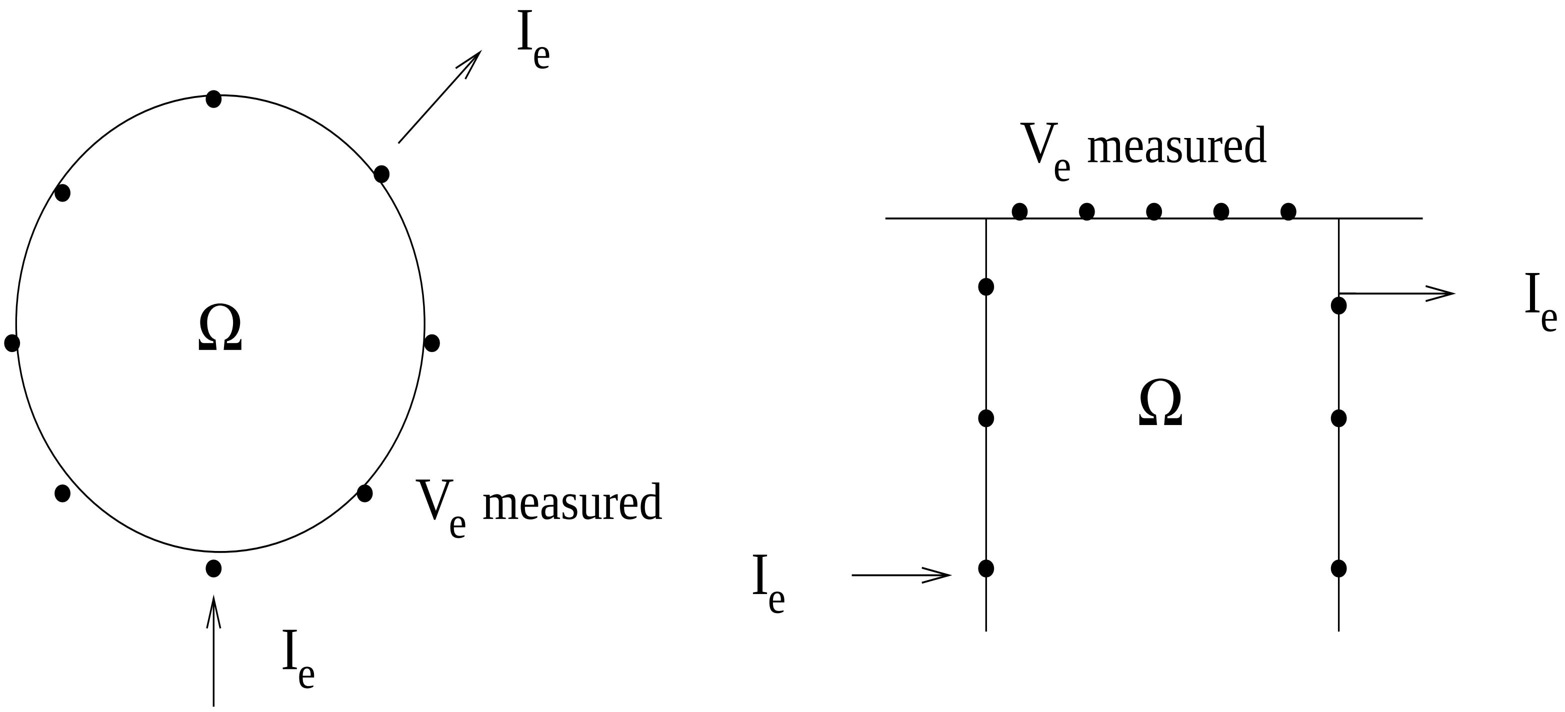
EIT AND THE INVERSE CONDUCTIVITY PROBLEM

Electrical Impedance Tomography (EIT) is the inverse problem of determining the electrical conductivity in the interior of an object, Ω, given simultaneous measurements of direct electric currents and voltages on the boundary of the object.



EIT and the inverse conductivity problem

EIT applications:

- **medicine**
  - detection of pulmonary emboli
  - monitoring of heart functional blood flow
  - breast cancer detection
- **geophysics**
  - locating underground mineral deposits
  - detection of leaks in underground storage tanks
  - monitoring flows of injected fluids into the earth
- **nondestructive testing**
  - detection of corrosion and of small defects in metals: cracks or voids

EIT AND THE INVERSE CONDUCTIVITY PROBLEM

Let $\Omega \subset \mathbb{R}^n$, $n = 2, 3$ be a bounded domain and σ be an isotropic conductivity distribution ($0 < c < \sigma < \infty$).

If an electric current $j = \sigma \frac{\partial \Phi}{\partial n} \in H^{-\frac{1}{2}}(\partial\Omega)$ is applied then the induced electric potential $\Phi \in H^1(\Omega)$ satisfies:

$$\boxed{\nabla \cdot [\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})] = 0 \,, \quad \mathbf{x} \in \Omega}$$

Definition. *The ICP is to find $\sigma(\mathbf{x})$, $\mathbf{x} \in \Omega$, given the NtD map*

$$\begin{aligned} (\Lambda_\sigma)^{-1} : \mathcal{I} \quad &\longrightarrow \quad H^{1/2}(\partial\Omega) \\ j \quad &\longmapsto \quad \Phi|_{\partial\Omega} \,, \end{aligned}$$

where $\mathcal{I} = \{ j(\mathbf{x}) \in H^{-\frac{1}{2}}(\partial\Omega) : \int_{\partial\Omega} j(\mathbf{x}) d\mathbf{x} = 0 \}$.

Uniqueness of solutions

- the linearized problem

  A.P. Calderón. *On an inverse boundary value problem*. Seminar on Numerical Analysis and its Applications to Continuum Physics. Soc. Brasileira de Mathèmatica, Rio de Janeiro, (1980).

- in $n \geq 3$ dimensions for $\gamma \in W^{3/2,\infty}(\Omega)$

  L. Päivärinta, A. Panchenko and G. Uhlmann. *Complex geometric optics solutions for Lipschitz conductivities*. Rev. Mat. Iberoamericana **19**, (2003).

- in two dimensions $\sigma \in L^\infty(\Omega)$

  K. Astala and L. Päivärinta. *Calderón's inverse conductivity problem in the plane*. Ann. Math. **163**, (2006).

EIT AND THE INVERSE CONDUCTIVITY PROBLEM

Reconstruction methods:

- back-projection methods
- iterative methods
- factorization approaches
- integral equation methods

EIT AND THE INVERSE CONDUCTIVITY PROBLEM

Reconstruction methods:

- back-projection methods
- iterative methods
- factorization approaches
- integral equation methods

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INTEGRAL EQUATION FORMULATION

Assume $\Omega \subset \mathbb{R}^n$, $n = 2,\, 3$, an open bounded Lipshitz domain with boundary $\partial\Omega$ and $\sigma \in C^{0,1}(\overline{\Omega})$.

The inverse problem to solve is the following: find $\sigma(\mathbf{x})$, $\mathbf{x} \in \Omega$, satisfying

$$\nabla \cdot [\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})] = 0\,, \quad \mathbf{x} \in \Omega\,, \quad \Phi \in H^1(\Omega),$$

$$\Phi(\mathbf{x}), \quad \mathbf{x} \in \partial\Omega,$$

$$j(\mathbf{x}) = \sigma(\mathbf{x}) \frac{\partial \Phi(\mathbf{x})}{\partial n}, \quad \mathbf{x} \in \partial\Omega,$$

$$\sigma(\mathbf{x}), \quad \mathbf{x} \in \partial\Omega.$$

INTEGRAL EQUATION FORMULATION

$$\nabla \cdot [\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})] = 0 \,, \quad \mathbf{x} \in \Omega$$

$$\partial\Omega \; : \; \sigma, \;\; \Phi, \;\; \frac{\partial \Phi}{\partial n} \Rightarrow \sigma(\mathbf{x}), \;\; \mathbf{x} \in \Omega$$

INTEGRAL EQUATION FORMULATION

$$\nabla \cdot [\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})] = 0\,, \quad \mathbf{x} \in \Omega$$

$$\partial\Omega \; : \; \sigma, \;\; \Phi, \;\; \frac{\partial \Phi}{\partial n} \Rightarrow \sigma(\mathbf{x}), \;\; \mathbf{x} \in \Omega$$

$$\nabla \sigma(\mathbf{x}) \cdot \nabla \Phi(\mathbf{x}) + \sigma(\mathbf{x}) \Delta \Phi(\mathbf{x}) = 0$$

INTEGRAL EQUATION FORMULATION

$$\nabla \cdot [\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})] = 0 \,, \quad \mathbf{x} \in \Omega$$

$$\partial\Omega \; : \; \sigma, \;\; \Phi, \;\; \frac{\partial \Phi}{\partial n} \Rightarrow \sigma(\mathbf{x}), \;\; \mathbf{x} \in \Omega$$

$$\Delta \Phi(\mathbf{x}) = -\frac{\nabla \sigma(\mathbf{x})}{\sigma(\mathbf{x})} \cdot \nabla \Phi(\mathbf{x})$$

INTEGRAL EQUATION FORMULATION

$$\nabla \cdot [\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})] = 0 \,, \quad \mathbf{x} \in \Omega$$

$$\partial\Omega \, : \, \sigma, \;\; \Phi, \;\; \frac{\partial \Phi}{\partial n} \Rightarrow \sigma(\mathbf{x}), \;\; \mathbf{x} \in \Omega$$

$$\Delta \Phi(\mathbf{x}) = -\nabla \tilde{\sigma}(\mathbf{x}) \cdot \nabla \Phi(\mathbf{x})$$

$\tilde{\sigma}(\mathbf{x}) = \ln \sigma(\mathbf{x})$

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INTEGRAL EQUATION FORMULATION

$$\nabla \cdot [\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})] = 0 \,, \qquad \mathbf{x} \in \Omega$$

$$\partial\Omega \,:\, \sigma, \;\; \Phi, \;\; \frac{\partial \Phi}{\partial n} \Rightarrow \sigma(\mathbf{x}), \;\; \mathbf{x} \in \Omega$$

$$\Delta \Phi(\mathbf{x}) = -Y(\mathbf{x})$$

$$\tilde{\sigma}(\mathbf{x}) = \ln \sigma(\mathbf{x}) \;,\; Y(\mathbf{x}) = \nabla \tilde{\sigma}(\mathbf{x}) \cdot \nabla \Phi(\mathbf{x})$$

INTEGRAL EQUATION FORMULATION

$$\nabla \cdot [\sigma(\mathbf{x}) \nabla \Phi(\mathbf{x})] = 0\,, \quad \mathbf{x} \in \Omega$$

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$$\Delta\Phi(\mathbf{x}) = -Y(\mathbf{x})$$

$$\partial\Omega\ :\ \tilde{\sigma},\ \ \Phi,\ \ \frac{\partial \Phi}{\partial n} \Rightarrow \tilde{\sigma}(\mathbf{x}),\ \ \mathbf{x} \in \Omega$$

INTEGRAL EQUATION FORMULATION

Let $H : \Omega \times \Omega \longrightarrow \mathbb{R}$ satisfy the Helmholtz equation

$$\Delta_y H(\mathbf{x},\mathbf{y}) \; + \lambda \, H(\mathbf{x},\mathbf{y}) \; = \; 0 \, , \quad \mathbf{x} \, , \mathbf{y} \in \Omega \, , \;\; \lambda > 0$$

Let $\mathcal{F}$ be the set of functions $f : \Omega \longrightarrow \mathbb{R}$ which satisfy the Helmholtz equation

$$\Delta_y f(\mathbf{y}) \; + \lambda \, f(\mathbf{y}) \; = \; 0 \, , \quad \mathbf{y} \in \Omega \, , \;\; \lambda > 0$$

Let $\mathcal{G}_0(\mathbf{x},\mathbf{y})$ be the free space Green's function

$$\Delta_y \mathcal{G}_0(\mathbf{x},\mathbf{y}) \; + \lambda \, \mathcal{G}_0(\mathbf{x},\mathbf{y}) \; = \; - \, \delta(\mathbf{x} - \mathbf{y}) \, , \quad \mathbf{x} \, , \mathbf{y} \in \Omega$$

Green's second identity

$$\int_\Omega (u \Delta v - v \Delta u) = \int_{\partial\Omega} \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n} \right)$$

INTEGRAL EQUATION FORMULATION

$$0 = \zeta(\mathbf{x}) - \int_{\Omega} d\mathbf{y}\ H(\mathbf{x}, \mathbf{y})(Y(\mathbf{y}) - \lambda\Phi(\mathbf{y}))$$

$$\zeta(\mathbf{x}) = \int_{\partial\Omega} d\mathbf{y}\ \left(\Phi(\mathbf{y})\frac{\partial H}{\partial n}(\mathbf{x}, \mathbf{y}) - H(\mathbf{x}, \mathbf{y})\frac{\partial\Phi(\mathbf{y})}{\partial n}\right)$$

INTEGRAL EQUATION FORMULATION

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$$\Phi(\mathbf{x}) = \zeta_0(\mathbf{x}) - \int_{\Omega} d\mathbf{y}\, \mathcal{G}_0(\mathbf{x},\mathbf{y})(Y(\mathbf{y}) - \lambda\Phi(\mathbf{y}))$$

$$\zeta_0(\mathbf{x}) = \int_{\partial\Omega} d\mathbf{y}\; \left(\mathcal{G}_0(\mathbf{x},\mathbf{y})\frac{\partial \Phi(\mathbf{y})}{\partial n} - \Phi(\mathbf{y})\frac{\partial \mathcal{G}_0}{\partial n}(\mathbf{x},\mathbf{y})\right)$$

INTEGRAL EQUATION FORMULATION

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$$X(\mathbf{y}) = Y(\mathbf{y}) - \lambda\Phi(\mathbf{y})$$

INTEGRAL EQUATION FORMULATION

$$0 = \zeta(\mathbf{x}) - \int_{\Omega} d\mathbf{y}\ H(\mathbf{x},\mathbf{y})X(\mathbf{y})$$

$$\zeta(\mathbf{x}) = \int_{\partial\Omega} d\mathbf{y}\ \left(\Phi(\mathbf{y})\frac{\partial H}{\partial n}(\mathbf{x},\mathbf{y}) - H(\mathbf{x},\mathbf{y})\frac{\partial\Phi(\mathbf{y})}{\partial n}\right)$$

$$\Phi(\mathbf{x}) = \zeta_0(\mathbf{x}) - \int_{\Omega} d\mathbf{y}\, \mathcal{G}_0(\mathbf{x},\mathbf{y})X(\mathbf{y})$$

$$\zeta_0(\mathbf{x}) = \int_{\partial\Omega} d\mathbf{y}\ \left(\mathcal{G}_0(\mathbf{x},\mathbf{y})\frac{\partial\Phi(\mathbf{y})}{\partial n} - \Phi(\mathbf{y})\frac{\partial\mathcal{G}_0}{\partial n}(\mathbf{x},\mathbf{y})\right)$$

$$X(\mathbf{y}) = Y(\mathbf{y}) - \lambda\Phi(\mathbf{y})$$

INTEGRAL EQUATION FORMULATION

- Solve the linear problem $\int_{\Omega} d\mathbf{y}\ H(\mathbf{x},\mathbf{y})X(\mathbf{y}) = \zeta(\mathbf{x})$

INTEGRAL EQUATION FORMULATION

- Solve the linear problem $AX = \zeta$

INTEGRAL EQUATION FORMULATION

- Solve the linear problem $AX = \zeta$

$$\begin{aligned} A : L^2(\Omega) &\longrightarrow L^2(\Omega) \\ X &\longmapsto \int_\Omega d\mathbf{y}\ H(\mathbf{x},\mathbf{y}) X(\mathbf{y})\,. \end{aligned}$$

INTEGRAL EQUATION FORMULATION

- Solve the linear problem $AX = \zeta$

$$
\begin{aligned}
A : L^2(\Omega) &\longrightarrow L^2(\Omega) \\
X &\longmapsto \int_\Omega d\mathbf{y}\; H(\mathbf{x},\mathbf{y}) X(\mathbf{y}) \, .
\end{aligned}
$$

$$X(\mathbf{x}) \approx Y(\mathbf{x}) - \lambda \Phi(\mathbf{x}),$$

INTEGRAL EQUATION FORMULATION

- Solve the linear problem $AX = \zeta$

$$
\begin{array}{rcl}
A : L^2(\Omega) & \longrightarrow & L^2(\Omega) \\
X & \longmapsto & \int\limits_{\Omega} d\mathbf{y}\ H(\mathbf{x},\mathbf{y}) X(\mathbf{y})\,.
\end{array}
$$

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INTEGRAL EQUATION FORMULATION

- Solve the linear problem $AX = \zeta$

$$\begin{array}{rcl} A : L^2(\Omega) & \longrightarrow & L^2(\Omega) \\ X & \longmapsto & \int_\Omega d\mathbf{y}\ H(\mathbf{x},\mathbf{y})X(\mathbf{y}) \,. \end{array}$$

- $\Phi(\mathbf{x}) = \zeta_0(\mathbf{x}) - \int_\Omega d\mathbf{y}\, \mathcal{G}_0(\mathbf{x},\mathbf{y})X(\mathbf{y})$, $\mathbf{x} \in \Omega$

INTEGRAL EQUATION FORMULATION

- Solve the linear problem $AX = \zeta$

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- $\Phi(\mathbf{x}) = \zeta_0(\mathbf{x}) - \int_\Omega d\mathbf{y}\, \mathcal{G}_0(\mathbf{x},\mathbf{y})X(\mathbf{y}),\;\; \mathbf{x} \in \Omega$

- Compute $\tilde{\sigma}(\mathbf{x}) = \ln \sigma(\mathbf{x}),\;\; \mathbf{x} \in \Omega$, by solving

$$\nabla\tilde{\sigma}(\mathbf{x}) \cdot \nabla\Phi = -Y(\mathbf{x}),\;\; \text{subject to}\;\; \widetilde{\sigma(\mathbf{x})}\,,\;\; \mathbf{x} \in \partial\Omega\,.$$

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- Conclusions and future work

A TIKHONOV REGULARIZATION METHOD

$$AX = \zeta$$

$$\begin{array}{rcl} A : L^2(\Omega) & \longrightarrow & L^2(\Omega) \\ X & \longmapsto & \int\limits_{\Omega} d\mathbf{y}\ H(\mathbf{x},\mathbf{y})X(\mathbf{y})\,. \end{array}$$

A TIKHONOV REGULARIZATION METHOD

$$AX = \zeta$$

$$\begin{array}{rcl} A : L^2(\Omega) & \longrightarrow & L^2(\Omega) \\ X & \longmapsto & \int_\Omega d\mathbf{y}\ H(\mathbf{x},\mathbf{y})X(\mathbf{y})\,. \end{array}$$

$\|\zeta(\mathbf{x}) - AX_{reg}(\mathbf{x})\|_{L^2} \to \min$ subject to $\|X_{reg}(\mathbf{x}) - X_{mod}(\mathbf{x})\|_{L^2} \leq \delta$

A TIKHONOV REGULARIZATION METHOD

$$AX = \zeta$$

$$\begin{array}{rcl} A : L^2(\Omega) & \longrightarrow & L^2(\Omega) \\ X & \longmapsto & \int\limits_{\Omega} d\mathbf{y}\ H(\mathbf{x},\mathbf{y}) X(\mathbf{y})\,. \end{array}$$

$$\|\zeta(\mathbf{x}) - AX_{reg}(\mathbf{x})\|^2_{L^2} + \mu\left(\|X_{reg}(\mathbf{x}) - X_{mod}(\mathbf{x})\|^2_{L^2} - \delta^2\right) \rightarrow \min$$

A TIKHONOV REGULARIZATION METHOD

$$AX = \zeta$$

$$\begin{aligned} A : L^2(\Omega) &\longrightarrow L^2(\Omega) \\ X &\longmapsto \int_\Omega d\mathbf{y}\; H(\mathbf{x},\mathbf{y}) X(\mathbf{y}) \,. \end{aligned}$$

$$\|\zeta(\mathbf{x}) - AX_{reg}(\mathbf{x})\|^2_{L^2} + \mu \left(\|X_{reg}(\mathbf{x}) - X_{mod}(\mathbf{x})\|^2_{L^2} - \delta^2 \right) \to \min$$

$$X_{reg}(\mathbf{x}) = X_{mod}(\mathbf{x}) + \frac{1}{\mu} \int_\Omega d\mathbf{y} H(\mathbf{x},\mathbf{y}) \zeta(\mathbf{y}) - \frac{1}{\mu} \int_\Omega d\mathbf{y} H_2(\mathbf{x},\mathbf{y}) X_{reg}(\mathbf{y})$$

where $H_2(\mathbf{x},\mathbf{y}) = \int_\Omega d\mathbf{z}\, H(\mathbf{x},\mathbf{z}) H(\mathbf{z},\mathbf{y})\,.$

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A MOLLIFIER METHOD

$$AX = \zeta$$

$$\begin{aligned} A : L^2(\Omega) &\longrightarrow L^2(\Omega) \\ X &\longmapsto \int_\Omega d\mathbf{y}\; H(\mathbf{x}, \mathbf{y}) X(\mathbf{y}) \,. \end{aligned}$$

A MOLLIFIER METHOD

$$AX = \zeta$$

$$\begin{aligned} A : L^2(\Omega) &\longrightarrow L^2(\Omega) \\ X &\longmapsto \int_{\Omega} d\mathbf{y}\ H(\mathbf{x},\mathbf{y}) X(\mathbf{y})\,. \end{aligned}$$

Advantages of mollifier methods:

- Locally adapted resolution can be easily incorporated.
- Solve an operator equation for every reconstruction point $\mathbf{x}$.
- All the pointwise reconstruction vectors can be precomputed.
- The choice of the function H can be optimized.

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A MOLLIFIER METHOD

$$\boxed{AX = \zeta}$$

$$\int_{\Omega} d\mathbf{y}\, X(\mathbf{y})\, e_{\gamma}(\widetilde{\mathbf{y}}, \mathbf{y}) \rightarrow X(\widetilde{\mathbf{y}}), \text{ as } \gamma \rightarrow 0$$

$$e_{\gamma}(\widetilde{\mathbf{y}}, \mathbf{y}) = \frac{1}{|B(\widetilde{\mathbf{y}}, \gamma)|} \cdot \begin{cases} 1 & : \mathbf{y} \in B(\widetilde{\mathbf{y}}, \gamma) \\ 0 & : \text{otherwise} \end{cases} ,$$

$$B(\widetilde{\mathbf{y}}, \gamma) = \{\mathbf{y} : ||\widetilde{\mathbf{y}} - \mathbf{y}|| \leq \gamma\}$$

.

$$X_{\gamma}(\widetilde{\mathbf{y}}) = < e_{\gamma}(\widetilde{\mathbf{y}}, \cdot), X >_{L^2(\Omega)} ,$$

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A MOLLIFIER METHOD

$$\boxed{AX = \zeta}$$

$$\int\limits_{\Omega} d\mathbf{y}\, X(\mathbf{y})\, e_\gamma(\widetilde{\mathbf{y}}, \mathbf{y}) \rightarrow X(\widetilde{\mathbf{y}}), \ \text{ as } \ \gamma \rightarrow 0$$

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$$B(\widetilde{\mathbf{y}}, \gamma) = \{\mathbf{y} : \ ||\widetilde{\mathbf{y}} - \mathbf{y}|| \leq \gamma\}$$

.

$$X_\gamma(\widetilde{\mathbf{y}}) = < \tilde{e}_\gamma(\widetilde{\mathbf{y}}, \cdot), X >_{L^2(\Omega)} ,$$

where $\tilde{e}_\gamma \in R(A^*) \subset \mathcal{F}$.

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A MOLLIFIER METHOD

$$\boxed{AX = \zeta}$$

Theorem. *The mollified approximation to the solution* X *of the above equation at* $\widetilde{\mathbf{y}}$ *is given by*

$$\begin{aligned} X_\gamma(\widetilde{\mathbf{y}}) &= \; < \widetilde{e}_\gamma(\widetilde{\mathbf{y}}, \cdot),\, X >_{L^2(\Omega)} \\ &= \int\limits_{\partial\Omega} \Big(\frac{\partial \widetilde{e}_\gamma}{\partial n}(\widetilde{\mathbf{y}}, \mathbf{x}) \Phi(\mathbf{x}) - \widetilde{e}_\gamma(\widetilde{\mathbf{y}}, \mathbf{x}) \frac{\partial \Phi}{\partial n}(\mathbf{x}) \Big) d\mathbf{x} \;. \end{aligned}$$

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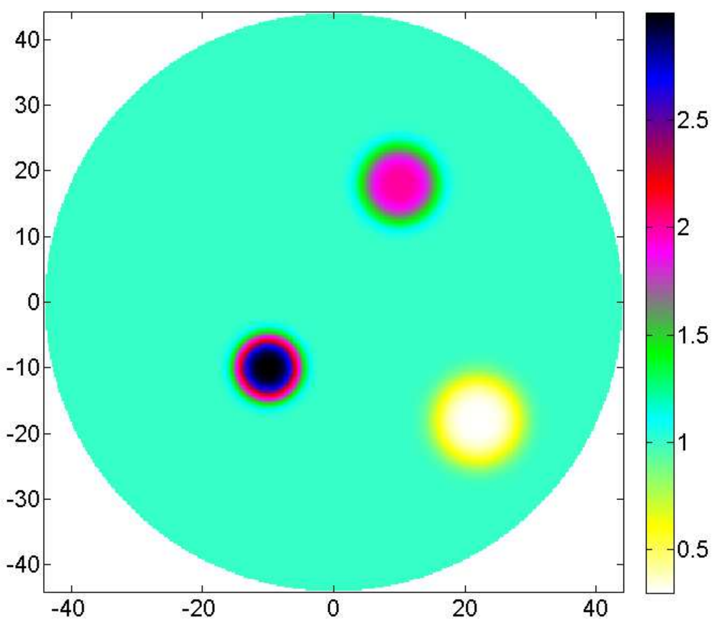
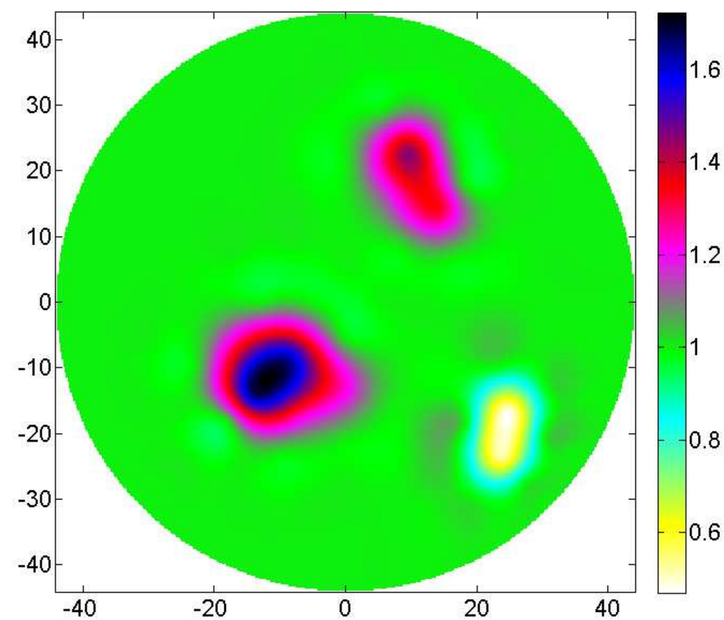
NUMERICAL EXAMPLES

Example. *The conductivity distribution consists of two off-centered high conductivity regions and a low conductivity region within a constant background We reconstructed conductivity for data with 2$\%$ random errors, regularization parameter $\gamma = 0.1$ and $\lambda = 1$.*



(a) The exact conductivity distribution.



(b) The reconstructed conductivity

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CONCLUSIONS

- Integral Equation formulation for reconstructing $\sigma \in C^{0,1}(\Omega)$
- The kernel of the Fredholm integral equation of the first kind satisfies the Helmholtz equation
- The approaches are not geometrically constrained
- The algorithms are quite stable with respect to the noise level in the data