

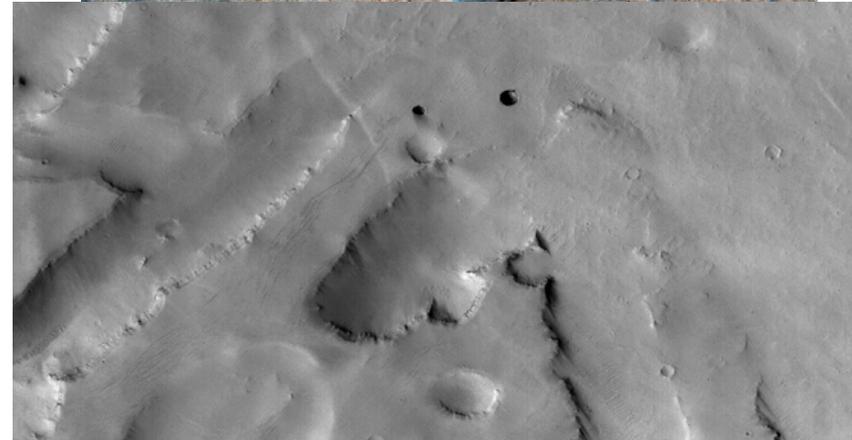
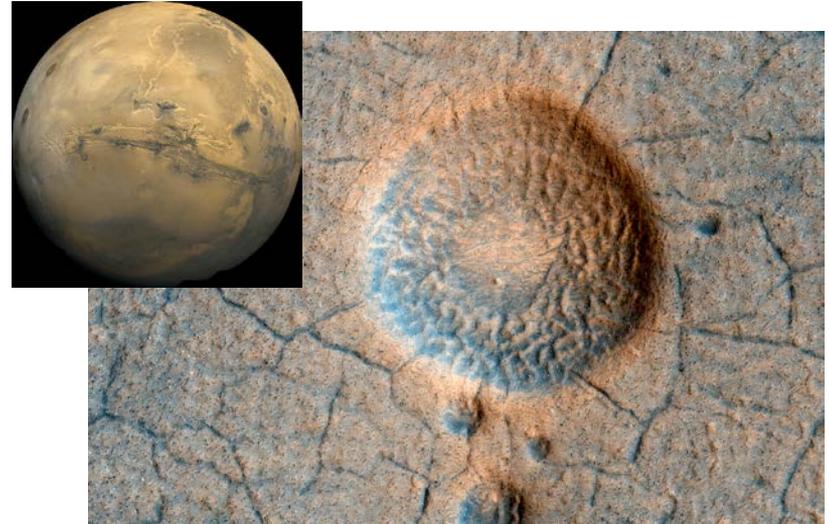
# Volume slicing with multi- directional muon radiography

Hiroiyuki K.M. Tanaka  
University of Tokyo

# Geological problems to be solved by muon radiography

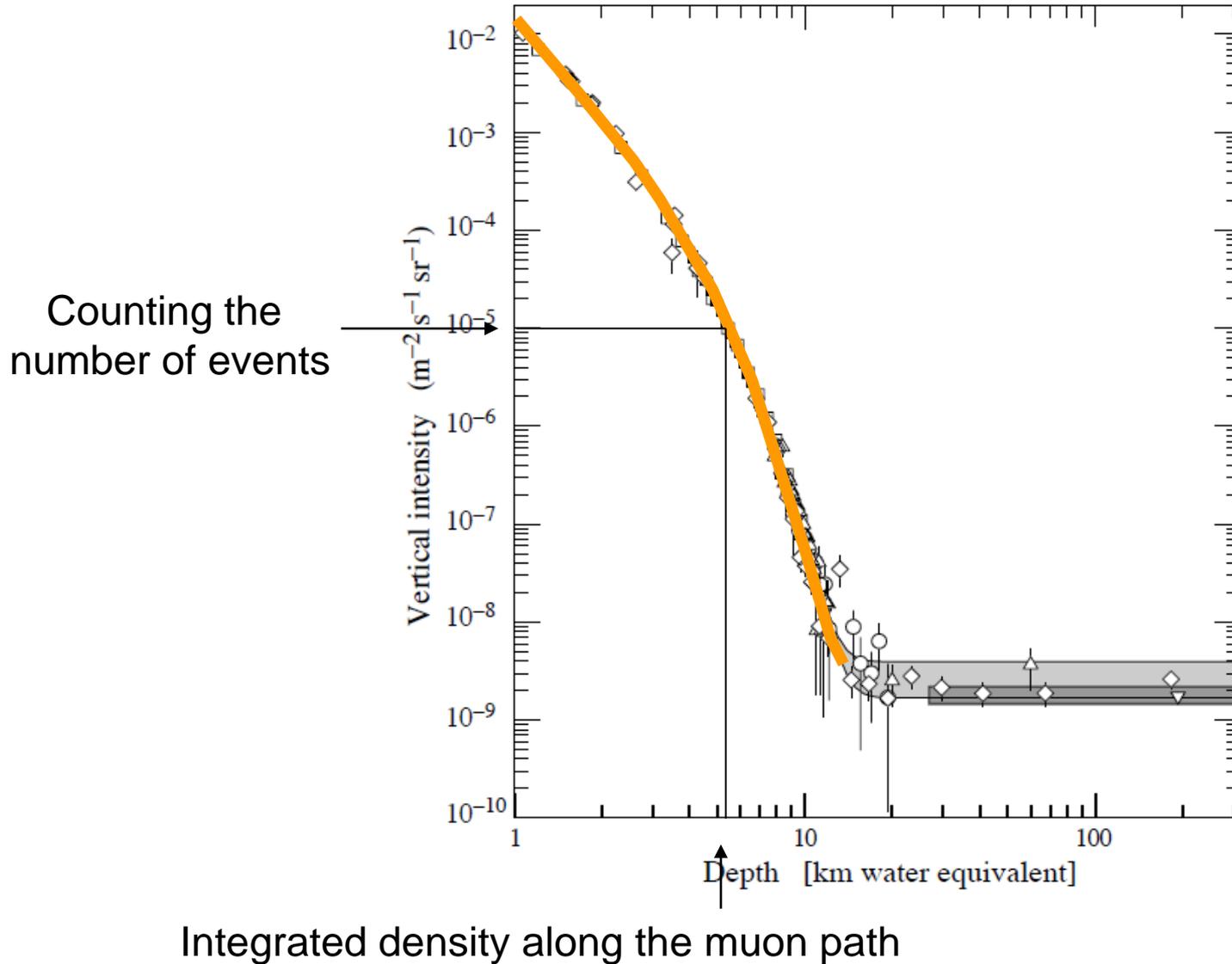


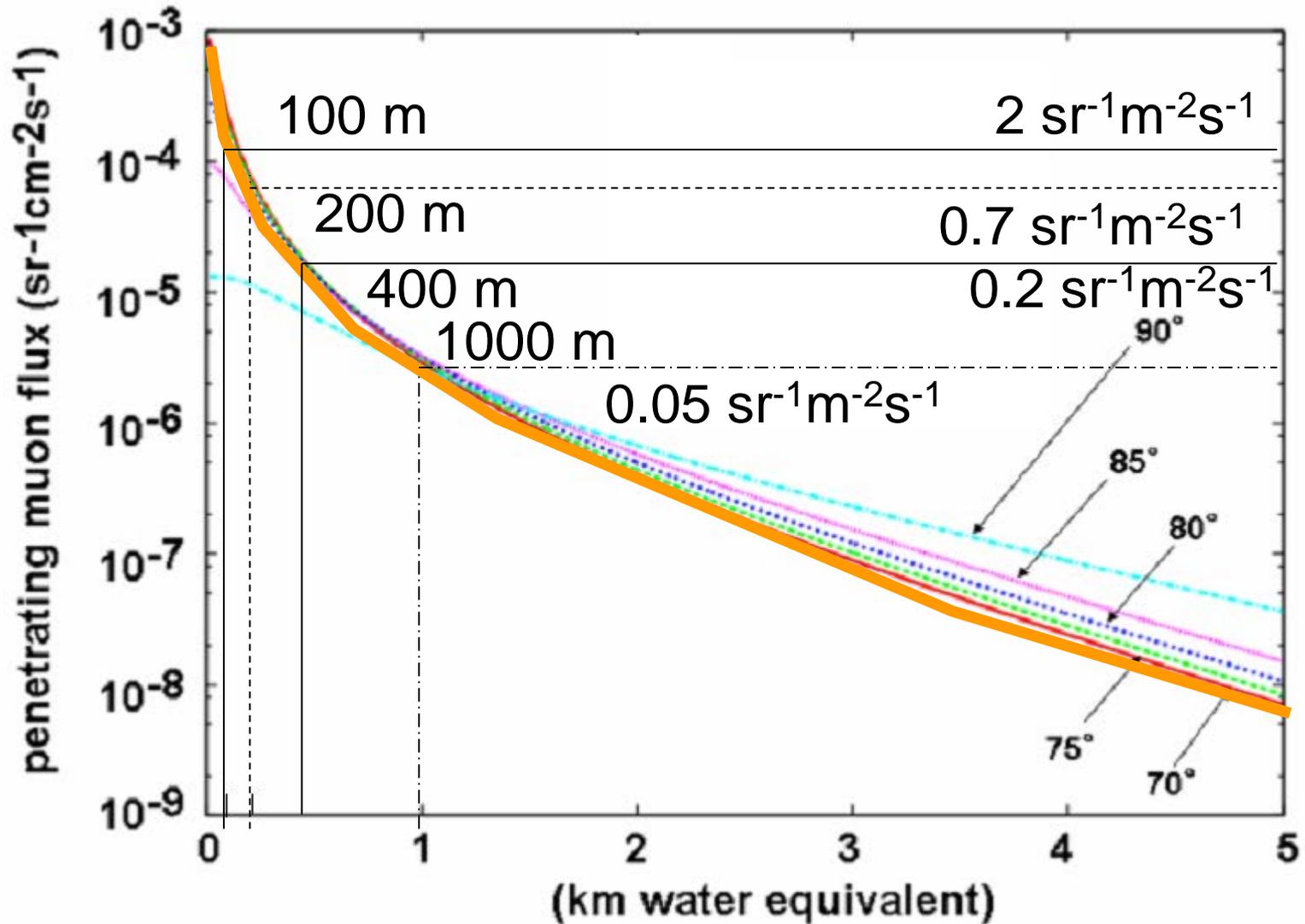
**Growing lavadome  
Degassing evolution  
Eruption dynamics**



- Short time scale
- Short time restriction
- Shape changes with time

# Muon radiography principle





Thickness 400 m 20 events/20min  
 Thickness 200 m 70 events/20min  
 Thickness 100m 210 events/20min

Density 50% off distinguishable

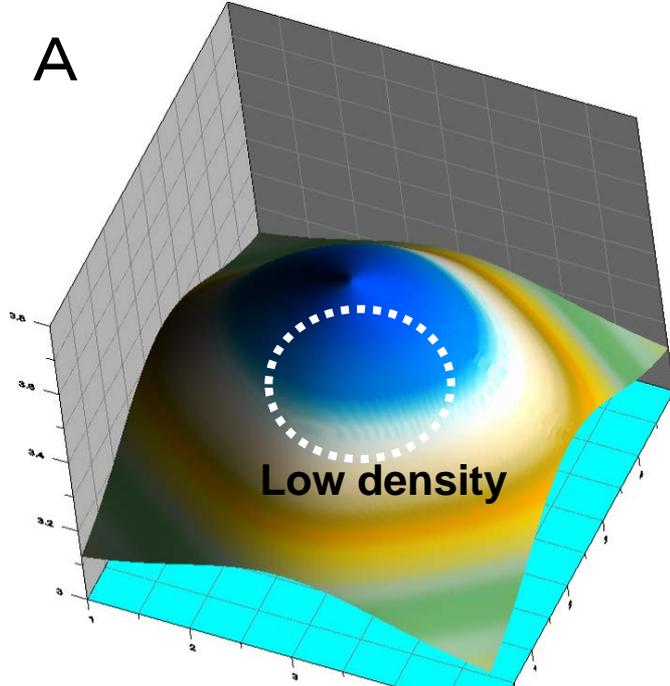
( $S=50 \times 50 \text{ cm}^2$   $L= 1.5 \text{ m}$ )

# However

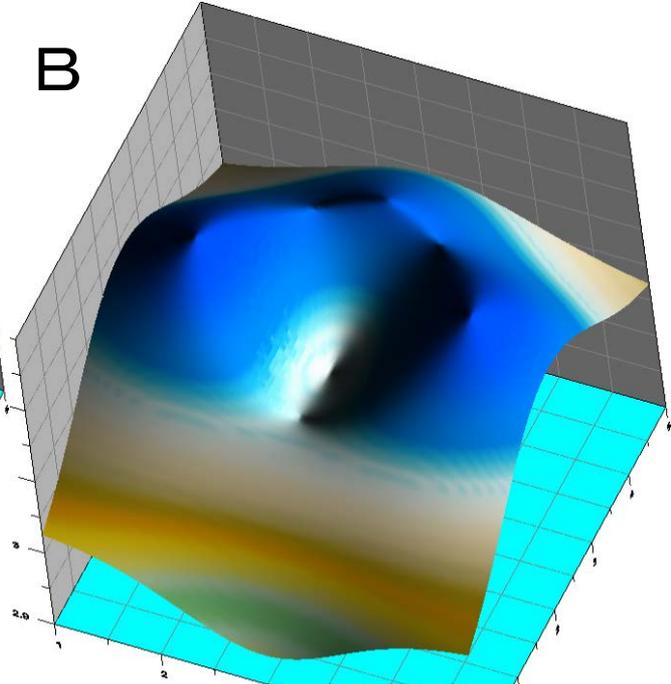
- There still has a problem
- If we cannot measure the density length



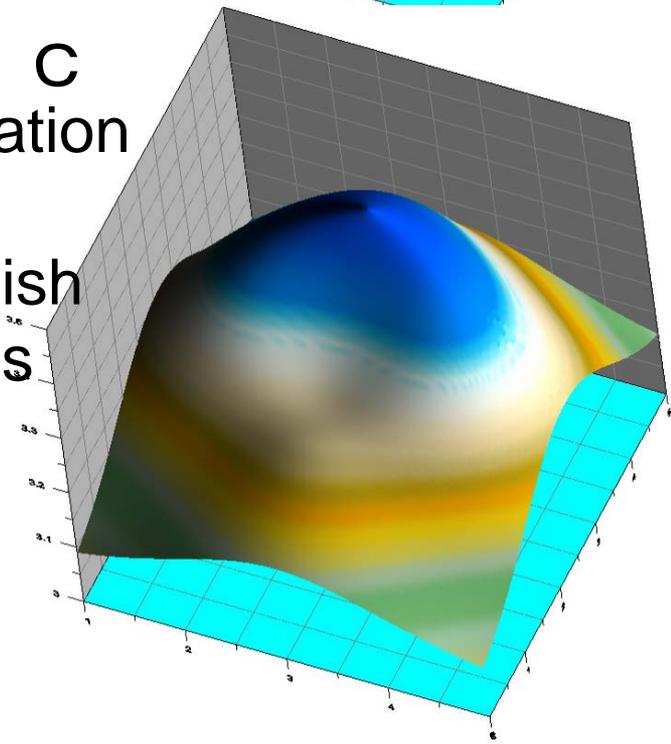
A



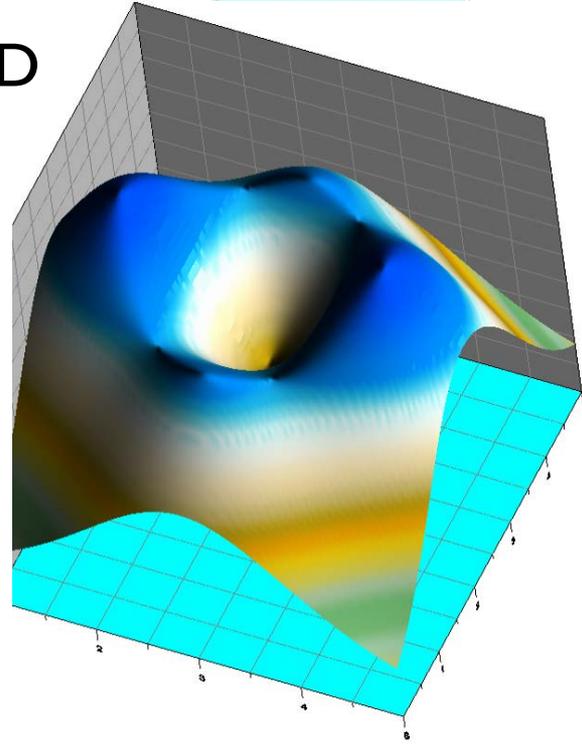
B



C



D

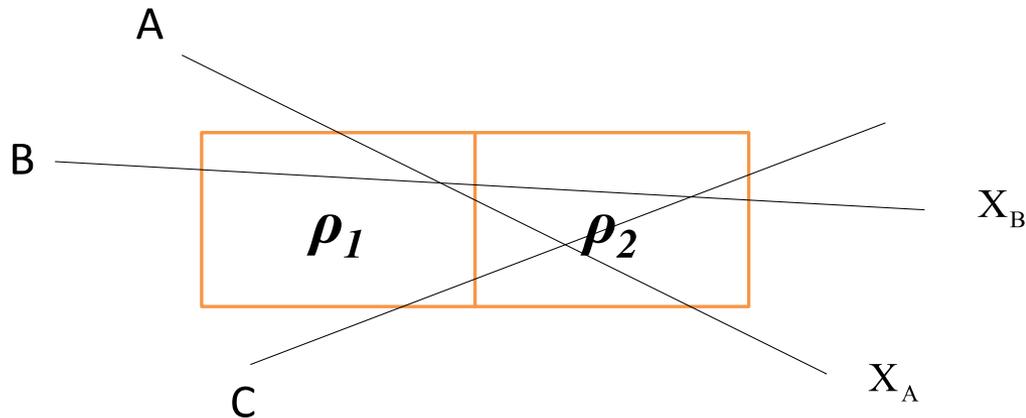


If there is no  
topographical information

We cannot distinguish  
From four patterns

# Method1

- Singular value decomposition



$$L = X\Lambda P^T$$

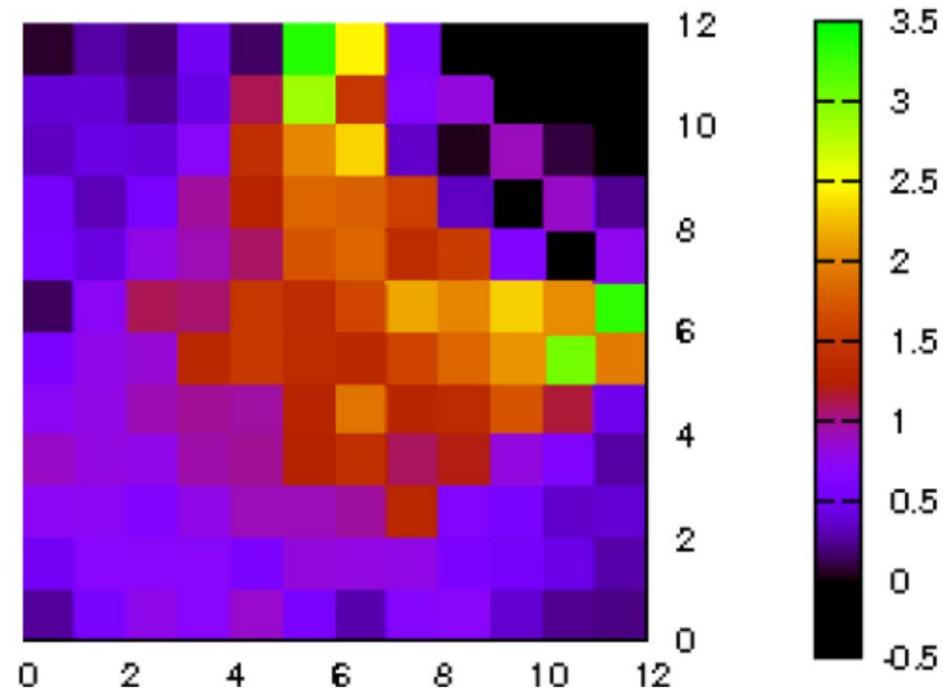
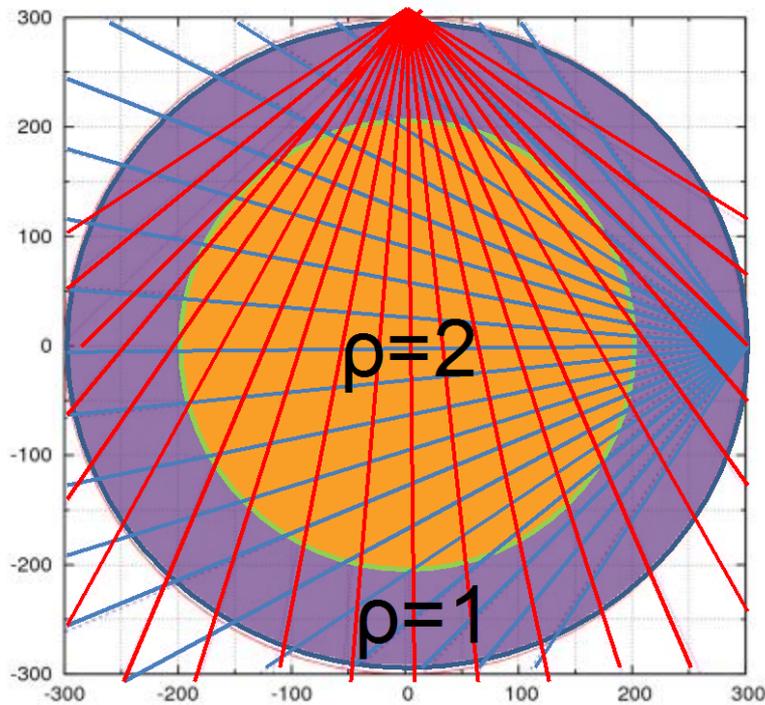
$$X = [x_1, x_2, x_3, \dots, x_N]$$

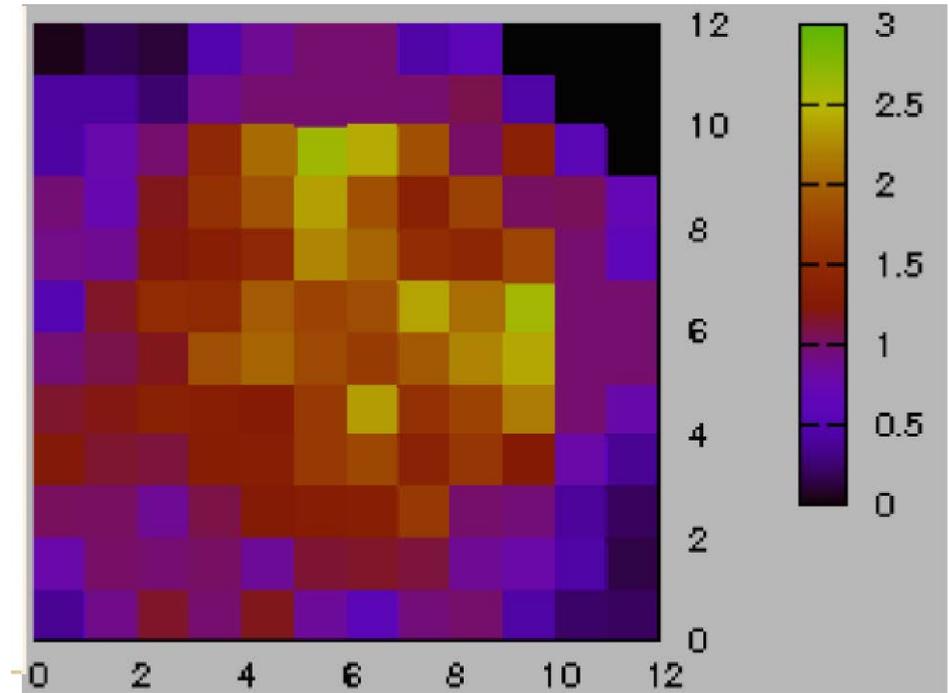
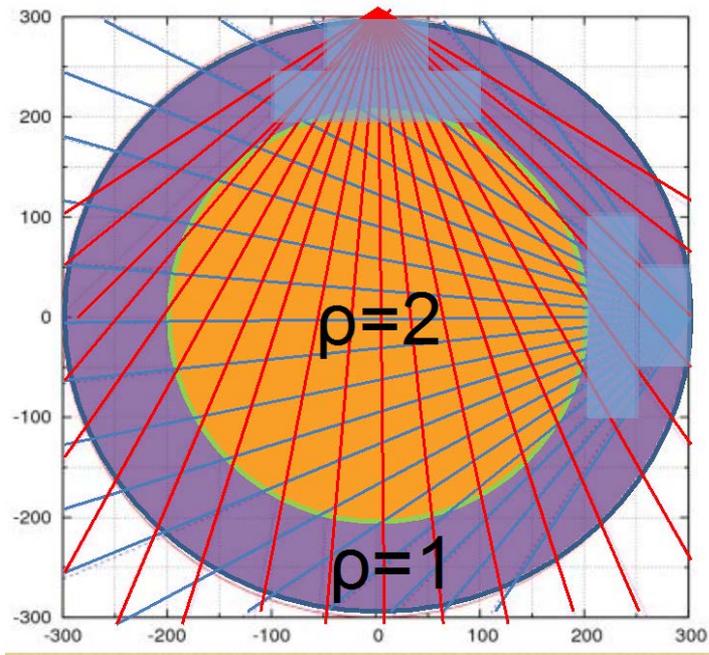
$$P = [\rho_1, \rho_2, \rho_3, \dots, \rho_M]$$

$$\Lambda = \underbrace{\begin{pmatrix} \lambda_1 & & 0 & 0 & \dots & 0 \\ & \ddots & & \vdots & \ddots & \vdots \\ 0 & & \lambda_p & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 \end{pmatrix}}_N \quad \left. \vphantom{\begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_p \\ \vdots \\ 0 \end{pmatrix}} \right\} M$$

Fan beam case

by Penrose





Pros: Exterior shape info is not necessary.

Cons: Density of the path lines affects the result

# Method 2

Damped least squares

$$\Phi(\mathbf{m}) = \mathbf{E} + \varepsilon^2 \mathbf{N}$$

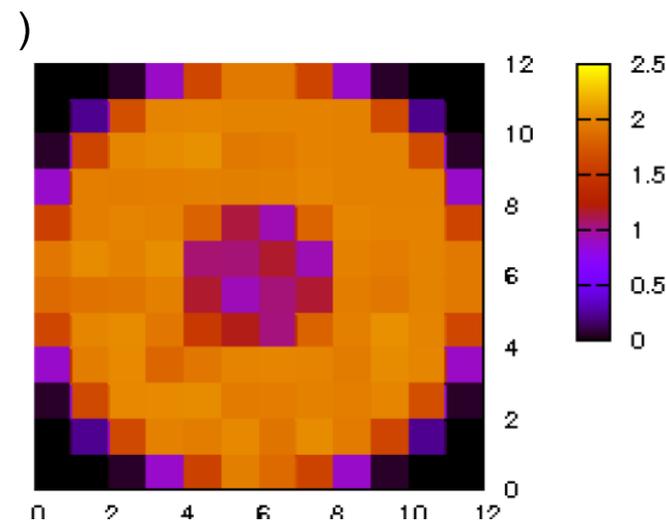
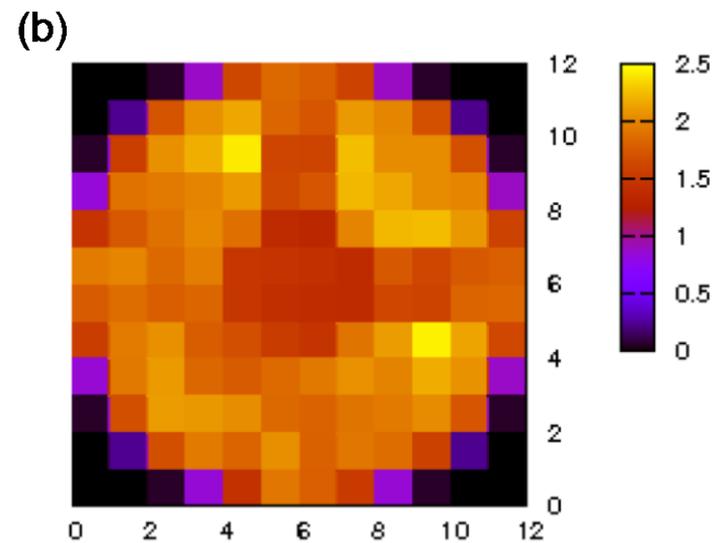
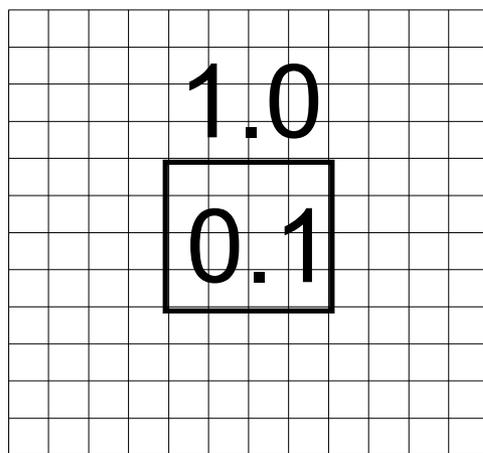
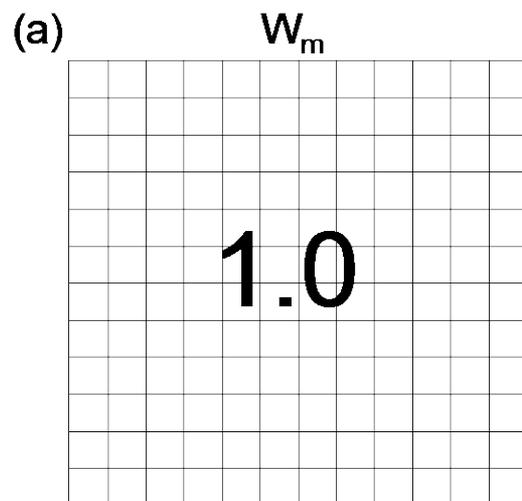
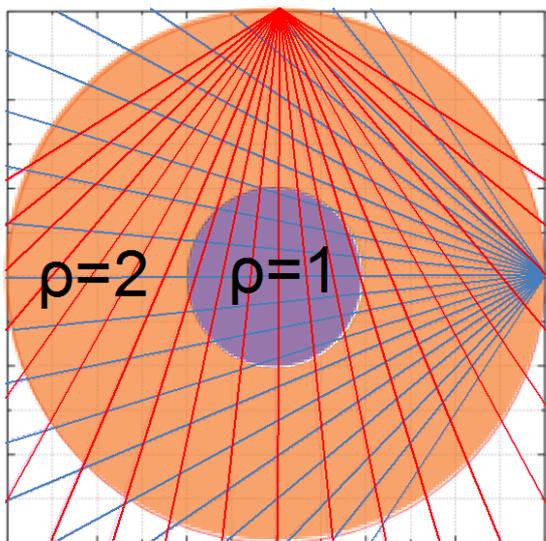
$$\mathbf{E} = (\mathbf{x} - \mathbf{L}\rho)^T \mathbf{W}_e (\mathbf{x} - \mathbf{L}\rho)$$

$$\mathbf{N} = (\rho - \langle \rho \rangle)^T \mathbf{W}_m (\rho - \langle \rho \rangle)$$

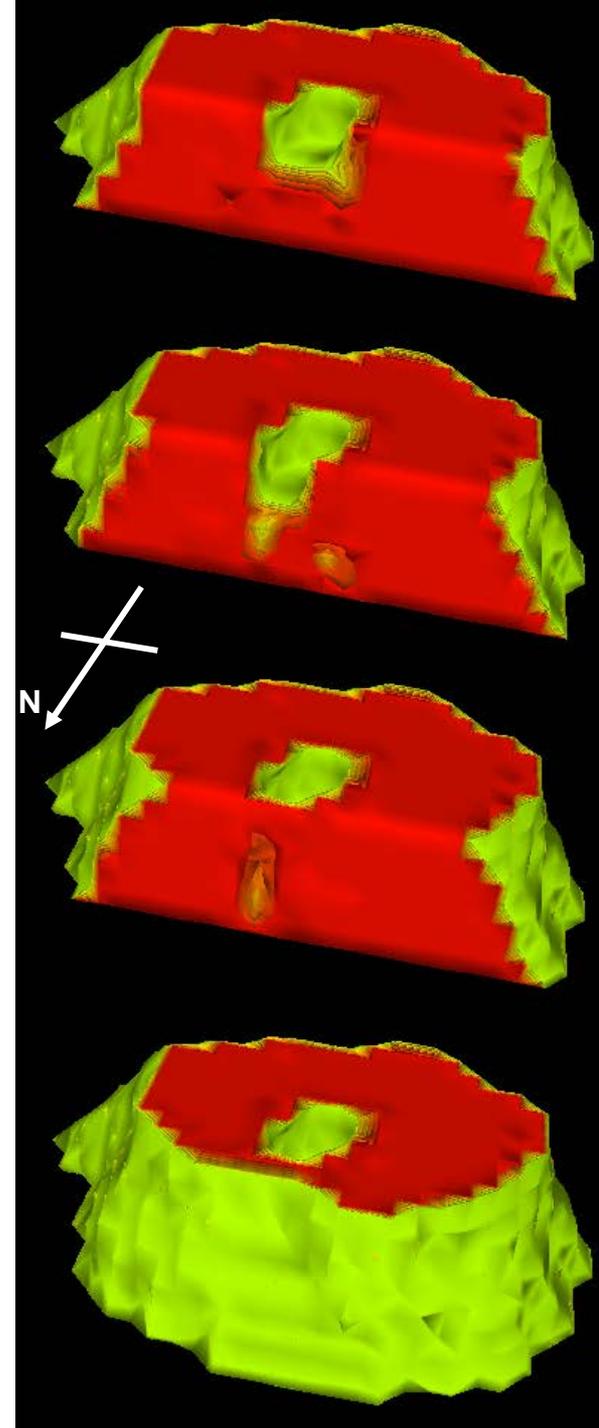
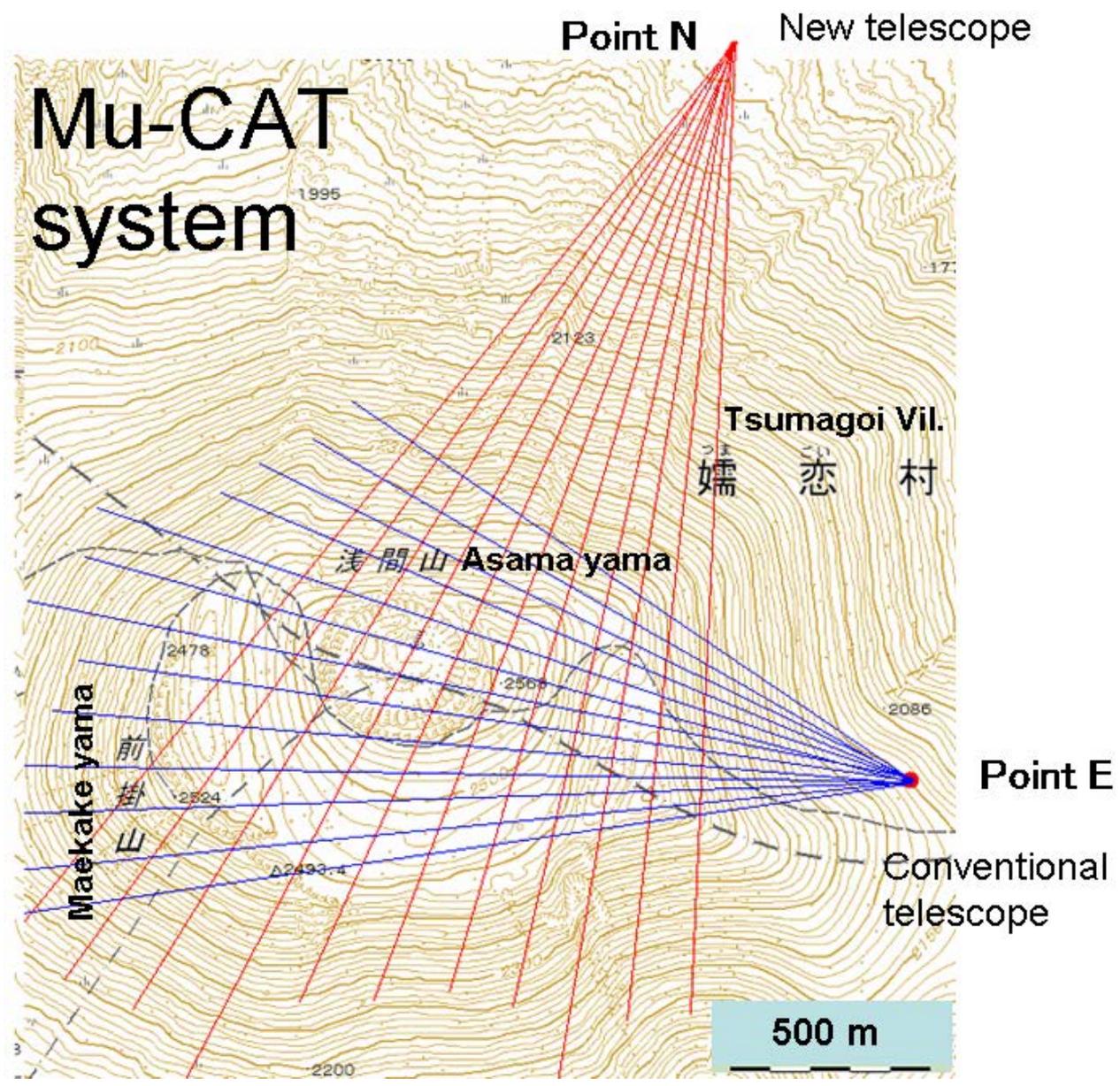
$\langle \rho \rangle$  is artificially given

Pros: Less distortion

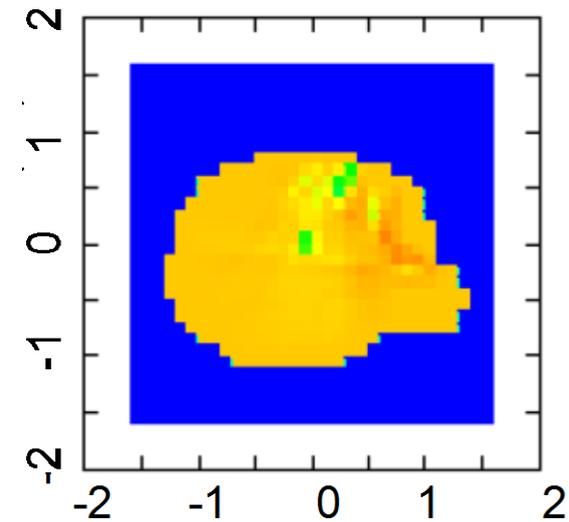
Cons: Exterior shape info is necessary



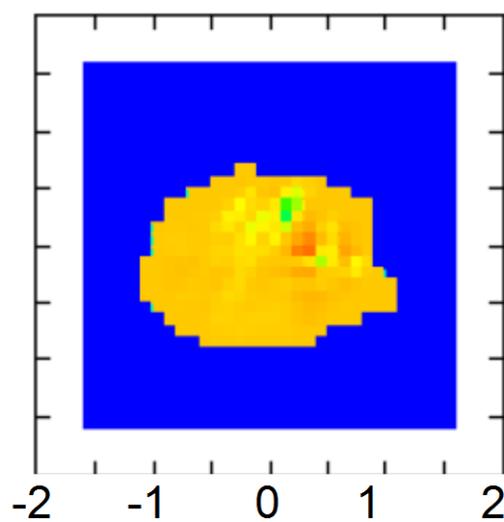
# Mu-CAT system



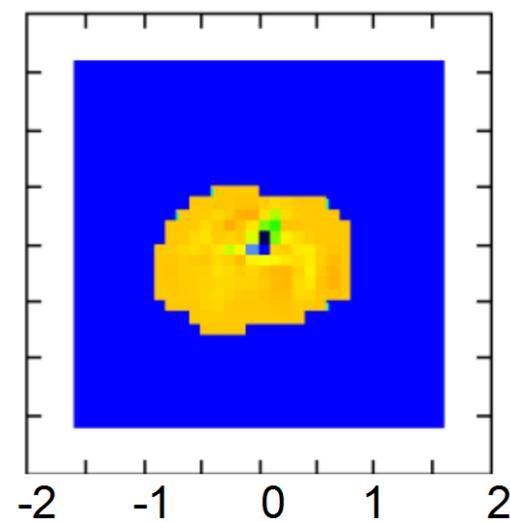
2170-2270m (2×2)



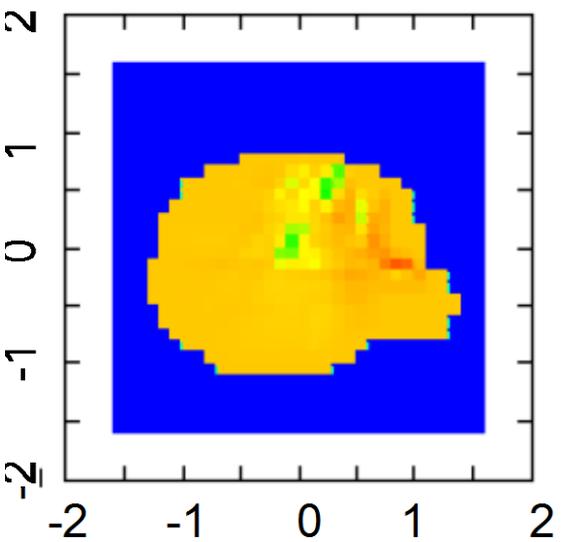
(b) 2270-2370m (2×2)



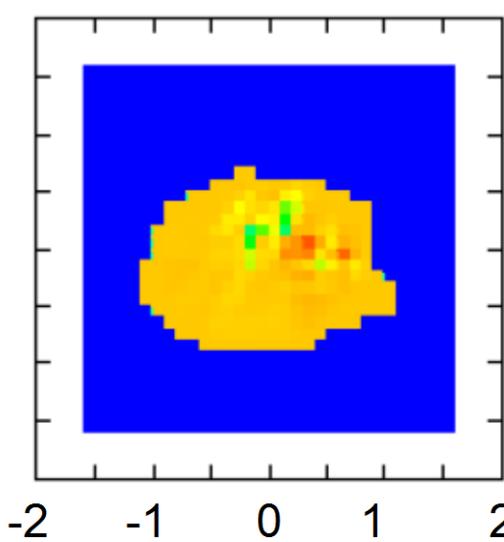
(c) 2370-2470m (2×2)



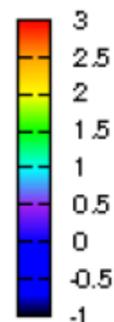
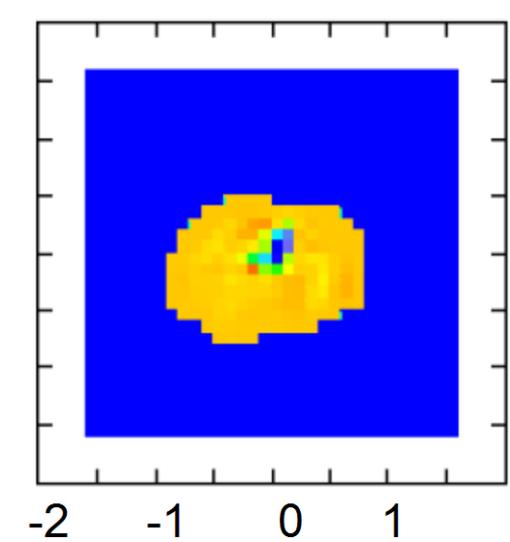
2170-2270m (4×4)



(e) 2270-2370m (4×4)



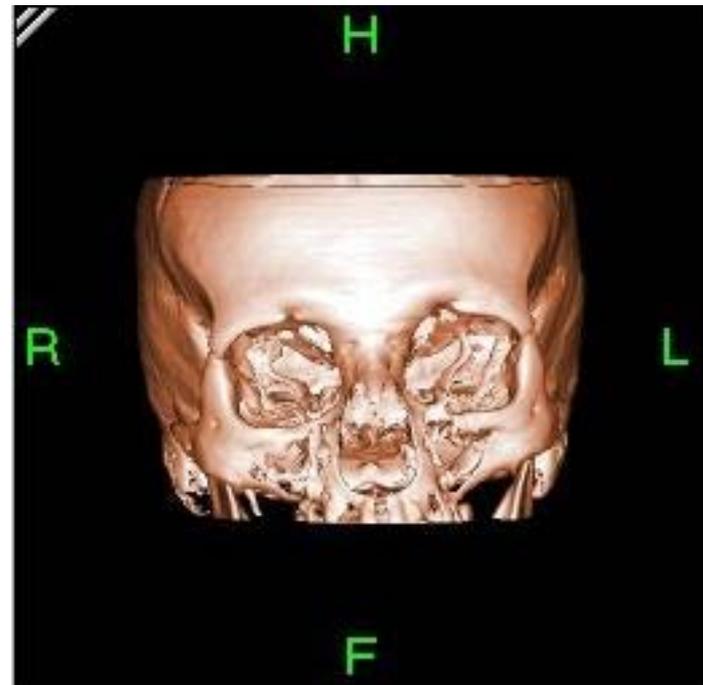
(f) 2370-2470m (4×4)



# Method 3

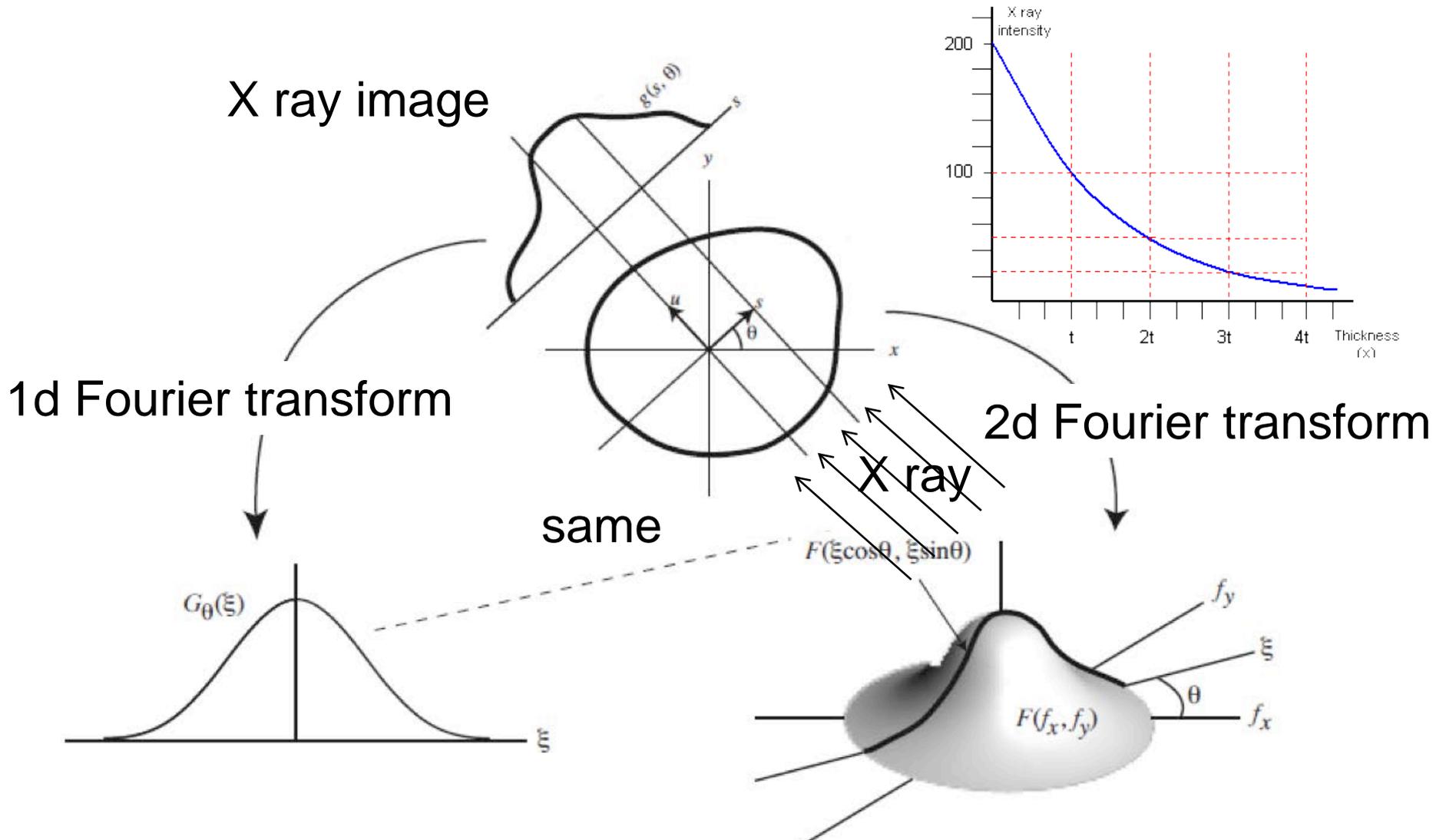
We do not need  
the exterior geometry

Why?

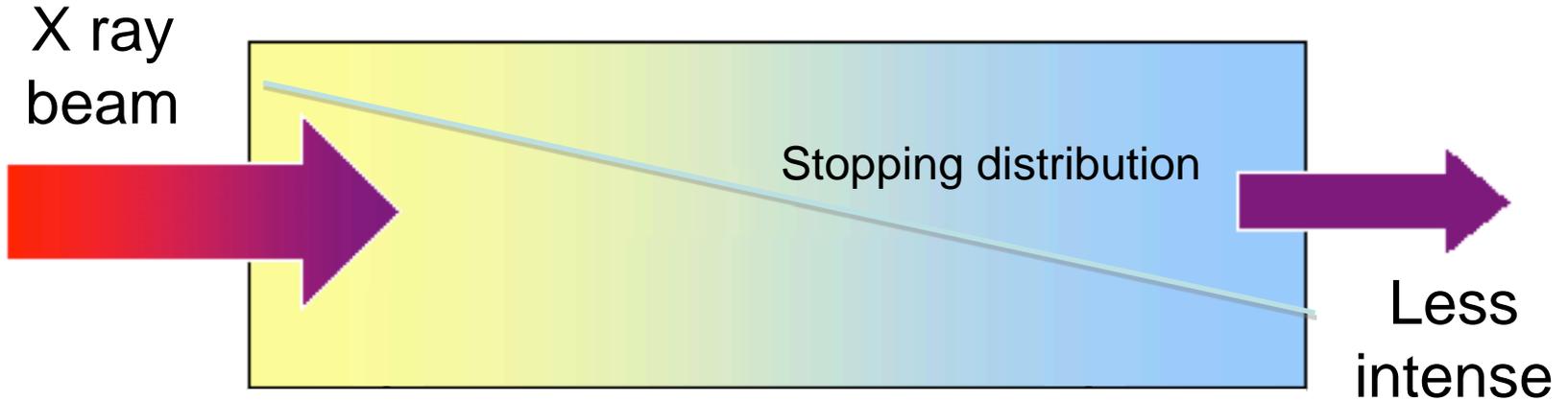


# Radon's theory

A value  $\rho(x_1, y_1)$  in a 2d function  $f(x, y)$  is uniquely determined from an integration of  $f(x, y)$  along arbitrary lines that cross  $(x_1, y_1)$

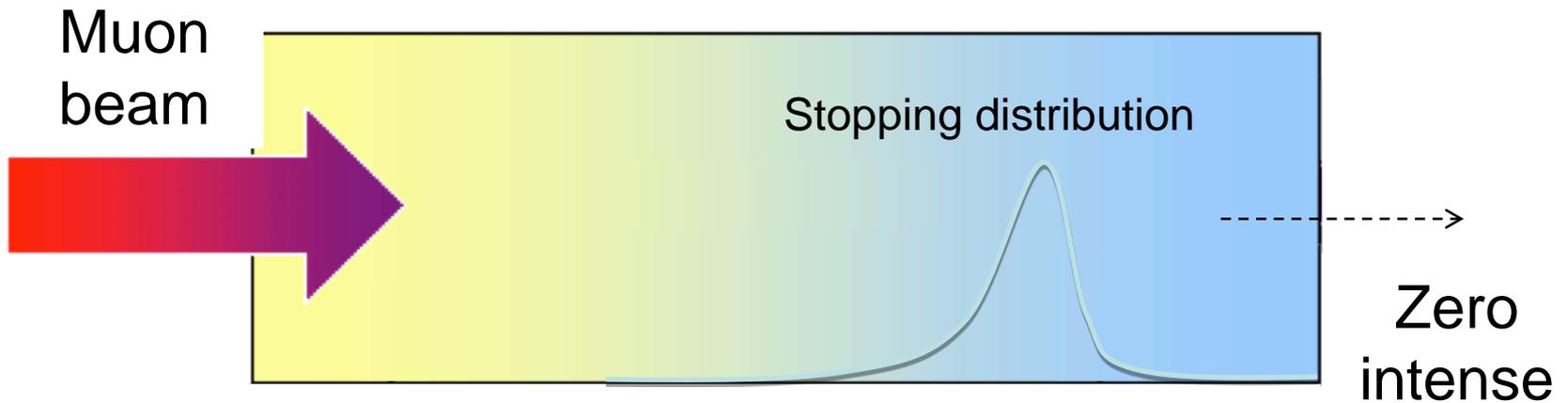


## X ray absorption

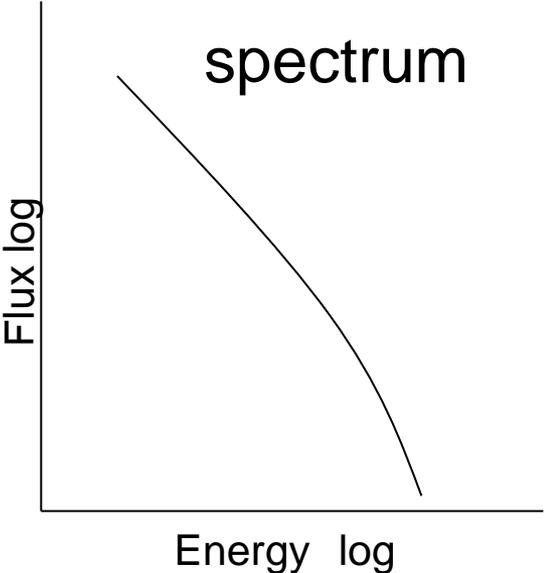


80 kV have a half value thickness of about 3 mm

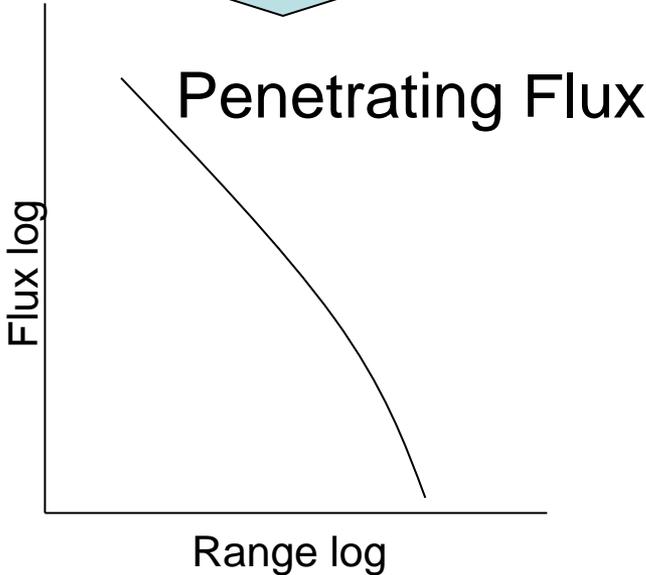
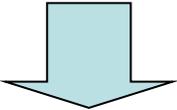
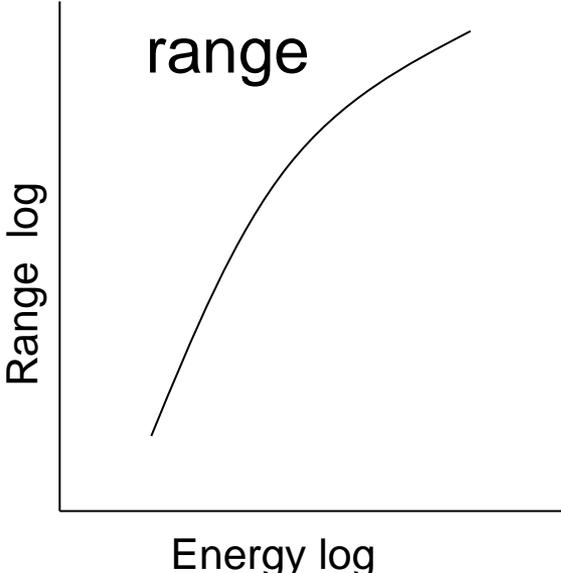
## Muon absorption



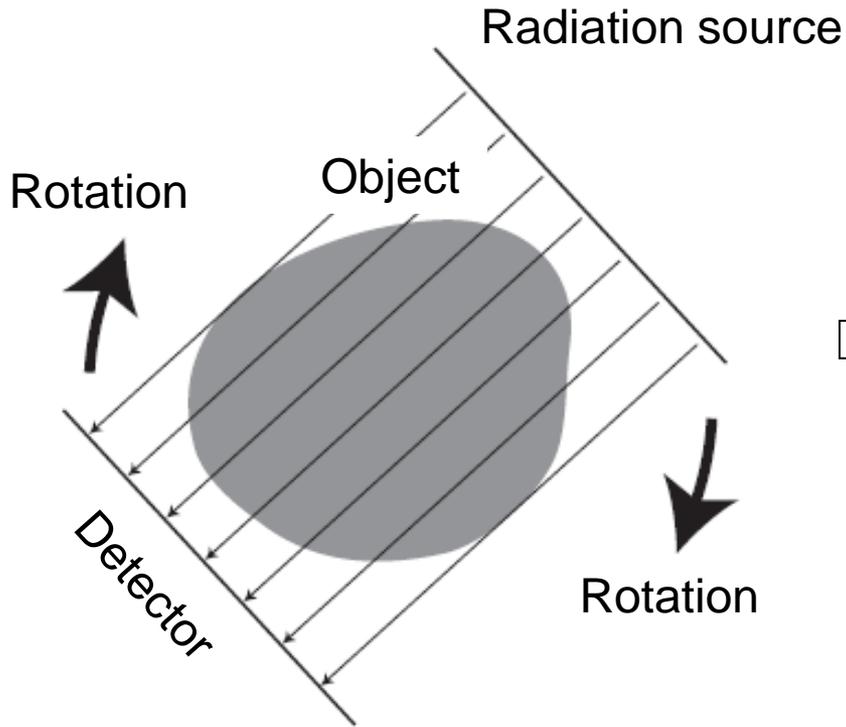
# Cosmic ray muons



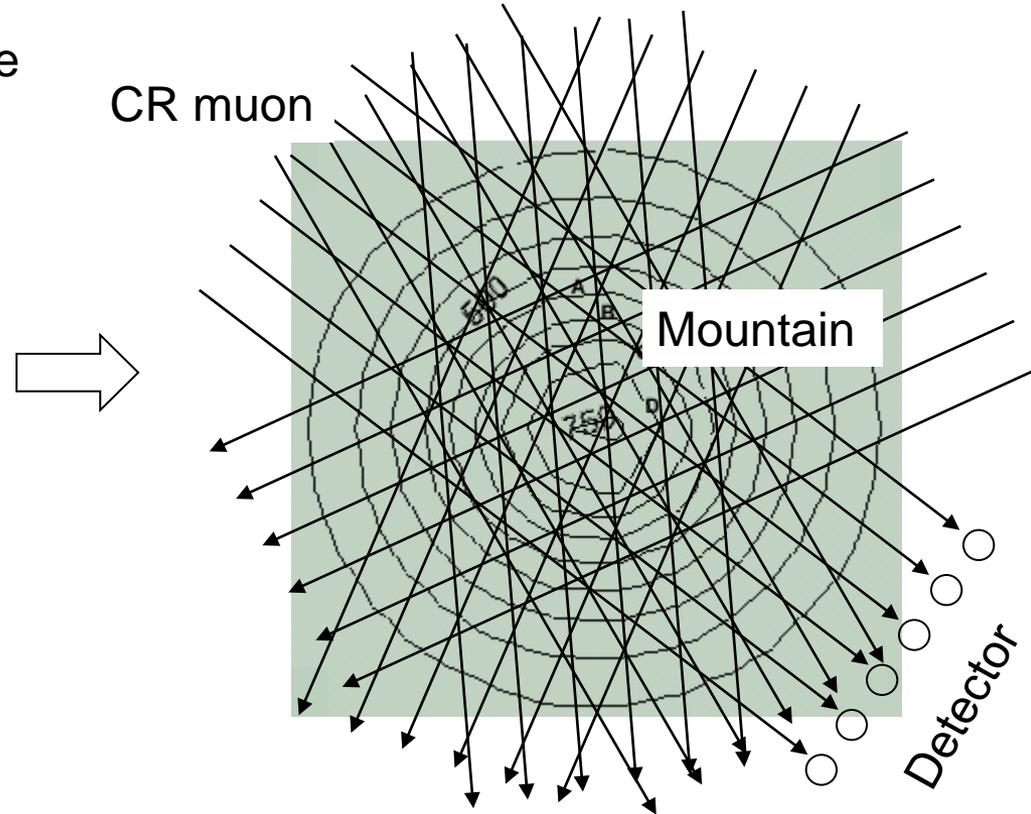
+



## X ray tomography



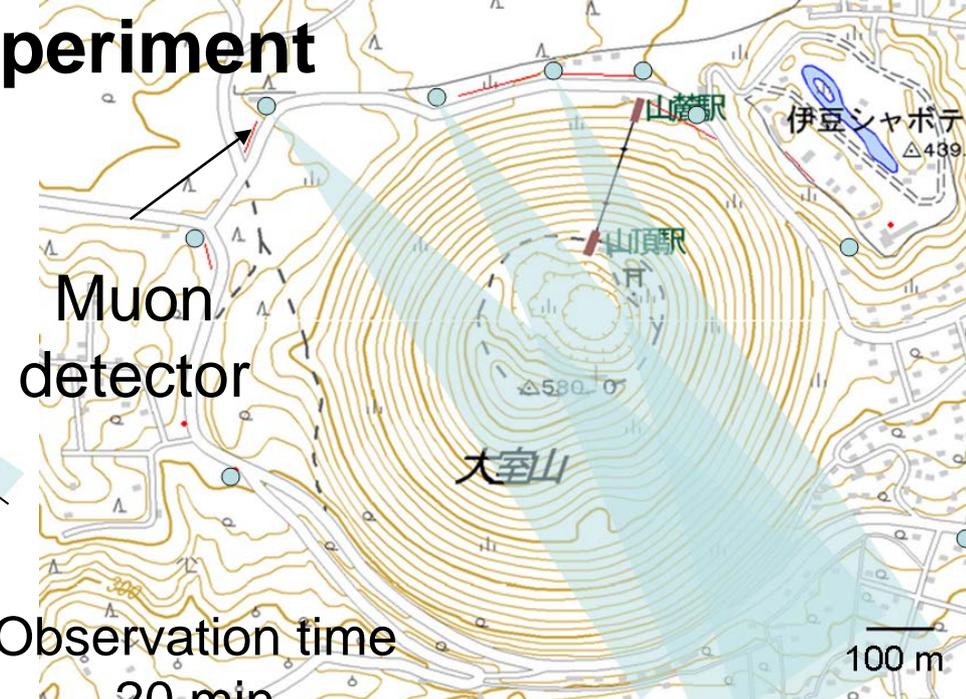
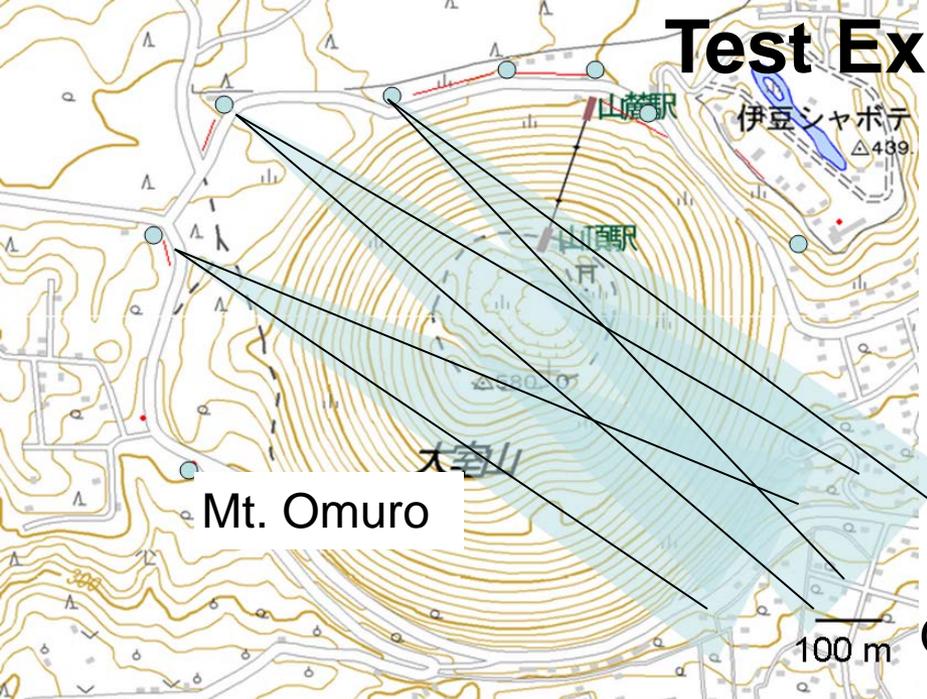
## Muon tomography



Exterior information is not necessary

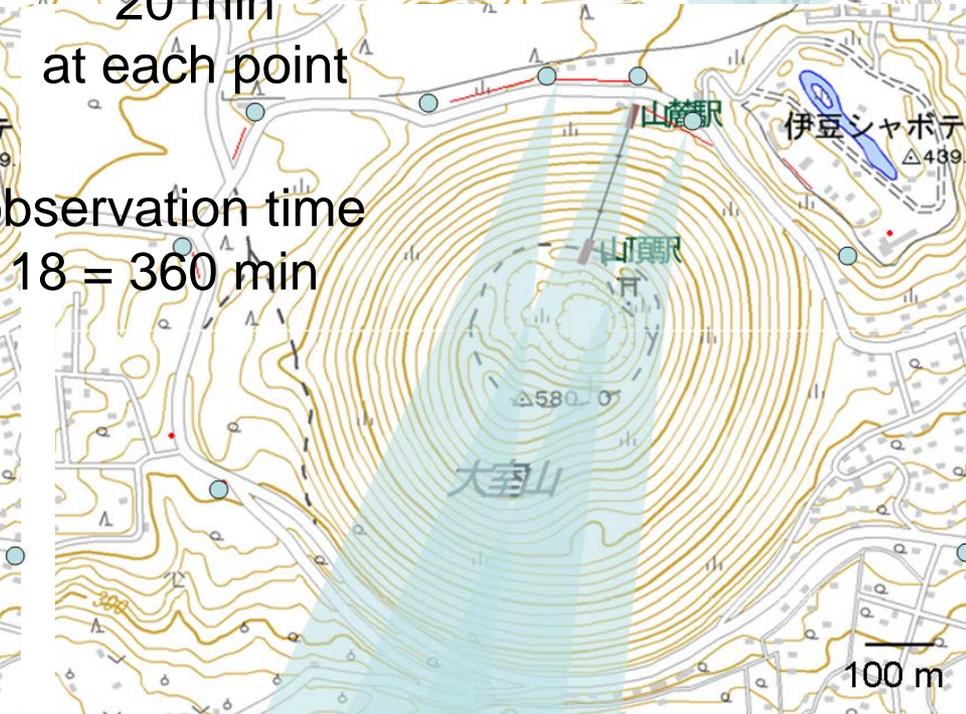
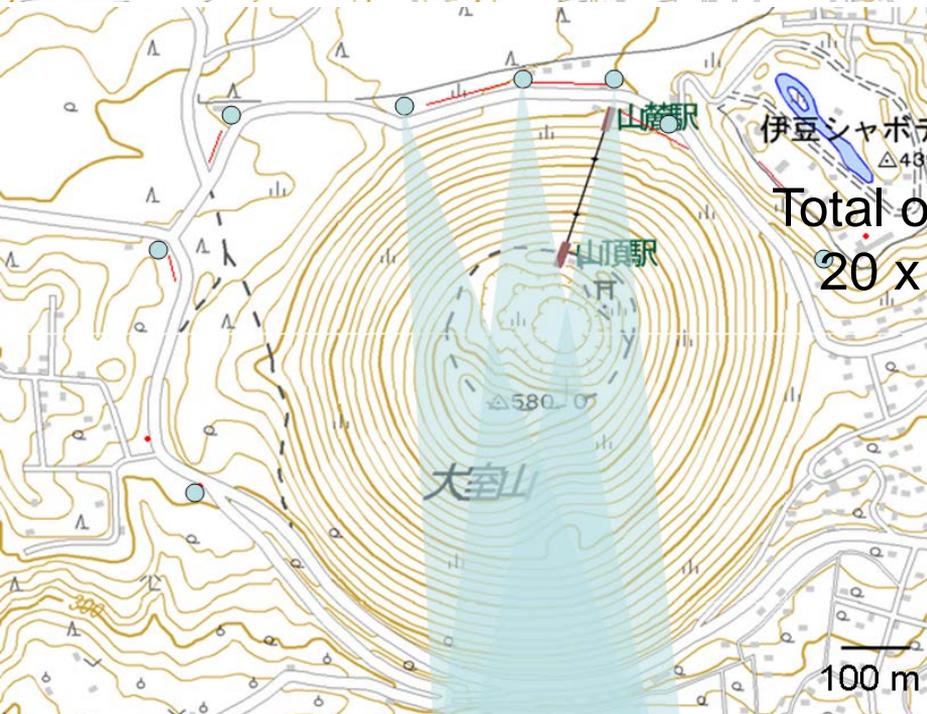


# Test Experiment



Muon detector

Observation time  
20 min  
at each point

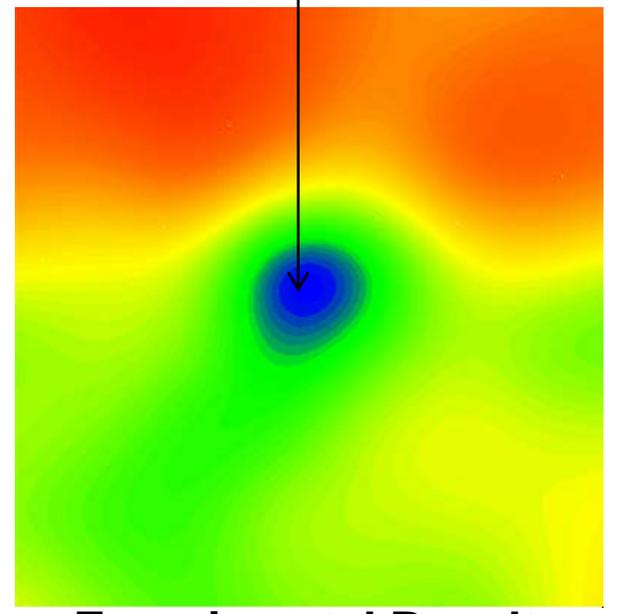
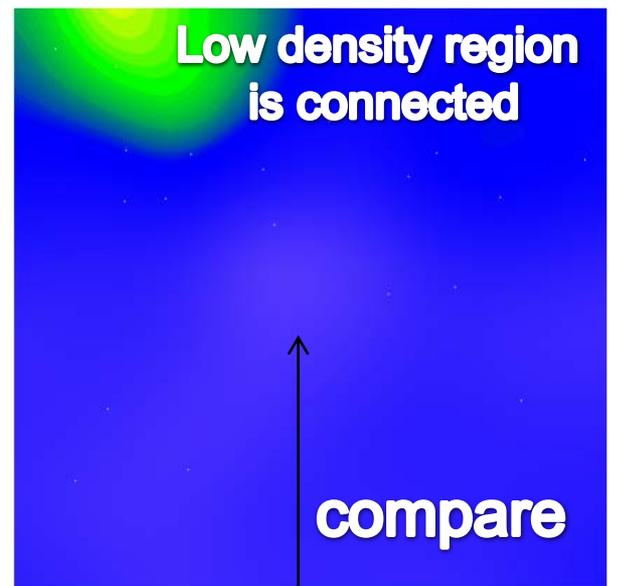
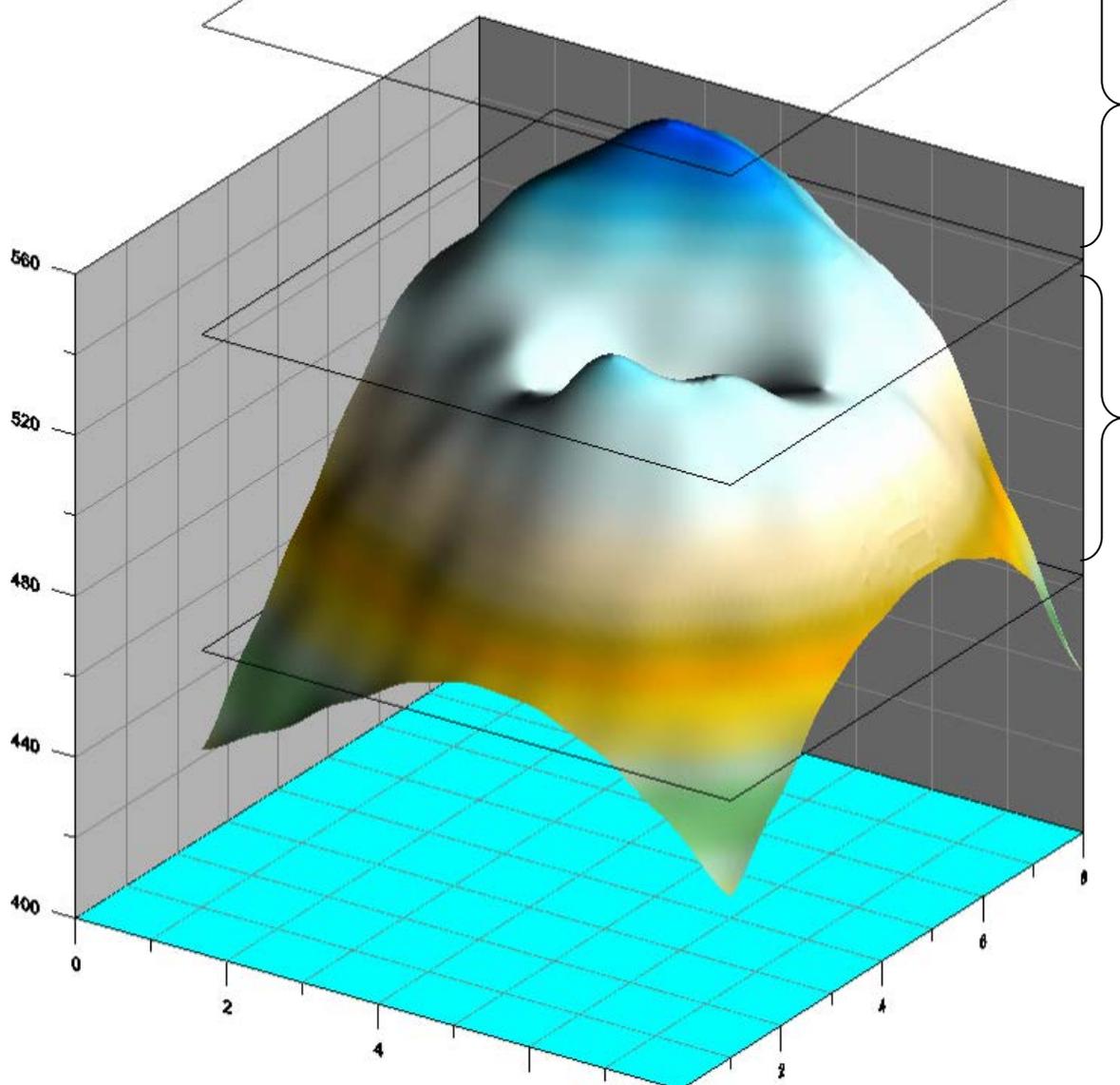


Total observation time  
 $20 \times 18 = 360$  min

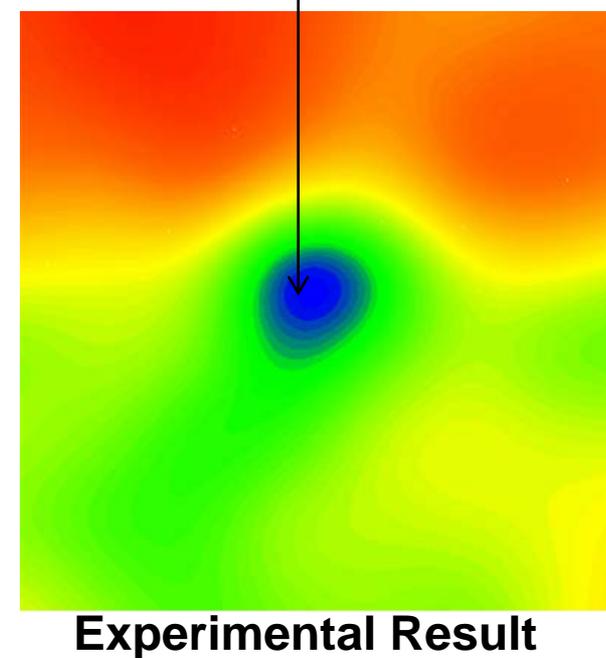
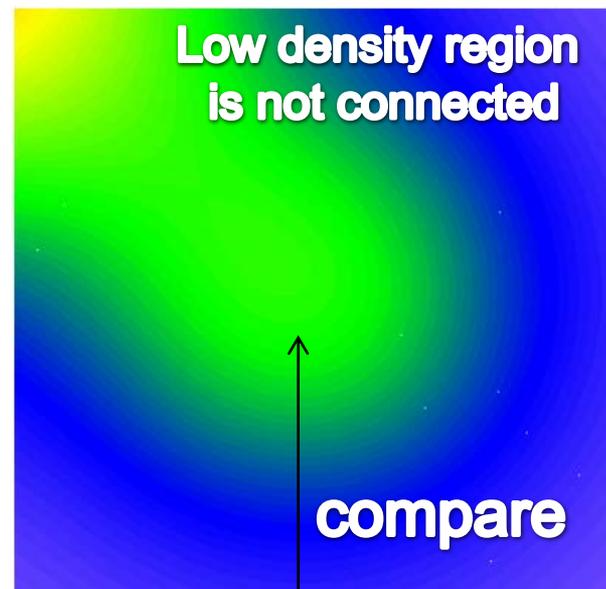
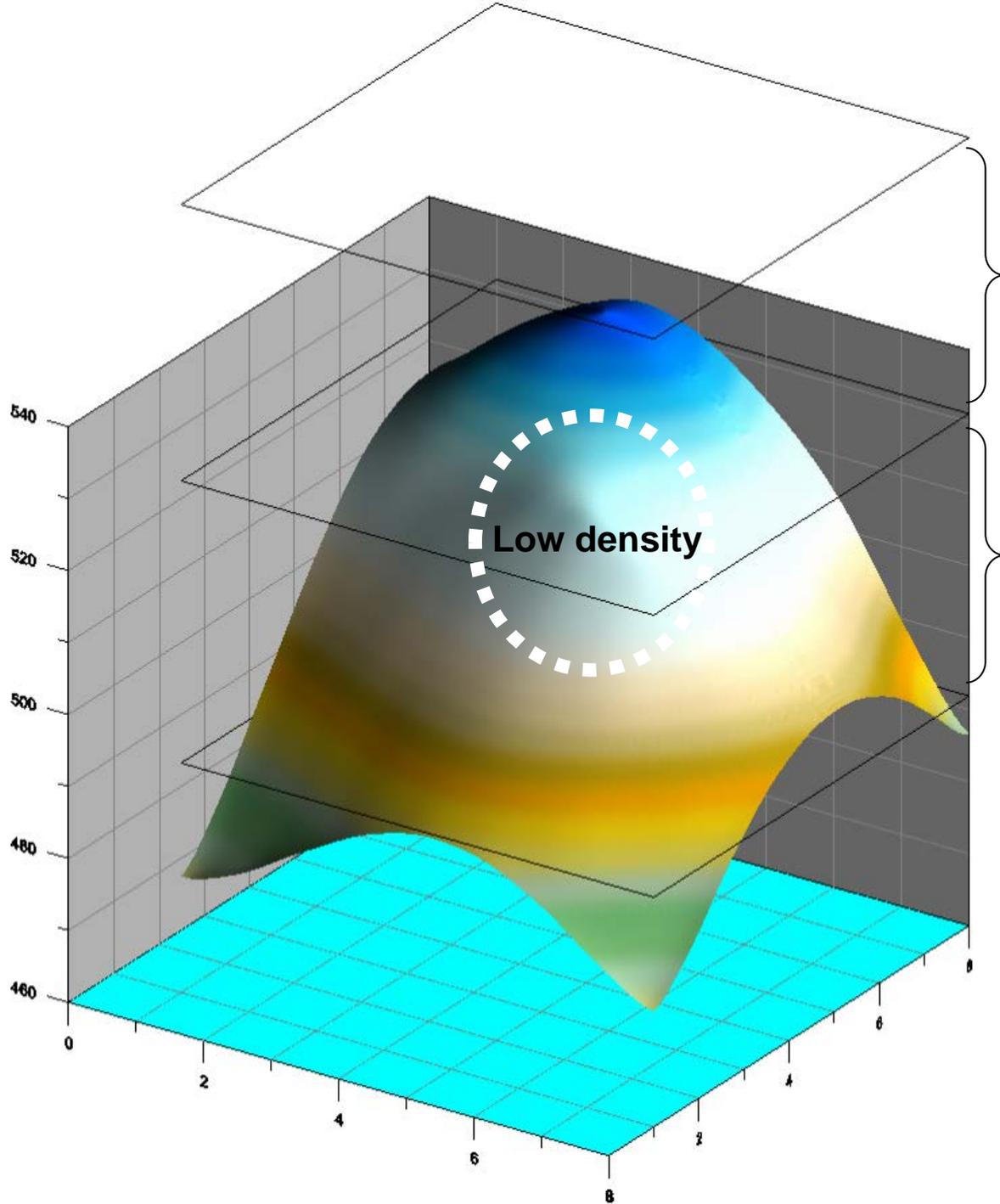


# Discussion

## Distinguish A from D



Making two slices and compare



# Conclusion

- Regular radon transform could be applied to the mobile muon spectrometry.
- Only 20 minute observation time is necessary for each point.