QCD

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LAPTH

Outline

- QCD as a field theory between quark and gluon

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- QCD as a field theory between quark and gluon
- Applications for proton-proton collider

Part 1

QCD as a field theory between quark and gluon

QCD Lagrangian : classical level

QCD : gauge theory based on the group SU(3)

$$\mathcal{L}_{QCD}(x) = -\frac{1}{4} F^a_{\mu\nu}(x) F_a^{\mu\nu}(x) + \bar{\psi}_j(x) \left(i \not{D}_{ji} - m \right) \psi_i(x)$$

with

$$D^\mu_{ij} = \partial^\mu \delta_{ij} + i\, g\, (T^c)_{ij}\, A^{c\,\mu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$$

Matter field $\psi_i(x) \rightarrow$ quarks; i : color index $i = 1, \cdots, 3$
Gauge field $A^a_\nu(x) \rightarrow$ gluons; a : color index $i = 1, \cdots 8$
$T^c \rightarrow$ 3x3 matrices : generators of the SU(3) group

$$[T^a, T^b] = i\, f^{abc}\, T^C$$

Gauge transformation : classical level

$$\psi_i'(x) \to U_{ij}\,\psi_j(x)$$

with $U = \exp(-i\,T^a\,\theta^a)$,
θ^a : parameters which may depend on x
$\theta^a \ll 1$, $U \simeq 1 - i\,T^a\,\theta^a$

$$A'^{\,a}_{\mu} \to A^a_\mu + f^{abc}\,\theta^b\,A^c_\mu - \frac{1}{g}\,\partial_\mu\,\theta^a$$

QCD Lagrangian : quantum level

$$\begin{aligned}\mathcal{L}_{QCD}(x) &= -\frac{1}{4}\,F^a_{\mu\nu}(x)F_a^{\mu\nu}(x) + \bar{\psi}_j(x)\,\left(i\not{D}_{ji} - m\right)\psi_i(x) \\ &\quad -\frac{1}{2\xi}\left(\partial_\mu A_a^\mu(x)\right)^2 + \partial_\mu \eta^{a\dagger}\left(D^\mu_{ab}\eta^b\right)\end{aligned}$$

with

$$D^\mu_{ab} = \partial^\mu\,\delta_{ab} + i\,g\,\left(F^c\right)_{ab}\,A^{c\,\mu}.$$

Whatever the method we used for quantization (canonical, functional integral), we have to eleminate the freedom of the gauge : covariant (Lorentz) gauge $\partial_\mu A_a^\mu(x) = 0$

QCD Lagrangian : quantum level

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$$-\frac{1}{2\xi} \left(\partial_\mu A^\mu_a(x)\right)^2 + \partial_\mu \eta^{a\dagger} \left(D^\mu_{ab} \eta^b\right)$$

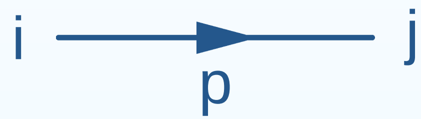
with

$$D^\mu_{ab} = \partial^\mu \delta_{ab} + i\, g\, (F^c)_{ab}\, A^{c\,\mu}.$$

Using Lorentz gauge, the S matrix elements do not verify the gauge invariance because internal gluons propagate non physical polarisation states $\longrightarrow$ extra particles ghost $\eta(x)$ whose role is to restore gauge invariance

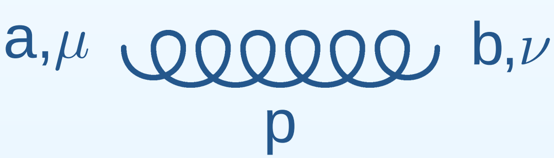
Feynman rules I

Quark propagator:



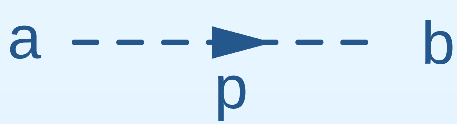
$$\frac{i\,\delta^{ij}}{\not p - m + i\,\lambda}$$

Gluon propagator:



$$\frac{-i\,\delta^{ab}}{p^2 + i\,\lambda}\left(g^{\mu\,\nu} - (1-\xi)\,\frac{p^\mu\,p^\nu}{p^2 + i\,\lambda}\right)$$

Ghost propagator:



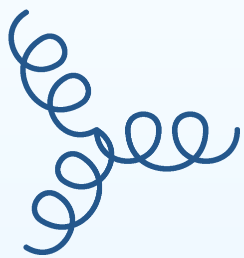
$$\frac{i\,\delta^{ab}}{p^2 + i\,\lambda}$$

Feynman rules II

Vertex gluon-gluon-gluon (**all momentum are incoming**)

b, β

q



c, γ

r

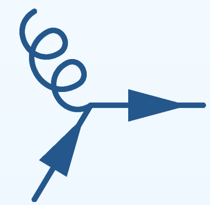
$$-g\, f^{abc}\,\left[g^{\alpha\,\beta}\,(p-q)^{\gamma} + g^{\beta\,\gamma}\,(q-r)^{\alpha} + g^{\gamma\,\alpha}\,(r-p)^{\beta}\right]$$

a, α

p

Vertex quark-quark-gluon

a,μ

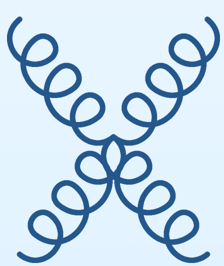


j $\quad -i\,g\,(T^{a})_{ji}\,\gamma^{\mu}$

i

Vertex gluon-gluon-gluon-gluon

a, α b, β

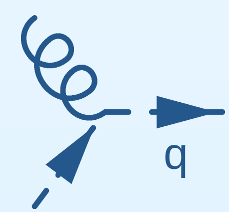


c, γ d, δ

$$\begin{aligned}&-ig^{2}\, f^{eac}\, f^{ebd}\,(g^{\alpha\,\beta}\, g^{\gamma\,\delta} - g^{\alpha\,\delta}\, g^{\beta\,\gamma})\\ &-ig^{2}\, f^{ead}\, f^{ebc}\,(g^{\alpha\,\beta}\, g^{\gamma\,\delta} - g^{\alpha\,\gamma}\, g^{\beta\,\delta})\\ &-ig^{2}\, f^{eab}\, f^{ecd}\,(g^{\alpha\,\gamma}\, g^{\beta\,\delta} - g^{\alpha\,\delta}\, g^{\beta\,\gamma})\end{aligned}$$

Vertex ghost-ghost-gluon

a,μ

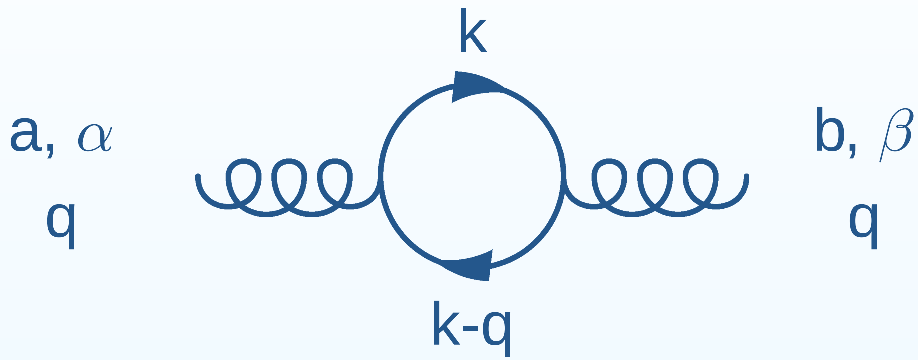


c $\quad g\,f^{abc}\,q^{\mu}$

q

b

Renormalisation I



$$\mathcal{P}^{(1)}_{\mu\nu}(q) \simeq \int \frac{d^4k}{(2\pi)^n} Tr\left[\gamma_\mu \frac{\not{k}}{(k^2 + i\lambda)} \gamma_\nu \frac{(\not{k} - \not{q})}{((k-q)^2 + i\lambda)}\right]$$

$$\int d^4k \, \frac{k^\mu \, k^\nu}{k^4} \sim \int_o^\infty d\bar{k} \, \bar{k} \to \infty$$

Renormalisation II

What does that mean ?

We are getting a contribution from intermediate states involving $q\ \bar{q}$ pairs but the energy of these intermediate states is arbitrarily high! We have no idea what the interaction of gluons with arbitrarily high momentum quarks is, this contribution is silly.

We made the assumption, at the very beginning, that the $q - g$ interaction is point-like. But we cannot test at such high energies that the interaction $q - g$ is like that

Renormalisation III

What can we do ?

Renormalisation III

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Firstly, give a meaning to the expression of $\mathcal{P}^{(1)}_{\mu\nu}(q)$ by regularising the integral.

- cut-off $\int_0^\Lambda d\bar{k}\, f(\bar{k}) = F(\Lambda) \longrightarrow$ breaks some symetries of the Lagragian
- dimensional regularisation $d^4k \longrightarrow d^nk$ or $\bar{k}^3\, d\bar{k} \longrightarrow \bar{k}^{n-1}\, d\bar{k} \longrightarrow F(\epsilon)$ with $n = 4 - 2\,\epsilon$

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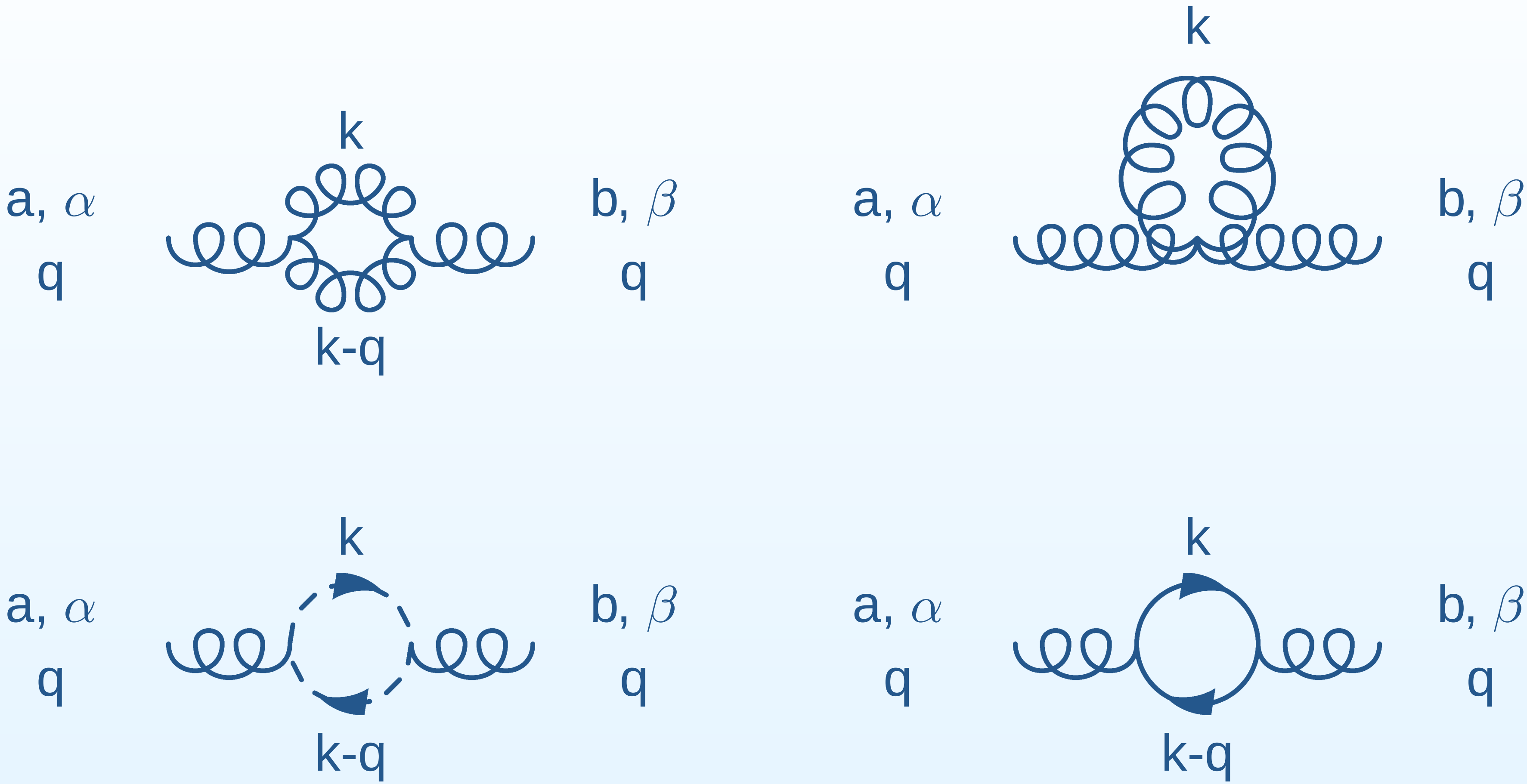
Secondly, change the normalisation of the fields and the vertices (and the masses) of the Lagrangian (they are free .....) in such a way that the regulator dependence is absorbed by this normalisation

Example


k
a, α
b, β
q
q
k-q
k
a, α
b, β
q
q
k
a, α
b, β
q
q
k-q
k
a, α
b, β
q
q
k-q

Example II

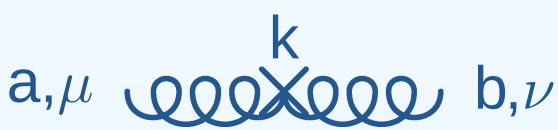
$$
\begin{aligned}
\mathcal{P}^{(1)\,gg}_{\mu\nu}(q) &= \frac{1}{\epsilon} N\,\delta^{ab}\,K(\epsilon)\left[q^2\,g^{\mu\,\nu}\left(\frac{19}{12}+\frac{\epsilon}{18}\right)-q^{\mu}\,q^{\nu}\left(\frac{11}{6}+\frac{\epsilon}{18}\right)\right] \\
\mathcal{P}^{(1)\,ggg}_{\mu\nu}(q) &= 0 \\
\mathcal{P}^{(1)\,GG}_{\mu\nu}(q) &= \frac{1}{\epsilon} N\,\delta^{ab}\,K(\epsilon)\left[q^2\,g^{\mu\,\nu}\left(\frac{1}{12}+\frac{\epsilon}{18}\right)-q^{\mu}\,q^{\nu}\left(-\frac{1}{6}+\frac{\epsilon}{18}\right)\right] \\
\mathcal{P}^{(1)\,qq}_{\mu\nu}(q) &= -\frac{1}{\epsilon} T_F\,\delta^{ab}\,K(\epsilon)\left[q^2\,g^{\mu\,\nu}\left(\frac{4}{3}-\frac{4\,\epsilon}{9}\right)-q^{\mu}\,q^{\nu}\left(\frac{4}{3}-\frac{4\,\epsilon}{9}\right)\right]
\end{aligned}
$$

The ghost are necessary to have the transversality : $q^{\mu}\,q^{\nu}\mathcal{P}^{(1)}_{\mu\nu}(q) = 0$ as expected because the gluon is a massless spin 1 particle

Example III

arbitrarily normalisation for $A^{\mu}(x)$: Z_3
new term in the Lagrangian

$$-\frac{1}{4}\,(Z_3^{(1)}-1)\,\delta^{ab}\,(\partial_\mu\,A^a_\nu-\partial_\nu\,A^a_\mu)\,(\partial_\mu\,A^b_\nu-\partial_\nu\,A^b_\mu)$$



$$-\,i\,(Z_3^{(1)}-1)\,\delta^{ab}\left(k^2\,g^{\alpha\beta}-k^{\alpha}k^{\beta}\right)$$

$$i\mathcal{P}^{(1)\;tot}_{\alpha\beta,ab}\longrightarrow i\,\mathcal{P}^{(1)\;tot}_{\alpha\beta,ab}-i\,(Z_3^{(1)}-1)\,\delta^{ab}\left(q^2\,g^{\alpha\beta}-q^{\alpha}q^{\beta}\right)$$

In the $\overline{MS}$ scheme:

$$(Z_3^{(1)}-1)=\frac{\alpha_s}{4\,\pi}\left(\frac{1}{\varepsilon_{uv}}+\ln(4\,\pi)-\gamma\right)\left(N\,\frac{5}{3}-T_F\,\frac{4}{3}\right)$$

Renormalisation IV

This procedure works at all orders in α_s for QCD

$$
\begin{array}{lcllcl}
Z_3 & : & A(x) & Z_1^F & : & \psi(x)\,\bar\psi(x)\,A(x) \\
Z_2 & : & \psi(x) & Z_1 & : & A(x)\,A(x)\,A(x) \\
\tilde Z_3 & : & \eta(x) & \tilde Z_1 & : & A(x)\,\eta(x)\,\eta(x) \\
 & & & Z_4 & : & A(x)\,A(x)\,A(x)\,A(x)
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of course not, we can absorb with the regulator dependence some finite terms. They define the renormalisation scheme

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of course not, we can absorb with the regulator dependence some finite terms. They define the renormalisation scheme
A new energy scale μ has now appeared.

$$
\begin{array}{ll}
[\mathcal{L}] = 4 & [g] = 0 \\
[\mathcal{L}] = n & [g] = 2 - \frac{n}{2}
\end{array}
$$

$g \longrightarrow \tilde{g}\,\mu^{2-n/2}$ with $[\tilde{g}] = 0$ and $[\mu] = 1$

Renormalisation V

Whatever the way we proceed, after the steps regularisation and renormalisation, a energy scale appear in the Lagrangian

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The physical observables (S matrix elements) must not depend on the choice of this scale μ or on the renormalisation scheme.

They obey to differential equations (renomalisation group equation). Fulfilling these equations amounts to the same thing than introducing an effective coupling constant

Running coupling constant I

This effective coupling constant obeys to the following equation:

$$\frac{d\,\alpha_s(t)}{dt} = \beta(\alpha_s(t)) \quad \text{with} \quad t = \ln(\mu^2/\mu_0^2) \quad \text{and} \quad \alpha_s = \frac{g^2}{4\,\pi}$$

and

$$\beta(\alpha_s(t)) = -b\,\alpha_s(t)^2\,(1 + b'\,\alpha_s(t) + \cdots) \quad \text{with} \quad b = \frac{11\,N - 2\,N_F}{12\,\pi}$$

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Solving this equation (keeping the first term for β function):

$$t = \int_{\alpha_s(0)}^{\alpha_s(t)} \frac{dx}{\beta(x)} = \frac{1}{b}\left(\frac{1}{\alpha_s(t)} - \frac{1}{\alpha_s(0)}\right)$$

$$\alpha_s(t) = \frac{\alpha_s(0)}{1 + b\,t\,\alpha_s(0)}$$

Running coupling constant II

$b > 0$ if $N_F \leq 16 \rightarrow d\,\alpha_s(t)/dt \leq 0$. So $\alpha_s(t) \searrow$ when $t \nearrow$

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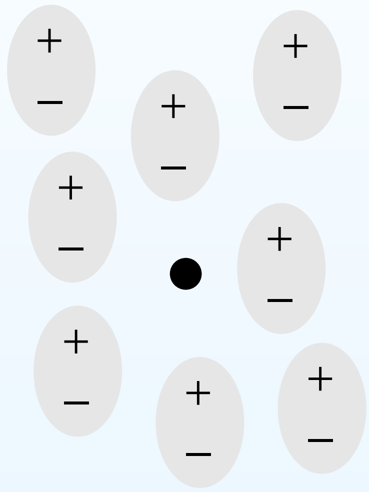

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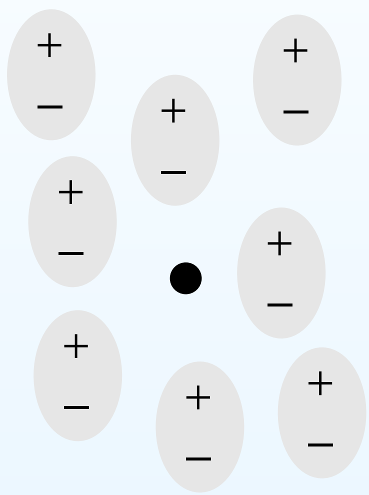
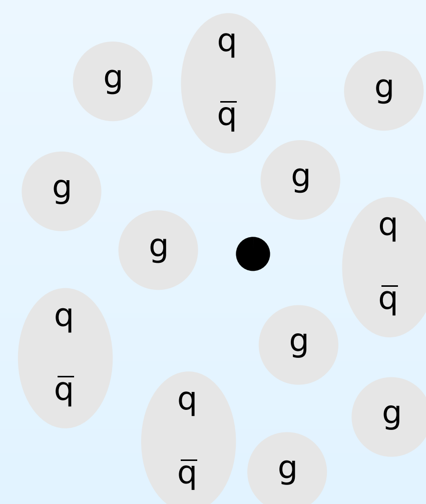


In QED, $b < 0$ screening effect : an electric charge is screened by the virtual $\pm$ charge in the vacuum

In QCD, $b > 0$ anti-screening effect : an color charge is screened by the virtual $q\,\bar{q}$ but anti-screened by the g in the vacuum



Running coupling constant III

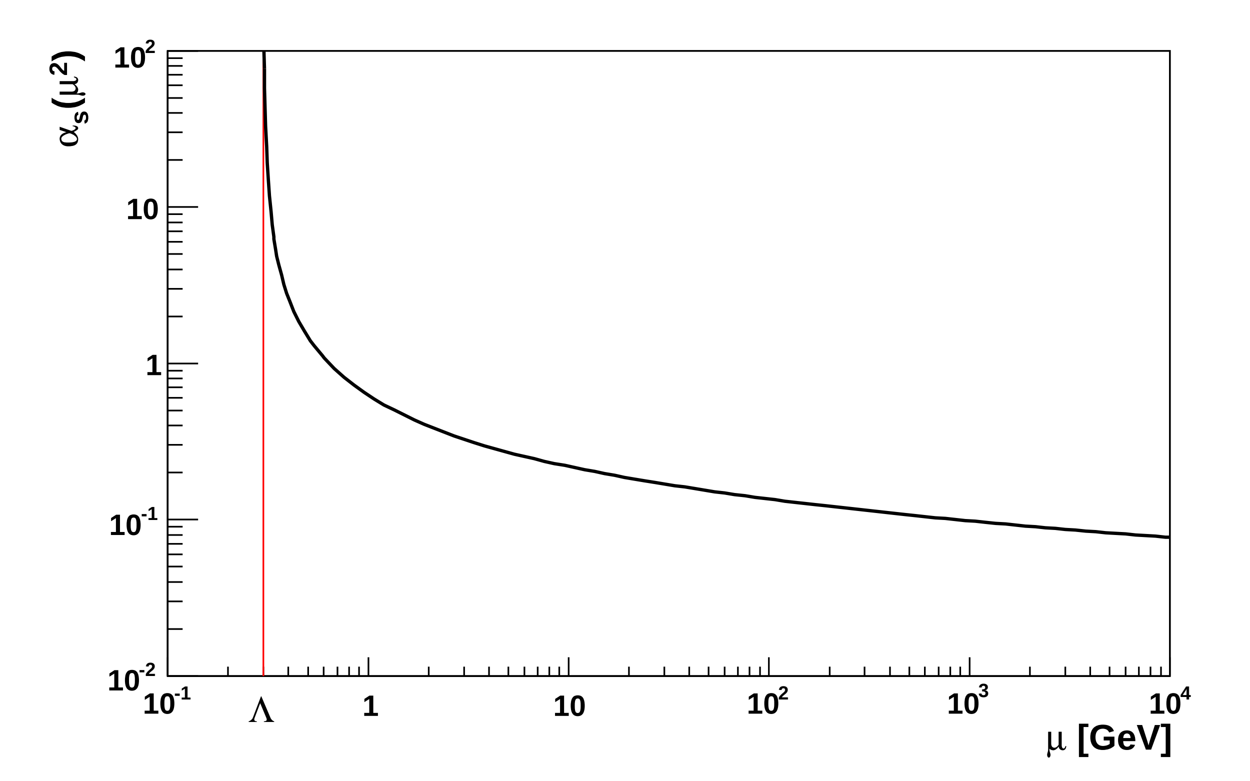
we define a parameter Λ such that:

$$\ln\left(\frac{\mu^2}{\Lambda^2}\right) = -\int_{\infty}^{\alpha_s(\mu^2)} \frac{dx}{\beta(x)}$$

$$\alpha_s(\mu^2) = \frac{1}{b\,\ln\left(\frac{\mu^2}{\Lambda^2}\right)} \quad \rightarrow \quad \mu^2 = \Lambda^2 \; \alpha_s(\Lambda^2) = \infty$$

Λ : a scale which separate perturbative and non perturbative regime (Λ depends on the renormalisation scale)

Running coupling constant IV



$$
\begin{aligned}
R &= K\,\alpha_s(Q^2) \\
&= K\,\alpha_s(\mu^2)\,\left[1 - \alpha_s(\mu^2)\,b\,t + \alpha_s^2(\mu^2)\,(b\,t)^2 + \cdots\right]
\end{aligned}
$$

Part 2

Applications for proton-proton collider

Parton model I

How to relate partonic cross section to hadronic cross section?

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Proton : granular structure
the granules (partons) are:

- hard point like
- almost free (but nevertheless confined)

partons = quarks and gluons

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The parton model (Feynman)

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When a sufficiently high momentum transfer reaction takes place, the projectile, be it a lepton or a parton inside a hadron, sees the target as made up of almost free constituents and is scattered by a single, free, effectively massless constituent.

Parton model II

$$\sigma_{PP\to X} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1\, dx_2\, F_i^P(x_1, M^2)\, F_j^P(x_2, M^2)\, \hat{\sigma}_{ij\to X}(\mu^2, M^2)$$

$$\frac{dF_i^P(x, M^2)}{d\ln(M^2)} = \frac{\alpha_s(M^2)}{2\,\pi} \sum_j \int_0^1 \frac{dy}{y}\, P_{ij}\left(\frac{x}{y}, \alpha_s(M^2)\right) F_i^P(y, M^2)$$

$$\begin{aligned} P_{ij}\left(x, \alpha_s(M^2)\right) &= P_{ij}^{(0)}(x) + \frac{\alpha_s(M^2)}{2\,\pi} P_{ij}^{(1)}(x) + \cdots \\ \hat{\sigma}_{ij} &= \hat{\sigma}_{ij}^{(0)} + \frac{\alpha_s(M^2)}{2\,\pi} \hat{\sigma}_{ij}^{(1)} + \cdots \end{aligned}$$

Partonic densities $F_i^P(x, M^2)$: number of parton i inside the proton P carrying a fraction of momentum x of its parent proton at the scale M^2

PDF I

The evolution equation is a matricial equation:

$$\frac{d}{dt}\begin{pmatrix} F_q^P(x,t) \\ F_g^P(x,t) \end{pmatrix} = \frac{\alpha_s(t)}{2\,\pi}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}^{(0)}(y) & P_{qg}^{(0)}(y) \\ P_{gq}^{(0)}(y) & P_{gg}^{(0)}(y) \end{pmatrix}\begin{pmatrix} F_q^P(x/y,t) \\ F_g^P(x/y,t) \end{pmatrix}$$

which can be written as

$$\frac{d}{dt}V(t) = \frac{\alpha_s(t)}{2\,\pi}\Gamma\otimes V(t) \qquad t = \ln\left(M^2/M_0^2\right)$$

with

$$V(t) = \begin{pmatrix} F_q^P(x,t) \\ F_g^P(x,t) \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} P_{qq}^{(0)}(y) & P_{qg}^{(0)}(y) \\ P_{gq}^{(0)}(y) & P_{gg}^{(0)}(y) \end{pmatrix}$$

PDF II

$$\begin{aligned}
P_{qq}^{(0)}(y) &= C_F\left[\frac{1+y^2}{(1-y)_+}+\frac{3}{2}\,\delta(1-y)\right]\\
P_{qg}^{(0)}(y) &= \frac{N_F}{2}\,\frac{y^2+(1-y)^2}{y}\\
P_{gq}^{(0)}(y) &= C_F\left[\frac{1+(1-y)^2}{y}\right]\\
P_{gg}^{(0)}(y) &= 2\,N\left[\frac{1}{(1-y)_+}+\frac{1-y}{y}+y\,(1-y)\right]+\delta(1-y)\,\frac{b}{2\,\pi}
\end{aligned}$$

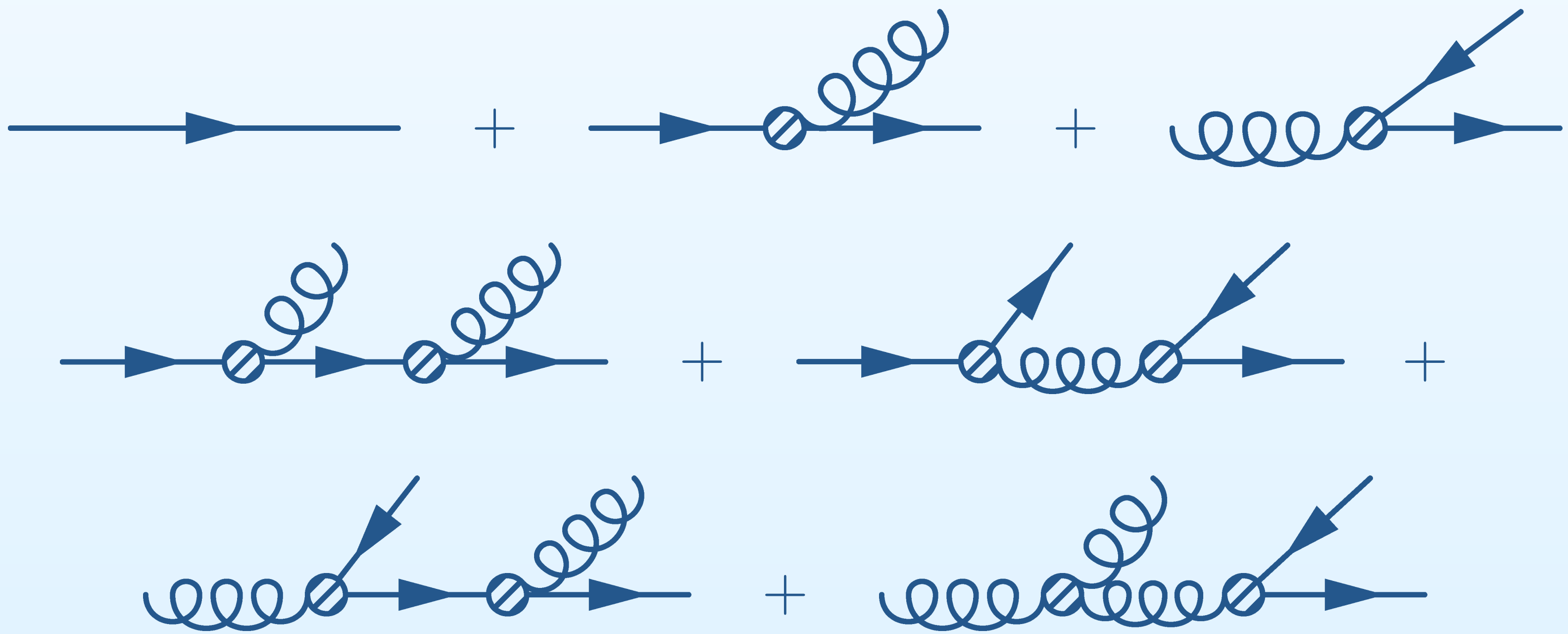
with $C_F = 4/3$ and

$$\begin{aligned}
\int_0^1 dx\,\frac{f(x)}{(1-x)_+} &= \int_0^1 dx\,\frac{f(x)-f(1)}{1-x}\\
\int_0^1 dx\,f(x)\,\delta(1-x) &= f(1)
\end{aligned}$$

PDF III

expanding in α_s the solution:

$$
\begin{aligned}
V(t) \quad &= \quad V(0) + \frac{\alpha_s(0)}{2\,\pi}\, t\,\Gamma \otimes V(0) \\
&\quad + \left(\frac{\alpha_s(0)}{2\,\pi}\right)^2 t^2 \left[\frac{1}{2}\,\Gamma \otimes \Gamma - \pi\, b\, \Gamma\right] \otimes V(0) + \cdots
\end{aligned}
$$



PDF IV

How to extract the non perturbative part $F_i^P(x, M_0^2)$?

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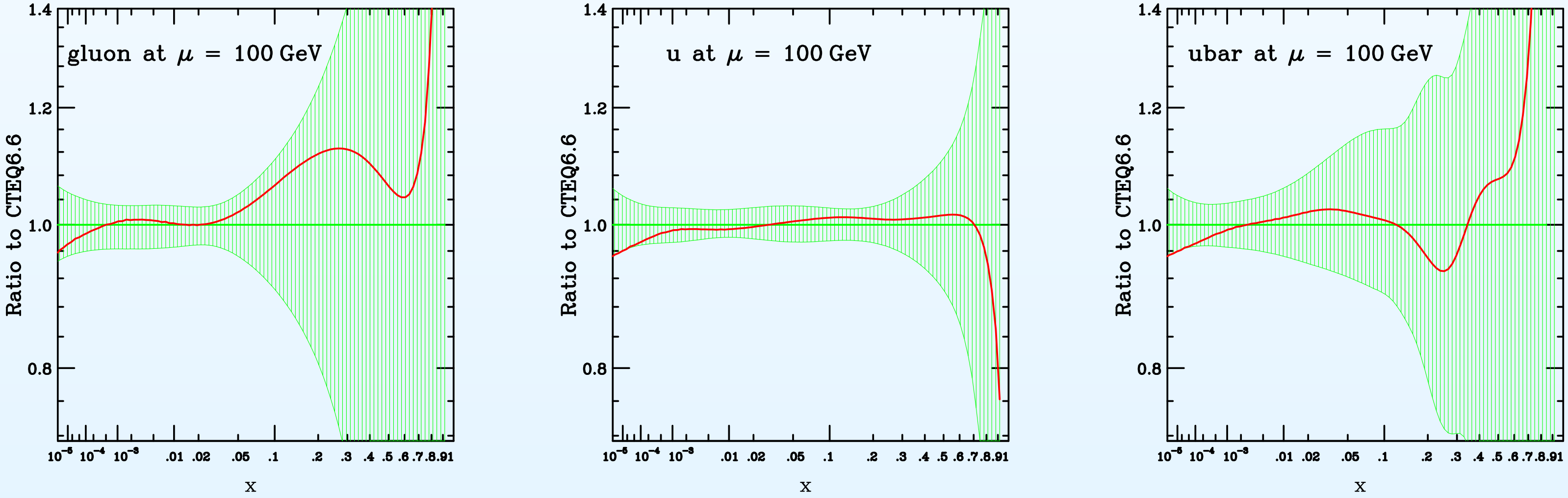
Fits to different type of cross section : DIS ($e^- + P \to e^- + X$), Drell-Yann ($P + P(\bar{P}) \to l + \bar{l} + X$), W/Z, jets, ...
http://hep.pa.msu.edu/cteq/public/cteq6.html



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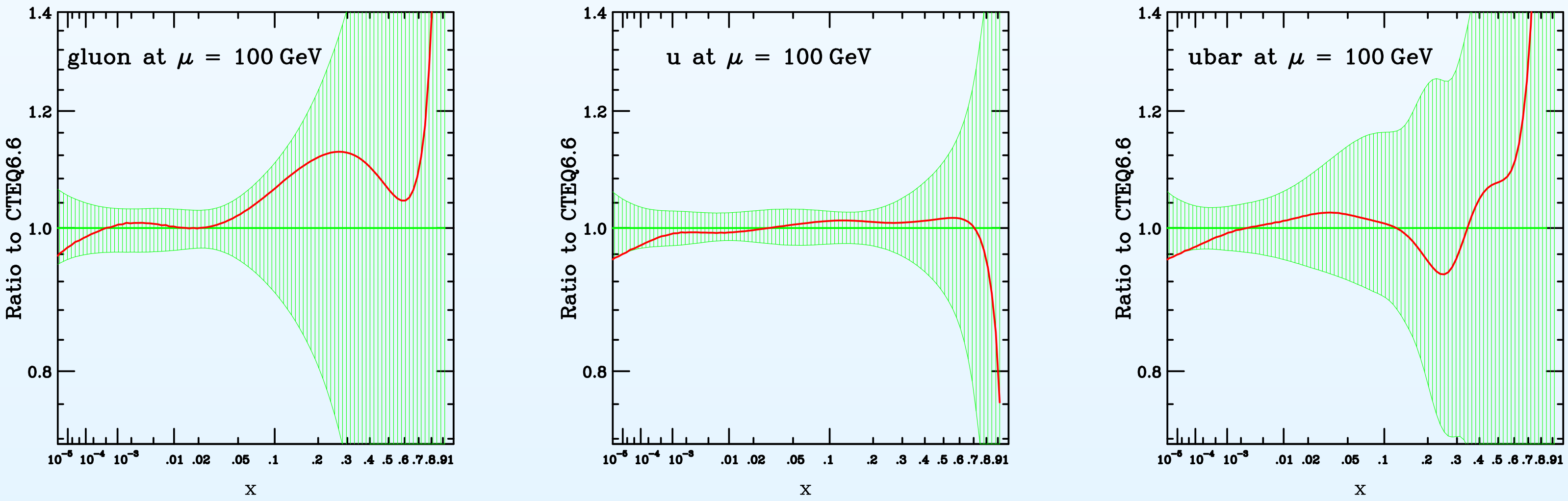
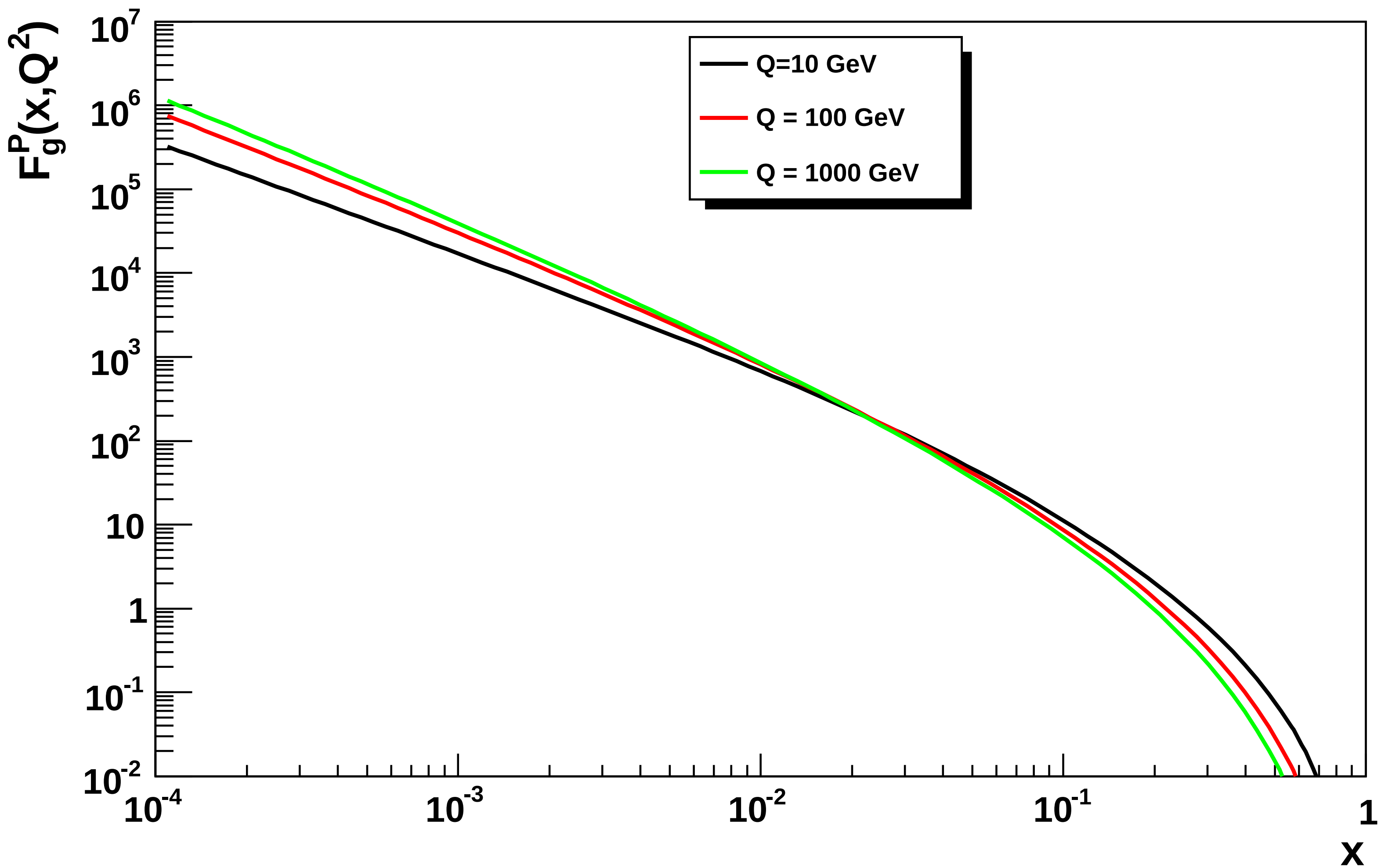


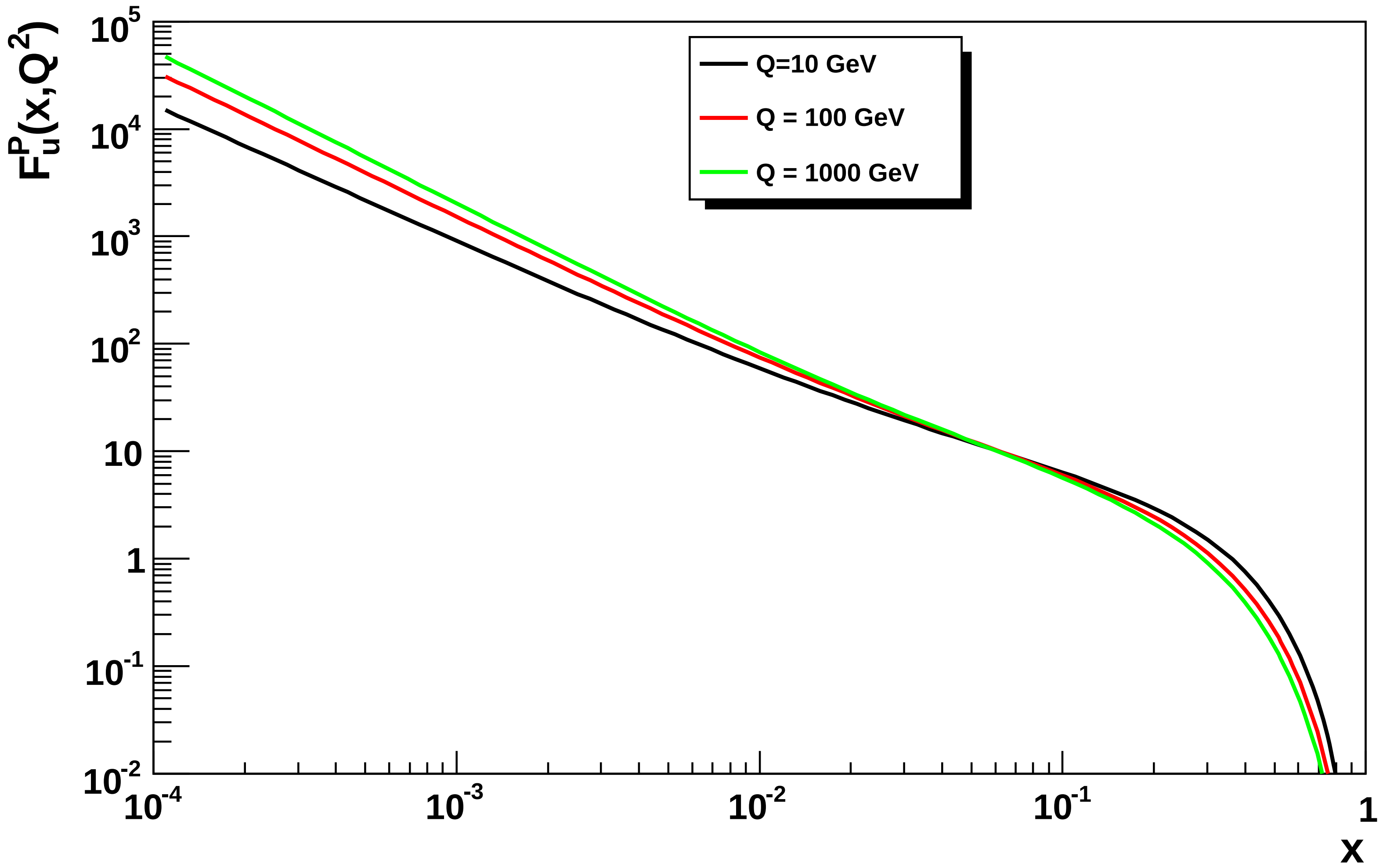
Data base: Cteq, MRS, ...
http://projects.hepforge.org/lhapdf

PDF V



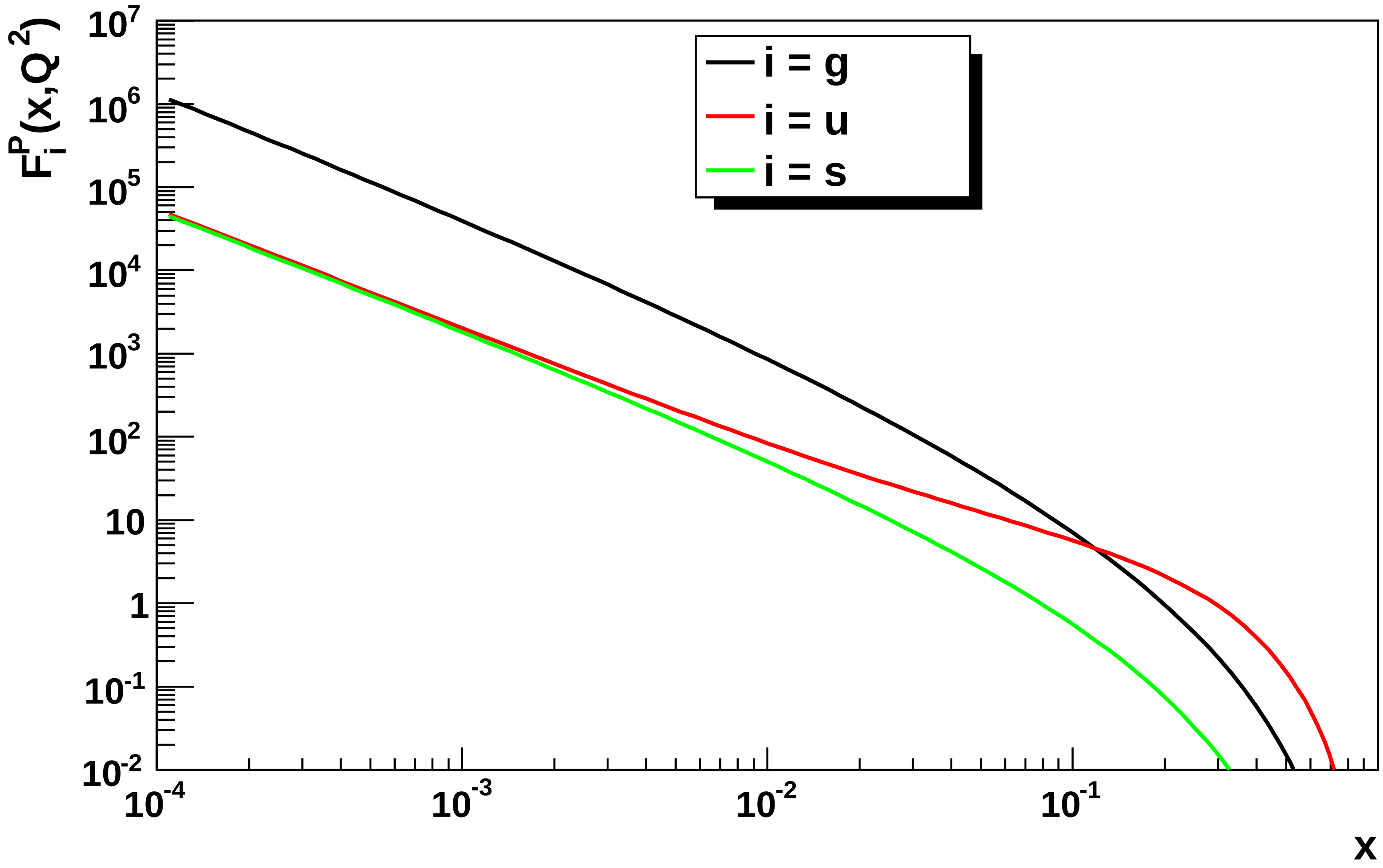
$F_g^P(x,Q^2)$
10^7
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
Q=10 GeV
Q = 100 GeV
Q = 1000 GeV

PDF V



$F_u^P(x,Q^2)$
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
Q=10 GeV
Q = 100 GeV
Q = 1000 GeV

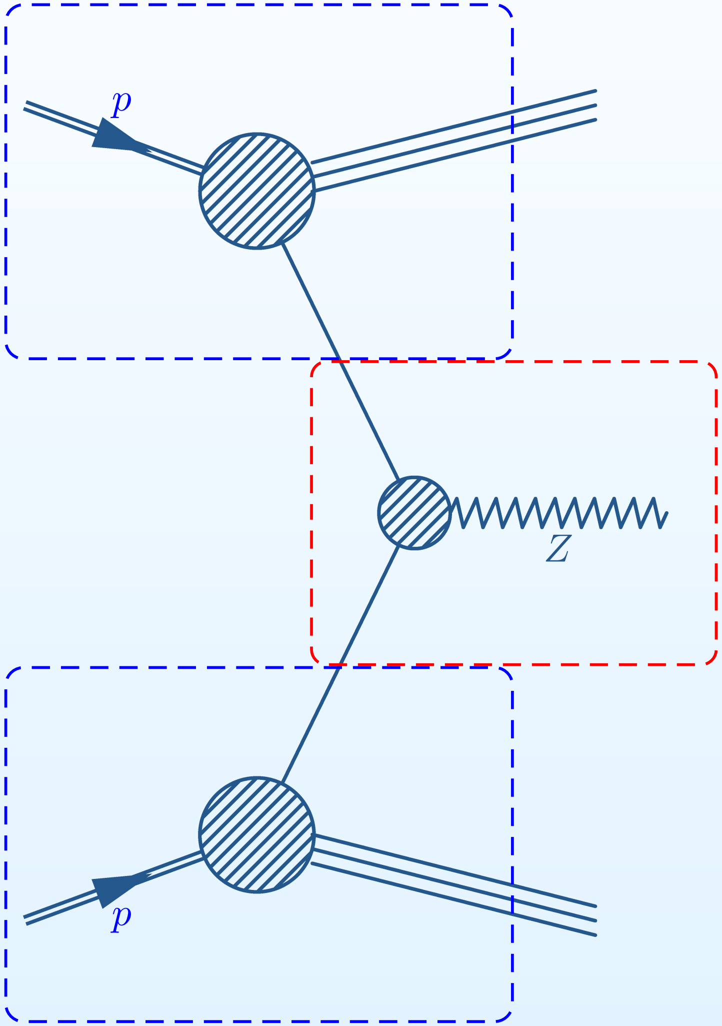
PDF V



$F_i^P(x,Q^2)$
10^7
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x
i = g
i = u
i = s

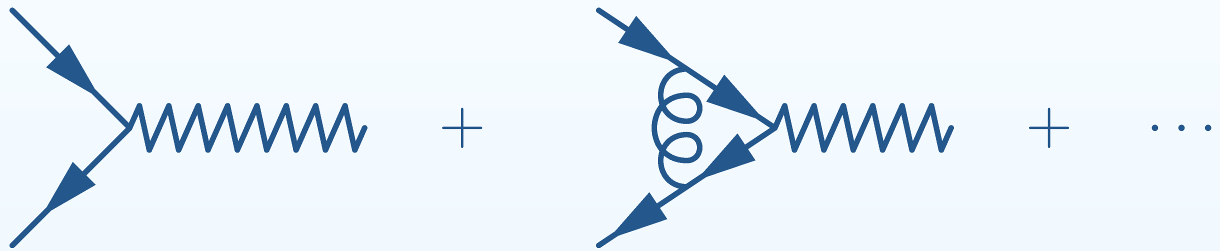
Z production

Inclusive reaction:
$PP \rightarrow Z + X$
The typical hard scale is M_Z
The pdf are evolved at the scale $M = M_Z$

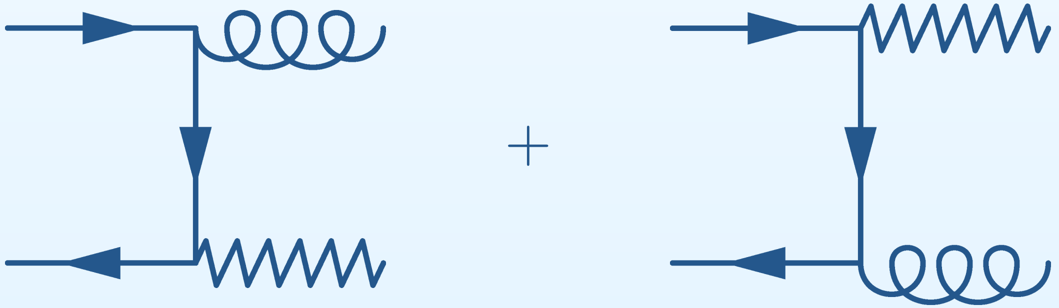


Z production II

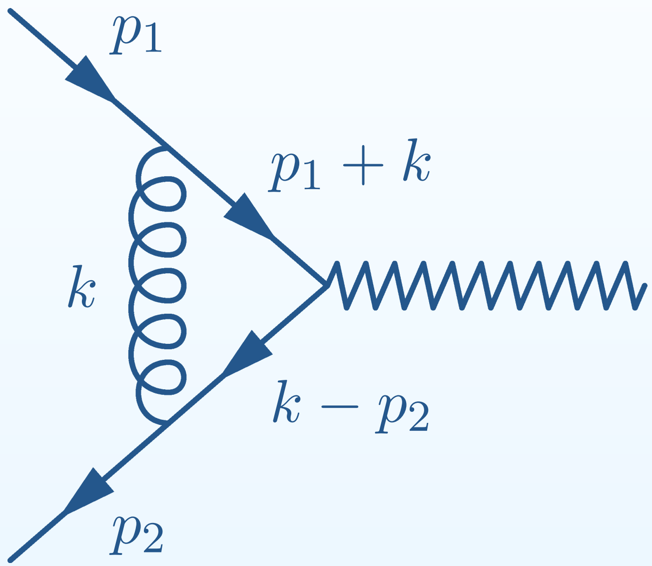
Order α_s corrections : one-loop diagrams to $q\bar{q} \longrightarrow Z$ (order α_s^2 but interference term)



tree diagrams for the reaction $q\bar{q} \longrightarrow Zg$

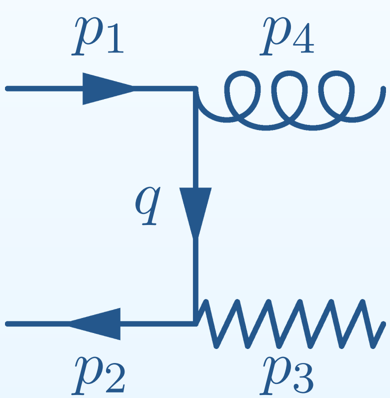


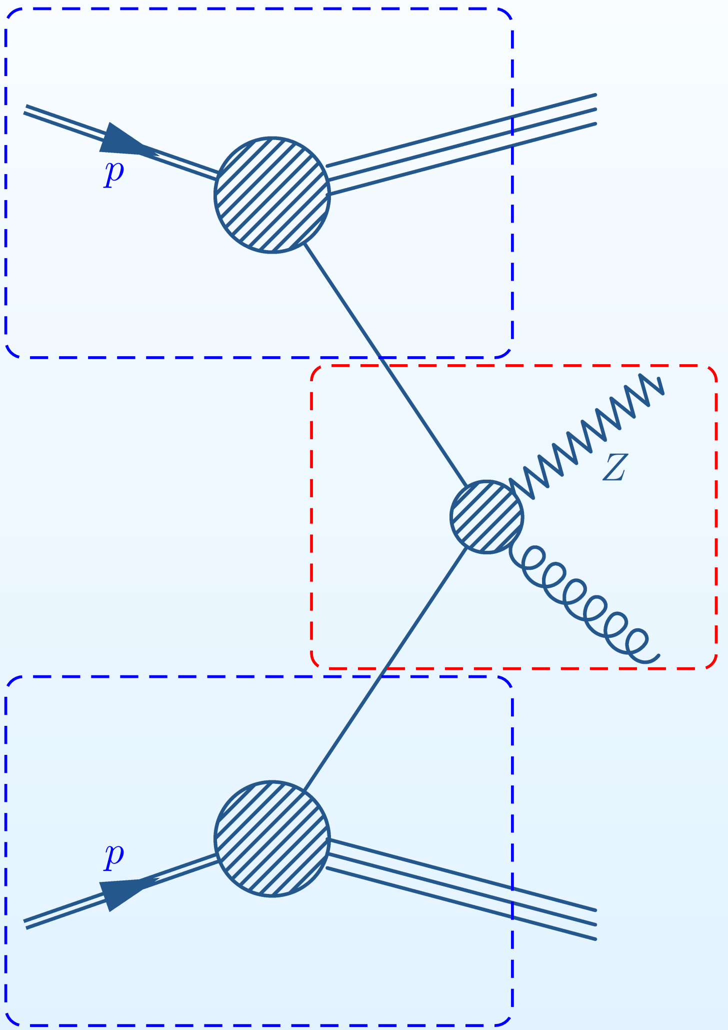
Infra red problem



divergence when $k \to 0$: soft divergence (long distance) every time a gluon is exchanged
different from the UV divergence (short distance)
cancel with another soft divergence coming from the integration the real emission over the phase space of the extra-gluon.

Z production III





colinear singularity :

$$q^2 = (p_1 - p_4)^2 = m^2 - 2\,p_1.p_4$$

Colinear singularity I

$$\hat{\sigma} \propto |T_1 + T_2|^2 \propto 1/(p1.p4\, p2.p4)$$

$$\begin{aligned} p1.p4 &= E\, E_4 \left(1 - \frac{\sqrt{E^2 - m^2}}{E} \cos(\theta)\right) \\ &\simeq E\, E_4 \left(1 - \left[1 - \frac{1}{2}\frac{m^2}{E^2}\right] \cos(\theta)\right) \end{aligned}$$

In the CM $\vec{p}_1 + \vec{p}_2 = \vec{0}$

$p_1 = (E, 0, 0, \bar{p})$

$p_2 = (E, 0, 0, -\bar{p})$

with $E^2 - \bar{p}^2 = m^2$

$p_4 = (E_4, \vec{p}_{T4}, E_4 \cos(\theta))$

$$\begin{aligned} p1.p4 &\simeq E\, E_4\, (1 - A \cos(\theta)) \\ p2.p4 &\simeq E\, E_4\, (1 + A \cos(\theta)) \end{aligned}$$

Phase space measure $\int \frac{d^3 p_4}{2\, E_4} \longrightarrow E_4\, dE_4\, d\cos(\theta)\, d\phi$

$$\int_{-1}^{1} d\cos(\theta)\, \frac{1}{1 - A \cos(\theta)} = -\frac{1}{A} \left(\ln(1 - A) - \ln(1 + A)\right)$$

Colinear singularity II

we get : ($E \simeq M_Z$)

$$\ln\left(\frac{m^2}{M_Z^2}\right) P^{(0)} \otimes \sigma_{q\bar{q}\to Z}$$

which can be written as:

$$\boxed{\ln\left(\frac{m^2}{M_0^2}\right) P_{qq}^{(0)} \otimes \sigma_{q\bar{q}\to Z}} + \boxed{\ln\left(\frac{M_0^2}{M_Z^2}\right) P_{qq}^{(0)} \otimes \sigma_{q\bar{q}\to Z}}$$

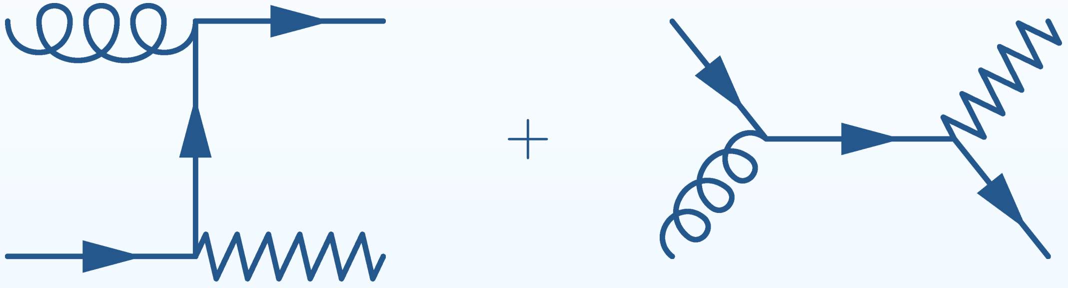
$$\boxed{F_q^P(x) \longrightarrow F_q^p(x, M_0^2)}$$

hard process

with $M_0 \sim 1$ GeV

Z Production IV

tree diagrams for the reaction $qg \to Zq$



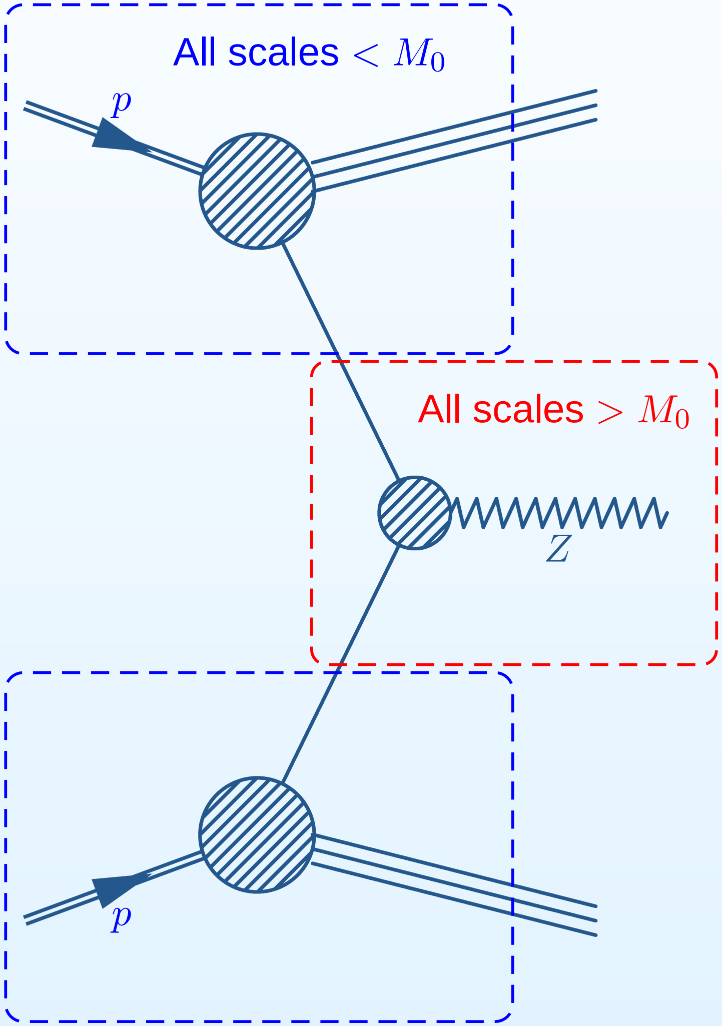
No soft divergence, but colinear one:

$$\boxed{\ln\left(\frac{m^2}{M_0^2}\right)\; P_{gq}^{(0)} \otimes \sigma_{q\bar{q}\to Z}} \quad + \quad \boxed{\ln\left(\frac{M_0^2}{M_Z^2}\right)\; P_{gq}^{(0)} \otimes \sigma_{q\bar{q}\to Z}}$$

$$\boxed{F_g^P(x) \longrightarrow F_g^p(x, M_0^2)}$$

hard process

Factorisation



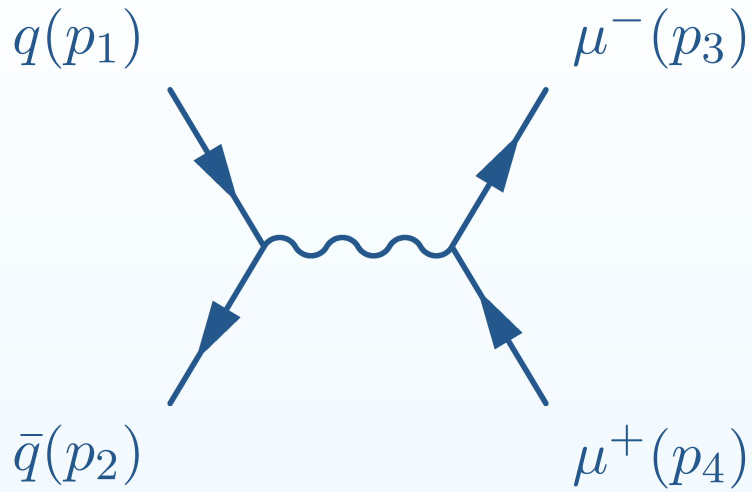
Soft logarithms

For example:

1) $M_Z \sim \sqrt{S} \quad x_1\,,\ x_2 \sim 1$
$\alpha_s\, ln(1 - x_1) \sim 1$
in evolution equation, change $\alpha_s(M^2)$ in $\alpha_s((1-y)\, M^2)$

2) $p_{tZ} \sim 0$, $M_Z < \sqrt{S}$

Compute I



$$d\hat{\sigma} \quad = \quad \frac{1}{4\,p_1.p_2}\,(2\,\pi)^4\,\delta^4(p_1+p_2-p_3-p_4)\,\frac{d^3\vec{p}_3}{(2\,\pi)^3\,2\,E_3}\,\frac{d^3\vec{p}_4}{(2\,\pi)^3\,2\,E_4}|M_1|^2$$

$$\begin{aligned} M_1 \quad &= \quad i\,e^2\,Q_f\,\bar{v}_{i,l_2}(p_2)\,\gamma_\mu\,u_{i,l_1}(p_1)\left(g^{\mu\,\nu}-(1-\xi)\,\frac{k^\mu\,k^\nu}{k^2+i\,\lambda}\right)\frac{1}{k^2+i\,\lambda} \\ &\quad \times\,\bar{u}_{l_3}(p_3)\,\gamma_\nu\,v_{l_4}(p_4) \end{aligned}$$

with $k = p_1 + p_2 = p_3 + p_4$

Compute II

The term $k^\mu\, k^\nu$ does not contribute

$$M_1 = i\, e^2\, Q_f\, \bar{v}_{i,l_2}(p_2)\, \gamma_\mu\, u_{i,l_1}(p_1)\, \frac{1}{k^2}\, \bar{u}_{l_3}(p_3)\, \gamma_\mu\, v_{l_4}(p_4)$$

We have to compute:

$$|\overline{M_1}|^2 = \frac{1}{4\, N^2} \sum_{color} \sum_{l_1=1}^{2} \sum_{l_2=1}^{2} \sum_{l_3=1}^{2} \sum_{l_4=1}^{2} |M_1|^2$$

$$\begin{aligned} |\overline{M_1}|^2 \quad &= \quad \frac{e^4\, Q_f^2}{4\, N^2\, k^4} \sum_{color} \sum_{l_1=1}^{2} \sum_{l_2=1}^{2} \sum_{l_3=1}^{2} \sum_{l_4=1}^{2} \Bigg[\bar{v}^{\alpha}_{i,l_2}(p_2)\, \gamma_\mu^{\alpha\,\beta}\, u^{\beta}_{i,l_1}(p_1) \\ &\times \left(\bar{v}^{\alpha'}_{j,l_2}(p_2)\, \gamma_\nu^{\alpha'\,\beta'}\, u^{\beta'}_{j,l_1}(p_1) \right)^\dagger \bar{u}^{\gamma}_{l_3}(p_3)\, \gamma_\mu^{\gamma\,\delta}\, v^{\delta}_{l_4}(p_4) \left(\bar{u}^{\gamma'}_{l_3}(p_3)\, \gamma_\nu^{\gamma'\,\delta'}\, v^{\delta'}_{l_4}(p_4) \right)^\dagger \Bigg] \end{aligned}$$

Compute III

the indices α, β, γ and δ are the spinorial indices.

$$\left(\bar{v}_{i,l_2}(p_2)\,\gamma_\nu\,u_{i,l_1}(p_1)\right)^\dagger = \bar{u}_{i,l_1}(p_1)\,\gamma_\nu\,v_{i,l_2}(p_2)$$

$$\begin{aligned}|\overline{M_1}|^2 &= \frac{e^4\,Q_f^2}{4\,N^2\,k^4}\sum_{color}\sum_{l_j=1,\,j=1..4}^{2}\Big[\bar{v}^{\alpha}_{i,l_2}(p_2)\,\gamma_\mu^{\alpha\,\beta}\,u^{\beta}_{i,l_1}(p_1)\bar{u}^{\beta'}_{j,l_1}(p_1)\,\gamma_\nu^{\beta'\,\alpha'}\,v^{\alpha'}_{j,l_2}(p_2)\\ &\quad\times\bar{u}^{\gamma}_{l_3}(p_3)\,\gamma_\mu^{\gamma\,\delta}\,v^{\delta}_{l_4}(p_4)\bar{v}^{\delta'}_{l_4}(p_4)\,\gamma_\nu^{\delta'\,\gamma'}\,u^{\gamma'}_{l_3}(p_3)\Big]\end{aligned}$$

$$\begin{aligned}\sum_{l=1}^{2}u^{\alpha}_{i,l}(p)\,\bar{u}^{\beta}_{j,l}(p) &= \Lambda^{+\,\alpha\beta}\,\delta_{ij}\\ \sum_{l=1}^{2}v^{\alpha}_{i,l}(p)\,\bar{v}^{\beta}_{j,l}(p) &= -\Lambda^{-\,\alpha\beta}\,\delta_{ij}\end{aligned}$$

$$\Lambda^{\pm}(p) = (\pm\not{p}+m)$$

Compute IV

We get:

$$|\overline{M_1}|^2 = \frac{e^4\, Q_f^2}{4\, N^2\, k^4}\, N\, Tr\left[\not{p}_2\, \gamma_\mu\, \not{p}_1\, \gamma_\nu\right] Tr\left[\not{p}_3\, \gamma_\mu\, \not{p}_4\, \gamma_\nu\right]$$

If A_1,A_2,...A_n are 4-vectors:

$$\begin{aligned}
Tr\left[\not{A}_1\, \not{A}_2\right] &= 4\, A_1.A_2 \\
Tr\left[\not{A}_1\, \not{A}_2\, \not{A}_3\, \not{A}_4\right] &= 4\, (A_1.A_2\; A_3.A_4 + A_1.A_4\; A_2.A_3 - A_1.A_3\; A_2.A_4) \\
Tr\left[\not{A}_1\, \not{A}_2\, \cdots\, \not{A}_{2n}\right] &= A_1.A_2\, Tr\left[\not{A}_3\, \not{A}_4\, \cdots\, \not{A}_{2n}\right] - A_1.A_3\, Tr\left[\not{A}_2\, \not{A}_4\, \cdots\, \not{A}_{2n}\right] \\
&\quad + \cdots + A_1.A_{2n}\, Tr\left[\not{A}_2\, \not{A}_3\, \cdots\, \not{A}_{2n-1}\right]
\end{aligned}$$

Compute V

With these formula, we can compute the trace over γ matrices and get:

$$|\overline{M_1}|^2 = \frac{8\, e^4\, Q_f^2}{N\, k^4} \left\{ p_2.p_3\, p_1.p_4 + p_2.p_4\, p_1.p_3 \right\}$$

We want to look the μ^-, We will compute the differential cross section:

$$\frac{E_3\, d\hat{\sigma}}{d^3\vec{p}_3} = \frac{1}{4\, p_1.p_2} (2\,\pi)^4\, \delta^4(p_1 + p_2 - p_3 - p_4)\, \frac{1}{2\,(2\,\pi)^3}\, \frac{d^3\vec{p}_4}{(2\,\pi)^3\, 2\, E_4} |M_1|^2$$

$$\int \frac{d^3\vec{p}_4}{2\, E_4} = \int d^4p_4\, \delta^+(p_4^2)$$

$$\begin{aligned} \frac{E_3\, d\hat{\sigma}}{d^3\vec{p}_3} &= \frac{1}{4\, p_1.p_2}\, \frac{1}{2\,(2\,\pi)^2}\, \delta^+(p_4^2) |M_1|^2 \\ &= \frac{2\,\alpha\, Q_f^2}{N\, \hat{s}}\, \delta^+(\hat{s} + \hat{t} + \hat{u}) \left(\frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2} \right) \end{aligned}$$

with:

$$\begin{aligned} \hat{s} &= (p_1 + p_2)^2 \\ \hat{t} &= (p_1 - p_3)^2 \\ \hat{u} &= (p_2 - p_3)^2 \end{aligned}$$

Compute VI

At the hadronic level:

$$\frac{E_3\, d\sigma_{PP\to\mu^+\mu^-}}{d^3\vec{p}_3} = \sum_{i=u,d,s,c,\ldots} \int_0^1 dx_1\, dx_2$$
$$\times \left[F_{q_i}^P(x_1, M^2)\, F_{\bar{q}_i}^P(x_2, M^2)\, \frac{E_3\, d\hat{\sigma}_{q_i\bar{q}_i\to\mu^+\mu^-}}{d^3\vec{p}_3} \right.$$
$$\left. + F_{\bar{q}_i}^P(x_1, M^2)\, F_{q_i}^P(x_2, M^2)\, \frac{E_3\, d\hat{\sigma}_{\bar{q}_i q_i\to\mu^+\mu^-}}{d^3\vec{p}_3} \right]$$

$$\begin{aligned}
p_1 &= x_1 \frac{\sqrt{S}}{2} (1, 0, 0, 1) \\
p_2 &= x_2 \frac{\sqrt{S}}{2} (1, 0, 0, -1) \\
p_3 &= p_t \left(\cosh(y_3), \vec{a}, \sinh(y_3)\right) \\
p_4 &= p_t \left(\cosh(y_4), -\vec{a}, \sinh(y_4)\right)
\end{aligned}$$

Compute VII

$$\int dx_1\, \delta^+(x_1\, x_2\, S - x_1\, \sqrt{S}\, p_t\, \exp(-y_3) - x_2\, \sqrt{S}\, p_t\, \exp(y_3))\, f(x_1) =$$

$$\frac{1}{x_2\, S - \sqrt{S}\, p_t\, \exp(-y_3)}\, f\left(\frac{x_2\, p_t\, \exp(y_3)}{x_2\, \sqrt{S} - p_t\, \exp(-y_3)}\right)$$

$$\begin{aligned}\frac{E_3\, d\sigma_{PP\to\mu^+\mu^-}}{d^3\vec{p}_3} &= \frac{2\,\alpha\, \exp(-y_3)}{N\, S\, p_t\, \sqrt{S}} \sum_{i=u,d,s,c,\ldots} \int_{x_{2min}}^{1} \frac{dx_2}{x_2^2} \\ &\times \left[F_{q_i}^P(x_1, M^2)\, F_{\bar{q}_i}^P(x_2, M^2) + F_{\bar{q}_i}^P(x_1, M^2)\, F_{q_i}^P(x_2, M^2)\right] \\ &\times Q_i^2 \left(\frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2}\right)\end{aligned}$$

with :

$$x_{2min} = \frac{p_t\, \exp(-y_3)}{\sqrt{S} - p_t\, \exp(y_3)}$$

Jets I

Jet : bunch of hadrons flying in the same direction depositing it energy in localised place

Jet definition

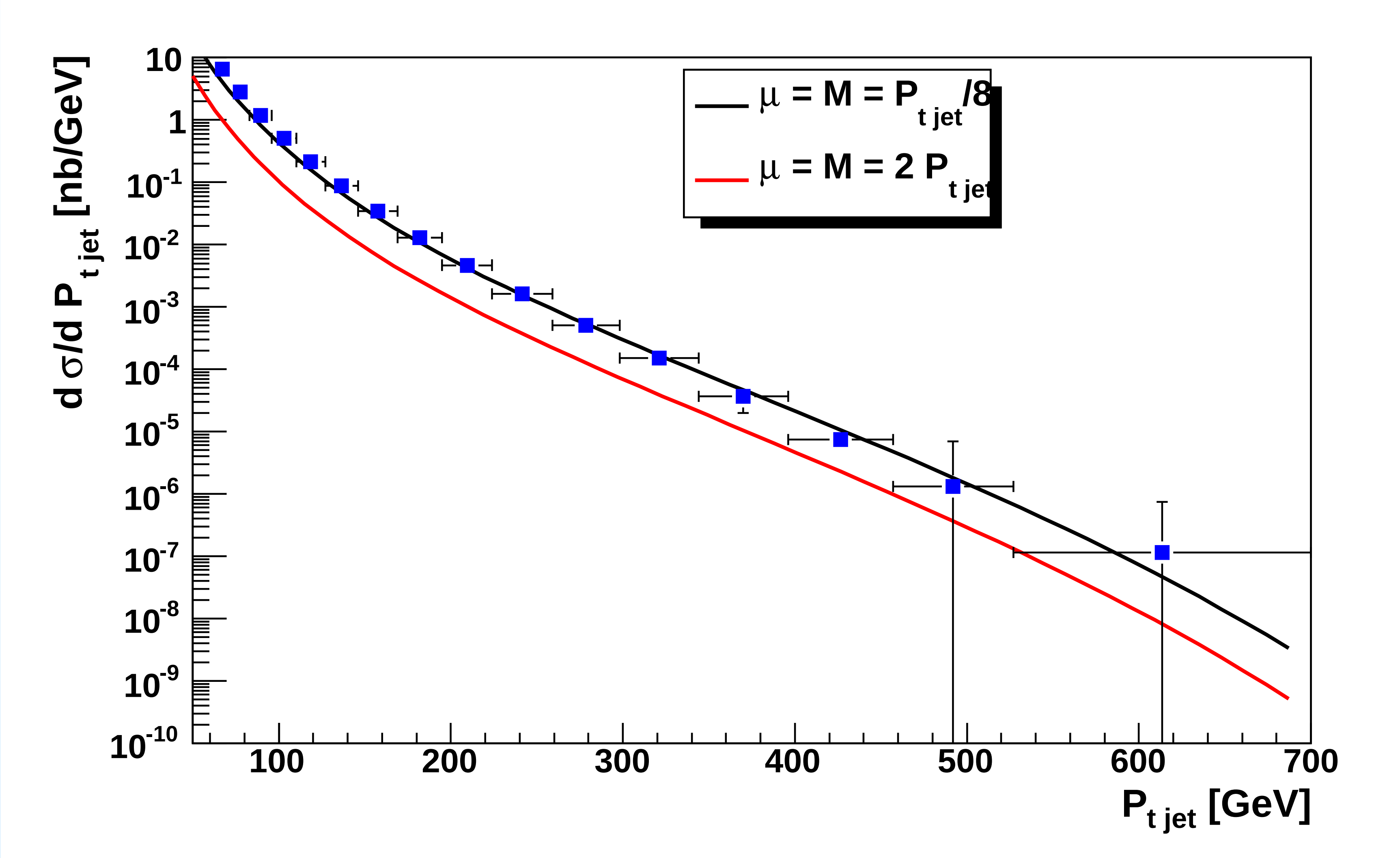
- What is the criterion for a hadron/parton to belong to a jet
- How to build the jet kinematics from the hadron/parton kinematics

Jet II

At partonic level parton = jet
partonic reactions:

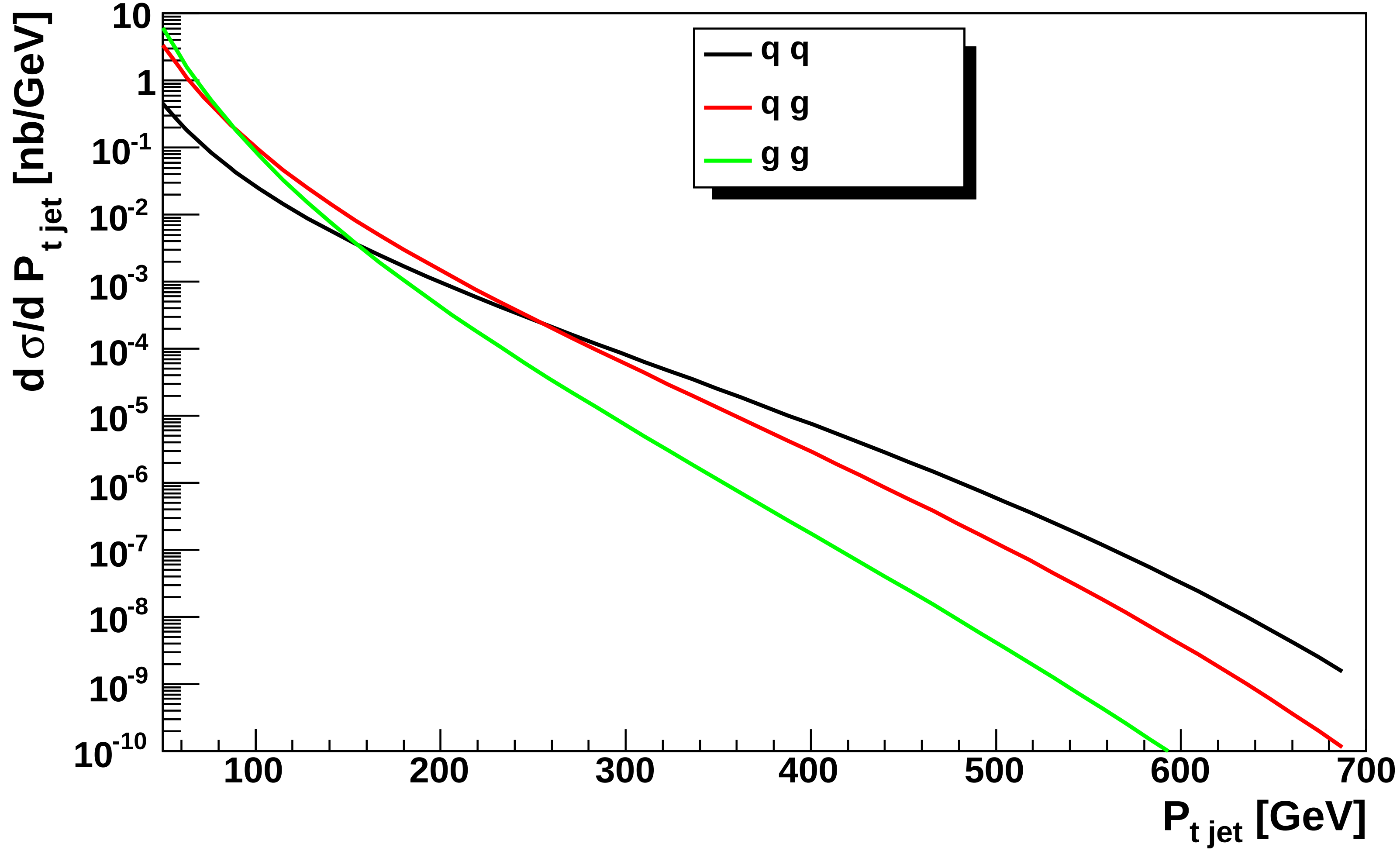
$$
\begin{array}{rcll}
q_i + q_k & \longrightarrow & jet + X & q_i + q_k \longrightarrow q_i + q_k \\
q_i + \bar{q}_k & \longrightarrow & jet + X & q_i + \bar{q}_k \longrightarrow q_i + \bar{q}_k \\
q_i + q_i & \longrightarrow & jet + X & q_i + q_i \longrightarrow q_i + q_i \\
q_i + \bar{q}_i & \longrightarrow & jet + X & \left\{ \begin{array}{l} q_i + \bar{q}_i \longrightarrow q_i + \bar{q}_i \\ q_i + \bar{q}_i \longrightarrow q_k + \bar{q}_k \ (\times N_F - 1) \\ q_i + \bar{q}_i \longrightarrow g + g \end{array} \right. \\
q_i + g & \longrightarrow & jet + X & q_i + g \longrightarrow q_i + g \\
g + g & \longrightarrow & jet + X & \left\{ \begin{array}{l} g + g \longrightarrow q_i + \bar{q}_i \ (\times N_F) \\ g + g \longrightarrow g + g \end{array} \right.
\end{array}
$$

Jet III



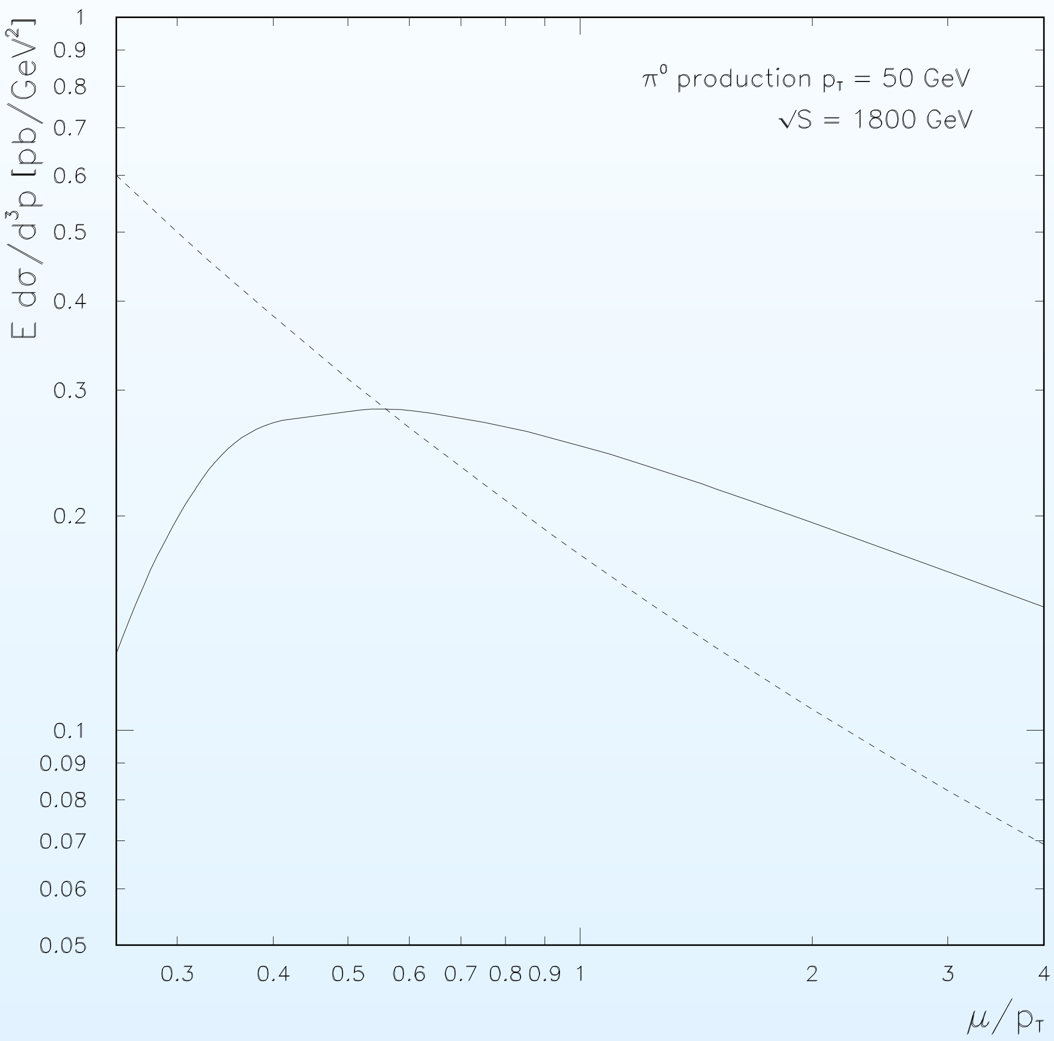
μ = M = P t jet /8
μ = M = 2 P t jet
d σ/d P t jet [nb/GeV]
P t jet [GeV]
10
1
10-1
10-2
10-3
10-4
10-5
10-6
10-7
10-8
10-9
10-10
100
200
300
400
500
600
700

Jet III



q q
q g
g g
$d\sigma/dP_{t\,jet}$ [nb/GeV]
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
100
200
300
400
500
600
700
$P_{t\,jet}$ [GeV]

Scale dependence



Jet IV

A jet algorithm must be :

- infrared and collinear safe

soft emission should not change jets
collinear splitting should not change jets

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  soft emission should not change jets
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- identically defined at parton and hadron level (so that perturbative calculations can be compared to experiments)

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  collinear splitting should not change jets

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- not too sensitive to hadronisation, underlying event, pile-up (because we are not very good at modeling non-perturbative stuff)

Jet IV

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- infrared and collinear safe
  soft emission should not change jets
  collinear splitting should not change jets
- identically defined at parton and hadron level (so that perturbative calculations can be compared to experiments)
- not too sensitive to hadronisation, underlying event, pile-up (because we are not very good at modeling non-perturbative stuff)
- realistically applicable at detector level (e.g. not too slow)

K_t algorithm

- calculate the distances between the particles :
  $d_{ij} = \min(p_{ti}, p_{tj}) \frac{\Delta y_{ij}^2 + \phi_{ij}^2}{R^2}$

K_t algorithm

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- Combine particles with smallest distance or, if d_{iB} is smallest, call it a jet
- Find again smallest distance and repeat procedure until no particles are left