Gravitational Waves associated with Gamma-ray Bursts

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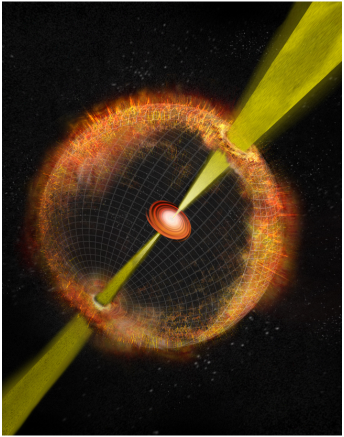
Seminar at LAPP



Credit: Bill Saxton, NRAO/AUI/NSF

GR $\rightarrow$ Gravitational Waves

- General Relativity: space-time is a Riemann space

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu,$$

  with the metric created by the matter/energy (Einstein's equation)

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Gravitational Waves (GW) $\rightarrow$ usually seen as linear limit of GR

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \ll 1 \quad \eta_{\mu\nu} \text{– flat metric}$$

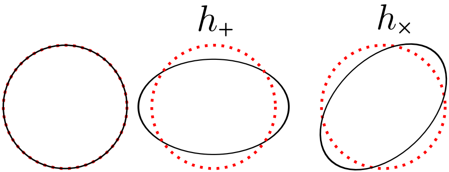
$$\Rightarrow \Box h_{\mu\nu} = 0$$

  Waves propagating at speed of light
- Tensorial waves $\rightarrow$ 10 degrees of freedom (symmetric tensor)
  gauge freedom $\rightarrow$ 2 polarizations

$$h_{\mu\nu} = h_+ A_{\mu\nu} + h_\times B_{\mu\nu}$$

Gravitational wave polarization

- $h_{\mu\nu} = h_+ A_{\mu\nu} + h_\times B_{\mu\nu}$
- Both polarization are transverse
- Deform circle of free masses into an ellipse



$\Rightarrow$ GW observation $\leftrightarrow$ measure relative variation in two orthogonal lengths

Gravitational source quadrupolar approximation

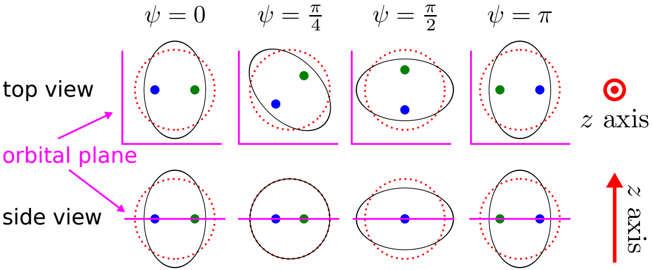
Approximation: far field + slow moving source

- Mass distribution quadrupolar moment

$$I_{ij} = \int (x_i x_j - \tfrac{1}{3}\delta_{ij}\delta_{km}x^k x^m)\rho(x)\mathrm{d}^3x.$$

- Source of gravitational waves

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad \longrightarrow \quad h^{TT}_{jk} = \frac{2G}{rc^4}\underbrace{P_{jkmn}}_{\text{projection}}\ddot{I}^{mn}(t - \tfrac{r}{c}),$$



circular polarization
$h_+ = \pm i h_\times$

linear polarization
$h_+ \neq 0, \quad h_\times = 0$

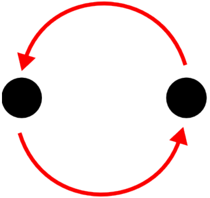
Gravitational wave source: luminosity

Luminosity (radiated power) in GW

$$\mathcal{L}_{\mathrm{GW}} = \frac{G}{5c^5}\langle \dddot{I}_{\mu\nu}\dddot{I}^{\mu\nu}\rangle = \frac{c^5}{G}\underbrace{\epsilon^2}_{\text{asymmetric}}\underbrace{\left(\frac{R_s}{R}\right)^2}_{\text{compact}}\underbrace{\left(\frac{v}{c}\right)^6}_{\text{relativistic}}$$

- Potentially huge power: $c^5/G \sim 10^{53}$ W
- Good sources are:
  - asymmetric $\rightarrow$ $\epsilon = \frac{I_{xx}-I_{yy}}{I_{zz}} \sim 1$
  - compact $\rightarrow$ size R is near the Schwartzschild radius R_s
  - relativistic $v \sim c$
- Example of terrestrial production
  - 10 ton bar $\rightarrow$ not compact
  - 50 rotation per second, size 1 m $\rightarrow$ non relativistic
  - $\Rightarrow$ GW amplitude $h \sim 10^{-35}$, flux $\sim 10^{-31}\,\mathrm{W\,m^{-2}}$
- $\Rightarrow$ Astrophysical sources
  - neutron star merger at 10 Mpc $\rightarrow$ (a few dozen galaxies)
  - GW amplitude $h \sim 10^{-21}$, flux $\sim 10^{-3}\,\mathrm{W\,m^{-2}}$ $\rightarrow$ possible
- NB: Earth radius $\times\, 10^{-21} \sim$ atomic nucleus radius

Indirect observation of GWs



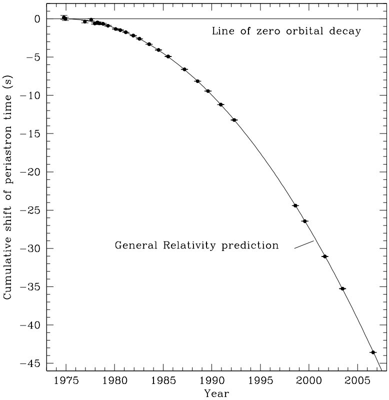
Indirect observation of gravitational radiation

- Neutron star binary $\rightarrow \mathcal{L}_{GW}$

$\Rightarrow$ Energy loss of the binary neutron star system

- Orbital period measured through Doppler effects on radio pulses
- Follows GR with $\sim 10^{-3}$ precision

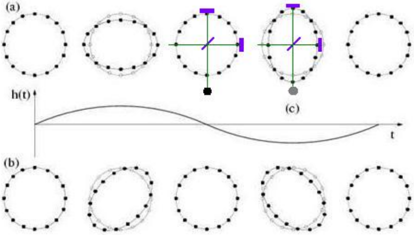
"double" pulsar PSR1913+16



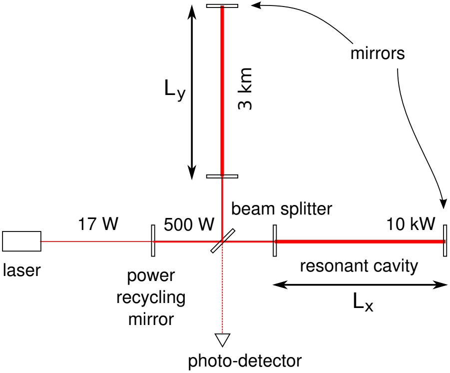
Hulse-Taylor Nobel Prize 1993

Observation principles

Look at the relative variation of two orthogonal length ⇒ Michelson interferometer



basic optical layout of a GW detector

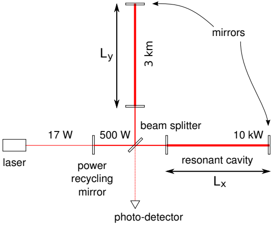


- Suspended mirrors (horizontally free masses)

Observation principles

- Phase shift between light reflected by the two cavities

$$\Delta\varphi_{\mathrm{GW}} \propto \frac{L_x - L_y}{L_0} = h_+(t)$$



- Output light power at dark fringe working point

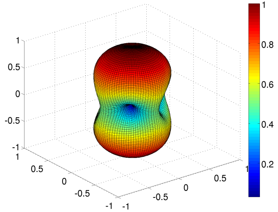
$$\frac{P_{\mathrm{out}}(\Delta\varphi_{\mathrm{GW}})}{P_{\mathrm{out}}(0)} \simeq 1 + K\Delta\varphi_{\mathrm{GW}}$$

- Data after calibration

$$d(t) = \frac{\Delta L}{L} = \underbrace{F_+(\theta,\phi)h_+(t) + F_\times(\theta,\phi)h_\times(t)}_{s(t)} + n(t)$$

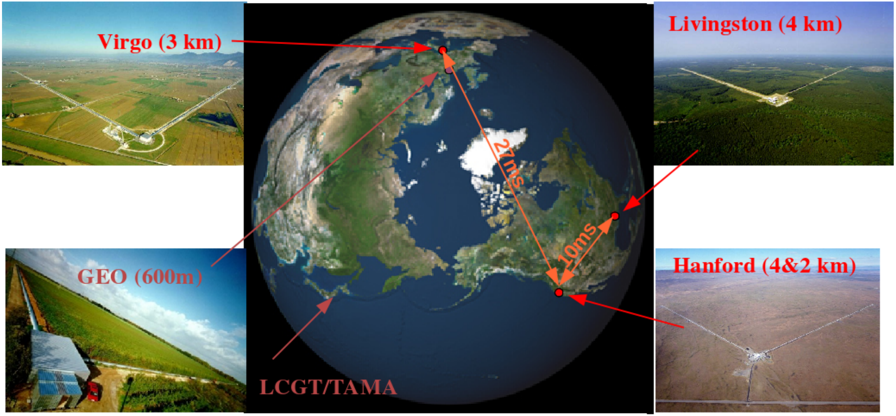
  - θ, ϕ – direction of the source of GW
  - $h_+(t), h_\times(t)$ – GW amplitude
  - $F_+, F_\times$ – geometric antenna factors, $\leftrightarrow$ detector sensitive to only one polarization
  - $s(t)$ – signal
  - $n(t)$ – noise

$$\sqrt{F_+(\theta,\phi)^2 + F_\times(\theta,\phi)^2}$$



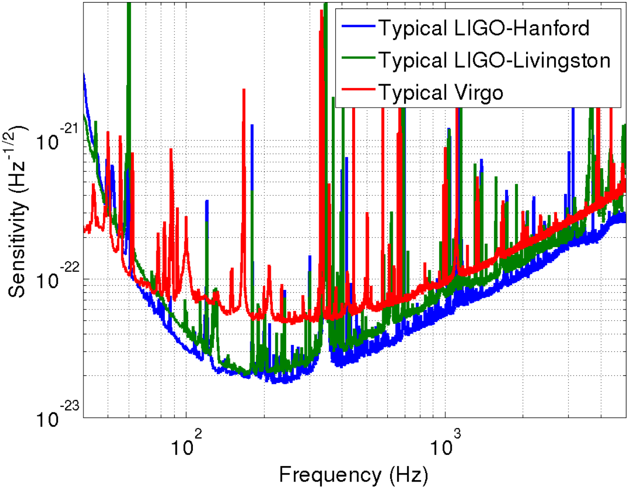
non directional detector

A network of detectors



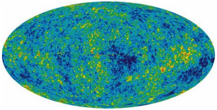
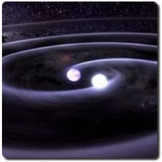
- GW same everywhere but propagation delayed
- 3 detectors → sky localization by triangulation

A network of detectors – 2009/2010



- Most sensitive for GW in [50, 500] Hz band

4 families of potential GW signal morphologies

	precisely modeled	uncertain form
permanent	Deformed rotating neutron stars 	Incoherent sum of unresolved sources Primordial GW background 
transient	Cosmic strings cusps, kinks Coalescence of neutron stars or black holes 	Star quakes, Non spherically symmetric stellar collapse, ... 

4 families of potential GW signal morphologies

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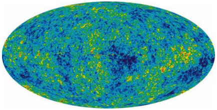
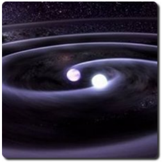
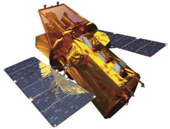
permanent	Deformed rotating neutron stars 	Incoherent sum of unresolved sources Primordial GW background 
transient	Cosmic strings cusps, kinks Coalescence of neutron stars or black holes 	Star quakes, Non spherically symmetric stellar collapse, ... 

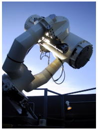
Multi-messenger observations with GWs

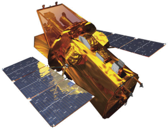
- Additional information for GW search ⇒ detect weaker GWs
- Confirmation of GW candidates ⇒ increased detection confidence
- Richer understanding of astrophysical sources



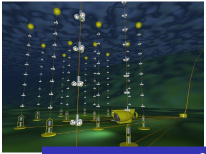
External trigger

EM followup of GW triggers





Joint analysis

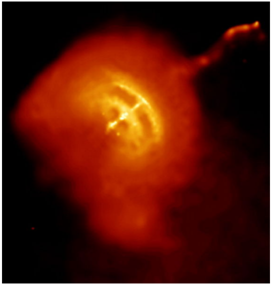


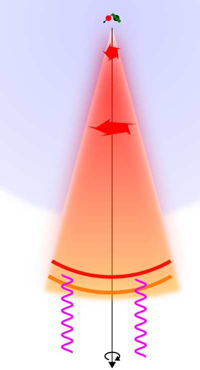
Some multi-messenger results unrelated to GRBs

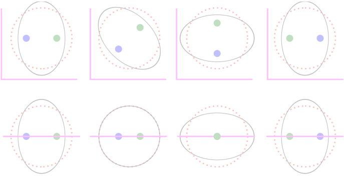
- Young pulsars (neutron stars)
  - Crab (SN 1054)
  - Vela (SN $\sim 10^4$ yr ago)

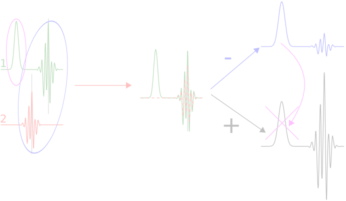
  spin frequency is precisely observed in radio
- The rotation period is decreasing
  $\longrightarrow$ loss of rotational energy
- LIGO 2005-2007: less than 2% of Crab energy loss is due to GW emission (Abbott et al., 2010)
- Virgo 2009-2010: less than 40% of Vela energy loss is due to GW emission (Abadie et al., 2011a)
- Without any radio observation the limits on energy loss higher by $\sim 10^2 - 10^3$ (Abadie et al., 2011b)

$\Rightarrow$ EM observation enhance GW searches sensitivity





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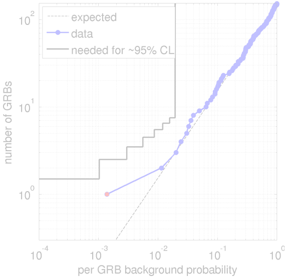
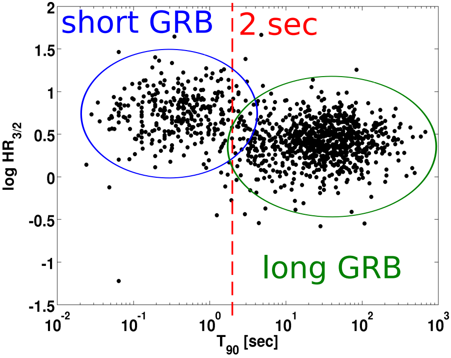

expected
data
needed for ~95% CL
number of GRBs
per GRB background probability

Gamma-ray bursts

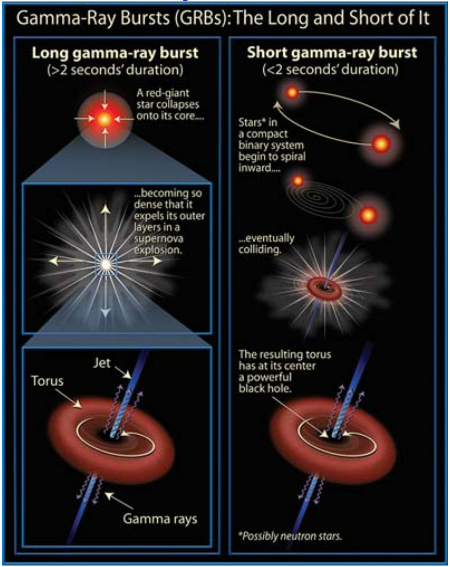
- Observational definition $\rightarrow$ a burst of γ-rays (10 keV – 1 MeV)
- Discovered in the 70's by nuclear bomb test surveillance satellites



BATSE 4B catalog

- T_{90} - duration of 90% of photon counts ($\sim$ 15 – 300 keV)
- Two observational populations:
  - short-hard GRBs $T_{90} \lesssim 2$ s spectrum peaks at higher energy
  - long-soft GRBs $T_{90} \gtrsim 2$ s spectrum peaks at lower energy

Gamma-ray burst models



credit: Ute Kraus

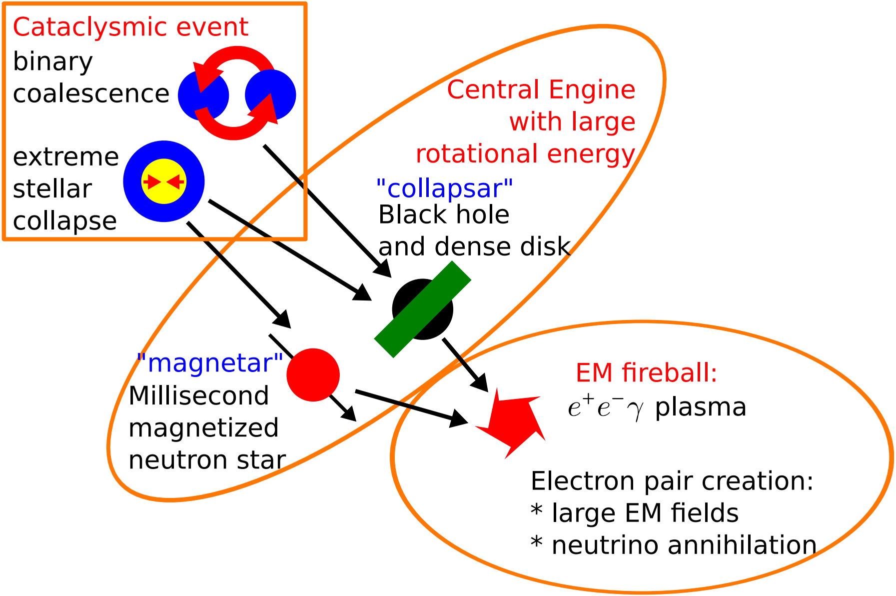
- Long GRBs

⇒ Massive rapidly spinning star collapse and explosion

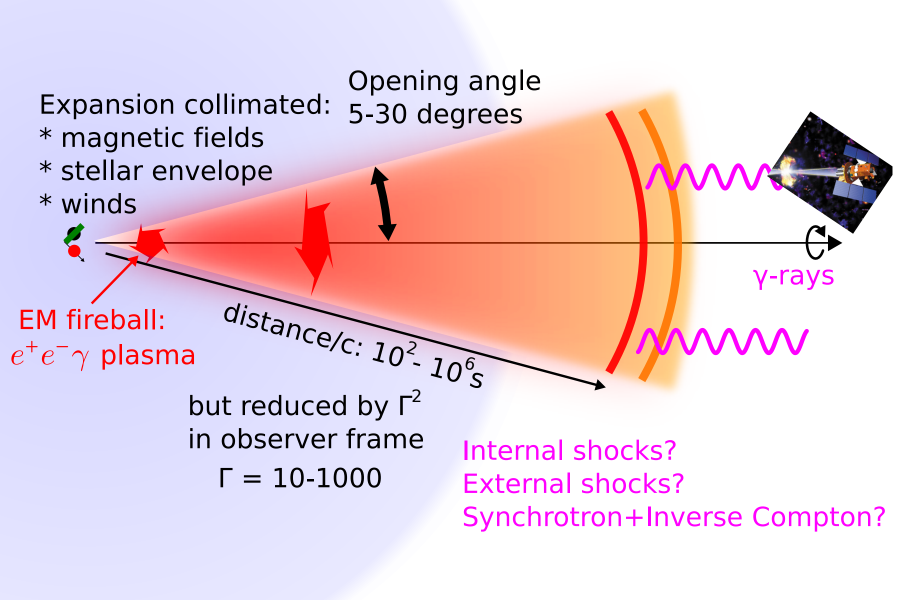
- Short GRBs

⇒ Coalescence of a neutron star and a compact object
  - small fraction is actually neutron star quakes ($\lesssim 15\%$)
- Both cases: asymmetric, compact, relativistic
  ⇒ good GW source
- Measured gamma emission:
  $\sim 10^{51}\,\text{erg} = 10^{-3}\,M_\odot c^2$
- Problem:
  typical distance $\sim 10\,\text{Gpc}$
  but some closer

EM emission - standard fireball model



EM emission - standard fireball model



What might we learn from GW-GRB observation

Models for short/long GRBs remain uncertain

- long GRBs
  - localization in star forming regions
  - associations with supernova
  - but also some long GRBs with strong limits on supernova ($< 10^{-3}$ typical luminosity)
- short GRBs
  - localization in galaxies with old stellar population
  - lack of supernova
  - observational confirmation weaker than for long GRBs

Potential lessons from GW-GRB detection

- Confirm the binary coalescence model for short GRBs
- Learn more about progenitor of long GRBs
  - black hole or magnetar?
- Precise measurement of GW speed, $\Delta v/c \sim 10^{-16}$
- Measure of Hubble's constant, distance $\leftrightarrow$ redshift relation

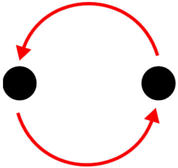
Astrophysical inputs & analysis strategy

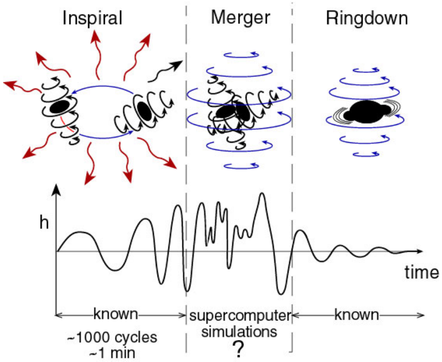
Goal: Find GW associated with GRBs

- What to look for?
  - GW signal waveform
  - GW signal amplitude
  - GW signal polarization
- Where to look for?
  - GRB sky localization
  - Timing between GRB trigger and GW trigger
    $\Rightarrow$ Understand both EM and GW emission
- Is it worthwhile to search?
  - GRB progenitors distance distribution
  - Is it better than blind (all-sky, all-time) search?

GW emission - coalescence scenario

- Binary system of two compact objects (NSNS or NSBH)





- Lose energy by GW radiation
- GW emission enters sensitive band ($\gtrsim 50\,\mathrm{Hz}$) $< 50\,\mathrm{s}$ before coalescence
- GW at merger, ringdown $\rightarrow$ high frequency ($M_{\mathrm{BH}} \lesssim 20\,\mathrm{M}_\odot$) $\rightarrow$ low SNR

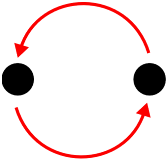
GW emission - coalescence scenario

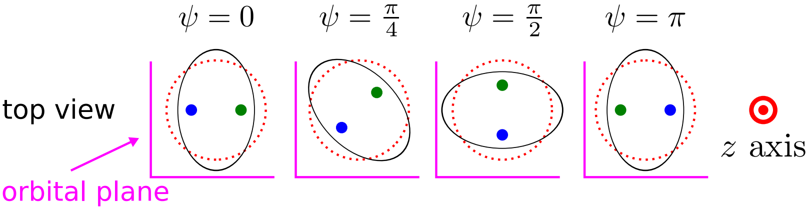
- GRB central engine formed in $\lesssim 1$ s

$\Rightarrow$ GW emission $[-50, 0]$ s prior to central engine formation

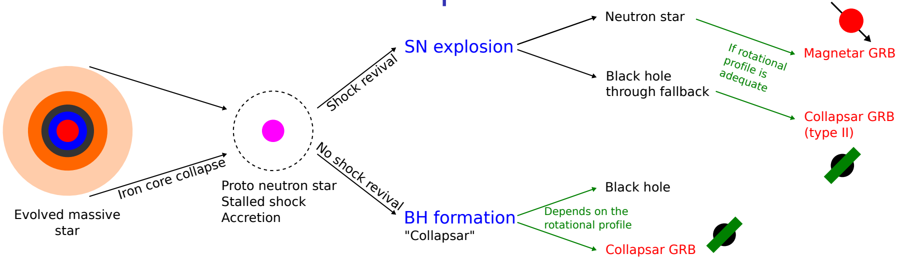
- GRB observed $\rightarrow$ rotation axis points at observer

$\Rightarrow$ GW well known and circularly polarized
up to inclination of 60°$\rightarrow$ loose constraint
(jet opening angle $\lesssim$ 30°)





GW emission - stellar collapse scenario



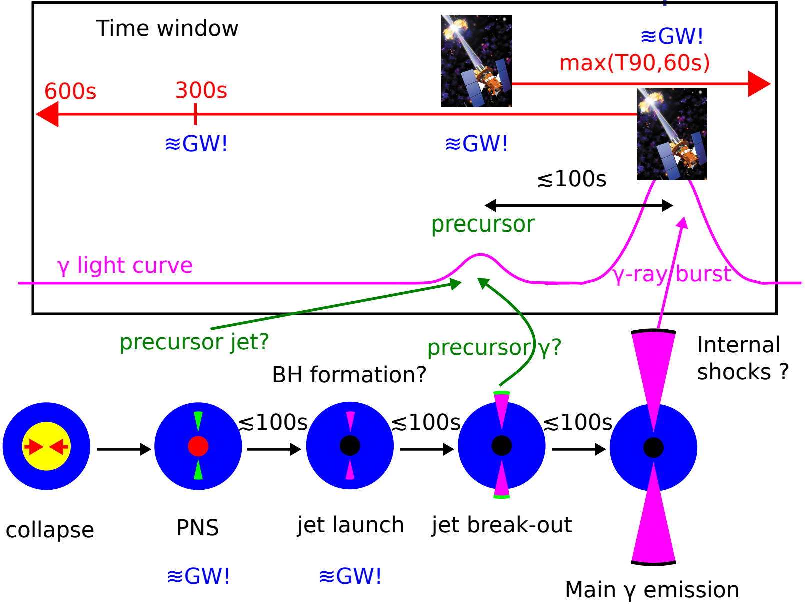
- Magnetar central engine / Proto neutron star
  - bar mode instability in the star
  - neutron star core fragmentation
- Black hole and accretion disk
  - Disk fragmentation
  - Disk precession

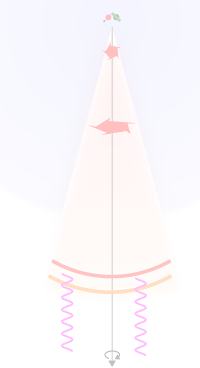
$\Rightarrow$ circular polarization along rotation axis

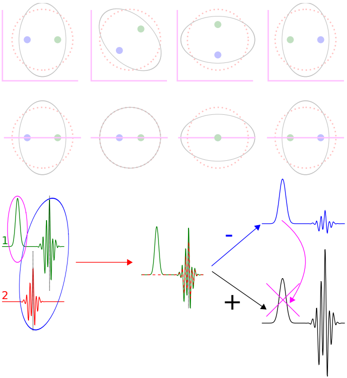
$\Rightarrow$ Emitted GW energy $\lesssim 10^{-2}\,M_{\odot}c^2$

- Other emission mechanism but no prospects for extra-galactic reach
  - Out of frequency band (Neutrino, normal modes, ...)
  - Too small amplitude (Core bounce, SASI, ...)

GW vs GRB time of arrival - stellar collapse







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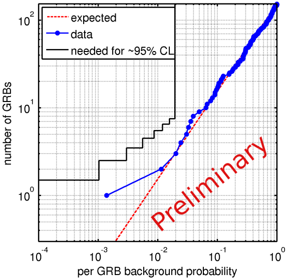
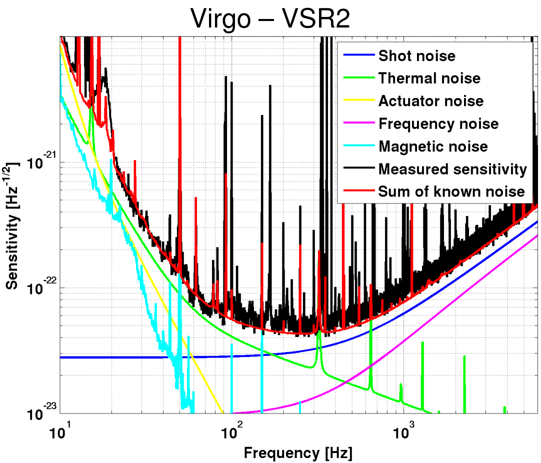


expected
data
needed for ~95% CL
number of GRBs
per GRB background probability
Preliminary

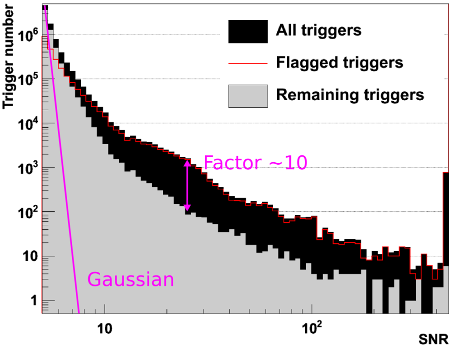
Noise - colored spectrum, “Gaussian” part



- Ground (seismic) motion → mirrors motion ($\lesssim$ 10 Hz)
- Thermal noise → mirrors surface/position fluctuation (10 – few 100Hz)
- Shot noise → photons counting at photo-diode ($\gtrsim$ few 100 Hz)
- Technical noise (control, scattered light, ...)

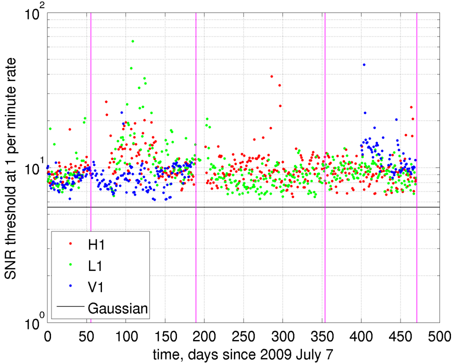
Data processing → poorly modeled glitches

non-Gaussian



- Long tail of loud glitches
- ∼ 90% of them understood

non-stationary

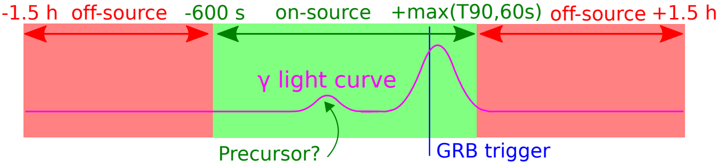


- Loudness at given rate is not stationary
- Usually a factor 2 above Gaussian expectation

Two complementary searches

- Broad in scope – covers most possibilities
  - "burst" searching method – any signal shapes
  - Limited to 60 – 500 Hz band, $\lesssim 1$ s duration
  - Assumes circular polarization
  - Loose time coincidence between γ-rays and GW $T_\gamma - T_{\mathrm{GW}} \in [-600, \max(T_{90}, 60)]$ s
  - $\Rightarrow$ Preliminary results
- Focused on short GRBs – binary coalescence
  - Inspiral waveform templates, NS-NS and NS-BH
  - Tight time coincidence between γ-rays and GW $T_\gamma - T_{\mathrm{GW}} \in [-5, 1]$ s
  - More sensitive to inspiral signals by factor ~ 2
  - $\Rightarrow$ Results not released yet
- Focus here on GW burst search

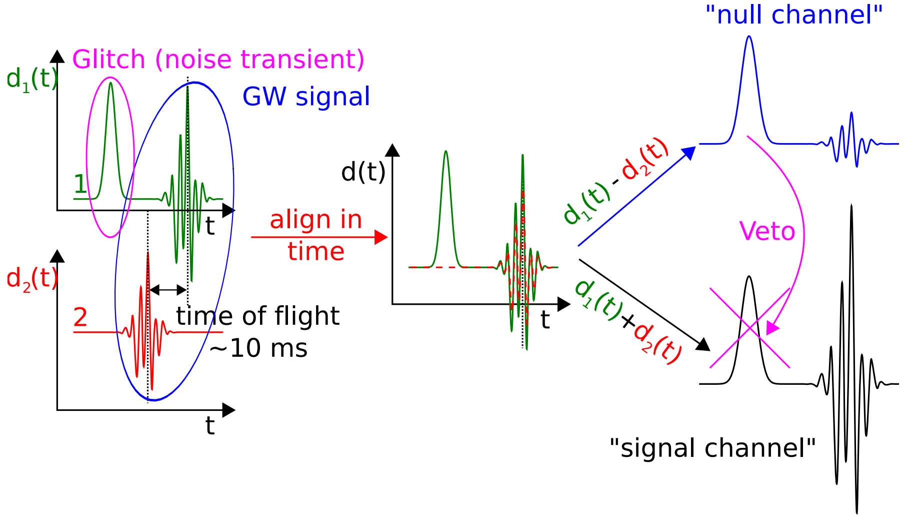
GRB triggered GW burst search



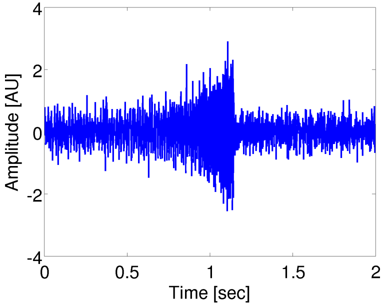
- Known position and time
  - Position → simplify coherent analysis
    - time delays between detectors constrained by sky location box
  - Reduced time → reduced background

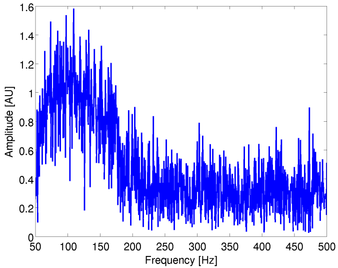
  ⇒ Better sensitivity by a factor ~ 2 wrt to all-sky/all-time search
- On-source data
  - Search for potential GW events
- Off-source data, time slides
  - Measurement of event background distribution
- Result of search
  - Background probability of most interesting on-source event
- Repeated independently for each GRB

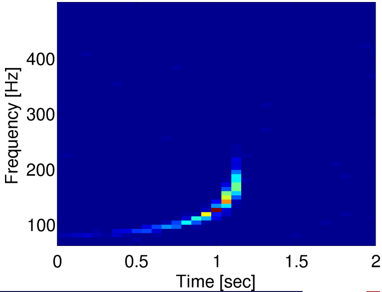
Combine data (add/substract) from several detectors



Excess wrt Gaussian noise → Time frequency maps



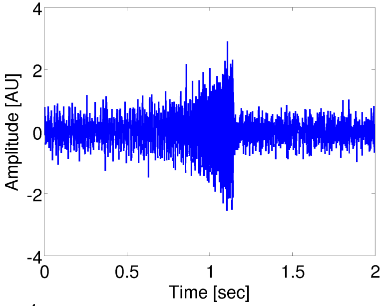


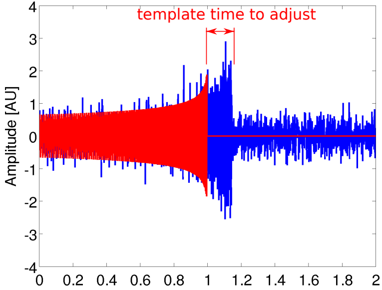


- Burst search
  - Concentrate signal energy in a small number of pixels
  - Sum energy over clusters of “loud” pixels

⇒ Ranking statistic

Excess wrt Gaussian noise → match with templates

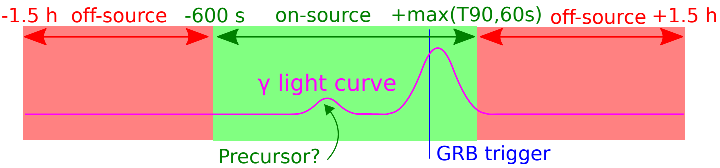




- Coalescence search
  - Adjust template time, parameters (masses, ...)
  - Sum coherently energy using waveform template
  - Check that residual is consistent with Gaussian noise (χ^2)

$\Rightarrow$ Ranking statistic

Background estimation



Goal: generate background sample

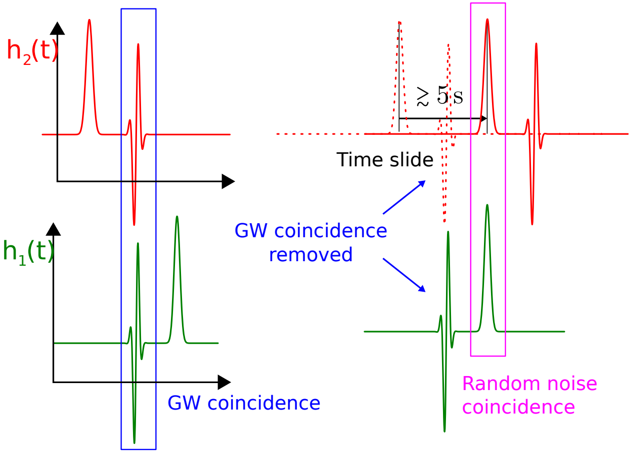
- Large non-Gaussian tail ⇒ no Monte Carlo generation

⇒ Need to use data to estimate background

- GW expected in the on-source region
- off-source region should contain only background

Time slides

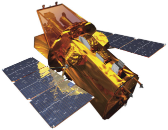
Question: how often random coincidence of glitches produce events that look like GWs?

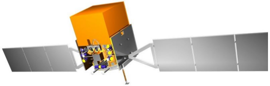


- Repeat shifts of $\gtrsim 5\,\mathrm{s}$ $\Rightarrow$ more (roughly) independent noise realizations
- We use $\sim$ 800 shifts

Data sample

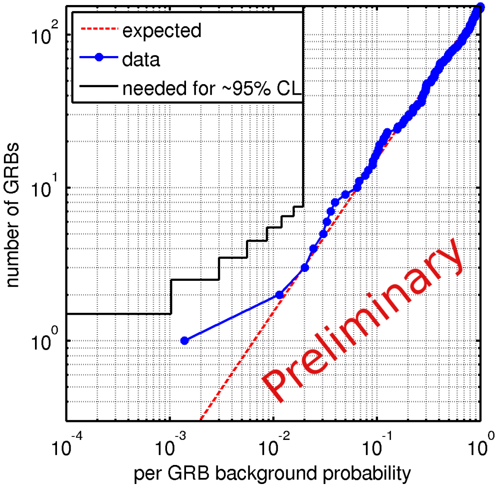
- July 2009 – October 2010
- Network of three GW detectors
  - LIGO Hanford
  - LIGO Livingston
  - Virgo, Italy
- 407 GRBs observed by γ-ray satellites
  Gamma-ray burst Coordinates Network
  - Swift
  - Fermi
  - ...
- 153 GRBs with good data from at least two GW detectors





Search for GW burst – no detection

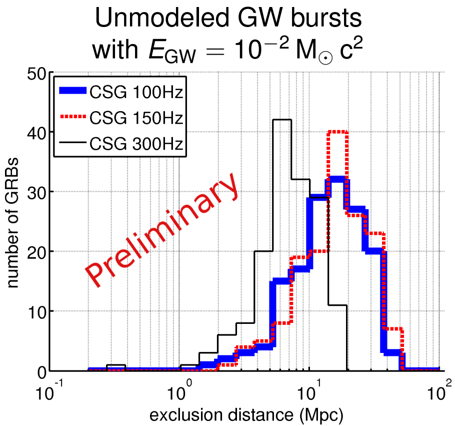
Distribution of per GRB background probabilities for 153 GRBs

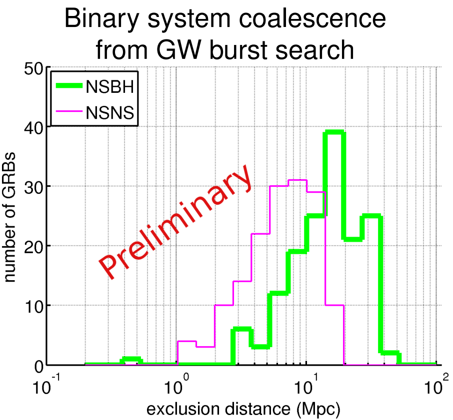


⇒ consistent with uniform distribution

GW burst non detection consequences

GRB progenitor distance exclusion





	burst 100Hz	burst 150Hz	NSBH	burst 300Hz	NSNS
median (Mpc)	15	17	16	7	7

Binary coalescence detection still possible by inspiral search

GRB070201 / GRB051103

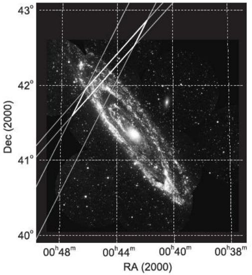
Significant previous non detections

- Short GRBs,
  - GRB070201 sky location overlap with M31, (Andromeda 770 kpc)
  - GRB051103 sky location overlap with M81 ($\sim$ 3.6 Mpc)
- no GW found

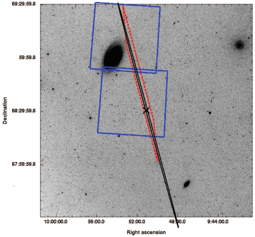
  $\Rightarrow$ Binary coalescence in M31 excluded at >99% confidence level (Abbott et al., 2008)

  $\Rightarrow$ Binary coalescence in M81 excluded at 98% confidence level (Abadie et al., 2012)
- Compatible with
  - Neutron star quake in M31/M81 (Soft gamma-repeater)
  - Coalescence in galaxy behind M31/M81

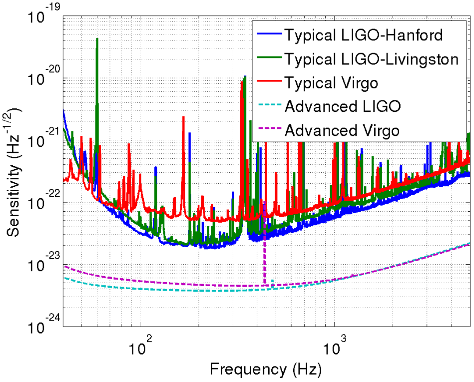
GRB070201 error box (Mazets et al., 2008)



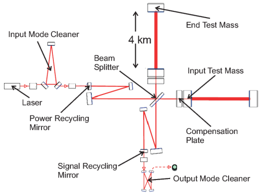
GRB051103 error box (Hurley et al., 2010)



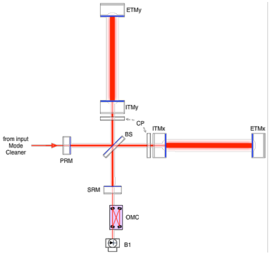
Network of “Advanced” detectors



Advanced LIGO



Advanced Virgo



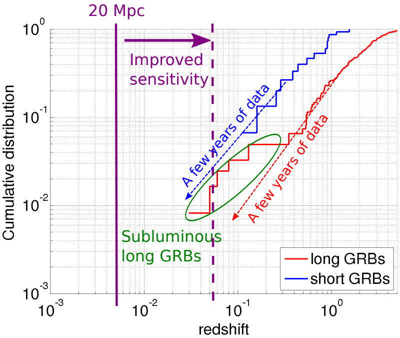
- 3 Advanced LIGO / Advanced Virgo → 2015
- factor ∼ 10 improvement in sensitivity
- factor ∼ 10^3 improvement in volume within reach
- Reaching design sensitivity will take a few years

- KAGRA (LCGT, Japan) started construction → 5 detectors ∼ 2018

Expectations

- Number of GRBs expected within $\sim 17\,\text{Mpc}$, 0.5 year of coincident data

  (Leonor et al., 2009)
- Different population of GRBs
  - Long GRBs:
    $\rho \sim 0.5\,\text{Gpc}^{-3}\text{yr}^{-1} \rightarrow N \sim 4 \times 10^{-6}$
  - Sub-luminous GRBs:
    $\rho \sim 500\,\text{Gpc}^{-3}\text{yr}^{-1} \rightarrow N \sim 4 \times 10^{-3}$
  - Short GRBs:
    $\rho \sim 10\,\text{Gpc}^{-3}\text{yr}^{-1} \rightarrow N \sim 10^{-4}$



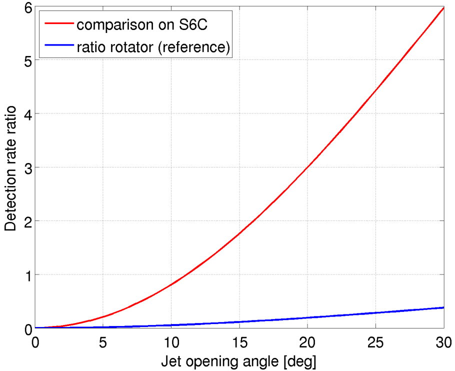
Swift GRBs

- Improvements
  - Short GRBs: CBC search sensitivity better by factor ~ 2
  - All cases: advanced detectors improve sensitivity by ~ 10
- Prospects for advanced detectors

$\Rightarrow$ Both short and sub-luminous GRB: $N \sim 1$ within range

Relevance of triggered search vs all-sky search

- Triggered search misses progenitors beaming away from Earth
- Triggered search is more sensitive



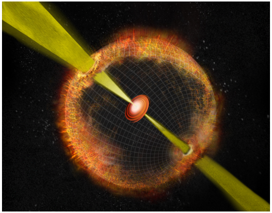
$\Rightarrow$ interesting even for small jet opening angles

- Reference: fraction found by all-sky search with γ-ray counterpart

$\Rightarrow$ Two approaches see (mostly) independent events

Summary

- Long and short GRBs progenitors may produce large amounts of GWs
- No associated GW detection to date
- Some relevant exclusions: GRB070201, GRB051103
- Good prospects for first detection with advanced detectors $\gtrsim$ 2015
- Joint GW-γ observation should determine the nature of GRB central engine



References

B. P. Abbott et al., Astrophys. J. **713**, 671 (2010).

J. Abadie et al., Astrophys. J. **737**, 93 (2011a).

J. Abadie et al., Astrophys. J. **737**, 93 (2011b).

S. Kobayashi and P. Mészáros, Astrophys. J. **589**, 861 (2003).

A. L. Piro and E. Pfahl, Astrophys. J. **658**, 1173 (2007).

G. E. Romero, M. M. Reynoso, and H. R. Christiansen, Astron. Astrophys. **524**, A4 (2010).

M. Shibata, K. Shigeyuki, and E. Yoshiharu, Mon. Not. R. Astron. Soc. **343**, 619 (2003).

M. B. Davies, A. King, S. Rosswog, and G. Wynn, Astrophys. J. Lett. **579**, L63 (2002).

B. P. Abbott et al., Astrophys. J **681**, 1419 (2008).

J. Abadie et al., arXiv:1201.4413 (2012), lIGO-P1000097.

E. P. Mazets et al., Astrophys. J. **680**, 545 (2008).

H. Hurley et al., Mon. Not. R. Astron. Soc. **403**, 342 (2010).

I. Leonor et al., Class. Quantum. Grav **26**, 204017 (2009).

Signal to noise ratio – SNR

- Perfectly known signal $s(t) \leftrightarrow \tilde{s}(f)$

$$\mathrm{SNR}^2_{\mathrm{optimal}} = 2 \int_{-\infty}^{\infty} \frac{|\tilde{s}(f)|^2}{A(|f|)^2} \mathrm{d}f$$

- Whitened signal/data

$$\tilde{d}^w(f) = \tilde{d}(f) \times \frac{\sqrt{2}}{A(|f|)}$$

- Matched filtering: scalar product between template and data

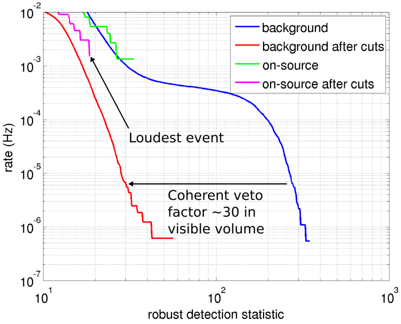
$$\mathrm{SNR} = \int_{-\infty}^{\infty} \tilde{s}^w(f)^* \tilde{d}^w(f) \mathrm{d}f \Big/ \sqrt{\int_{-\infty}^{\infty} |\tilde{s}^w(f)|^2 \mathrm{d}f}$$

- $d(f) = n(f)$ – SNR normally distributed
- $d(f) = s(f) + n(f)$ – distribution mean is shifted by $\mathrm{SNR}_{\mathrm{optimal}}$

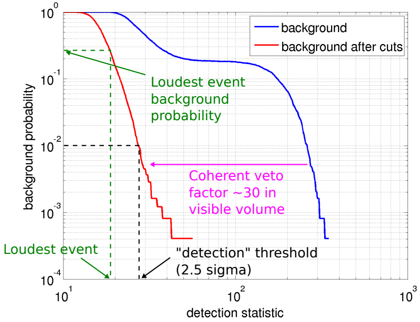
$\Rightarrow$ $\mathrm{SNR}_{\mathrm{optimal}}$ – detectability in perfect conditions

Background estimation

Event rate above threshold



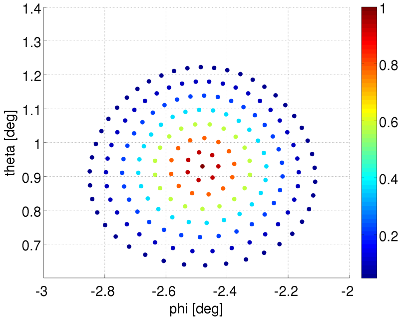
background probability in window



Background probability $\simeq$ background event rate $\times$ time window length

Loudest event in on-source window $\Rightarrow$ Effective clustering over the window

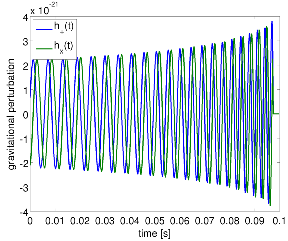
Sky position error box



$$L(\mathbf{d}|\circlearrowright, \sigma_h) = \frac{|\mathbf{e}^{\circlearrowright} \cdot \mathbf{d}|^2}{1 + 1/(\sigma_h |\mathbf{f}^{\circlearrowright}|)^2} - \log(1 + \sigma_h^2 |\mathbf{f}^{\circlearrowright}|^2),$$

$$L(\mathbf{d}|\text{circular}) = 2 \log \sum_{\sigma_h \in \mathcal{A}} \frac{\max\left\{\exp\left[\frac{1}{2} L(\mathbf{d}|\circlearrowright, \sigma_h)\right], \exp\left[\frac{1}{2} L(\mathbf{d}|\circlearrowleft, \sigma_h)\right]\right\}}{|\mathcal{A}|}.$$

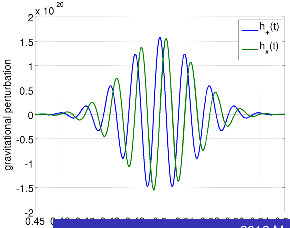
Sensitivity estimation - signal models



- Compact object coalescence (inspiral)
  - BH-NS: $m_{\text{BH}} = 10 \pm 6\,\text{M}_\odot$
    $m_{\text{NS}} = 1.4 \pm 0.4\,\text{M}_\odot$
  - NS-NS: $m_{\text{NS}} = 1.4 \pm 0.2\,\text{M}_\odot$
- Extreme stellar collapse
  - Ad-hoc model to sample parameter space – sine-Gaussian

$$\begin{bmatrix} h_+(t+t_0) \\ h_\times(t+t_0) \end{bmatrix} = A_0 \begin{bmatrix} \cos(2\pi f_0 t)(1+\cos^2\iota) \\ \underbrace{\sin(2\pi f_0 t)}_{\text{rotation}}\ \underbrace{2\cos\iota}_{\text{inclination}} \end{bmatrix} \underbrace{\exp\left[-\frac{(2\pi f_0 t)^2}{2Q^2}\right]}_{\text{envelope}}$$

  - $f_0 = 2f_{\text{rot}} = 100, 150, 300\,\text{Hz},\ Q = 9$
  - $E_{\text{GW}} = 10^{-2}\,\text{M}_\odot c^2$, distance $\rightarrow A_0$
- Nuisance parameters $\rightarrow$ jitter injections
  - Sky localization error
  - Calibration errors
  - System inclination ι



Astrophysical framework

- GRB progenitors (CBC, hypernova,...) are standard GW sirens $\Rightarrow$ fixed $E_{\rm GW}$
- Uniform distribution in space
- Rotator GW emission pattern (binary, bar mode, ...)
  - face on $\rightarrow$ circular polarization
  - edge on $\rightarrow$ linear polarization
  - inclination ι $\rightarrow$ elliptical

$$\begin{pmatrix} h_+ \\ h_\times \end{pmatrix} \propto \begin{pmatrix} 1+\cos^2\iota \\ 2\cos\iota \end{pmatrix}$$

  - GW power flux dependence on ι, slight GW beaming

$$F(\iota) = \frac{(2\cos\iota)^2 + (1+\cos^2\iota)^2}{8}$$

- γ-ray emission in a cone around rotation axis, top hat emission
  - two sided jet $\rightarrow$ $\iota \in [0, \pi/2]$
  - jet of opening angle θ_j (thought to be in $5-30°$ range)
  - $\iota < \theta_j$ $\rightarrow$ GRB detected on Earth
  - $\iota > \theta_j$ $\rightarrow$ progenitor dark in γ-rays (missed by exttrig search)

Theoretical comparison

Issue

- All-sky searches for GW from all progenitors
- Exttrig searches only for progenitors with $\iota < \theta_j$

$\Rightarrow$ Gain in sensitivity $\leftrightarrow$ loss in GW source density rate

Analysis toy model

- Forget about ITF antenna patterns
- Analysis detection based on a sharp threshold on h_{rss}
  - At given inclination ι analysis efficiency drops from 1 to 0 at horizon distance $r(\iota)$
  - Simple dependence on inclination: $r(\iota) = \sqrt{F(\iota)}r(0)$
- Hopefully $r_{\text{exttrig}}(\iota) > r_{\text{all-sky}}(\iota)$ for $\iota < \theta_j$
- Effective search volume, marginalize over inclination
  - For all-sky

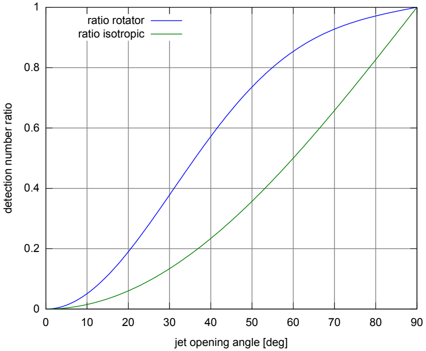
$$V_{\text{all-sky}} = \int_{\iota=0}^{\pi/2} \frac{4\pi}{3} r^3_{\text{all-sky}}(\iota)\sin(\iota)\mathrm{d}\iota$$

  - For exttrig

$$V_{\text{exttrig}} = \int_{\iota=0}^{\theta_j} \frac{4\pi}{3} r^3_{\text{exttrig}}(\iota)\sin(\iota)\mathrm{d}\iota$$

Detection rate ratio

- Detection rate ratio $R(\theta_j)$ for equal horizons: $r_{\text{all-sky}} = r_{\text{exttrig}}$



$$R(\theta_j) = \frac{V_{\text{exttrig}}(\theta_j)}{V_{\text{all-sky}}}$$

$$r(\iota) = \sqrt{F(\iota)}\,r(0)$$

$$F_{\text{rotator}}(\iota) = \frac{(2\cos\iota)^2 + (1+\cos^2\iota)^2}{8}$$

$$F_{\text{isotropic}} = 1$$

- For other sensitivity ratio, multiply curve by $(r_{\text{exttrig}}/r_{\text{all-sky}})^3$

$\Rightarrow$ GW beaming helps the exttrig approach by a factor ~ 3 (in the small opening angle limit)

$\Rightarrow$ If GRBs are very beamed exttrig search is useless