Framework
Diffusion equation
Standard deviation of the flux
Variance is infinite but confidence levels are finite
Catalog of sources

Can the local sources of cosmic rays explain the proton flux ?

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Motivations

Indirect detection of dark matter

- \rightarrow Predict the signal
- ightarrow Understand the background

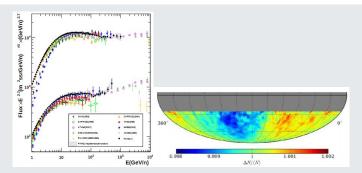


Figure: Proton Flux as observed by PAMELA Colaboration. Adriani et al arxiv:1103.4055,Cosmic rays anisotropy as observed by IceCube Toscano et al arxiv:1110.207



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Diffusion Model

The propagation of charged particles in the interstellar medium is determined by the structure of the galactic magnetic field.

- Regular component (around 2μ gauss)
- Stochastic component
- → Fluctuations of the magnetic field on every scale

Turbulence ~ Larmor Radius

Slightly turbulent regime

→ Diffusion process

Defined by a diffusion coefficient

$$D = D_0 \beta R^{\delta}$$

 D_0 Related to the amplitude of turbulence δ Related to the kind of magnetic turbulence



The diffusion equation

We solve a diffusion equation taking into account :

- diffusion (diffusion coefficient ? anisotropic ? inhomogeneous ?)
- escape (boundary conditions, geometry of diffusion volume)
- spallations (creation/destruction, cross-sections, interstellar medium)
- energy losses
- diffusive reaccerelation
- galactic wind (convection)
- Sources (distribution, spectra)



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Diffusion parameters

- D_0 and δ
- L height of the diffusive halo
- Vc convective wind
- Distribution of sources (supernova, sometimes we use pulsars distribution)
- Explosion rate of supernova



Previous work

The results will be presented for three benchmark sets of diffusion parameters, consistent with the energy dependance of the B/C ratio

Table: Diffusion parameters for the three benchmark models

model	$D_0 (\mathrm{kpc} \cdot \mathrm{My}^{-1})$	δ	L (kpc)	$V_c \; ({ m km \cdot s^{-1}})$
min	0.0016	0.85	1	13.5
med	0.0112	0.7	4	12
max	0.0765	0.46	15	5



The myriad model

We treat supernova explosions as point-like events Production rate of protons given by :

$$q_{\text{acc}}(\mathbf{x}_{S}, t_{S}) = \sum_{n \in \mathscr{P}} q_{n} \, \delta^{3}(\mathbf{x}_{S} - \mathbf{x}_{n}) \, \delta(t_{S} - t_{n}) , \qquad (1)$$

Each source n belonging to the population $\mathscr P$ of supernovae contributes a factor q_n at position $\mathbf x_n$ and time t_n .

ightarrow Infinite set of realisation possible in the range $t_{
m min} < t_n < t_{
m max}$ and $R_{
m min} < r_n < R_{
m max}$

Is it possible that a statistical variation of the flux gives rise to observed structures?



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Diffusion equation

$$\frac{\partial \psi}{\partial t} + \partial_z (V_C \psi) - K \triangle \psi = \sum_i q_i \, \delta^3(\mathbf{x}_S - \mathbf{x}_i) \, \delta(t_S - t_i) - 2 \, h \, \delta(z_S) \, \Gamma_p \, \psi(\mathbf{x}_S, t_S) . \tag{2}$$

$$\psi(\mathbf{x},t) = \int_{-\infty}^{t} dt_{S} \int_{\mathrm{DH}} d^{3}\mathbf{x}_{S} \,\mathcal{G}_{p}(\mathbf{x},t \leftarrow \mathbf{x}_{S},t_{S}) \, q_{\mathrm{acc}}(\mathbf{x}_{S},t_{S}) , \qquad (3)$$



Diffusion equation

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Mean value

$$\left\langle \mathscr{G}_{\textit{N}} \right\rangle = \textit{NA} \int 2\pi \, r_{\textit{s}} dr_{\textit{s}} \int d\theta_{\textit{s}} \int dz_{\textit{s}} \int dt_{\textit{s}} f_{\theta}(\theta) f_{r}(r_{\textit{s}}) f_{z}(z_{\textit{s}}) f_{t}(t_{\textit{s}}) \mathscr{G}_{\textit{N}}\left(r_{\textit{s}}, \theta_{\textit{s}}, t_{\textit{s}}\right)$$

The source distribution in space are those from Yusifov and Küçük (2004)



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Variance of the propagator

The second order momentum of the distribution diverges if we chose $R_{\min}=0$ and $t_{\min}=0$

$$\langle \mathcal{G}_1^2 \rangle \sim \frac{A}{4\pi D} \sum_{n,m} \langle \mathcal{G}_z^2 \rangle_{n,m} \ln R_{\min}$$
 (4)

how bad is it?

Does it means that the actual value of the flux had a disturbingly large probability to be very far from the mean value.



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Regularisations

- Regularisation using a cut-off (Blasi et Amato (2011))
- Confidence intervals
- Catalog

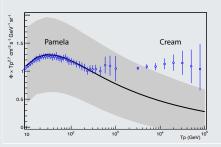


Regularisation introducing a cut off

Allowing sources to explode in the solar neighborhood is not physical : we put a cut-off on the ages of sources.

We choose $t_{min} = 100$ yr, physical limit of the diffusion at high energies.

ightarrow At 10 5 GeV, solutions of the propagation equation for sources youngers than 100 yr are non physical because of the non-relativistic nature of the equation



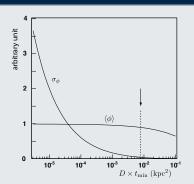
Variance of the flux computed with a 100 yr cut-off



Energy dependant cut-off

Work of Blasi et Amato(2011) $t_{\rm min}=R_{\rm max}/\sqrt{4\nu D(E)}$ corresponds to the limit for wich sources contributes very weakly to the flux

Influence of the cut-off



The value of $\sigma_{\phi}/\langle \phi \rangle$ depends sensitively on the chosen cut-off $t_{\rm min}$. For a very low value of $t_{\rm min}$, the standard deviation is very large and vice versa



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This confusing situation, where some rare events have a very small contribution to the mean, but give rise to a very large standard deviation, is not uncommon in physics (Cauchy distribution).

 \rightarrow The distribution of fluxes is such that the central limit theorem is not valid in its usual form.



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Very simple framework

We computed the probability for a flux from one source to be higher than a given value $\boldsymbol{\Phi}$ reads

$$P(>\Phi)=rac{\langle\phi
angle^2}{4\Phi^2}\,\Theta\left(\Phi-rac{1}{2}\langle\phi
angle
ight)$$

The probability that $\Phi > 10\langle \phi \rangle$ is only 1/400

For N source the effect is even more important. Considering 100 sources, $P(>2\langle\Phi\rangle)=2.510^{-3}$



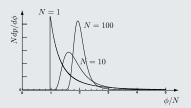
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Probability distribution of Φ/N for N=1, 10 and 100 sources.



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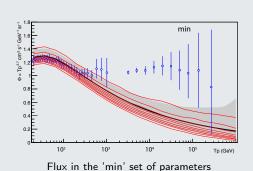
Quantiles

Using this information, we rather draw 68% confidence intervals, we also show 10% quantiles as a good estimator of theoretical uncertainties. (Monte-Carlo simulation)

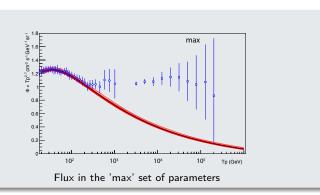


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The flux is very unlikely to be far from the mean value

The spread of the distribution is very sensitive to the thickness 'L' of the diffusive halo

Changing the cut-off t_{min} will only change extremal deciles.



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We know local sources

There are strong arguments to think that sources responsible for spread quantiles or large variance, are located in a region nearby the Sun. Following Delahaye et al. (2010), we have made use of two catalogs:

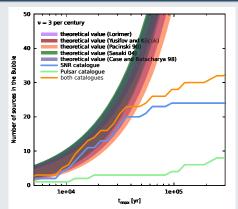
- Green catalog of supernova (Green (2009))
- ATNF pulsar catalog (Manchester (2005))

These catalogs are used to describe sources younger than $10^5\ \rm yr$ in a region of radius 2 kpc around the sun.

We created a new statistical set exluding this region, \rightarrow **no divergence**



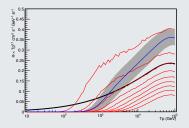
Are these sources representative of the local environement?



Theoretical number of sources and sources from the catalog considering an explosion rate of 3 supernova per century in a 2kpc region arround solar system.



Is the catalog probable?



We compare the flux from the known sources to the flux from a random set of population.

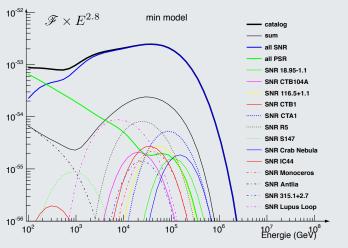
 \rightarrow The flux from the catalog is in the 68% confidence interval, including its theoretical uncertainties.

The flux from the local sources is very unlikely to produce important fluctuations at $200 \,\, \text{GeV}$



Framework

Contributions of the fluxes





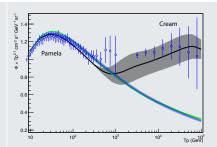


Variing the explosion rate

We draw the flux considering the catalog and with an explosion rate of 1 supernova per century in the galaxy.

In this assumption : the local rate of explosion is higher than the mean explosion rate in the galaxy

This result is not shown a statistical variation but an effect of local propagation parameters



We now draw the flux including the catalog for th'min' and 'max' benchmarks parameters considering an explosion rate of 1 supernova per century in the galaxy.



conclusion

- Use of confidence intervall rather than variance to compute the theoretical uncertainty of the flux
- The catalog is the best way to get rid of all divergence
- Only under certain assumptions (small L, small explosion rate) the confidence intervall are high enough to allow sufficiently big fluctuations
- Local diffusion parameters seem to be a good way to provide an explanation to the bump.

