

Absolute luminosity measurements at LHCb



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25/11/2011

- 1 Introduction: Luminosity**
- 2 LHC & LHCb**
- 3 Relative luminosity measurement**
- 4 Absolute luminosity measurement**
 - Beam intensity measurement
 - van der Meer scan method
 - Beam-gas imaging method

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INTRODUCTION

What is luminosity?

- **Interaction rate = Luminosity \times Cross-section** ($R = L \times \sigma$)
- Luminosity for two identical Gaussian bunches, colliding head-on and without offset:

$$L = \frac{N_1 N_2 f}{4\pi \sigma_x \sigma_y}$$

- $N_{1,2}$: bunch populations
- f : revolution frequency
- $\sigma_{x,y}$: transverse widths

Why do we need to know it?

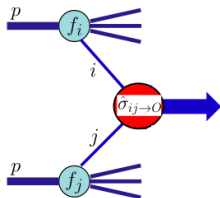
- Measure particle production cross-sections
- Quantify the collider performance

RELEVANCE OF ABSOLUTE LUMINOSITY MEASUREMENTS (1)

Theoretical description

- Hadron collisions \rightarrow QCD
 - Factorization
 - Calculation of hard-scattering cross-sections

$$\sigma_{pp \rightarrow O} = \int_0^1 dx_1 dx_2 \times \sum_{i,j} f_i(x_1, Q^2) f_j(x_2, Q^2) \hat{\sigma}_{ij \rightarrow O}$$

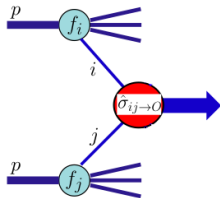


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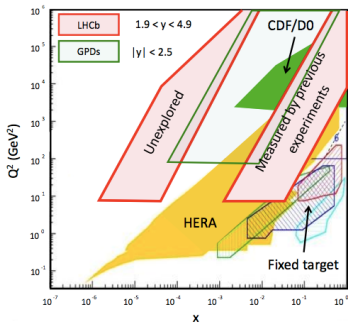
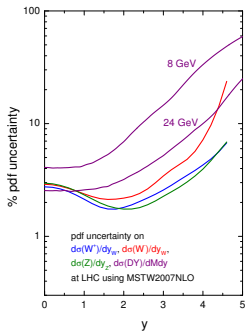
Measure cross-sections

- Test models of heavy flavor production (use knowledge of PDFs)
- Improve PDFs (use well-known partonic processes)

RELEVANCE OF ABSOLUTE LUMINOSITY MEASUREMENTS (2)

Improve knowledge of PDFs

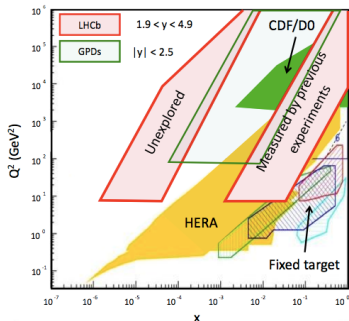
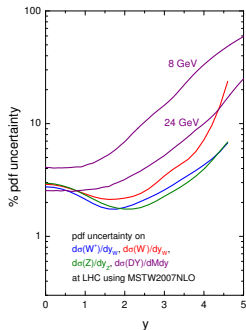
- W/Z and low-mass Drell-Yan production
 - At large rapidity, the production cross-section uncertainty is dominated by PDFs
 - Cross-section measurements at LHCb can constrain PDFs



RELEVANCE OF ABSOLUTE LUMINOSITY MEASUREMENTS (2)

Improve knowledge of PDFs

- W/Z and low-mass Drell-Yan production
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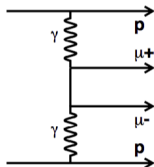
Partonic cross-section for Z production known to $\sim 2\%$
 Defines a target for the knowledge of the absolute luminosity: 2%

METHODS FOR MEASURING THE LUMINOSITY: *Indirect*

Do not rely on the knowledge of the beam parameters

Using a “reference” reaction (σ_{ref})

- $L = R_{\text{ref}}/\sigma_{\text{ref}}$
- Previously measured cross-section
- Precisely calculable process, *e.g.* $pp \rightarrow pp\mu^+\mu^-$
 - At LHCb with 1 fb^{-1} the expected precision is $\sim 2\%$
[J. Anderson, CERN-THESIS-2009-020]



Optical theorem and Coulomb interference region

- Measure elastic pp scattering at low momentum transfer
 - Determine total pp cross-section and luminosity
- Used in TOTEM/CMS and ALFA/ATLAS
 - Expected ultimate precision $\sim 1 - 3\%$ [TOTEM TDR, CERN-LHCC-2004-002],
[ATLAS-ALFA, CERN-LHCC-2008-004]

Indirect methods not discussed in the rest of this talk

METHODS FOR MEASURING THE LUMINOSITY: *Direct*

Determine L from beam parameters

- General formula for one pair of colliding bunches:

$$L = N_1 N_2 f 2c \cos^2 \alpha \int \rho_1(\mathbf{x}, t) \rho_2(\mathbf{x}, t) d^3x dt$$

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- Bunch revolution frequency: 11.245 kHz

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-
- Measurement of bunch populations is essential for all methods
 - Different approaches for the measurement of the Beam Overlap Integral
 - Wire scan method (not discussed) [J. Bosser et al., NIM A 235 (1985)]
 - **van der Meer scan method** [S. van der Meer, CERN-ISR-PO-68-31]
 - **Beam-gas imaging method** [M. Ferro-Luzzi NIM A 553 3 (2005)]

1 Introduction: Luminosity

2 LHC & LHCb

3 Relative luminosity measurement

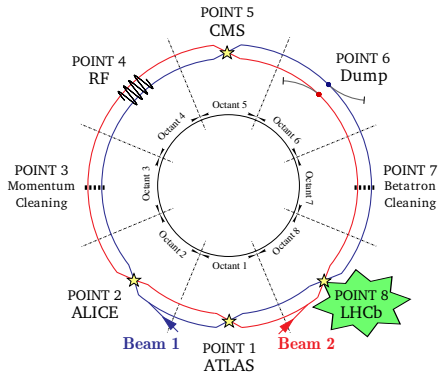
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LARGE HADRON COLLIDER

LHC

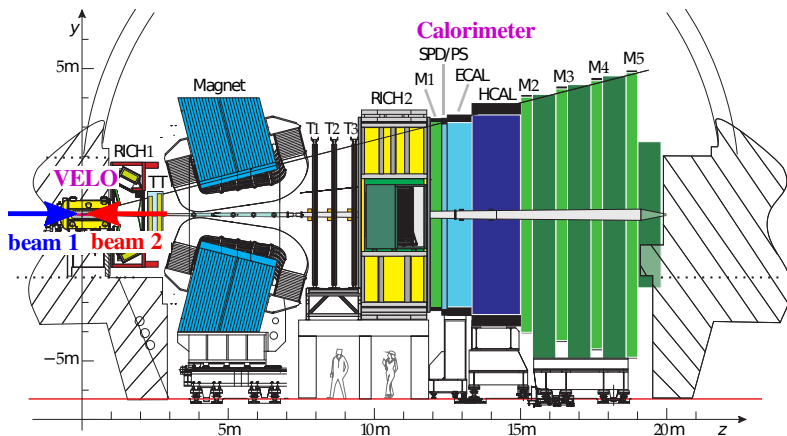
- 27-km accelerator and collider
- Separate vacuum pipes for the two beams
- 4 collision points
- Beam instrumentation at Point 4



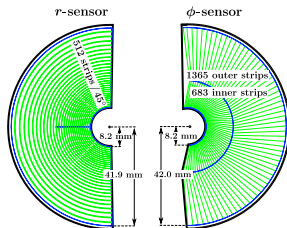
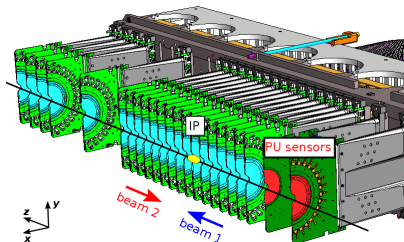
Parameter	2010	Nominal
Beam energy [TeV]	3.5	7
β^* [m]	2/3.5 (all IP)	0.55/10
Norm. emittance [$\mu\text{m rad}$]	2 – 3.5	3.75
Maximum $n_{\text{coll}}/\text{IP}$	348	2808
Bunch intensity [protons]	$2 \times 10^{10} - 1.2 \times 10^{11}$	1.15×10^{11}
Max. peak luminosity [$\text{cm}^{-2} \text{s}^{-1}$]	2×10^{32}	10^{34}

LHCb DETECTOR

- Dedicated to b -physics
- Single arm spectrometer, covers $\eta \in [2, 5]$
- Design luminosity: $2 \times 10^{32} \text{cm}^{-2} \text{s}^{-1}$ (2fb^{-1} per year)
 - Low probability for multiple interactions per bunch-crossing
- Vertex LOcator (VELO) and Calorimeter are essential for the luminosity measurement



VERTEX LOCATOR AND CALORIMETER SYSTEM

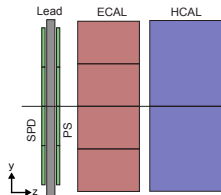


VELO

- 21 stations with r - and ϕ -measuring sensors and 2 “Pile-Up” (PU) stations
- Good acceptance for displaced vertices

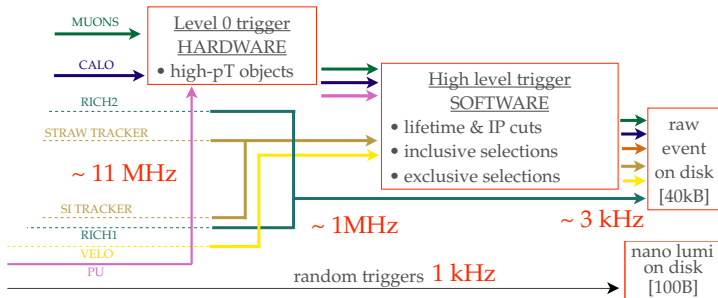
Calorimeter

- Four detectors located at $z = 12 - 15$ m
- SPD (Scintillating Pad Detector)
- PS (PreShower)
- ECAL and HCAL



LHCb TRIGGER

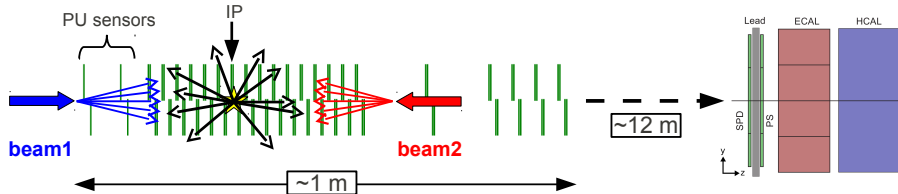
- L0: custom electronics (40 MHz)
 - Reduce visible interaction rate 11 MHz \rightarrow 1 MHz
- HLT: farm of 2000 multi-processor boxes
 - Select 3 kHz of events to be written to disk



Triggers for lumi measurement

- Acquire 1 kHz of random triggers for the measurement of the relative luminosity
- Dedicated beam-gas selections in L0 and HLT

BEAM-GAS TRIGGER



Types of BX (bunch crossings)

- bb : bunches of beam1 and beam2 are filled
- be : bunch of beam1 is filled, bunch of beam2 is empty
- eb : bunch of beam1 is empty, bunch of beam2 is filled
- ee : bunches of beam1 and beam2 are empty

L0 beam-gas

- Only in non-bb BX
- Exploit the directionality of the interaction products
- Use hit multiplicity in PU and transverse energy in HCAL

HLT beam-gas

- All types of BX
 - In bb BX search for beam-gas overlapped with pp ; veto lumi region
- Look for VELO tracks coming from the same point in z (pseudo-vertex)
 - z within 1.5–2 m from IP

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MEASUREMENT OF THE RELATIVE LUMINOSITY (1)

For two colliding bunches: $L = \frac{f\mu_{\text{vis}}}{\sigma_{\text{vis}}}$

- Valid for an arbitrary process
- The *absolute* luminosity calibration determines σ_{vis}
 - It is the result of a comprehensive analysis over short periods of data taking
- Measuring the *relative* luminosity means measuring μ_{vis}
 - Needs to be done in all data-taking periods

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Measuring μ_{vis}

- Different methods exist to determine μ_{vis} from “luminosity counters” (multiplicity of fired channels in a given detector, reconstructed tracks, etc.)
- LHCb uses the “zero-count” method
 - Assuming that the number of visible pp interactions follows a Poisson distribution:
 $\mu_{\text{vis}} = -\ln P(0)$
 - $P(0)$ is determined from a sample of randomly triggered events

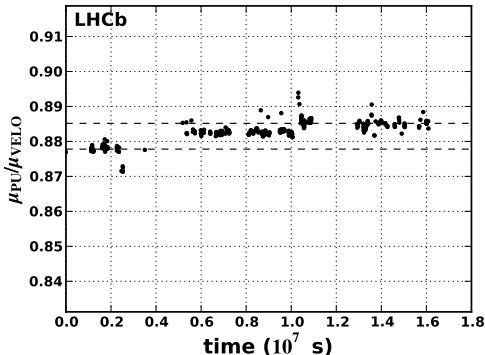
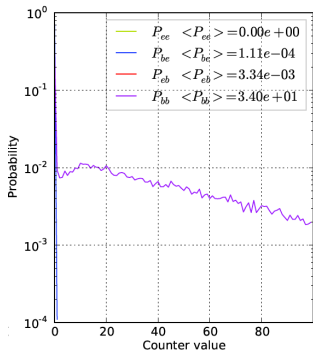
MEASUREMENT OF THE RELATIVE LUMINOSITY (2)

Choosing lumi counters

- Stability over time
- Possibility to be measured easily online

In 2010 LHCb used

- zero-count method and number of VELO tracks
 - “Empty” events defined as having 0 or 1 VELO track
 - Counter stability determined by comparing μ_{VELO} with μ_{vis} obtained from other counters



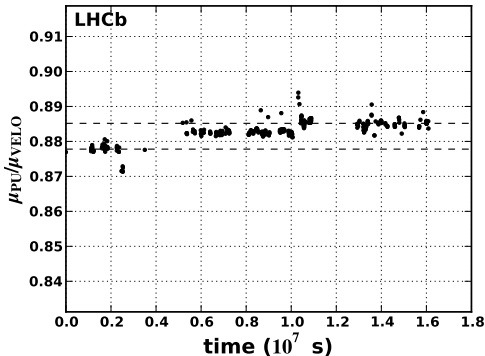
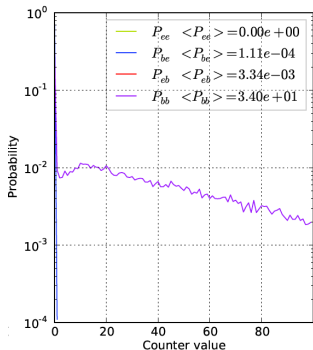
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Systematic error of the relative luminosity measurement in 2010: 0.7%

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 - Beam intensity measurement
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Rate of visible
interactions

Luminosity

Visible pp
cross-section

$$\text{Rate}_{\text{vis}} = n_{\text{bunches}} \cdot N_1 N_2 \cdot f \cdot 2c \cdot \text{Beam Overlap Integral} \times \sigma_{\text{vis}}$$

Measurement of bunch intensities

BUNCH CURRENT ANALYSIS – INTRODUCTION

DCCT (DC Current Transformer):

- Measures the total current in each of the two LHC beams N_{tot}
- Calibrated with a precise current source
- Noise $\mathcal{O}(10^9)$ charges (nominal bunch: 1.15×10^{11} charges)

FBCT (Fast Beam Current Transformer):

- Relative measurement of the current in all 25-ns bunch slots S_i ($i = 1, \dots, 3564$)
- Noise $\mathcal{O}(10^8)$ charges

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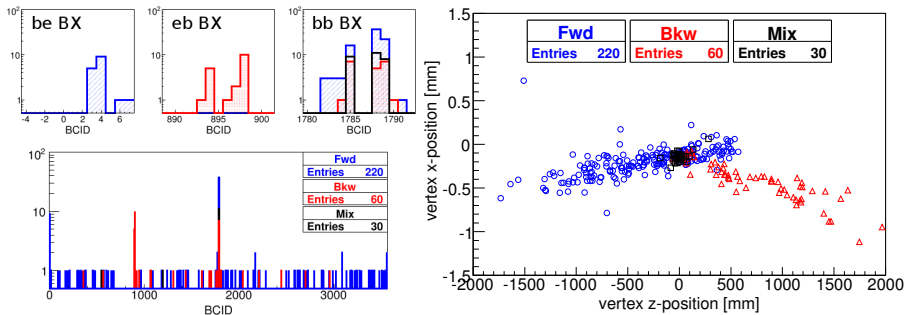
Estimation of the bunch populations

- For each beam define a common scale factor:

$$a = \frac{N_{\text{tot}} - N_{\text{ghost}}}{\sum_{i \in \text{filled slots}} S_i}$$

- Ghost charge N_{ghost} : occupies nominally-empty slots
 - Visible to DCCT, but not to FBCT
 - **In 2010 measured by LHCb**
- Individual bunch populations: $N_i = a S_i$

- Rate of beam-gas interactions \propto beam current
- Determined ghost charge from ratio of events in ee and be/eb crossings
 - Forward, Backward and Mixed vertices
 - BCID (Bunch Crossing ID)



- The typical number of beam-gas interactions used in the measurement (per beam):
 - be/eb BX: $\mathcal{O}(10^4)$
 - ee BX: $\mathcal{O}(10^2)$ (large fraction is concentrated near the filled bunches)

GHOST CHARGE MEASUREMENT (2)

- Calculation of the ghost charge fraction in beam1 (similarly for beam2)

$$f_{\text{ghost},1} = \frac{N_{\text{ghost},1}}{N_{\text{tot},1}} = \frac{F_{\text{ee}}}{F_{\text{be}}} \times \frac{\sum_{i \in \text{be}} S_i}{\sum_{i=1}^n S_i}$$

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 - ▶ Ratio of the number of forward vertices in ee and be crossings
 - ▶ Beam1 current in be crossings compared to the total current in beam1

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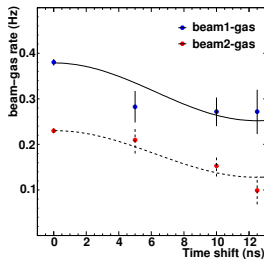
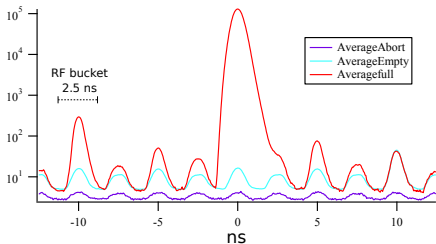
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- This method has intrinsic time granularity of 25 ns
- A systematic error arises due to
 - Unknown distribution of the ghost charge inside the 25 ns bunch slots
 - Non-uniform efficiency of the LHCb trigger with respect to timing shifts

charge distribution in a 25 ns bunch slot

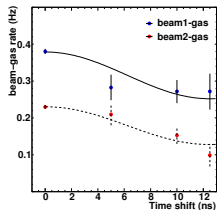


GHOST CHARGE MEASUREMENT (3)

Systematic error due to unknown distribution in the 25 ns slots

- Method 1: Average of two “extreme” cases
 - Case 1: All charge is in the central RF bucket \rightarrow trig.eff. = 100%
 - Case 2: Charge is equally distributed in the 5, 10 and 12.5 ns RF buckets
- Method 2: Assume uniform distribution in all RF buckets
 - Fit the measured trigger efficiency to the sum of a cosine and a constant

	Beam1	Beam2
5, 10, 12.5 ns average	0.73	0.67
$\epsilon_{\text{average}}$	0.86 ± 0.14	0.84 ± 0.16
ϵ_{fit}	0.83 ± 0.04	0.78 ± 0.04

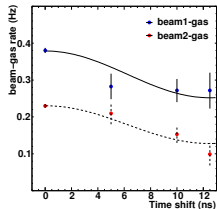


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- To apply the correction for the ghost charge calculate:

$$f_g = \left(1 - \frac{f_{g1} \pm \Delta f_{g1}}{\epsilon_{\text{average1}}}\right) \times \left(1 - \frac{f_{g2} \pm \Delta f_{g2}}{\epsilon_{\text{average2}}}\right)$$

The typical values of the corrections is 1%
 Assume systematic equal to half of the correction, *i.e.* $\sim 0.5\%$

BUNCH CURRENT ANALYSIS – RESULTS

Analysis performed by the Bunch Current Normalization WG

- Results for the May and October 2010 data taking periods are documented in [\[CERN-ATS-Note-2011-004\]](#), [\[CERN-ATS-Note-2011-016\]](#)
- Used by all experiments

Uncertainties

Source	Uncertainty on $N_1 N_2$	Estimation method
DCCT baseline offset	Up to a few % for “low-intensity” fills	DCCT measurements over long periods (hours) before and after fills. Min-to-max of noise defines the error.
DCCT absolute scale	2.7%	Scale reproducibility in three absolute calibrations in 2010. Peak to peak variations transformed to 68% CL.
FBCT offset	2 – 3%	Comparison to ATLAS BPTX.
Ghost charge	$\sim 0.5\%$	LHCb measurement of beam-gas interactions in ee crossings.

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FBCT offset	2 – 3%	Comparison to ATLAS BPTX.
Ghost charge	~ 0.5%	LHCb measurement of beam-gas interactions in ee crossings.

**These four uncertainties are uncorrelated
The total error on the bunch current product
depends on the fill**

Rate of visible
interactions

Luminosity

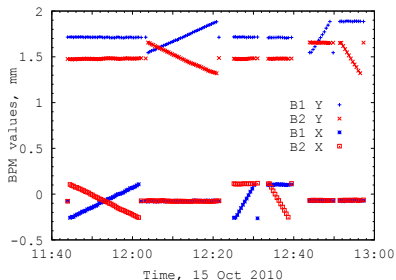
Visible pp
cross-section

$$\boxed{\text{Rate}_{\text{vis}}} = \boxed{n_{\text{bunches}} \cdot N_1 N_2 \cdot f \cdot 2c \cdot \text{Beam Overlap Integral}} \times \boxed{\sigma_{\text{vis}}}$$

Absolute luminosity determination with the
van der Meer method

VAN DER MEER METHOD – INTRODUCTION

- Measure interaction rate while separating the two beams
- One “scan” = consecutive separations in each transverse direction



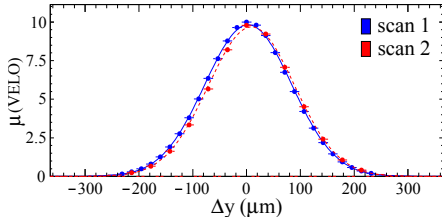
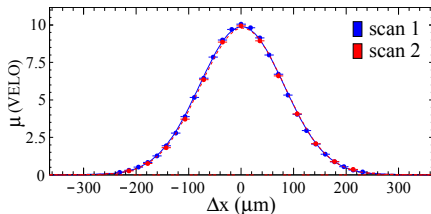
- For a single bunch pair:

$$\sigma_{\text{vis}} = \frac{\int \mu_{\text{vis}}(\Delta_x, \Delta_{y_0}) d\Delta_x \int \mu_{\text{vis}}(\Delta_{x_0}, \Delta_y) d\Delta_y}{N_1 N_2 \cos \alpha \mu_{\text{vis}}(\Delta_{x_0}, \Delta_{y_0})}$$

- ▶ $\mu_{\text{vis}}(\Delta_x, \Delta_y)$: average number of interactions per crossing as function of the separation in x and y
 - ▶ $\Delta_{x_0}, \Delta_{y_0}$: beam separation at the nominal (“working”) point
 - ▶ $N_{1,2}$: bunch intensities
 - ▶ α : half crossing angle
- LHCb scans in 2010: April and **October**

VDM SCANS – CROSS-SECTION DETERMINATION

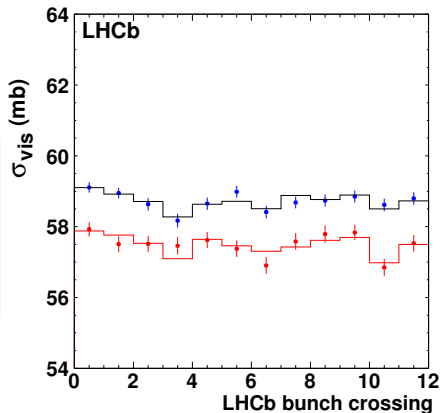
- “VDM profiles” summed over all bunches



- Fit the VDM profiles with a double Gaussian
- Fit simultaneously the x and y profiles, constrain $\mu_{\text{vis}}(\Delta_{x_0}, \Delta_{y_0})$ to be the same
- Length scale calibration: use VELO vertex measurements to cross-check absolute beam displacement provided by LHC

VDM SCANS – RESULTS (1)

- Results from the two scans in October:
 - indicated with circles
 - $\sim 2\%$ discrepancy between the two scans, included as a systematic
- Fit the results of all 12 colliding bunches
 - Use FBCT offsets as free parameters
 - Fit functions are indicated with lines



VDM SCANS – RESULTS (2)

Largest systematic errors

Source	April (%)	Oct (%)
DCCT scale	2.7	2.7
DCCT offset and FBCT	4.9	0.2
Ghost charge	0.08	0.15
Working point stability	–	0.4
Length scale	2	1
x - y coupling	–	0.3
Difference between scans	4.4	2.1

		relative uncertainty (%)		
	σ_{VELO} (mb)	total	systematic	statistical
April	59.7	7.5	7.4	0.9
October	<u>58.35</u>	3.64	3.64	0.09

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Better precision in October: result retained as a final 2010 VDM luminosity calibration

Rate of visible
interactions

Luminosity

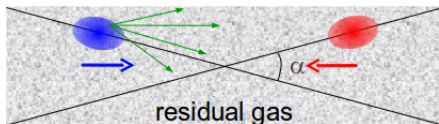
Visible pp
cross-section

$$\boxed{\text{Rate}_{\text{vis}}} = \boxed{n_{\text{bunches}} \cdot N_1 N_2 \cdot f \cdot 2c \cdot \text{Beam Overlap Integral}} \times \boxed{\sigma_{\text{vis}}}$$

Absolute luminosity determination with the
beam-gas imaging method

BEAM-GAS IMAGING METHOD (BGI) [M. Ferro-Luzzi NIM A 553 3 (2005)]

$$L = N_1 N_2 f 2c \int \rho_1(\mathbf{x}, t) \rho_2(\mathbf{x}, t) d^3x dt$$



- Residual gas near interaction point used as a beam visualizing medium
 - Reconstruct beam-gas interaction vertices to measure
 - beam crossing angles
 - positions and shapes of the colliding bunches
 - Determine the Beam Overlap Integral of the colliding bunches
-
- **Use the copious and better measured pp vertices to improve the precision on the beam overlap**

Strength with respect to the VDM method

- Avoid effects related to beam displacement
- Can be applied during physics fills

BGI method requires

- Vertex resolution better or comparable to the beam size ($\mathcal{O}(30) \mu\text{m}$ in x/y)
- Sufficient residual pressure and good acceptance for beam-gas interactions

MEASUREMENT OF THE Beam Overlap Integral WITH THE BGI METHOD

- Assuming two Gaussian bunches (transverse widths σ_{ij} , $i = 1, 2$, $j = x, y$):

$$L = \frac{f N_1 N_2}{2\pi \sqrt{(\sigma_{1x}^2 + \sigma_{2x}^2)(\sigma_{1y}^2 + \sigma_{2y}^2)}} \times C_{\text{offset}} \times C_{\text{angle}}$$

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- Measure bunch widths
 - Measure the beam angles
 - Determine primary vertex resolution

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- Measure bunch widths
 - Measure the beam angles
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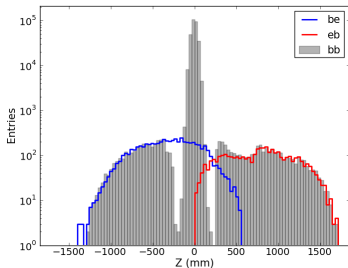
The analysis is applied for each individual colliding bunch pair

DATA SELECTION

- Select fills on the basis of stability and availability of all data

Fill	n_{tot}	n_{coll}	N	T (h)	#ev 1	#ev 2
1089	2	1	$2 \cdot 10^{10}$	15	1270	720
1090	2	1	$2 \cdot 10^{10}$	4	400	300
1101	4	2	$2 \cdot 10^{10}$	6	730	400
1104A	6	3	$2 \cdot 10^{10}$	5	510	350
1104B	6	3	$2 \cdot 10^{10}$	5	520	350
1117	6	3	$2 \cdot 10^{10}$	6	700	500
1118	6	3	$2 \cdot 10^{10}$	5	500	400
1122	13	8	$2 \cdot 10^{10}$	3	300	250

- Online: dedicated beam-gas trigger



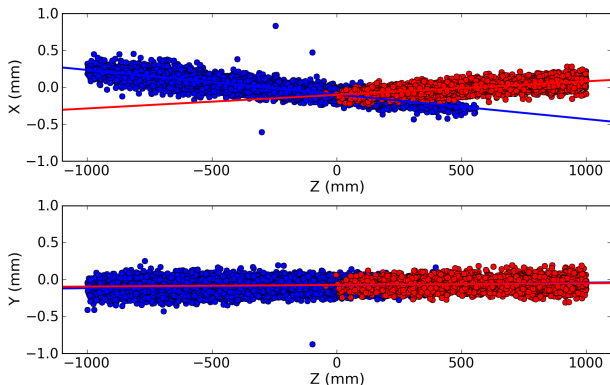
- Offline vertex reconstruction and selection:

	Type of Interaction		
	beam1-gas	beam2-gas	pp
Type of BX	be/bb	be/bb	bb
Only Fwd/Bkwd	Fwd	Bkwd	–
Min. tracks/vtx	11		21
z position [mm]	$[-1000, 500]$	$[0, 1000]$	$[-150, 150]$
Δr to beam [mm]	2		

- In bb crossings veto luminous region (± 250 mm)
- Reduced z range for measuring beam widths

MEASUREMENT OF THE BEAM ANGLES

- Beam angles are determined from straight-line fits to the beam-gas vertices in be and eb crossings



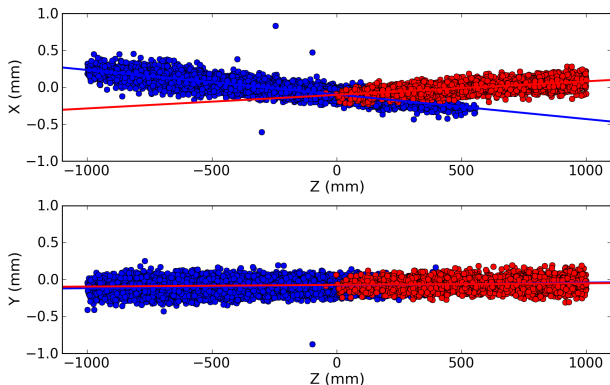
- Fill 1117:

Plane	Variable	$ b1 - b2 $	$ b1 - b2 $ expected
X-Z	offset [μm]	5 ± 3	0
	angle [μrad]	515 ± 5	540
Y-Z	offset [μm]	7 ± 3	0
	angle [μrad]	15 ± 5	0

MEASUREMENT OF THE BEAM ANGLES

- Beam angles are determined from straight-line fits to the beam-gas vertices in be and eb crossings

- Project beams onto a plane perpendicular to trajectory to determine width



- Fill 1117:

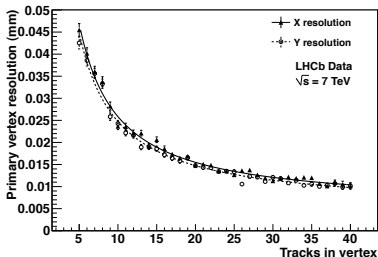
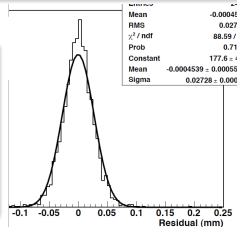
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X-Z	offset [μm]	5 ± 3	0
	angle [μrad]	515 ± 5	540
Y-Z	offset [μm]	7 ± 3	0
	angle [μrad]	15 ± 5	0

PRIMARY VERTEX RESOLUTION: DEPENDENCE ON THE NUMBER OF TRACKS

- The measured vertex distribution is a convolution of the true width and the resolution
- Parametrize resolution as function of N_{Tr} (number of tracks in the vertex) and z

Track-splitting method

- Split tracks into two collections (at random)
- Reconstruct two vertices (require $N_{Tr1} = N_{Tr2}$)
- For each N_{Tr} fill histograms with x and y residuals
- Fit to a Gaussian to determine the resolution for each N_{Tr}

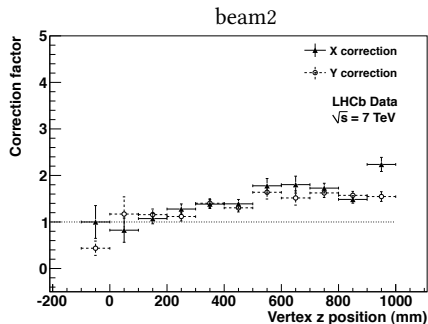
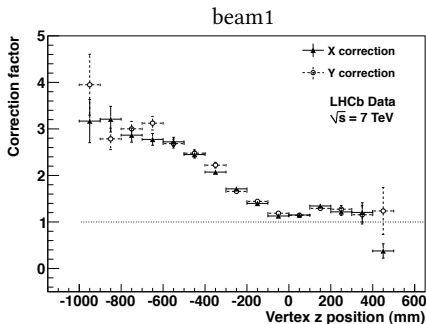


- The N_{Tr} dependence of the vertex resolution is parametrized as $\sigma_{res} = \frac{A}{N_{Tr}^B} + C$

	x	y
$A \text{ (mm)}$	0.215 ± 0.020	0.202 ± 0.018
B	1.023 ± 0.054	1.008 ± 0.053
$C (10^{-3} \text{ mm})$	5.463 ± 0.675	4.875 ± 0.645

PRIMARY VERTEX RESOLUTION: DEPENDENCE ON THE z POSITION

- A similar procedure is used with beam-gas vertices in be and eb crossings
- $N_{\text{Tr}1}$ can be different from $N_{\text{Tr}2}$
- Determine the pull $P_z = \frac{\xi_1 - \xi_2}{\sqrt{\sigma_{N_{\text{Tr}1}}^2 + \sigma_{N_{\text{Tr}2}}^2}}$ ($\xi = x, y$)
- Depending on z fill P_z into different histograms
 - The width of the fitted Gaussian determines the z correction factor



RESOLUTION UNFOLDING

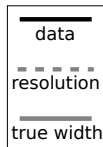
- The effect of the vertex resolution is parametrized as a sum of 6 Gaussians:

$$R(x) = \sum_{n=1}^6 c_n g_n(x; \sigma_n)$$

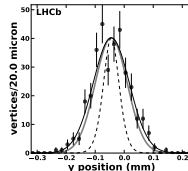
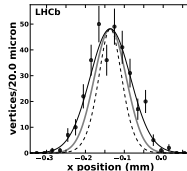
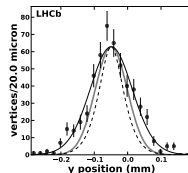
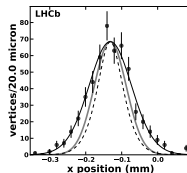
- The amplitudes c_n and the widths σ_n are determined separately for each vertex distribution

- The true bunch profiles are obtained by deconvolving the vertex resolution from the measured vertex distribution
- The effect of the vertex resolution for pp events is much smaller than for beam-gas interactions

beam1 →



beam2 →



The knowledge of resolution is essential

CONSISTENCY CHECK: RESOLUTION UNFOLDING

RATIO OF BEAM WIDTHS MEASURED CLOSE AND AWAY FROM $z = 0$ (be and eb CROSSINGS)

beam1

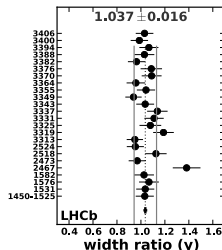
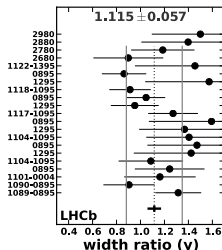
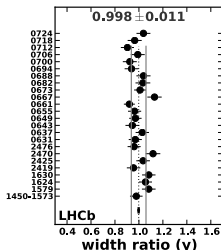
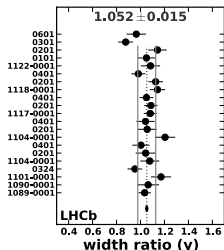
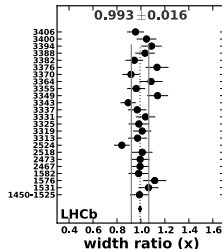
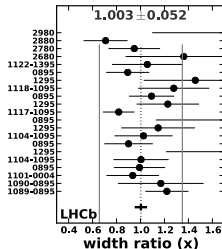
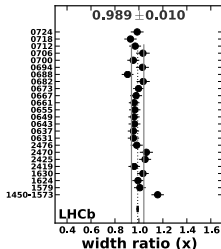
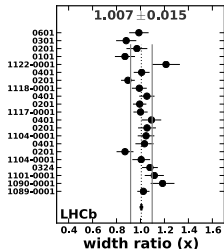
beam2

May data

October data

May data

October data



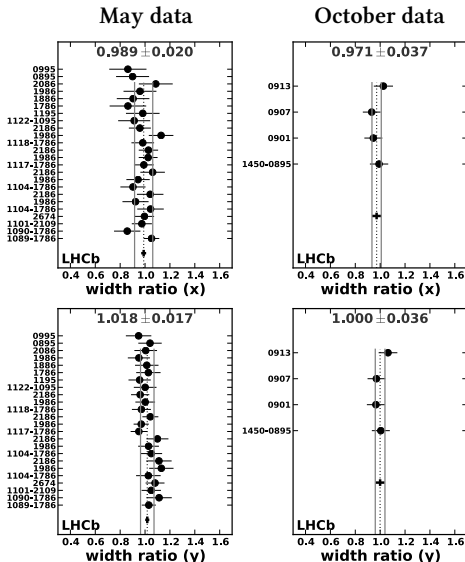
CONSISTENCY CHECK: RESOLUTION UNFOLDING

RATIO OF LUMI REGION WIDTHS MEASURED WITH pp AND BEAM-GAS INTERACTIONS
(bb CROSSINGS)

- The product of two Gaussians with widths σ_1 and σ_2 is a Gaussian with width

$$\sigma_{\otimes}^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} \quad (1)$$

- For each transverse coordinate measure the bunch widths with beam-gas interactions away from $z = 0$
- Compare the luminous region width obtained with Eq. (1) and the one measured with pp interactions

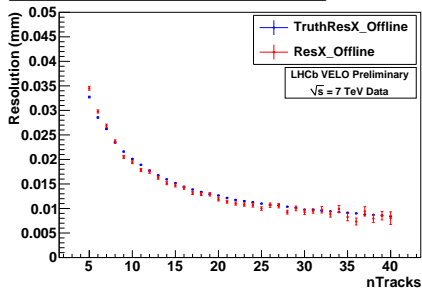


CROSS-CHECK OF TRACK-SPLITTING METHOD WITH MC

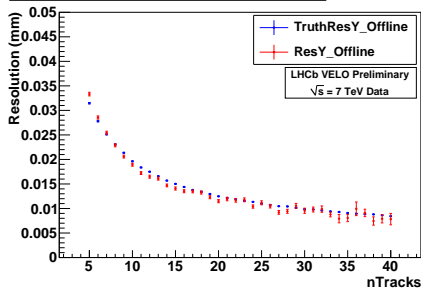
- Using MC simulated events, the N_{Tr} parametrization obtained with the track-splitting method is compared to the resolution obtained from MC-truth
- Consistency within 5%

pp events – X resolution

Comparison of true and split methods - offline x

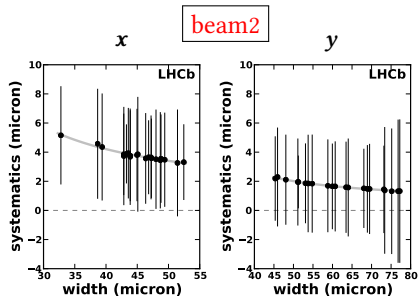
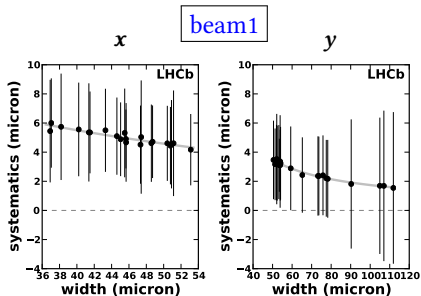
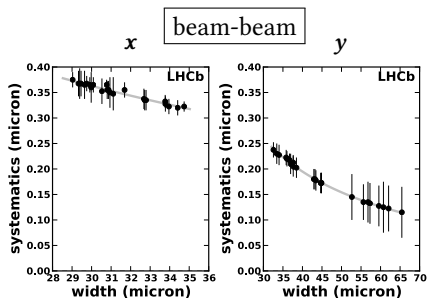
*pp* events – Y resolution

Comparison of true and split methods - offline y



SYSTEMATIC ERROR FROM RESOLUTION UNCERTAINTY

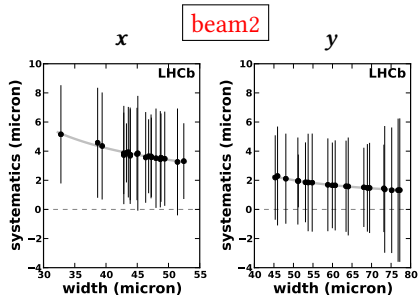
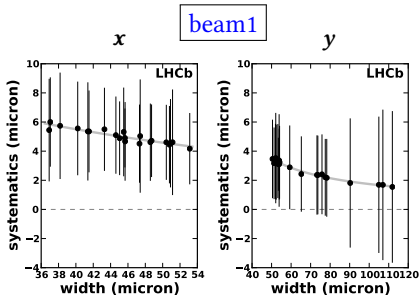
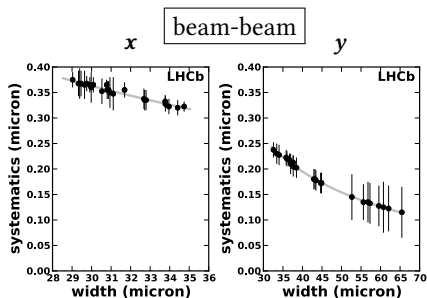
- Parametrize error on widths of lumi region and beams
 - Determine the width twice using the upper and the lower limits of the resolution error (vary N_{Tr} and z resolution coherently)
 - Error = difference between the two widths



SYSTEMATIC ERROR FROM RESOLUTION UNCERTAINTY

- Parametrize error on widths of lumi region and beams
 - Determine the width twice using the upper and the lower limits of the resolution error (vary N_{Tr} and z resolution coherently)
 - Error = difference between the two widths

Error on cross-section depends on the beam widths. On average it is 2.5%



ADDITIONAL FACTORS

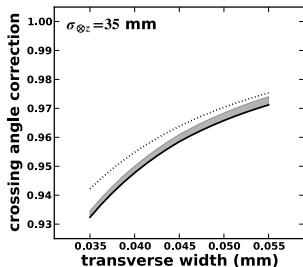
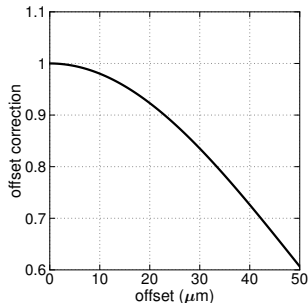
$$L = \frac{f N_1 N_2}{2\pi \sqrt{(\sigma_{1x}^2 + \sigma_{2x}^2)(\sigma_{1y}^2 + \sigma_{2y}^2)}} \times C_{\text{offset}} \times C_{\text{angle}}$$

Transverse offset of the beams

- Per transverse coordinate: $C_{\text{offset}} = \exp\left(-\frac{1}{2} \frac{\Delta_\mu^2}{\sigma_1^2 + \sigma_2^2}\right)$

Crossing angle

- In the presence of a crossing angle, the Beam Overlap Integral depends on the bunch lengths σ_{1z} and σ_{2z}
- $C_{\text{angle}} = [1 + \tan^2 \alpha (\sigma_{1z}^2 + \sigma_{2z}^2) / (\sigma_{1x}^2 + \sigma_{2x}^2)]^{-\frac{1}{2}}$
- However, σ_{1z} and σ_{2z} are not directly measured
- Workaround: use an equation relating the luminous length $\sigma_{\otimes z}$ with $\sigma_{1z}^2 + \sigma_{2z}^2$



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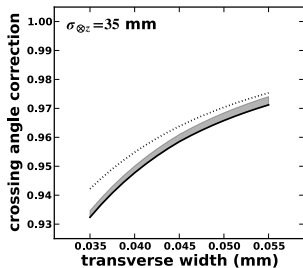
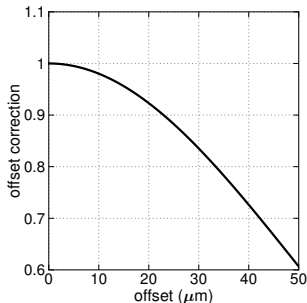
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- However, σ_{1z} and σ_{2z} are not directly measured
- Workaround: use an equation relating the luminous length $\sigma_{\otimes z}$ with $\sigma_{1z}^2 + \sigma_{2z}^2$

In the fills used in this analysis $C_{\text{angle}} \sim 0.95$
 Systematic error: 1%



SYSTEMATIC ERRORS

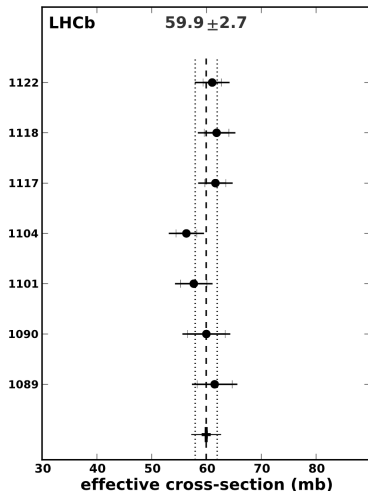
- Seven independent measurements of an *effective reference cross-section*
 - one measurement per fill
- Correlations for the averaging:
 - u : uncorrelated between bunches and between fills
 - f : correlated between bunches, uncorrelated between fills
 - c : correlated between bunches and between fills

relative cross-section errors [%]

	correl.	average
Statistics	u	0.96
Overlap syst		3.35
Crossing angle	c	1.00
Width syst		3.20
Resolution syst	c	2.56
Trend syst	c	1.00
Bias syst	c	1.61
Beam normalisation		2.88
DCCT scale	c	2.70
DCCT baseline	f	0.10
Ghost charge	f	0.19
FBCT offset	f	0.91
Relative lumi	c	0.50
Weight		1.38
Total Systematics		4.45
Uncorrelated syst	f	0.93
Correlated syst	c	4.33
Total		4.55
Excluding norm		3.63

RESULTS

Results of BGI method

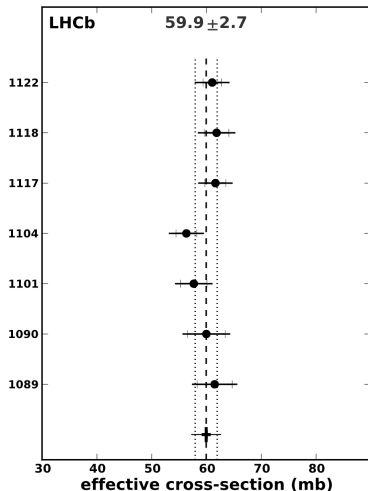
 σ_{VELO} [mb]

Combination of VDM and BGI results

	Average	VDM	BGI
Cross-section σ_{VELO} [mb]	58.8	58.4	59.9
DCCT scale uncertainty [%]	2.7	2.7	2.7
Uncorrelated uncertainty [%]	2.0	2.4	3.7
Cross-section uncertainty [%]	3.4	3.6	4.6

RESULTS

Results of BGI method

 σ_{VELO} [mb]

Combination of VDM and BGI results

	Average	VDM	BGI
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DCCT scale uncertainty [%]	2.7	2.7	2.7
Uncorrelated uncertainty [%]	2.0	2.4	3.7
Cross-section uncertainty [%]	3.4	3.6	4.6

Overall precision of the absolute
luminosity normalization: 3.4%

CONCLUSION

Two methods were applied for the absolute luminosity normalization of the data collected by LHCb in 2010

- van der Meer scans in dedicated fills in April and October (precision 3.6%)
- Beam-gas imaging during physics fills in May and using the residual gas in the beam vacuum pipe (precision 4.6%)
 - **Demonstrating the potential of the BGI method**
- Error on the integrated luminosity: 3.5%
 - 3.4% from the determination of the absolute scale
 - 0.7% from the measurement of the relative luminosity
- The luminosity is *not* the dominating uncertainty in most of the physics cross-section measurements

CONCLUSION

Two methods were applied for the absolute luminosity normalization of the data collected by LHCb in 2010

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 - 3.4% from the determination of the absolute scale
 - 0.7% from the measurement of the relative luminosity
- The luminosity is *not* the dominating uncertainty in most of the physics cross-section measurements

Ways to improve the precision (goal: 2%)

- Beam intensity: improve uncertainty on DCCT scale
- VDM: systematic studies of non-reproducibility
- BGI: apply in a fill with optimal conditions (broader beams and higher gas pressure)

Backup Slides

RESOLUTION UNFOLDING

- It is assumed that the effect of the vertex resolution can be parametrized as a superposition of Gaussian functions $g_n(x; \sigma_n)$ centered at zero:

$$R(x) = \sum_{n=1}^N c_n g_n(x; \sigma_n)$$

- Estimate the total effective resolution for a given vertex distribution
 - Fill a 1-d histogram with the expected resolution of each vertex (using the N_{Tr} and z parametrization)
 - Divide the histogram in N bins
 - The number of entries in each bin determine the weight coefficient c_n
 - The average resolution in each bin gives the corresponding value of σ_n
 - Six bins are sufficient to give a good description of the resolution

RESOLUTION UNFOLDING

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$$R(x) = \sum_{n=1}^N c_n g_n(x; \sigma_n)$$

- The physical beam shape is assumed to be described by a Gaussian with amplitude a , position μ and width σ : $f(x; a, \mu, \sigma)$
- The measured vertex distribution $M(x)$ is a convolution of the physical beam shape and the resolution function $R(x)$:

$$M(x) = \int_{-\infty}^{+\infty} \sum_{n=1}^N c_n g_n(x - t; \sigma_n) f(t; a, \mu, \sigma) dt \quad (2)$$

- Using the basic algebraic properties of the convolution and defining $\sigma_n^* = \sqrt{\sigma_n^2 + \sigma^2}$ this equation can be rewritten as:

$$M(x) = \sum_{n=1}^N c_n f_n(x; a, \mu, \sigma_n^*), \quad (3)$$

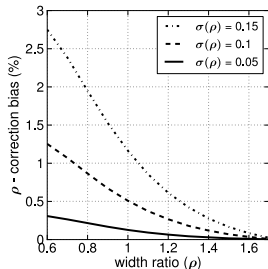
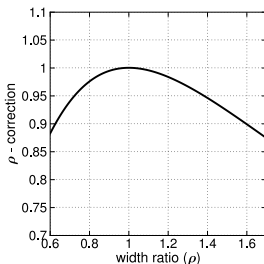
- The physical beam position, μ , and width, σ , are obtained by fitting $M(x)$ to the data

BIAS DUE TO UNEQUAL BEAM SIZES AND BEAM OFFSETS

- Using the second lumi-region constraint (Eq. (4)) and introducing $\boxed{\rho = \sigma_2/\sigma_1}$ it can be shown that: $\sigma_{\otimes} = \frac{\rho\sigma_1}{\sqrt{1+\rho^2}}$
- Then, *per transverse coordinate*, the Beam Overlap Integral is proportional to $\frac{\rho}{(1+\rho^2)\sigma_{\otimes}}$
 - This form is used for the calculation of the Beam Overlap Integral
 - The value of σ_{\otimes} is the best measured quantity entering the Beam Overlap Integral
 - The measurement of ρ is associated with a larger statistical uncertainty

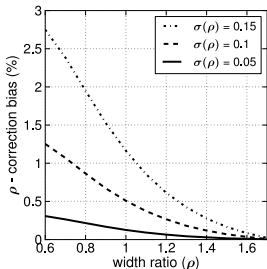
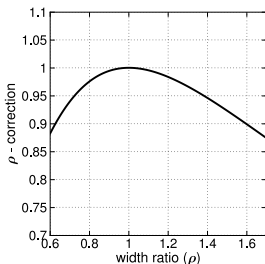
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- This expression has an extremum at $\rho = 1$ and can only take values smaller than unity
- The precision of measuring ρ (about 10 – 15%) is similar to its difference from unity
- The experimental estimate of the ρ -correction factor $2\rho/(1+\rho^2)$ is *biased* towards smaller values



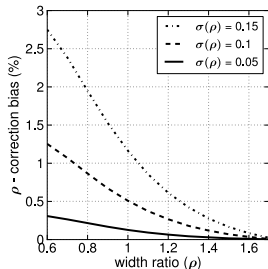
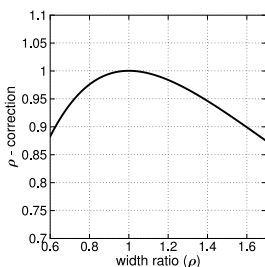
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 - Systematic uncertainty: half of the correction
- A similar situation occurs for the transverse-offset correction



LUMI REGION CONSTRAINTS

- For each transverse coordinate we have relations between the widths and the positions of the two bunches (index 1, 2), and their luminous region (index \otimes)

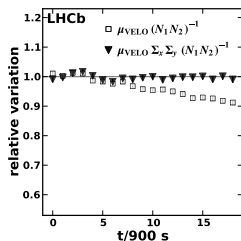
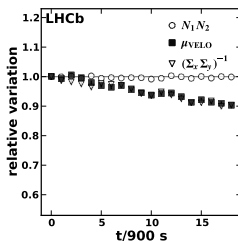
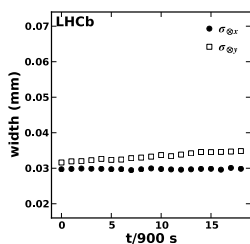
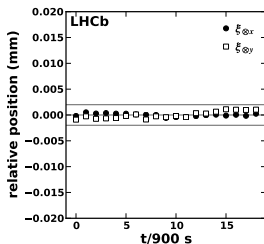
- *lumi region constraints*

$$\mu_{\otimes} = \frac{\mu_1 \sigma_2^2 + \mu_2 \sigma_1^2}{\sigma_1^2 + \sigma_2^2} \quad \text{and} \quad \sigma_{\otimes}^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} \quad (4)$$

- The precision of the measured properties of the luminous region is higher than for the separate beams
- Employ the lumi region constraints to fit 4 parameters using 6 measurements (fit parameters are $\sigma_{1,2}$ and $\mu_{1,2}$): **Significant improvement of the precision**
- An alternative choice of the fit parameters makes the corresponding luminosity error propagation easier without changing the result (the motivation for this choice will become clear later):
 - $\sqrt{\sigma_1^2 + \sigma_2^2}$, σ_2/σ_1 , $\Delta_\mu = \mu_1 - \mu_2$ and μ_{\otimes}

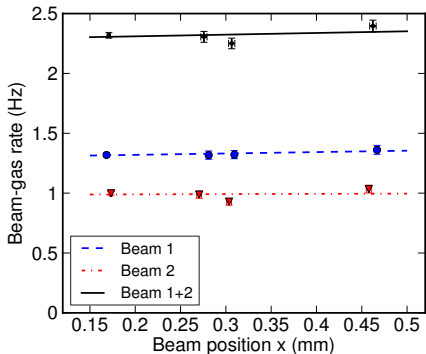
TIME DEPENDENCE AND STABILITY

- The beam stability is essential, given the long integration times for collecting sufficient beam-gas statistics
 - Use only fills where the variation of the beam intensity and emittance are smooth
- Time variation of the transverse position and width of the luminous region of one colliding bunch pair
- The time variation of quantities determining the interaction rate of the same bunch pair
- The average position and size of the luminous region is used together with the beam profiles measured in the same period with beam-gas interactions
 - Systematic error of 1% is assigned to account for the variations



GAS PRESSURE GRADIENT

- In the BGI method, the gas in the beam-vacuum pipe is used to obtain an image of the transverse profile of the beams
- In case of gas inhomogeneity in the transverse plane the beam image will be distorted
- A measurement of the gas homogeneity is performed by displacing the beams and recording the rate of beam-gas interactions at these different beam positions
- An upper limit on the gradient of the interaction rate: 0.62 Hz/mm at 95% CL, compared to a rate of 2.14 ± 0.05 Hz observed with the beam at its nominal position
- With the measured limit on the gradient, the maximum relative effect on the Beam Overlap Integral is estimated to be 4.2×10^{-4}
- In practice, the effect would be smaller, as the beam widths enter the Beam Overlap Integral measurement through $\rho = \sigma_2/\sigma_1$



BGI RESULTS

		average	1089	1090	1101	1104	1117	1118	1122
Cross-section (mb)		<u>59.94</u>	61.49	59.97	57.67	56.33	61.63	61.84	61.04
Statistics	<i>u</i>	0.96	4.06	4.73	3.09	2.56	1.89	2.66	1.82
Overlap syst		3.35	3.33	3.58	3.21	3.70	3.00	3.15	3.49
Crossing angle	<i>c</i>	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Width syst		3.20	3.18	3.43	3.05	3.56	2.83	2.99	3.34
Resolution syst	<i>c</i>	2.56	2.79	2.74	2.54	2.86	2.37	2.47	2.44
Trend syst	<i>c</i>	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Bias syst	<i>c</i>	1.61	1.14	1.81	1.35	1.89	1.19	1.35	2.05
Beam normalisation		2.88	4.21	4.21	3.91	3.48	3.65	3.67	3.37
DCCT scale	<i>c</i>	2.70	2.70	2.70	2.70	2.70	2.70	2.70	2.70
DCCT baseline	<i>f</i>	0.10	0.97	1.01	0.43	0.29	0.29	0.29	0.14
Ghost charge	<i>f</i>	0.19	0.70	0.65	1.00	0.60	0.38	0.55	0.35
FBCT offset	<i>f</i>	0.91	3.00	3.00	2.61	2.10	2.41	2.41	1.98
Relative lumi	<i>c</i>	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Weight		1.38	5.19	5.73	4.19	3.38	3.10	3.64	2.72
Total Systematics		4.45	5.39	5.55	5.08	5.11	4.75	4.87	4.88
Uncorrelated syst	<i>f</i>	0.93	3.23	3.23	2.83	2.20	2.46	2.49	2.02
Correlated syst	<i>c</i>	4.33	4.32	4.51	4.22	4.61	4.07	4.18	4.44
Total		4.55	6.75	7.29	5.95	5.71	5.11	5.55	5.20
Excluding norm		3.63	6.17	6.75	5.28	5.01	4.31	4.82	4.42