NEW SPECTRA IN THE HEIDI MODELS

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We study the so-called HEIDI models, which are renormalizable extensions of the standard model with a higher-dimensional scalar singlet field. We compare their predictions with the recent results from the LHC. We show that the data can easily be described by the HEIDI models. However more data are necessary in order to distinguish these models from the standard model. A particular prediction is that the width of the Higgs could be in the GeV range. Such a width could be difficult to establish at the LHC. Suggestions for experiments beyond the LHC are made.

1 Introduction

The standard model is in very good agreement with the data. With the exception of the Higgs sector all particles of the model have been seen and so far there is no evidence for new physics. There appear to be no new fermions, vectorbosons or new forces. The results from the experiments at the LHC, presented in this conference, have put strong constraints on the possibility of new physics. As there are strong theoretical arguments, that the standard model is the only possible low-energy theory for the vectorbosons and the fermions\textsuperscript{1,2,3}, this is not surprising.

This leaves only the Higgs-sector for a possible discovery. In order to determine what one should look for, let us discuss the situation before the presentation on 13 december 2011 at CERN. What was known for sure is, that massive vectorbosons exist with non-abelian couplings. For quantum mechanics to be valid this implies that a Higgs field has to exist\textsuperscript{4,5}, otherwise one violates unitarity. Since quantum field theory is correct, this implies that the Higgs field must satisfy the Källén-Lehmann representation\textsuperscript{6,7}, so it has a spectral density. Because of renormalizability the integral over the spectral density must be one. Without this condition the precision electroweak data cannot be described properly. Actually the precision electroweak data show that the spectral density is concentrated in the low mass region; the Higgs boson is light. This much is basically certain about the Higgs sector. Everything else is conjecture. In particular the idea, that there is a single Higgs particle delta-peak in the spectrum is an assumption, for which there is no basis in theory or experiment. As the Higgs field appears in the theory in a different way from the other fields, it is not unreasonable to assume that the Higgs field has a non-trivial spectral density. It is somewhat ironic that this possibility was overlooked\textsuperscript{8} in the 1960’s, when the Higgs particle was introduced, as in this period there was a lot of research on the related subject of dispersion relations. The scientific goal regarding the electroweak symmetry breaking is therefore to measure the Källén-Lehmann spectral density of the Higgs field. Under circumstances this might be quite difficult at the LHC and one would
have to think about a Higgs-factory beyond the LHC. In the following we introduce the HEIDI models, which form an elegant construction allowing for non-trivial spectral densities with a minimum number of parameters. Then we compare the models with the new results from the LHC and discuss future prospects.

2 Lagrangian and propagator

The fact that propagators in quantum field theory are described by a Källén-Lehmann spectral density follows from fundamental principles of quantum mechanics and relativity and is mathematically rigorously true. Normally however one ignores this, since most fields correspond directly to single particles and one takes a Breit-Wigner approximation for the propagator. The general idea of a spectral density is somewhat abstract. In order to clarify the idea and make contact with more usual descriptions I will below give Lagrangians, where such a Källén-Lehmann representation arises naturally. First we start with a discrete spectrum and then generalize to a (partly) continuous spectrum. The basic ideas have been presented in Moriond before.

2.1 The Hill model

The simplest extension of the standard model, with only two extra parameters is the Hill model. One adds to the standard model a single scalar field $H$ with the following Lagrangian.

$$\mathcal{L} = -\frac{1}{2}(D_\mu \Phi)\dagger (D^\mu \Phi) - \frac{\lambda_1}{8}(\Phi\dagger \Phi - f_1^2)^2 - \frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda_2}{8}(2f_2 H - \Phi\dagger \Phi)^2. \quad (1)$$

Here $\Phi$ is the standard model Higgs doublet. Important for the structure of the theory is that the extra singlet has no self-couplings. Somewhat surprisingly this is consistent with renormalizability. After diagonalizing the Lagrangian one ends up with two particles similar to the standard model Higgs particle, however with the couplings to the standard model particles reduced by a common factor, so the branching ratios do not change. An alternative way to describe these features is to use a modified propagator for the Higgs field, containing two poles.

$$D_{HH}(k^2) = \frac{\sin^2 \alpha}{k^2 + m^2_+} + \frac{\cos^2 \alpha}{k^2 + m^2_-} \quad (2)$$

Such a description is sufficient to study Higgs signals (interaction basis).

The generalization to more fields is straightforward. One introduces $n$ Higgses $H_i$ with reduced couplings $g_i$ to the standard model particles. After diagonalizing one finds the sum rule:

$$\sum g_i^2 = g_{\text{Standard model}}^2 \quad (3)$$

This can straightforwardly be generalized to a continuum density $\rho(s)$, with the relation:

$$\int \rho(s)ds = 1 \quad (4)$$

The density $\rho(s)$ is the Källén Lehmann density, which has here been constructed from a tree level Lagrangian.

2.2 HEIDI models

Given enough fields one can construct an arbitrary spectral density for the Higgs field, however this involves an infinite number of parameters as well, which is not very satisfactory. The question is whether there is a more elegant way. Indeed there is. Because the singlet fields have
no self-couplings all their interactions are superrenormalizable in four dimensions. Therefore one can take a higher dimensional field for the Higgs field without spoiling renormalizability. This explains the name HEIDI (German for high-D, compare with SUSY for supersymmetry). One can go up to six-dimensional fields without spoiling renormalizability and can even postulate fields with a fractional dimension. With this procedure one still has only a few parameters, but can get a fairly broad range of spectra. For deriving the propagator one assumes a torus compactification giving rise to winding modes $H_i$; however we will take the continuum limit in the end.

In terms of the modes $H_i$ the Lagrangian is the following:

$$L = -\frac{1}{2} D_\mu \Phi^\dagger D^\mu \Phi - \frac{M_0^2}{4} \Phi^\dagger \Phi - \frac{\lambda}{8} (\Phi^\dagger \Phi)^2 - \frac{1}{2} \sum (\partial_\mu H_k)^2 - \frac{g}{2} \Phi^\dagger \Phi \sum H_k - \frac{\zeta}{2} \sum H_i H_j$$

(5)

$$m_k^2 = m^2 + m_\gamma^2 \vec{k}^2,$$

where $\vec{k}$ is a $\gamma$-dimensional vector, $m_\gamma = 2\pi/L$ and $m$ a $d$-dimensional mass term for the field $H$.

In terms of continuum fields the last two terms can be described by the following action:

$$S = \int d^{4+\gamma}x \prod_{i=1}^{\gamma} \delta(x_{4+i}) \left( g_B H(x) \Phi^\dagger \Phi - \zeta_B H(x) H(x) \right)$$

(6)

### 2.3 Propagators

Minimizing the potential, diagonalizing and taking the continuum limit, one finds the following propagator:

$$D_{HH}(q^2) = \left( q^2 + M^2 - \frac{\mu^{8-d}}{(q^2 + m^2)^{\frac{a_d}{2}} \pm \nu^{6-d}} \right)^{-1}$$

(7)

Here $M$ is a four-dimensional mass, $m$ a higher dimensional mass, $\mu$ a high-to-four-dimensional mixing scale and $\nu$ a brane mass, mixing the higher-dimensional fields among themselves. This term is new in the analysis and is needed, because it is induced by renormalization in any case.

To go to six dimensions one has to make a limiting procedure and finds a propagator in the following form:

$$D_{HH}(q^2) = \left( q^2 + M^2 + \mu^2 \frac{\log((q^2 + m^2)/m^2)}{1 + \alpha_6 \log((q^2 + m^2)/m^2)} \right)^{-1}$$

(8)

Given the form of the propagator it is straightforward to find the spectral density, by taking the imaginary part. There are some constraints on the parameters, while tachyons must be absent. Dependent on the parameters there are different possibilities. One can have zero, one or two delta-peaks, corresponding to ”particles”. At a mass greater than the location of the peak(s) there is a continuum, that starts at $s = m^2$.

### 3 Comparison with experiment

As reported in previous Moriond conferences, it is possible to compare the HEIDI models with Higgs search experiments. We first discuss the situation before the LHC and then see how the new data change the description compared to the previous results.
3.1 Before the LHC

Before the LHC data the HEIDI models were compared with the direct Higgs search at LEP-200. Within the pure standard model the absence of a clear signal led to a lower limit on the Higgs boson mass of 114.4 GeV at the 95% confidence level. Although no clear signal was found the data had some intriguing features, that could be interpreted as evidence for Higgs bosons beyond the standard model. There is a 2.3σ effect seen by all experiments at around 98 GeV. A somewhat less significant 1.7σ excess was seen around 115 GeV. Finally over the whole range $s^{1/2} > 100$ GeV the confidence level was less than expected from background. These features were taken as evidence for a spread-out Higgs-boson. The peak at 98 GeV was taken to correspond to the delta peak in the Källén-Lehmann density. The other excess data were interpreted as part of the continuum, that peaks around 115 GeV. Fitting the data with this interpretation led to the picture that the Higgs signal would be a very broad enhancement at low energies, that could escape detection at the LHC.

3.2 Interpretation of the LHC data

With the results for the Higgs search at the LHC the picture has changed. The excess at 115 GeV, that was present at LEP-200 has now disappeared; averaging between LEP, CMS and ATLAS one is very close to pure background. However the somewhat more significant peak at 98 GeV has not been affected by the LHC data, which are not sensitive in this range yet. Actually the ATLAS experiment even sees an excess in the 101 GeV bin, which might be related to the LEP peak. Of course this is not statistically very relevant, however we take it as a motivation to keep the LEP peak at 98 GeV in the analysis. This leaves us with the excess in the 116-130 GeV range. The interpretation of the data is not quite straightforward. Making a naive average of the CMS and ATLAS measurements one finds a picture that can be interpreted as a single peak, two peaks or a continuous signal with a bit of fluctuations. Clearly more data are necessary. We will leave the question of the precise form of the signal open and interpret the combined LEP and LHC data in the following way. There is no signal below 95 GeV, there is a 2.3σ signal at 98 GeV, there is no further signal below 116 GeV and the bulk of the spectrum is between 116 and 130 GeV. Allowing for uncertainties in the experiment we therefore impose the following conditions on the spectral density:

$$\begin{align*}
95 \text{ GeV} &< m_{\text{peak}} < 101 \text{ GeV}, \\
0.056 &< \frac{g_{98}^2}{g_{SM}^2} < 0.144 \\
m &> 116 \text{ GeV}, \\
\int_{(130)^2}^{\infty} \rho(s) ds &< 0.1
\end{align*}$$

3.3 Fits to the data

We first attempt to fit the data with one peak around 98 GeV plus a continuum. The results are shown in figures 1-4. The inner lines correspond to keeping a 10% peak at 98 GeV. For the outer lines we vary the location and strength of the peak. From the figures it is clear that one can fit the data easily without any particular fine-tuning of parameters.

How would the signal for a HEIDI Higgs differ from a standard model Higgs? First there is of course the small peak around 98 GeV. Another interesting effect is, that the continuum appears as a wide Higgs, with a somewhat weaker coupling than for a full Higgs particle peak. A typical spectrum is presented in figure 5 for a five-dimensional and a six-dimensional model. One sees that the width of the Higgs is much larger than in the standard model, it is roughly 2 GeV for the parameters in the figure. In principle one would like to use the data to distinguish between the two curves in the figure, but this might be difficult because of the experimental resolution at the LHC.
One can also fit the data with two peaks and a continuum, one peak at 98 GeV and the second in the range 116-130 GeV. In the experiment one can probably not separate the second peak from the continuum, as the continuum starts very close to the second peak. The situation is described in figure 6, where we plot the start of the continuum as a function of the strength of the second peak, together with the mass average over the combination of the second peak and the continuum. One would probably see a somewhat wider and asymmetric peak only.

Finally it is possible to have a pure continuum, but in this case one has to ignore the LEP peak, assuming that it is a statistical fluctuation.

4 Future prospects

It is clear that the present data are not enough to come to definite conclusions. Below we discuss some questions to be addressed in the near and far future.

4.1 Questions for the LHC

Even with the planned running of the LHC this year it is unlikely that one can distinguish the HEIDI models from the standard model in a definite way. What is clear from the above discussion is that the first priority should be to clearly establish the presence of a signal, which might be a bit weaker than a standard model Higgs signal. The present analysis should be extended to lower energies, in order to get to the LEP peak at 98 GeV. Combined data from ATLAS and CMS are highly desirable from the theoretical point of view. It would probably be possible to already start with a "model-independent" analysis for the spectral density $\rho(s)$. A somewhat crude first approach could be for instance to divide the range of 116-130 GeV in 7 bins of 2 GeV each and allow for a Higgs signal to be present in the bins for instance in steps of 1/6 of a Higgs signal. This would give rise to 1716 models, for each of which one can calculate a probability how well it fits the data.

After that, determining the branching ratios with some precision is important to confirm that one is dealing with a Higgs boson, that is similar to the standard model. However it is particularly important to determine the width of the Higgs boson, or more general its spectral density. The width should be studied in correlation with the strength of the Higgs signal, as also invisible decay could lead to a wide Higgs$^{16}$. However in this case the branching ratio to standard model particles is suppressed. The invisible decay particles could be candidates for dark matter$^{17}$. These latter two issues can probably be addressed with some accuracy only when the LHC reaches its full design parameters. Even then the LHC will not be able to determine the line shape of the Higgs boson to great accuracy, due to the resolution of the detectors.

4.2 A Higgs factory

Ultimately to determine the Källén-Lehmann density of the Higgs field precisely, the LHC is not an optimal machine. One will need a Higgs factory. There are essentially three options, all of them with some disadvantages. The first would be a muon-collider, where the Higgs is produced directly and one can make an energy scan to probe the resonance region as in LEP with the Z-boson. The disadvantage is that it is quite unclear if such a machine can be built, in particular it would be difficult to get a high enough luminosity. The second possibility would be a 250-300 GeV linear collider. The problem here is that one needs a high precision on the momenta of incoming and outgoing particles, as the spectrum has to be determined from the recoil in the process $e^+e^- \rightarrow ZH$. Beamstrahlung would probably reduce the resolution to an unacceptable level. However this should be studied in more detail. Most studies for a linear collider are focused on 500 GeV. It would be useful to study an optimal design for Higgs physics.
The problem of beamstrahlung can be largely avoided by going to a very large circular collider with a CM energy of 250-300 GeV. Synchrotron radiation drives up the radius of the ring and the largest collider proposed so far is the very large lepton collider at Fermilab, which could have a 230 km circumference. The limiting factor could be the precision with which one can measure the momentum of the outgoing leptons coming from the Z-boson. This is a detector problem, that also plays a role for the linear collider option, and needs careful study.

Although the civil engineering cost of such a large tunnel will be high, from the accelerator point of view a circular electron-positron collider is the easiest to build and could be achieved with current technology. In the longer term, a facility of this size would make a very natural stepping stone to study proton-proton collisions at very high energies, far beyond any LHC upgrade. A facility of this type would provide an exciting physics program for O(40) years and would require cooperation at an unprecedented level on the global scale. Given the long time scales involved, it would be interesting to explore the physics case for a $\sqrt{s} \sim 250 – 300$ GeV electron positron collider to fully explore a Higgs signal seen at the LHC, followed with a 50 – 100 TeV proton-proton collider to probe the high energy frontier.

5 Conclusion

The conclusion can be short: Higher dimensions may be hidden in the Higgs lineshape. Or somewhat more poetically:

Where is Heidi hiding?
Heidi is hidden
in the high-D Higgs Hill!

Acknowledgments

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References

8. F. Englert, private communication, this conference.
17. M. Pospelov, this conference.
Figure 1: Fit in 4.5 dimensions.

Figure 2: Fit in 5.0 dimensions.

Figure 3: Fit in 5.5 dimensions.
Figure 4: Fit in 6.0 dimensions.

Figure 5: Higgs line shape.

Figure 6: Two peak structure.