Theoretical update of $B$-mixing

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TeVatron gave us many presents, and then ...
Time evolution of a decaying particle: \[ B(t) = \exp \left[ -i m_B t - \frac{\Gamma_B}{2t} \right] \]
can be written as

\[
i \frac{d}{dt} \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix} = \left( \hat{M} - \frac{i}{2} \hat{\Gamma} \right) \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}
\]

**BUT:** In the neutral \( B \)-system transitions like \( B_{d,s} \rightarrow \bar{B}_{d,s} \) are possible due to weak interaction: **Box diagrams**
Mixing is a macroscopic quantum effect!

It was observed in

- $K^0$-system: 1950s (see text books, regeneration...)
- $B_d$-system: 1986
- $B_s$-system: 2006
- $D^0$-system: 2007

Strongly suppressed in the SM (due to virtual top-quarks)
New physics effects might be of comparable size

?Is QCD under control?
Mixing III

Time evolution of a decaying particle: 
\[ B(t) = \exp \left[ -im_B t - \frac{\Gamma_B}{2t} \right] \]
can be written as
\[ i \frac{d}{dt} \begin{pmatrix} \langle B(t) \rangle \\ \langle \bar{B}(t) \rangle \end{pmatrix} = \begin{pmatrix} \hat{M} - \frac{i}{2} \hat{\Gamma} \end{pmatrix} \begin{pmatrix} \langle B(t) \rangle \\ \langle \bar{B}(t) \rangle \end{pmatrix} \]

**BUT:** In the neutral \( B \)-system transitions like \( B_{d,s} \rightarrow \bar{B}_{d,s} \) are possible due to weak interaction: Box diagrams

\[ \Rightarrow \text{off-diagonal elements in } \hat{M}, \hat{\Gamma}: M_{12}, \Gamma_{12} \text{ (complex)} \]

Diagonalization of \( \hat{M}, \hat{\Gamma} \) gives the physical eigenstates \( B_H \) and \( B_L \) with the masses \( M_H, M_L \) and the decay rates \( \Gamma_H, \Gamma_L \)

**CP-odd:** \( B_H := pB + q\bar{B} \), **CP-even:** \( B_L := pB - q\bar{B} \) with \( |p|^2 + |q|^2 = 1 \)
Mixing IV

$|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \text{arg}(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference:** $\Delta M := M_H - M_L = 2|M_{12}| \left( 1 - \frac{1}{8} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} \sin^2 \phi + \ldots \right)$

  $|M_{12}|$ : heavy internal particles: t, SUSY, ...

- **Decay rate difference:** $\Delta \Gamma := \Gamma_L - \Gamma_H = 2|\Gamma_{12}| \cos \phi \left( 1 + \frac{1}{8} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} \sin^2 \phi + \ldots \right)$

  $|\Gamma_{12}|$ : light internal particles: u, c, ... (almost) no NP!!

- **Flavor specific/semileptonic CP asymmetries:**

  $\bar{B}_q \rightarrow f$ and $B_q \rightarrow \bar{f}$ forbidden

  No direct CP violation: $|\langle f | B_q \rangle| = |\langle \bar{f} | \bar{B}_q \rangle|$

  e.g. $B_s \rightarrow D_s^- \pi^+$ or $B_q \rightarrow X l \nu$ (semileptonic)

  \[ a_{sL} = a_{fL} = \frac{\Gamma(\bar{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow \bar{f})} = -2 \left( \left\| \frac{q}{p} \right\| - 1 \right) = \text{Im} \frac{\Gamma_{12}}{M_{12}} = \frac{\Delta \Gamma}{\Delta M} \tan \phi \]
The Mass Difference $\Delta M$

Calculating the box diagram with an internal top-quark yields

$$M_{12,q} = \frac{G_F^2}{12\pi^2} (V_{tq}^* V_{tb})^2 M_W^2 S_o(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

(Inami, Lim '81)

- Hadronic matrix element: \( \frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q} = \langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B_q \rangle \)
- Perturbative QCD corrections \( \hat{\eta}_B \) (Buras, Jamin, Weisz, '90)

Theory 1102.4274 vs. Experiment : HFAG 11

$$\Delta M_d = 0.543 \pm 0.091 \, ps^{-1} \quad \Delta M_d = 0.507 \pm 0.004 \, ps^{-1}$$

ALEPH, CDF, D0, DELPHI, L3,
OPAL, BABAR, BELLE, ARGUS, CLEO

$$\Delta M_s = 17.30 \pm 2.6 \, ps^{-1} \quad \Delta M_s = 17.70 \pm 0.12 \, ps^{-1}$$

CDF, D0, LHCb

Important bounds on the unitarity triangle and new physics
Determination of $\Gamma_{12}$

Sensitive to real intermediate states $\Rightarrow$ much more complicated than $M_{12}$

1. OPE I: Integrate out W: like $M_{12} \propto f_B^2 B$

2. OPE II: Heavy quark expansion $\Rightarrow$ $\Gamma_i^{(j)} \propto f_B^2 \sum C_k B_K$

\[
\Gamma_{12} = \left( \frac{\Lambda}{m_b} \right)^3 \left( \Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \ldots \right) + \left( \frac{\Lambda}{m_b} \right)^4 \left( \Gamma_4^{(0)} + \ldots \right) + \left( \frac{\Lambda}{m_b} \right)^5 \left( \Gamma_5^{(0)} + \ldots \right) + \ldots
\]


2006: A.L., Nierste

2007: Badin, Gabbiani, Petrov

\[
\Delta \Gamma_s = \Delta \Gamma_s^0 \left( 1 + \delta^{\text{Lattice}} + \delta^{\text{QCD}} + \delta^{\text{HQE}} \right)
\]

\[
= 0.142 \text{ ps}^{-1} \left( 1 - 0.14 - 0.06 - 0.19 \right)
\]
HQE under attack!

OPE II might be questionable - relies on quark hadron duality

- Mid 90’s: Missing Charm puzzle $n_c^{\text{Exp.}} < n_c^{\text{SM}}$, semi leptonic branching ratio
- Mid 90’s: $\Lambda_b$ lifetime is too short
- before 2003: $\tau_{B_s}/\tau_{B_d} \approx 0.94 \neq 1$
- 2010/2011: Di-muon asymmetry too large

Theory arguments for HQE

⇒ calculate corrections in all possible “directions”, to test convergence
  ⇒ $\Gamma_{12}$ seems to be ok!
⇒ test reliability of OPE II via lifetimes (no NP effects expected) “directions”, to test convergence
  ⇒ $\tau(B^+)/\tau(B_d)$ Experiment and theory agree within hadronic uncertainties
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- 2012: $n_{c}^{2011\text{PDG}} = 1.20 \pm 0.06$ vs. $n_{c}^{\text{SM}} = 1.20 \pm 0.04$
  
  Eberhardt, Krinner, A.L., Rauh in prep.

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- HFAG ’03 $\tau_{\Lambda_b} = 1.212 \pm 0.052$ ps$^{-1}$ $\rightarrow$ HFAG ’11 $\tau_{\Lambda_b} = 1.425 \pm 0.032$ ps$^{-1}$
  
  Shift by $4\sigma \Rightarrow$ Eagerly waiting for new LHCb results!!!

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The $B_s$ lifetime

Moriond 2012 LHCb vs SM A.L., Nierste 2011

\[
\frac{\tau_{B_s}^{\text{Exp}}}{\tau_{B_d}} = 1.001 \pm 0.014 \quad \frac{\tau_{B_s}^{\text{SM}}}{\tau_{B_d}} = 0.996...1.000
\]

■ 0.940 $\pm$ 0.014 would have been a desaster for SM = may be NP :-)

■ Update of effective lifetimes
  Fleischer et al used 1011.1096, 1109.1112, 1109.5115: $\tau_{B_s} = 1.477$ ps

<table>
<thead>
<tr>
<th></th>
<th>Exp.</th>
<th>SM-old</th>
<th>SM-new</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^{\text{Eff}}(K^+K^-)$</td>
<td>$1.44 \pm 0.10$</td>
<td>$1.390 \pm 0.032$</td>
<td>$1.43 \pm 0.03$</td>
</tr>
<tr>
<td>$\tau^{\text{Eff}}(\psi f_0)$</td>
<td>$1.70 \pm 0.12$</td>
<td>$1.582 \pm 0.036$</td>
<td>$1.63 \pm 0.03$</td>
</tr>
<tr>
<td>$\tau^{FS}$</td>
<td>$1.417 \pm 0.042$</td>
<td>$-$ $-$</td>
<td>$1.54 \pm 0.03$</td>
</tr>
</tbody>
</table>
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- 2010/2011: Di-muon asymmetry too large — Test $\Gamma_{12}$ with $\Delta \Gamma_s$!

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ΔΓ_s in NLO-QCD I

A brief history of theory predictions

<table>
<thead>
<tr>
<th>Year</th>
<th>Authors</th>
<th>ΔΓ_s expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>'81</td>
<td>Hagelin; Buras et al.</td>
<td>ΔΓ ∝ \mathcal{O}(0.15 \text{ ps}^{-1})</td>
</tr>
<tr>
<td>'93</td>
<td>Aleksan et al.</td>
<td>ΔΓ ∝ \mathcal{O}(0.10 \text{ ps}^{-1})</td>
</tr>
<tr>
<td>'96</td>
<td>Beneke, Buchalla, Dunietz</td>
<td>ΔΓ_s = (0.11^{+0.07}_{-0.06}) \text{ ps}^{-1}</td>
</tr>
<tr>
<td>'00</td>
<td>Beneke, A.L.</td>
<td>ΔΓ_s = (0.06 \pm 0.03) \text{ ps}^{-1}</td>
</tr>
<tr>
<td>'03</td>
<td>Ciuchini, et al</td>
<td>ΔΓ_s = (0.050 \pm 0.016) \text{ ps}^{-1}</td>
</tr>
<tr>
<td>'06</td>
<td>A.L., Nierste</td>
<td>ΔΓ_s = (0.096 \pm 0.036) \text{ ps}^{-1}</td>
</tr>
<tr>
<td>'11</td>
<td>A.L., Nierste</td>
<td>ΔΓ_s = (0.087 \pm 0.021) \text{ ps}^{-1}</td>
</tr>
</tbody>
</table>

Crucial dependence on non-perturbative parameters!

2011 \( f_{B_s} = 231 \pm 15 \text{ MeV} \) used.

Newer Results:
- 1110.4510 - HPQCD: \( f_{B_s} = 225 \pm 4 \text{ MeV} \) \( \Rightarrow \) \( ΔΓ_s = (0.083 \pm 0.017) \text{ ps}^{-1} \)
- 1112.3051 - Fermilab: \( f_{B_s} = 242 \pm 9.5 \text{ MeV} \) \( \Rightarrow \) \( ΔΓ_s = (0.095 \pm 0.021) \text{ ps}^{-1} \)
- 1201.3956 - chiral QM: \( f_{B_s} = 262\pm? \text{ MeV} \) \( \Rightarrow \) \( ΔΓ_s = (0.112\pm?) \text{ ps}^{-1} \)
### Improvement in theoretical accuracy

<table>
<thead>
<tr>
<th>$\Delta \Gamma_s^{\text{SM}}$</th>
<th>2011</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Value</td>
<td>$0.087 \text{ ps}^{-1}$</td>
<td>$0.096 \text{ ps}^{-1}$</td>
</tr>
<tr>
<td>$\delta(B_{R_2})$</td>
<td>$17.2%$</td>
<td>$15.7%$</td>
</tr>
<tr>
<td>$\delta(f_{B_s})$</td>
<td>$13.2%$</td>
<td>$33.4%$</td>
</tr>
<tr>
<td>$\delta(\mu)$</td>
<td>$7.8%$</td>
<td>$13.7%$</td>
</tr>
<tr>
<td>$\delta(B_{S,B_s})$</td>
<td>$4.8%$</td>
<td>$3.1%$</td>
</tr>
<tr>
<td>$\delta(B_{R_0})$</td>
<td>$3.4%$</td>
<td>$3.0%$</td>
</tr>
<tr>
<td>$\delta(V_{cb})$</td>
<td>$3.4%$</td>
<td>$4.9%$</td>
</tr>
<tr>
<td>$\delta(B_{B_s})$</td>
<td>$2.7%$</td>
<td>$6.6%$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$\sum \delta$</td>
<td>$24.5%$</td>
<td>$40.5%$</td>
</tr>
</tbody>
</table>
Finally $\Delta \Gamma_s$ is measured! (naive: $6.1\sigma$)

$$\Delta \Gamma_s^{\text{SM}} = (0.087 \pm 0.021) \text{ps}^{-1}$$

LHCb from $B_s \to J/\psi \phi$

LP 2011 $\Delta \Gamma_s = (0.123 \pm 0.031) \text{ps}^{-1}$ \quad \Rightarrow \quad \frac{\Delta \Gamma_s^{\text{Exp}}}{\Delta \Gamma_s^{\text{SM}}} = 1.41 \pm 0.50$

Moriond 2012 $\Delta \Gamma_s = (0.116 \pm 0.019) \text{ps}^{-1}$ \quad \Rightarrow \quad \frac{\Delta \Gamma_s^{\text{Exp}}}{\Delta \Gamma_s^{\text{SM}}} = 1.33 \pm 0.39$

- D0 $8\text{fb}^{-1}$ 1109.3166: $\Delta \Gamma_s = (0.163 \pm 0.065) \text{ps}^{-1}$
- CDF $9.6\text{fb}^{-1}$: Talk by G. Borissov $\Delta \Gamma_s < \Delta \Gamma_s^{\text{SM}}$
Finally $\Delta \Gamma_s$ is measured! (naive: $6.1\sigma$)

Get rid off the dependence on $f_{B_s}$ (No NP in $\Delta M$)

$$\frac{\Delta \Gamma_s}{\Delta M_s} = 10^{-4} \cdot \left[ 46.2 + 10.6 \frac{B_S'}{B} - \left( 13.2 \frac{B_{R_2}}{B} - 2.5 \frac{B_{R_0}}{B} + 1.2 \frac{B_R}{B} \right) \right]$$

$$= 0.0050 \pm 0.0010$$

HQE vs. Experiment

$$\left( \frac{\Delta \Gamma_s}{\Delta M_s} \right)_{\text{Exp}} / \left( \frac{\Delta \Gamma_s}{\Delta M_s} \right)_{\text{SM}} = 1.30 \pm 0.34$$

HQE works also for $\Gamma_{12}$!

How precise does it work? 30%? 10%?

Still more accurate data needed! TeVatron, LHCb, Super-B(elle)
\[ \Delta \Gamma_s^{\text{CP}} / \Gamma_s = 2 Br(B_s \to D_s^{(*)+} + D_s^{(*)-})? \]

- **1993 Aleksan; Le Yaouanc, Oliveira, Pene, Raynal:**
  The above equation holds in the limit: \( m_c \to \infty; m_b - 2m_c \to 0; N_c \to \infty \)
  Corresponds to negligible 3-body final state contributions to \( \Gamma_{12}^s \)

\[ \frac{\Delta \Gamma_s}{\Gamma_s} \propto \mathcal{O}(0.15) \]

- **1107.4325 Chua, Hou, Shen:** Reanalysis of the exclusive approach
  - 2-body final states contribute \( 0.100 \pm 0.030 \) to \( \Delta \Gamma / \Gamma \)
    Aleksan et al were lucky...
  - 3-body final states contribute about \( 0.06...0.08 \)
    This is comparable to 2-body final states! \( \Rightarrow \) bad approximation \( \Rightarrow \) test exp.

*We strongly discourage from the inclusion of \( Br(B_s \to D^{(*)+} + D^{(*)-}) \) in averages with \( \Delta \Gamma_s \) determined from clean methods.*

\[ \text{A.L., Nierste; hep-ph/0612167} \]
Semi leptonic CP-asymmetries $a_{fs}$ and $\Delta \Gamma_d$


\[
\begin{align*}
  a_{fs}^s &= (1.9 \pm 0.3) \cdot 10^{-5} & \phi_s &= 0.22^\circ \pm 0.06^\circ \\
  a_{fs}^d &= -(4.1 \pm 0.6) \cdot 10^{-4} & \phi_d &= -4.3^\circ \pm 1.4^\circ \\
  A_{sl}^b &= 0.406 a_{sl}^s + 0.594 a_{sl}^d &= (-2.3 \pm 0.4) \cdot 10^{-4}
\end{align*}
\]

Experimental bounds

\[
\begin{align*}
  a_{fs}^s &= (-1150 \pm 610) \cdot 10^{-5} & (HFAG 11) \\
  \phi_s &= -51.6^\circ \pm 12^\circ & (A.L., Nierste, CKMfitter, 1008.1593) \\
  &= -0.1^\circ \pm 5.0^\circ & LHCb Moriond 2012 \\
  a_{fs}^d &= -(49 \pm 38) \cdot 10^{-4} & (HFAG 11) \\
  \Delta \Gamma_d / \Gamma_d &= (-17 \pm 21) \cdot 10^{-3} & (Belle EPS 2011 ) \\
  A_{sl}^b &= -(7.87 \pm 1.72 \pm 0.93) \cdot 10^{-3} & (D0,1106.6308)
\end{align*}
\]

\[
A_{sl}^b(Exp.) / A_{sl}^b(Theory) = 34 \quad 3.9 - \sigma\text{-effect}
\]
New Physics in B-Mixing I

\[ \Gamma_{12,s} = \Gamma_{12,s}^{SM} ; \quad M_{12,s} = M_{12,s}^{SM} \cdot \Delta_s ; \quad \Delta_s = |\Delta_s| e^{i\phi_s^\Delta} \]
\[ \Delta_s = r_s^2 e^{2i\theta_s} = C_B s e^{2i\phi_{Bs}} = 1 + h_s e^{2i\sigma_s} \]

\[
\begin{align*}
\Delta M_s &= 2|M_{12,s}^{SM}| \cdot |\Delta_s| \\
\Delta \Gamma_s &= 2|\Gamma_{12,s}| \cdot \cos (\phi_s^{SM} + \phi_s^\Delta) \\
\frac{\Delta \Gamma_s}{\Delta M_s} &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{SM}|} \cdot \frac{\cos (\phi_s^{SM} + \phi_s^\Delta)}{|\Delta_s|} \\
a_{fs}^s &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{SM}|} \cdot \frac{\sin (\phi_s^{SM} + \phi_s^\Delta)}{|\Delta_s|} \\
\sin(\phi_s^{SM}) &\approx 1/240
\end{align*}
\]

For \( |\Delta_s| = 0.9 \) and \( \phi_s^\Delta = -\pi/4 \) one gets the following bounds in the complex \( \Delta \)-plane:
Combine all data till end of 2011 and neglect penguins

fit of $\Delta_d$ and $\Delta_s$ 1203.0238 (update of 1008.1593) soon v2!

Fits not so good anymore (LHCb vs. Dzero)

$B \rightarrow \tau\nu$ vs. $\sin 2\beta$ solved with $\phi_d^\Delta$ — No tension for $\epsilon_K$
The dimuon asymmetry

The central value of the $d\mu$ asymmetry is larger than *theoretically* possible!

$$A_{sl}^{\text{Max.}} \approx (0.594 \pm 0.022)(5.4 \pm 1.0) \cdot 10^{-3} \frac{\sin(\phi_d^{SM} + \phi_d^\Delta)}{|\Delta_d|}$$

$$+ (0.406 \pm 0.022)(5.0 \pm 1.1) \cdot 10^{-3} \frac{\sin(\phi_s^{SM} + \phi_s^\Delta)}{|\Delta_s|}$$

$$\approx (-3.1; -4.8[1\sigma]; -9.0[3\sigma]) \cdot 10^{-3}$$

$$A_{sl}^{D0} = (-7.8 \pm 2.0) \cdot 10^{-3}$$

Possible solutions:

- HQE violated by $O(200\% - 3300\%)$ now excluded!
- Huge new physics in $\Gamma_{12}$? - see talk by Uli Haisch
- Contradiction to $B_s \to J/\psi\phi$ from LHCb? - Penguins
- Stat. fluctuation (1.5 $\sigma$) of the D0 result? (Actual value is below -4.8 per mille?)

Independent measurements of semi leptonic asymmetries needed!
New physics in $\Gamma_{12}$?

- Large ($\mathcal{O}(200 - 3400\%)$) NP effects in $\Gamma_{12}$?

Why not seen somewhere else?

A new operator $b_s \rightarrow X$ with $M_x < M_B$ contributes not only to $a^s_{sl}$ but also to many more observables, e.g.:

- $\Gamma_3 \Rightarrow \left\{ \begin{array}{l} \tau(B_s)/\tau(B_d) \\ \Delta \Gamma_s \end{array} \right.$

- $\Gamma_0 \Rightarrow \left\{ \begin{array}{l} \tau(B_x) \\ B_{sl} \\ Br(b \rightarrow s \text{ no charm}) \end{array} \right.$

- ...

- A promising candidate for $X$ seems to be $\tau^+ + \tau^- \rightarrow \text{Uli Haisch.}$
Missing charm puzzle, e.g.
Bigi et al ’94; Bagan et al. ’94; Falk, Wise, Dunietz ’95, Neubert ’97... A.L.

Look at inclusive $b$-decay into 0, 1, 2 $c$-quarks
Define $r(x\text{ charm}) := \frac{\Gamma(b \to x\text{ charm})}{\Gamma_{sl}}$: $m_b^5 V_{cb}^2$ cancels; $\Gamma_{sl}$ seems safe
The average number of charm quarks per $b$-decay reads

$$n_c = 0 + [r(1c) + 2r(2c)] B_{sl}^{Exp.}$$
$$= 1 + [r(2c) - r(0c)] B_{sl}^{Exp.}$$
$$= 2 - [r(1c) + 2r(0c)] B_{sl}^{Exp.}$$

Buchalla, Dunietz, Yamamoto ’95

♦ $n_c^{Exp.} < n_c^{Theory}$ = missing charm puzzle
  May be enhanced $b \to s g$... Kagan ...

♦ latest Data from BaBar and CLEO agree within large uncertainties
  Recent and future experiments can do better!

♦ Any unknown, even invisible decay mode has an effect on $r(0, 1, 2 \text{ charm})$

!!! ⇒ Need new experimental values for $r(0c, 1c, 2c) = \Gamma_{0c,1c,2c}/\Gamma_{sl}$ and $B_{sl}!!!$
?New physics in $\Gamma_{12}$?

Step I: Forget about all the bounds and fit $\Delta \Gamma$, $a_{sl}$ and $\Delta M$:

\begin{align*}
\text{p-value} &
\end{align*}

Step II: Take your favourite model which gives new contributions to $\Gamma_{12}$

- Determine contributions to $\delta_d, \delta_s$
- Determine contributions to $\tau_{B_s}, n_c, \ldots$
- Exclude the model :-)

A.L, Nierste, CKMfitter 1203.0238
Angular analysis of $B_s \rightarrow J/\psi \phi$ at CDF, D0 and LHCb:

$$S_{\psi\phi}^{SM} = 0.0036 \pm 0.002 \rightarrow \sin \left( 2\beta_s - \phi_s^\Delta - \delta_s^{Peng,SM} - \delta_s^{Peng,NP} \right) = 0.002 \pm 0.087$$

Is this a contraction to the dimuon asymmetry?

Depends on the possible size of penguin contributions

- SM penguin are expected to be very small
  but see also Faller, Fleischer; Mannel 2008
- NP penguins might be larger

But: even small penguin contributions have a sizeable effect! A.L. 1106.3200
Wish-list for Experiments

a) Congratulations to LHCb for the first measurement of $\Delta \Gamma_s$!
   - Still more precision needed: LHCb, TeVatron, Super-B $B_s \to J/\psi \eta'$
   - Do not use $Br(B_s \to D_s^+(s^-) + D_s^-(s^-)) = \frac{\Delta \Gamma_{CP}}{2\Gamma}$ - check size of 3-body FS!

b) $\tau_{B_s} = (1.001 \pm 0.014)\tau_{B_d}$: strong constraint on NP and duality violation
   - Combine with other determinations of $\tau_{B_s}$: LHCb, ATLAS?, CMS?
   - $B_s$: Effective lifetimes, flavor specific lifetimes (2.x sigma deviation)
   - $\tau_{\Lambda_b}$, ...

c) Di muon asymmetry $A_{sl}^b$
   - HQE fails? No! At most $30 - 40\%$ — more precise test via $\tau(B_s), \Delta \Gamma_s,...$
   - NP acts in $\Gamma_{12}$? No! At most $40\%$! — More precise tests via $\tau(B_s), \Delta \Gamma_s, \Delta \Gamma_d, n_c, B_{sl}, r(0, 1, 2$ charm), $B_s \to \tau\tau, B \to K\tau\tau,...$
   - ???
   - Experimental cross-check via $a_{sl}^d$ and $a_{sl}^s$.

d) $\phi_s^{LHCb} \ll \phi_s^{A_{sl}}$ How large is the penguin pollution?
   - Even small penguins can be important!
   - Values for many penguin modes e.g. $B_s \to J/\psi K_s, K^0 \bar{K}^0, \phi\phi, \eta'\eta'$...
What to do list - Theory

Test of HQE with lifetimes

- $\tau_{B^+}/\tau_{B_d}$ and $\tau_{B_s}/\tau_{B_d}$ fits well $\Rightarrow$ currently no hints for deviations from HQE
- Precise non-perturbative matrix elements for 4-quark operators urgently needed
  
  *Beautiful Mesons and Baryons on the Lattice* ECT* Trento, 2-6 April 2012

- Perturbative improvements of lifetime predictions

Theoretical predictions for mixing observables

- Precise decay constants and Bag parameter for $\Delta M$
- Additional Bag parameters at dimension 6 and 7 for $\Gamma_{12}$
- $\alpha_s/m_b$ corrections for $\Gamma_{12}$
- $\alpha_s^2$ corrections for $\Gamma_{12}$

Theoretical predictions for charm mixing observables

- Push HQE to its limits
- Try to improve the exclusive approach

Update of theoretical predictions for inclusive rates
It is actually not bad, what the Grinch left for us

Expansion in $1/m_b$ works so well,
What does this tell about charm? $1/m_c \approx 3 \cdot 1/m_b$
CKM⁻: How large are Penguins? II

Many observables in the $B_s$ mixing system:

Elimination of $\Gamma_{12}^{\text{Theo}}$ via (No hint for incorrectness of $\Gamma_{12}^{\text{Theo}}$ except: $A_{s l}^b$ is $1.5\sigma$ above bound)

$$a_{s l}^s = - \frac{\Delta \Gamma}{\Delta M} \frac{S_{\psi \phi}}{\sqrt{1 - S_{\psi \phi}^2}} \cdot \delta$$

not possible at that simple level, because $\delta \neq 1$

$$\delta = \frac{\tan (\phi_s^{\text{SM}} + \phi_s^\Delta)}{\tan (-2\beta_s^{\text{SM}} + \phi_s^\Delta + \delta_{\text{peng},\text{SM}} + \delta_{\text{peng},\text{NP}})}$$

A.L. 1106.3200
Above relation can be used to determine $\delta_{s}^{\text{SM}} + \delta_{s}^{\text{NP}}$

To extract $\phi_{s}^{\Delta}$ one needs $\Gamma_{12}^{s,\text{SM}}$

$\delta_{s}^{\text{SM}} + \delta_{s}^{\text{NP}} = 10^\circ$
$\delta_{s}^{\text{SM}} + \delta_{s}^{\text{NP}} = 5^\circ$
$\delta_{s}^{\text{SM}} + \delta_{s}^{\text{NP}} = 2^\circ$
$\delta_{s}^{\text{SM}} + \delta_{s}^{\text{NP}} = 0^\circ$

$\phi_{s}^{\text{SM}} = 0.22^\circ \pm 0.06^\circ$
$-2\beta_{s} = (2.1 \pm 0.1)^\circ$

A.L. 1106.3200
Lifetimes: $\tau_{B^+}/\tau_{B_d}$ in NLO-QCD I

\[
\frac{\tau_1}{\tau_2} = 1 + \left( \frac{\Lambda}{m_b} \right)^3 \left( \Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \ldots \right) + \left( \frac{\Lambda}{m_b} \right)^4 \left( \Gamma_4^{(0)} + \ldots \right) + \ldots
\]


\[
\begin{bmatrix}
\tau(B^+) \\
\tau(B^+_d)
\end{bmatrix}_{\text{LO,NLO,HFAG10}} = 1.047 \pm 0.049 \leftrightarrow 1.063 \pm 0.027 \leftrightarrow 1.071 \pm 0.009
\]
Lifetimes: $\tau_{B^+}/\tau_{B_d}$ in NLO-QCD II

\[ \frac{\tau_1}{\tau_2} = 1 + \left( \frac{\Lambda}{m_b} \right)^3 \left( \Gamma_{3}^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_{3}^{(1)} + ... \right) + \left( \frac{\Lambda}{m_b} \right)^4 \left( \Gamma_{4}^{(0)} + ... \right) + ... \]


\[
\left[ \frac{\tau(B^+)}{\tau(B_d^0)} \right]_{\text{LO,NLO,HFAG11}} = 1.047 \pm 0.049 \leftrightarrow 1.044 \pm 0.024 \leftrightarrow 1.079 \pm 0.007
\]
Lifetimes: $\tau_{B^+}/\tau_{B_d}$ in NLO-QCD III

$$\frac{\tau_{B^+}}{\tau_{B_d}} - 1 = 0.0324 \left( \frac{f_B}{200\text{MeV}} \right)^2 \left[ (1.0 \pm 0.2)B_1 + (0.1 \pm 0.1)B_2 \right]$$

$$- (17.8 \pm 0.9)\epsilon_1 + (3.9 \pm 0.2)\epsilon_2 - 0.26$$

with non-perturbative input from Becirevic hep-ph/0110124

$$B_1 = 1.10 \pm 0.20$$

$$B_2 = 0.79 \pm 0.10$$

$$\epsilon_1 = -0.02 \pm 0.02$$

$$\epsilon_2 = 0.03 \pm 0.01$$

Update urgently needed!
Lifetimes: Lifetimes of heavy hadrons

- \( \tau(B^+)/\tau(B_d) \): HQE seems to fit, but we need urgently more precise hadronic matrix elements

\[
\frac{\tau(B_s)}{\tau(B_d)} = 0.996...1.000 \leftrightarrow 0.969 \pm 0.017 \quad \text{HFAG 2011}
\]
\[
A.L. 1102.4274 \leftrightarrow 1.004 \pm 0.018 \quad \text{LHCb-Conf2011-049}
\]

More data as well as non-perturbative matrix elements needed

- \( \tau(\Lambda_b), \tau(\Xi_b) \) and \( \tau(B_c) \): more data and further theory work (perturbative and non-perturbative) neccessary

- \( \tau(D) \), D-mixing: work in progress
  Bigi, Uraltsev 2001; Bobrowski, A.L., Riedl, Rohrwild 1002.4794; 1011.5608;
  Bobrowski, A.L. Nierste, Prill, to appear

It is not unplausible that HQE might give reasonable estimates
How large can the SM contribution to CP violation in $D^0$-$\bar{D}^0$ mixing be?

M. Bobrowski, A. Lenz, J. Riedl, J. Rohrwild

(Submitted on 25 Feb 2010)

We investigate the maximum size of CP-violating effects in $D$-mixing within the Standard Model (SM), using Heavy Quark Expansion (HQE) as theoretical working tool. For this purpose we determine the leading HQE contributions and also $\Delta \alpha_{\text{SM}}$ corrections as well as subleading $1/m_c$ corrections to the absorptive part of the mixing amplitude of neutral $D$ mesons. It turns out that these contributions to $\Sigma \Gamma_{[1,2]}$ do not vanish in the exact SUSY(3) limit. Moreover, while the leading HQE terms give a result for $\Sigma \Gamma_{[1,2]}$ orders of magnitude lower than the current experimental value, we do find a sizeable phase. In the literature it was suggested that higher order terms in the HQE could be much less affected by the severe GIM cancellations of the leading terms; it is even not excluded that these higher order terms can reproduce the experimental value of $\Sigma \Gamma$. If such an enhancement is realized in nature, the phase discovered in the leading HQE terms can have a sizeable effect. Therefore, we think that statements like [v1] "CP violating effects in $D$-mixing of the order of $10^{-5}$ to $10^{-2}$ are an unambiguous sign of new physics"--given our limited knowledge of the SM prediction--are premature. Finally, we give an example of a new physics model that can enhance the leading HQE terms to $\Sigma \Gamma_{[1,2]}$ by one to two orders of magnitude.