

Critical nucleus charge in a superstrong magnetic field: effect of screening.

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(<http://arxiv.org/abs/1112.1891>, accepted in Phys.Rev.D)

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Outline

- Critical charge
- Effect of screening
- Critical charge with the account of screening
- Conclusions

Pointlike Coulomb center with charge $Z > 137$ can not exist.

1945-1970

But if the finite size of a nucleus is taken into account then Dirac equation can be solved for $Z > 137$. At $Z = Z_{cr} \approx 170$ ground energy level reaches lower continuum ($\varepsilon = -m_e$) and spontaneous electron-positron pair production occurs. Electrons occupy ground level and one can observe two free positrons.

Critical charge in a magnetic field

V.N. Oraevskii, A.I. Rez, V.B. Semikoz, JETP, Vol. 45, No 3, p. 428 (March 1977)

Solution found for $B \gg m^2 e^3 Z^2$; $B \gg \frac{m^2}{e(Ze^2)^2}$

Critical charge:

$$\frac{B}{B_0} = 2(Z_{cr}e^2)^2 \exp\left(-\gamma + \frac{\pi - 2 \arg \Gamma(1 + 2iZ_{cr}e^2)}{Z_{cr}e^2}\right).$$

Here $B_0 = m_e^2/e = 4.41 \cdot 10^{13} G$, $e^2 = \alpha = 1/137$.

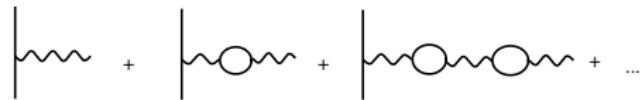
$$B/B_0 \quad 10^2 \quad 10^3 \quad 2 \cdot 10^4$$

$$Z_{cr} \quad 96 \quad 61 \quad 41$$

Effect of screening

(B.Machet, M.I.Vysotsky, Phys. Rev. D 83 (2011), 025022)

(A.E.Shabad, V.V.Usov, Phys. Rev. D. 77 (2008), 025001)

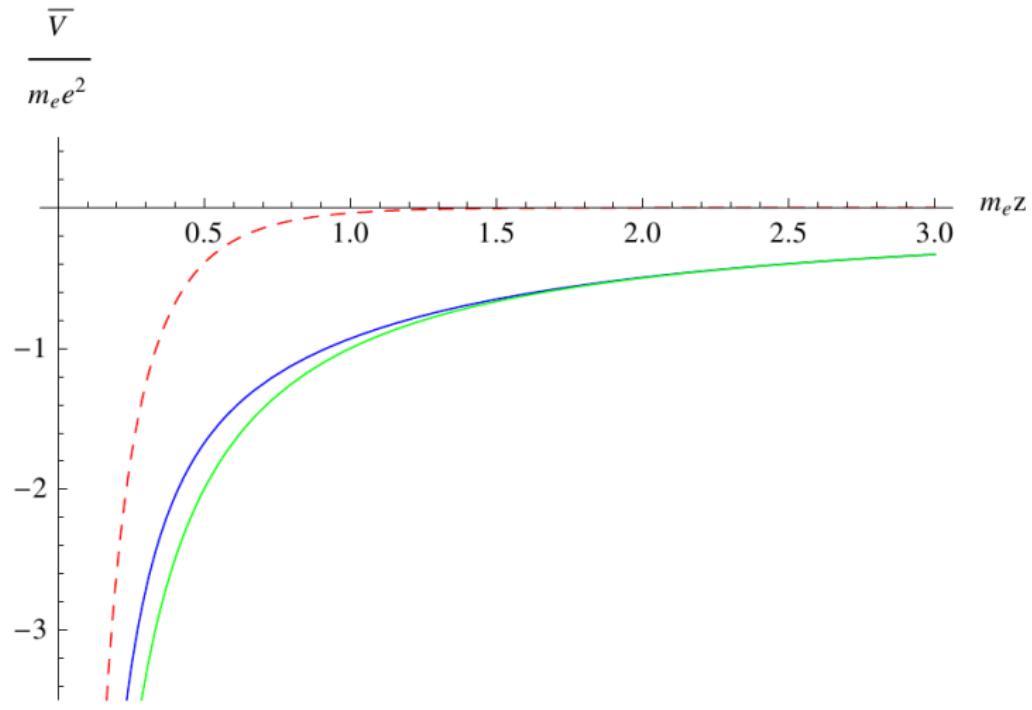


$$\Phi(z, \rho) = 4\pi e \int \frac{e^{i\bar{k}_\perp \bar{\rho} + ik_\parallel z} dk_\parallel d^2 k_\perp / (2\pi)^3}{k_\parallel^2 + k_\perp^2 + \frac{2e^3 B}{\pi} \exp(-\frac{k_\perp^2}{2eB}) \frac{k_\parallel^2}{6m_e^2 + k_\parallel^2}}$$



$$\Phi(z, 0) = \frac{e}{|z|} \left[1 - e^{-\sqrt{6m_e^2}|z|} + e^{-\sqrt{(2/\pi)e^3 B + 6m_e^2}|z|} \right] .$$

Effect of screening



Method used to solve Dirac equation

$$a_H = \frac{1}{\sqrt{eB}} \ll \frac{1}{m_e Z e^2} = a_B \Rightarrow B \gg m^2 e^3 Z^2$$

Then:

$$(\alpha(\mathbf{p} + e\mathbf{A}) + V + \beta)\psi = \varepsilon\psi$$



$$\frac{d^2\chi}{dz^2} + 2m_e(E - U)\chi = 0 ,$$

$$E = \frac{\varepsilon^2 - m_e^2}{2m_e},$$

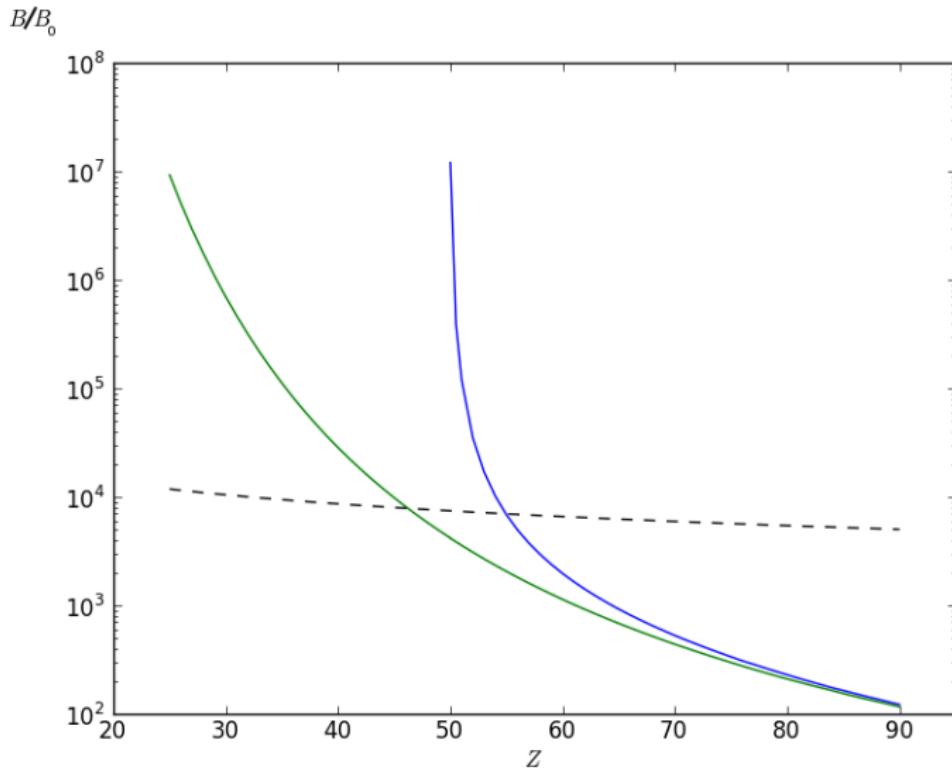
$$U = \frac{\varepsilon}{m_e}\bar{V} - \frac{1}{2m_e}\bar{V}^2 + \frac{\bar{V}''}{4m_e(\varepsilon + m_e - V)} + \frac{3/8(\bar{V}')^2}{m_e(\varepsilon + m_e - V)^2} ,$$

$$\bar{V} = \frac{1}{a_H^2} \int_0^\infty V(\sqrt{\rho^2 + z^2}) \exp\left(-\frac{\rho^2}{2a_H^2}\right) \rho d\rho$$

Results for $Z = 40$

$\frac{B}{B_0}$	ε/m_e (ORS formula)	ε/m_e (Numerical results)	ε/m_e (Numerical results with screening)
10^0	0.819	0.850	0.850
10^1	0.653	0.667	0.667
10^2	0.336	0.339	0.346
10^3	-0.158	-0.159	-0.0765
10^4	-0.758	-0.759	-0.376
$2 \cdot 10^4$	-0.926	-0.927	-0.423
...	at $B/B_0 \approx 2.85 \cdot 10^4$, $\varepsilon = -m_e$		
10^5	—	—	-0.488
10^6	—	—	-0.524
10^7	—	—	-0.535
10^8	—	—	-0.538

Critical charge with the account of screening



Conclusions

Because of screening only nucleus with $Z > 52$ becomes critical in a superstrong magnetic fields.

Thanks for attention!