Lattice QCD with physical quark masses

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for the

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(PLB 705:477,2011, JHEP 1108:148,2011;

PLB 701:265,2011; Science 322:1224,2008)

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Purpose of lattice QCD

- QCD fundamental objects: quarks and gluons
- QCD observed objects: protons, neutrons (π, K, ...)

! Huge discrepancy: not even the same particles observed as in the Lagrangean

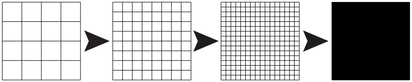
➛ Perturbation theory has no chance

- Need to solve low energy QCD to:
  - Compute hadronic and nuclear properties
    "people who love QCD"
    - Masses, decay widths, scattering lengths, thermodynamic properties, ...
  - Compute hadronic background
    "people who hate QCD"
    - Non-leptonic weak MEs, quark masses, g-2, ...

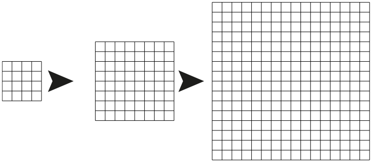
Lattice

Lattice QCD=QCD when

- Cutoff removed (continuum limit)



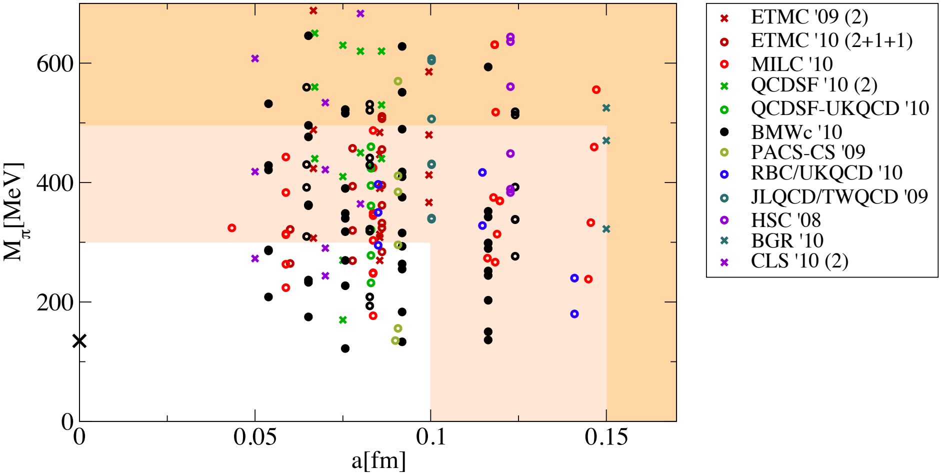
- Infinite volume limit taken



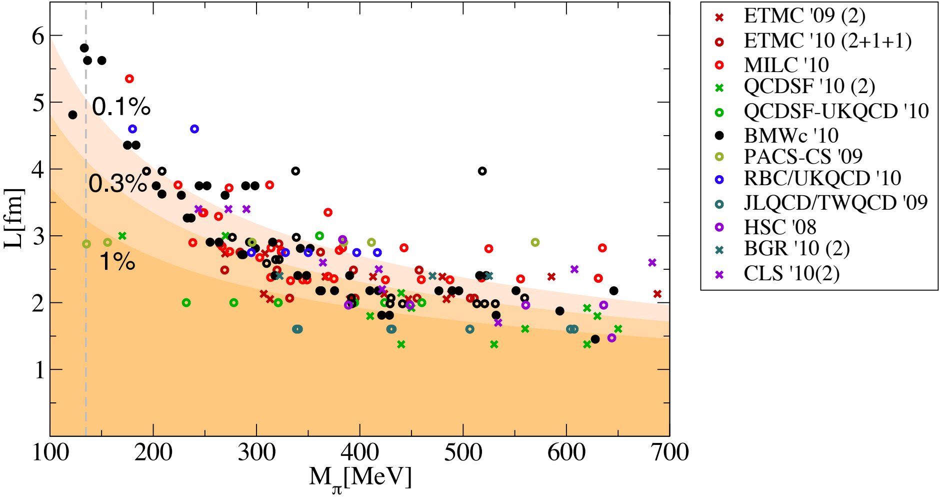
- At physical hadron masses (Especially π)
  - Numerically challenging to reach light quark masses

Statistical error from stochastic estimate of the path integral

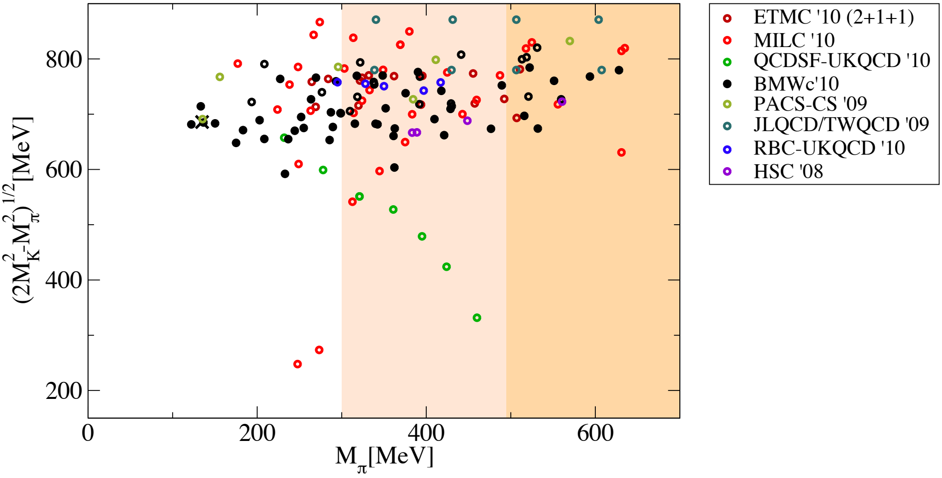
Landscape M_π vs. a



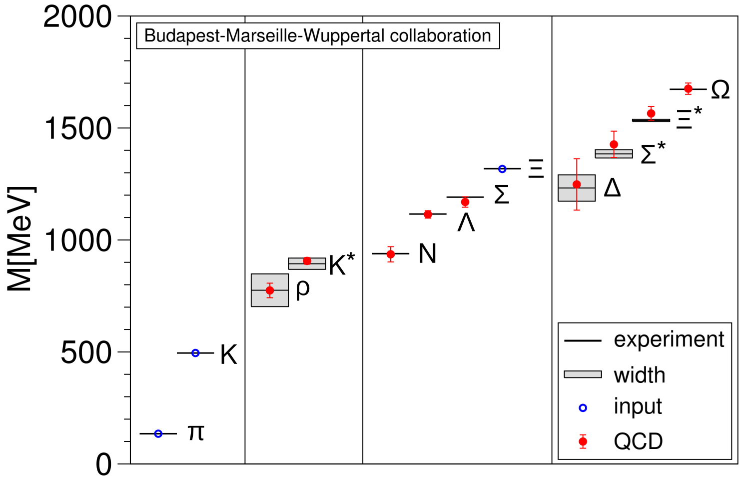
Landscape L vs. M_π



Landscape M_K vs. M_π



The light hadron spectrum



(Science 322:1224,2008)

Light quark masses

Goal:

- Compute light quark masses ab initio

Method:

- Go to the physical point
- Read off input quark masses and renormalize

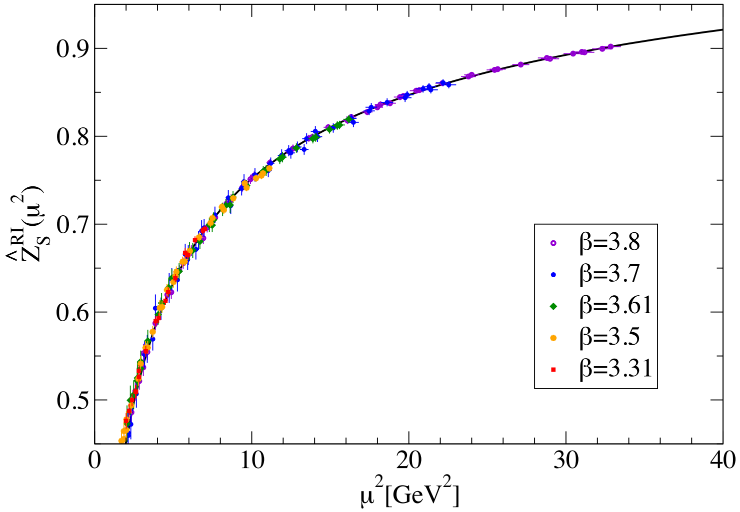
Challenge:

- Minimize and control all systematics
  - 2+1 dynamical fermion flavors
  - Physical quark masses
  - Continuum extrapolation
  - Infinite volume
  - Nonperturbative renormalization

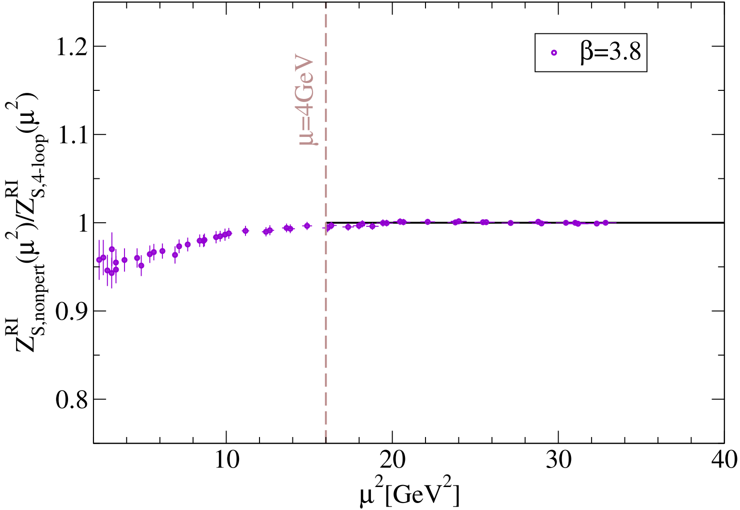
Renormalization strategy

- Goal:
  - Full nonperturbative renormalization
  - Optional accurate conversion to perturbative scheme
- Method:
  - We use RI-MOM scheme (Martinelli et. al., 1993)
    - $O(a)$ correction (Maillart, Niedermayer, 2008)
  - Compute m_q at low scale $\mu \ll 2\pi/a \sim 11 - 24$ GeV
    - $\mu = 2.1 GeV$
    - $\mu = 1.3 GeV$
  - Do continuum non-perturbative running to high scale $\mu' \gg \Lambda_{QCD}$
  - Further conversion in 4-loop PT

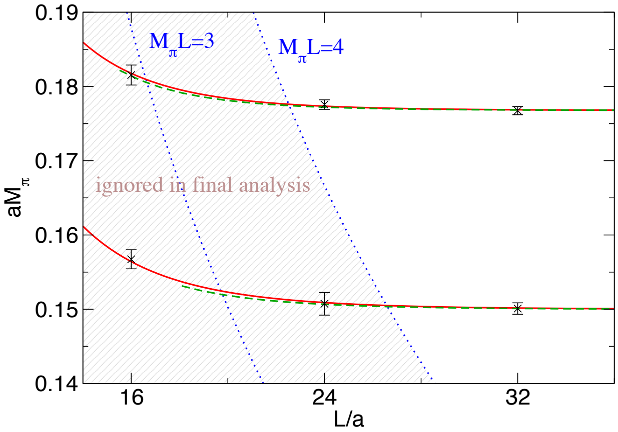
Nonperturbative running



Reaching the perturbative regime

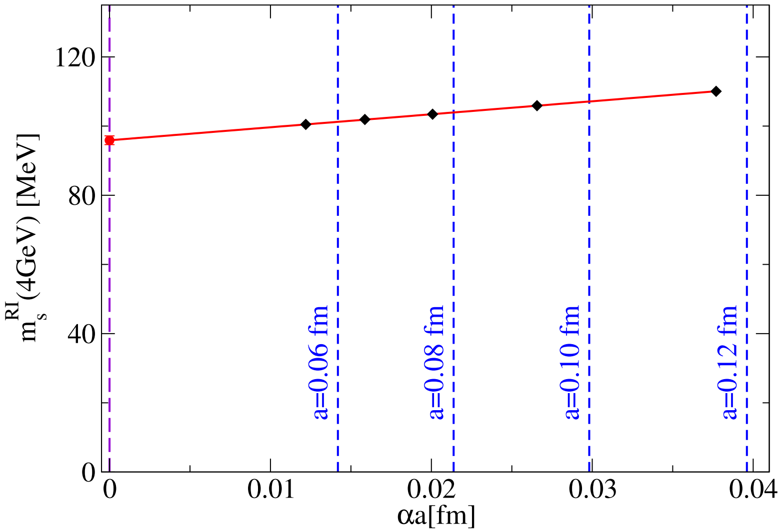


Tiny finite volume effects

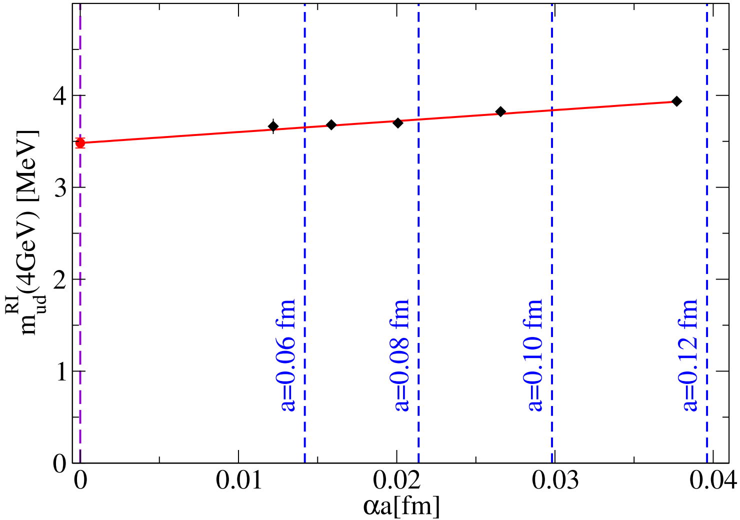


- FV effects tiny
- Dedicated FV runs
- Perfect agreement with FV χPT (Colangelo et. al. 2005)

Strange quark mass



Light quark masses



Individual m_u and m_d

- Goal:
  - Compute m_u and m_d separately
- Method:
  - Need QED and isospin breaking effects in principle
  - Alternative: use dispersive input -Q from $\eta \to \pi\pi\pi$

    $Q^2 = \frac{1}{2}\left(\frac{m_s}{m_{ud}}\right)^2 \frac{m_d - m_u}{m_{ud}}$
  - ✓ Transform precise m_s/m_{ud} into $(m_d - m_u)/m_{ud}$
  - We use the conservative $Q = 22.3(8)$ (Leutwyler, 2009)

Systematic error treatment

- Goal:
  - Reliably estimate total systematic error
- Method:
  - 288 full analyses (2000 bootstrap on each)
    - 2 plateaux regions
    - 2 continuum forms: $O(\alpha_s a)$, $O(a^2)$
    - 3 chiral forms: $2 \times SU(2)$, Taylor
    - 2 chiral ranges: $M_\pi < 340, 380$ MeV
    - 3 renormalization matching procedures
    - 2 NP continuum running forms
    - 2 scale setting procedures
  - All analyses weighted by fit quality
    - Mean gives final result
    - Stdev gives systematic error
  - Statistical error from 2000 bootstrap samples

Final result

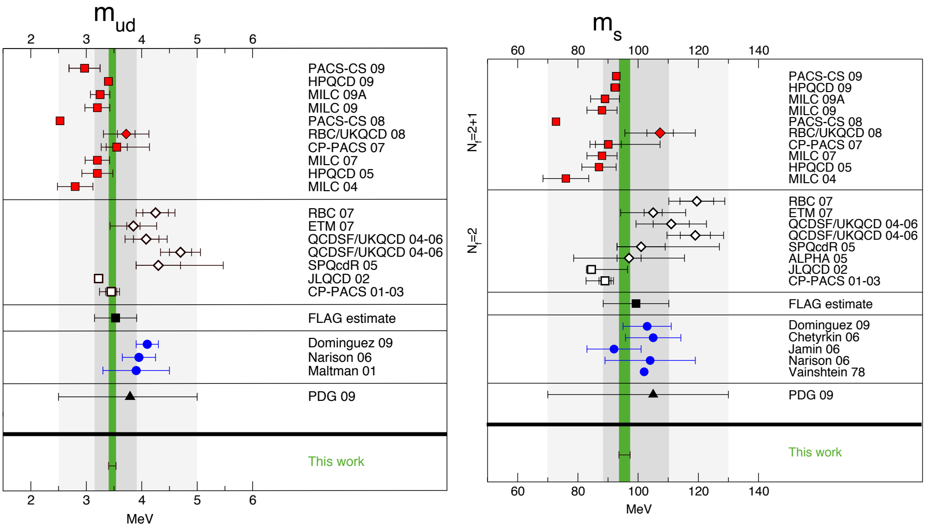
	RI @ 4 GeV	RGI	$\overline{\text{MS}}$ @ 2 GeV
m_s	96.4(1.1)(1.5)	127.3(1.5)(1.9)	95.5(1.1)(1.5)
m_{ud}	3.503(48)(49)	4.624(63)(64)	3.469(47)(48)
m_s/m_{ud}		27.53(20)(8)	
m_u	2.17(04)(10)	2.86(05)(13)	2.15(03)(10)
m_d	4.84(07)(12)	6.39(09)(15)	4.79(07)(12)

Relative error budget:

	stat.	plateau	scale set	mass	renorm.	cont.
m_s	0.702	0.148	0.004	0.064	0.061	0.691
m_{ud}	0.620	0.259	0.027	0.125	0.063	0.727
m_s/m_{ud}	0.921	0.200	0.078	0.125	—	0.301

(JHEP 1108:148,2011; PLB 701:265,2011)

Comparison


m_{ud}
2
3
4
5
6
PACS-CS 09
HPQCD 09
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS 07
MILC 07
HPQCD 05
MILC 04
RBC 07
ETM 07
QCDSF/UKQCD 04-06
QCDSF/UKQCD 04-06
SPQcdR 05
JLQCD 02
CP-PACS 01-03
FLAG estimate
Dominguez 09
Narison 06
Maltman 01
PDG 09
This work
MeV
m_s
60
80
100
120
140
$N_f=2+1$
$N_f=2$
PACS-CS 09
HPQCD 09
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS 07
MILC 07
HPQCD 05
MILC 04
RBC 07
ETM 07
QCDSF/UKQCD 04-06
QCDSF/UKQCD 04-06
SPQcdR 05
ALPHA 05
JLQCD 02
CP-PACS 01-03
FLAG estimate
Dominguez 09
Chetyrkin 06
Jamin 06
Narison 06
Vainshtein 78
PDG 09
This work
MeV

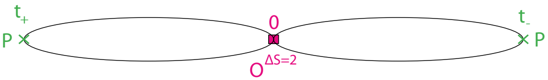
Standard model neutral K mixing

- Goal:
  - Check SM CP violation in neutral K system
- Method:
  - Compute effective weak matrix element $B_K = \frac{3\langle \bar{K} | O_{\Delta S=2} | K \rangle}{8 f_K^2 M_K^2}$
  - Relate kaon CP violation to CKM phase
    ➛from CKM unitarity: $\hat{B}_K = 0.89^{+0.16}_{-0.22}$ (CKMfitter, 2011)
- Challenge:
  - Minimize and control all systematics
    - 2+1 dynamical fermion flavors
    - Physical quark masses
    - Mixing of unphysical operators
    - Continuum
    - Infinite volume

Observable

Matrix element of the effective weak operator $\langle \bar{K}|O|K\rangle$:

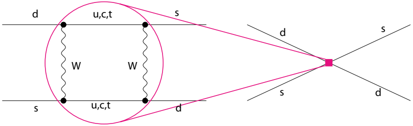
$$\langle P^\dagger(t_+)O(0)P(t_-)\rangle \xrightarrow{t_\pm\to\pm\infty} \frac{|\langle K|P|0\rangle|^2}{(2M_K)^2}\langle \bar{K}|O|K\rangle e^{-M_K(t_+-t_-)}$$



where

$$P = [\bar{s}d]_P = \bar{s}\gamma_5 d$$

$$O_{\Delta S=2} = [\bar{s}d]_{V-A}[\bar{s}d]_{V-A}$$

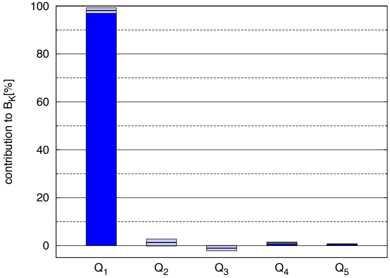


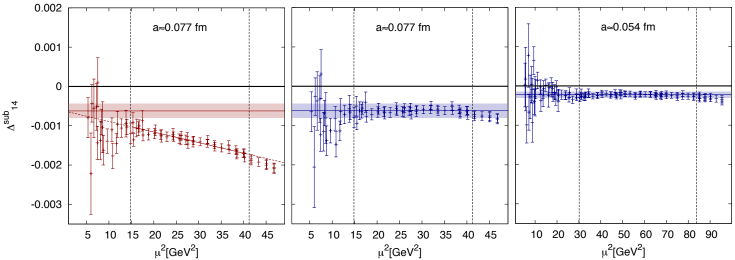
Norm from:

$$\langle P^\dagger(t)P(0)\rangle \xrightarrow{t\to\infty} \frac{|\langle K|P|0\rangle|^2}{2M_K}e^{-M_K t}$$

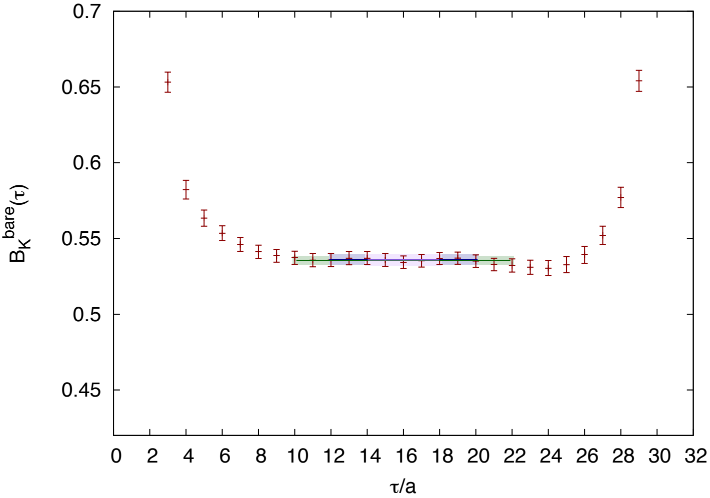
Unphysical operator mixing

- ☞ χSB induces mixing with 4 unphysical operators
- ☞ Mixing terms chirally enhanced
- ✓ Small even below physical m_π
- ✓ Good chirality of our action

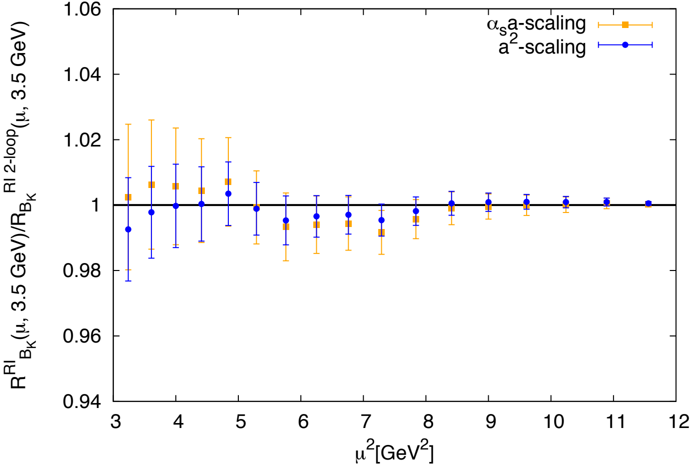




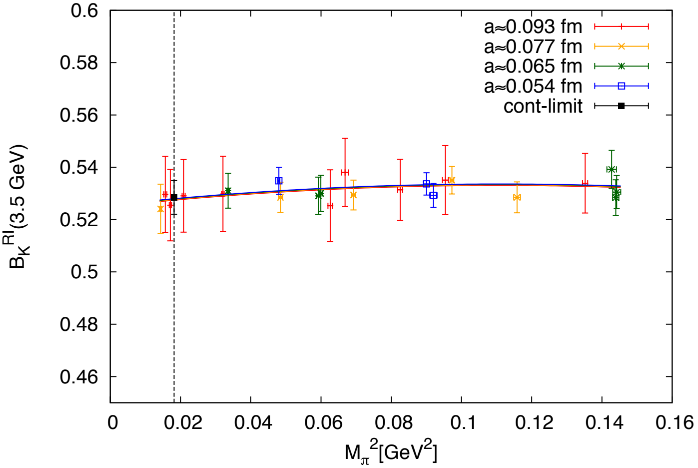
Signal


0.7
0.65
0.6
0.55
0.5
0.45
$B_K^{bare}(\tau)$
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
τ/a

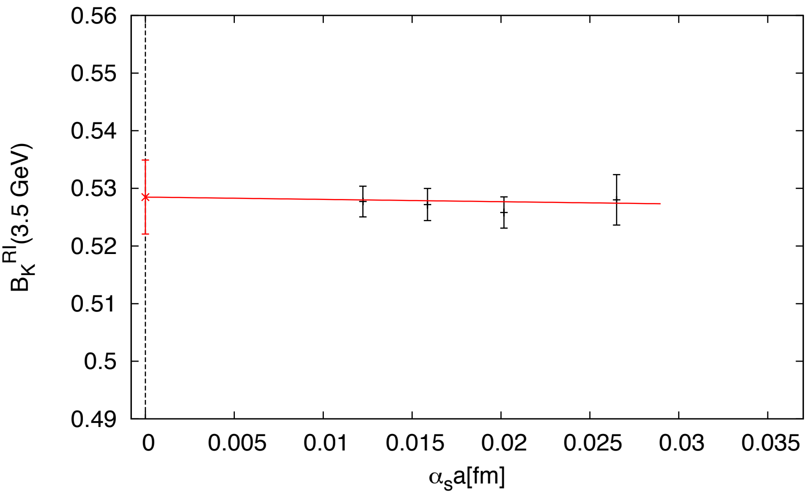
Running


α_sa-scaling
a^2-scaling
$R^{RI}_{B_K}(\mu, 3.5\text{ GeV})/R_{B_K}^{RI\ 2\text{-loop}}(\mu, 3.5\text{ GeV})$
μ^2[GeV^2]
1.06
1.04
1.02
1
0.98
0.96
0.94
3
4
5
6
7
8
9
10
11
12

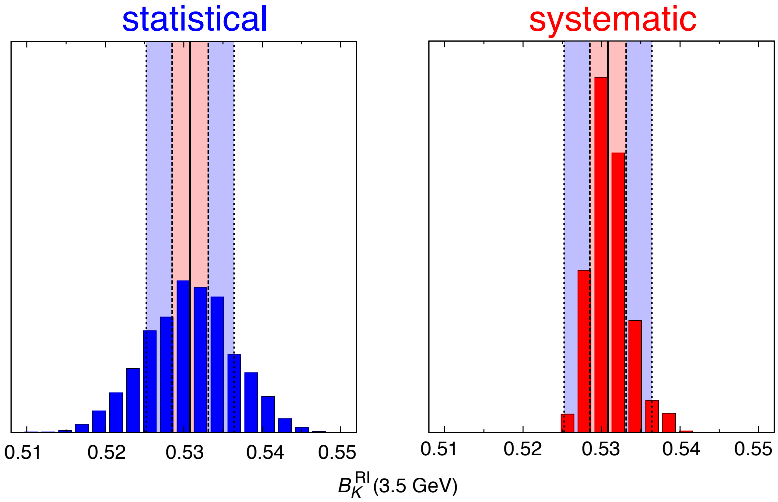
Physical point



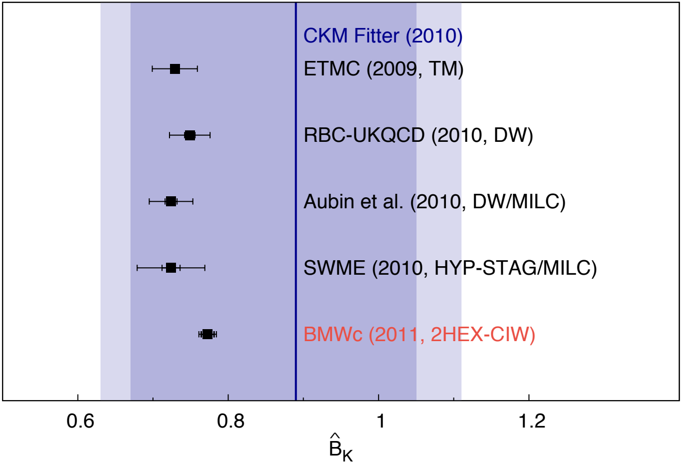
Continuum extrapolation



Errors


statistical
systematic
0.51
0.52
0.53
0.54
0.55
0.51
0.52
0.53
0.54
0.55
B_K^{RI}(3.5 GeV)

Comparison



(PLB 705:477,2011)

THANK YOU

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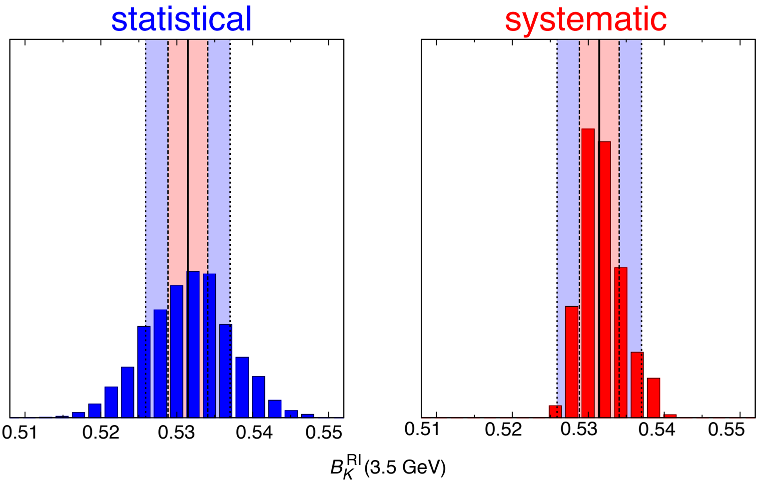
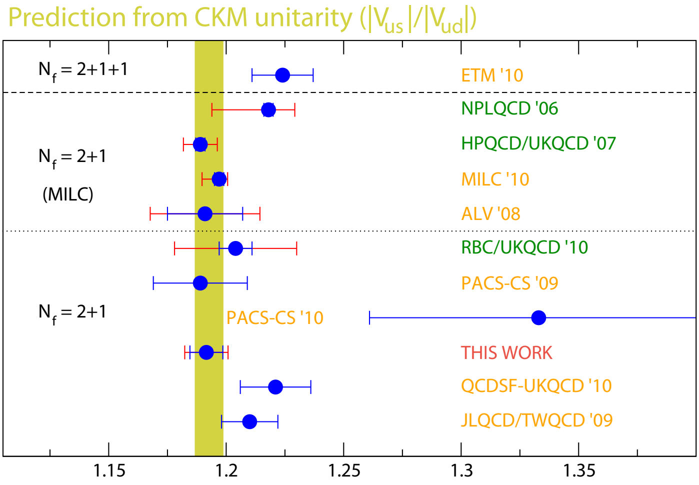
BACKUP

Errors - unit weight



Comparison


Prediction from CKM unitarity (|Vus|/|Vud|)
N_f = 2+1+1
ETM '10
NPLQCD '06
HPQCD/UKQCD '07
N_f = 2+1
(MILC)
MILC '10
ALV '08
RBC/UKQCD '10
PACS-CS '09
N_f = 2+1
PACS-CS '10
THIS WORK
QCDSF-UKQCD '10
JLQCD/TWQCD '09
1.15
1.2
1.25
1.3
1.35