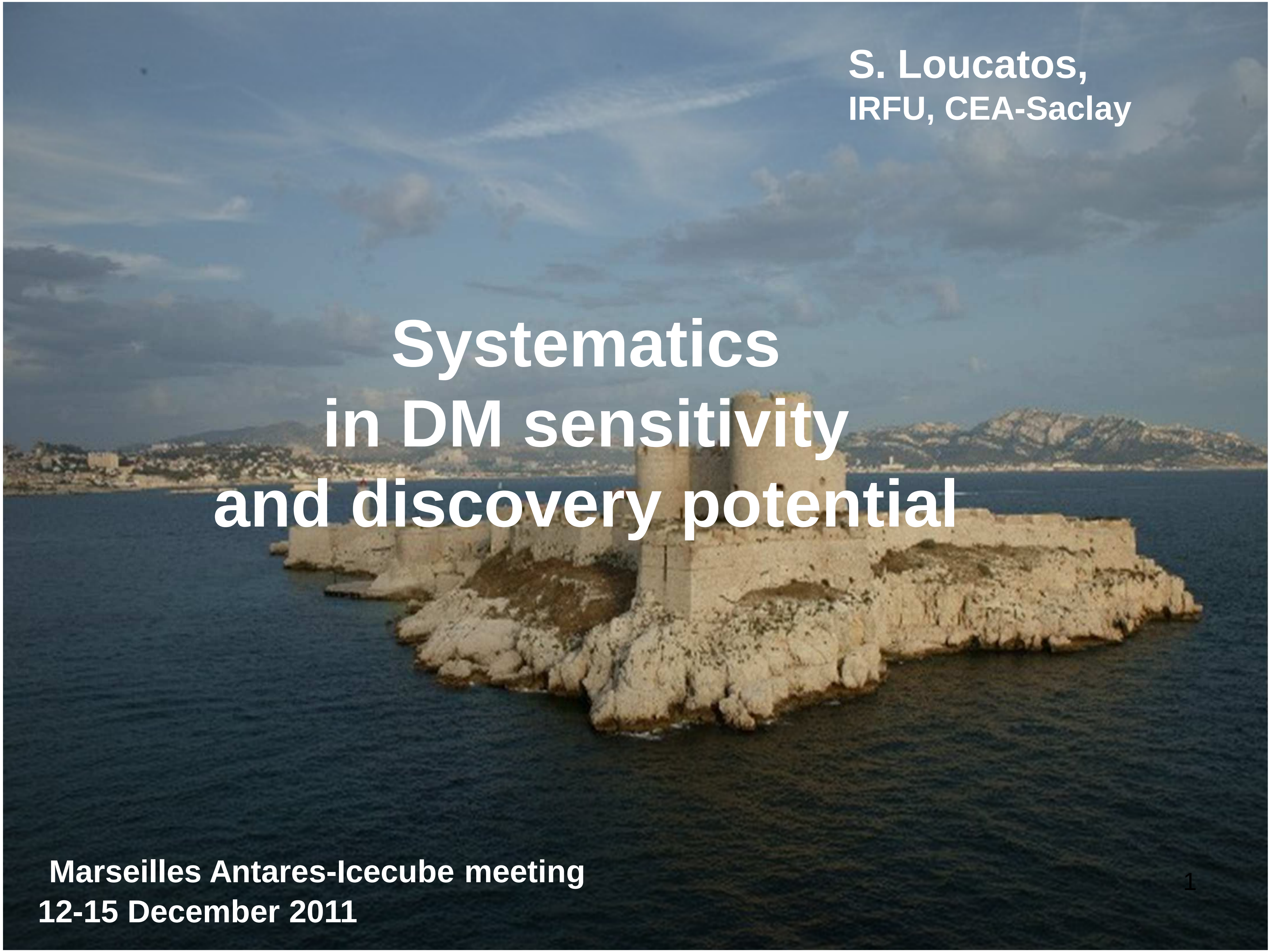

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Systematics in DM sensitivity and discovery potential

Marseilles Antares-Icecube meeting
12-15 December 2011

Outline

- Include systematics in the confidence interval.
  Methods:

  - simply shift the limit

  or introduce a probability distribution
  - pseudo experiments

  - profile likelihood
  - hybrid Bayesian, used here

- Evaluation of the DM analysis systematics

Profile likelihood

(tested, not used)

We illustrate the method of profile likelihood using the example of a search for a rare decay where the expected background is known only approximately. We will need the following notation. Assume that we observe x events in a suitably chosen signal region and a total of y events in the background region. Here the background region can be chosen fairly freely and need not be contiguous. Furthermore, the probability that a background event falls into the background region divided by the probability that it falls into the signal region is denoted by τ. For example, if we use two background regions of the same size as the signal region and assume the background distribution is flat we have $\tau = 2$. If the background rate is estimated from Monte Carlo, τ is the size of the Monte Carlo sample relative to the size of the data sample. Then a probability model for the data is given by

$$X \sim \mathrm{Pois}(\mu + b), \quad Y \sim \mathrm{Pois}(\tau b)$$

where μ is the signal rate, b is the background rate and Pois is the usual Poisson distribution. We will use large caps letters X, Y to denote random variables and small letters x, y to denote realizations (observed values) of these random variables.

We can assume X and Y to be independent and so

$$f(x, y|\mu, b) = \frac{(\mu + b)^x}{x!} \mathrm{e}^{-(\mu+b)} \cdot \frac{(\tau b)^y}{y!} \mathrm{e}^{-\tau b}.$$

The likelihood function is given by $L(\mu, b|x, y) = f(x, y|\mu, b)$. Maximizing over both μ and b we find the usual maximum likelihood estimators $(\widehat{\mu}, \widehat{b}) = (x - y/\tau, y/\tau)$. Fixing μ and maximizing over b alone yields

$$\hat{b}(\mu) = \frac{x + y - (1+\tau)\mu + \sqrt{(x + y - (1+\tau)\mu)^2 + 4(1+\tau)y\mu}}{2(1+\tau)}.$$

For other models it may not be possible to find $\hat{b}(\mu)$ analytically, in which case numerical methods need to be used. Now the profile likelihood function is given by

$$\lambda(\mu|x, y) = \frac{L(\mu, \hat{b}(\mu)|x, y)}{L(\widehat{\mu}, \widehat{b}|x, y)}$$

and according to the theory $-2 \log \lambda$ has an approximate χ^2 distribution with 1 degree of freedom.

Rolke et al,NIM A 551 (2005) 493)

Inclusion of systematics in confidence intervals

Numbers of expected background events from scrambled data . For an expected background n_b and an observed number of events n_{obs} →upper limit at 90% CL by Feldman Cousins method.

With systematics: the FC ordering is again used, but modified PDF: systematic uncertainties are theoretical (assumed Gauss-shaped *) uncertainties on

- background and
- signal efficiency

$$q(n)_{s+b} = \frac{1}{2\pi\sigma_b\sigma_\varepsilon}\int_0^\infty\int_0^\infty p(n)_{b'+\varepsilon' s} \times e^{-(b-b')^2/2\sigma_b^2}\, e^{-(1-\varepsilon')^2/2\sigma_\varepsilon^2}\, \mathrm{d}b'\, \mathrm{d}\varepsilon', \tag{1}$$

* log-normal better?

- Effect: enlargement of the confidence interval

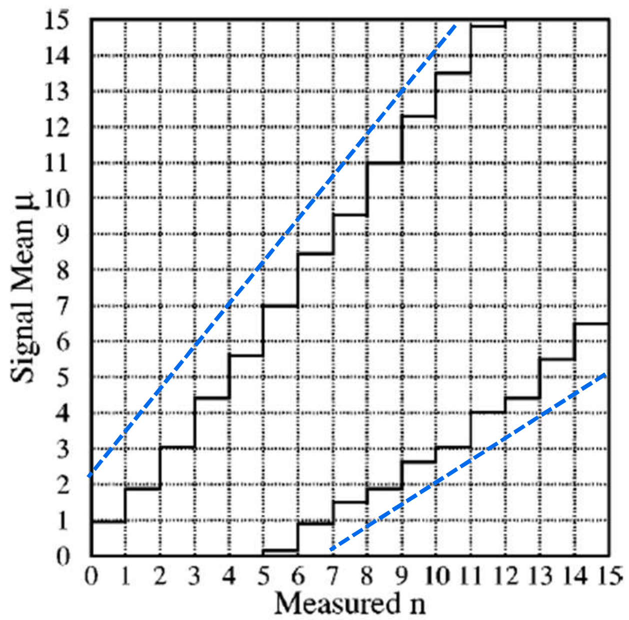


FIG. 7. Confidence belt based on our ordering principle, for 90% C.L. confidence intervals for unknown Poisson signal mean μ in the presence of a Poisson background with known mean $b = 3.0$.

Estimate systematics

- The uncertainty in efficiency ε (effective area) is the sum of uncertainties on 3 quantities: efficiency, resolution and pointing.
- The background *b* is considered perfectly known without uncertainty, taken from the scrambled data.

Estimate systematics

In the point source analysis using AAfit the systematics are:

i. 15% in track reconstruction due to OM efficiency uncertainty
ii. 15% in track reconstruction due to OM time resolution uncertainty (2 ns smearing)
iii. 15% uncertainty on angular resolution: .53± .08 degrees
iv. Absolute pointing uncertainty: 0.13° azimuthal rotation s 0.06° for the zenith

Items (iii) and (iv) enter as a signal efficiency uncertainty: equivalent to a larger PSF:

With cone cut in the binned search, some events will leak out of the cut. => if the resolution is 3° (median) and the cone cut is at 3°, the leakage effect can be significant.

An effect depending on the neutrino energy can also be expected, to be estimated.

Background flat around a source, taken from the scrambled data => a difference between the theoretical and real cone opening has no effect either.

Previous result (Bamberg): a 20% (15% $\oplus$ 15%) systematics equivalent in efficiency (effective area) was introduced, neglecting terms (iii) and (iv)

In order to evaluate the effect on efficiency of (iii) and (iv), the angular resolution uncertainty of BBfit has to be estimated.

The maximum possible effect can be first estimated introducing a strong uncertainty of 50% on the angular resolution.

Resolution (median) of Bbfit: between 2 and 4 degrees, depending on the neutrino energy.

Optimized half cone cuts (preliminry)

Half-Cone Cut°	50GeV	80.3GeV	100GeV	150GeV	200GeV	350GeV	500GeV	750GeV	1000GeV
Soft	5.8	6.8	6.8	6.1	5.0	5.1	4.6	4.2	4.2
$\text{Hard}_{\tau^+\tau^-}$	6.8	5.0	5.0	4.6	4.2	4.2	3.2	2.9	2.9
$\text{Hard}_{W^+W^-}$	-	4.2	5.0	4.7	4.2	3.2	2.9	2.9	2.9

Table 1: Optimized Half-Cone angle opening value by MRF for each Dark Matter M_χ.

Half-Cone Cut°	50GeV	80.3GeV	100GeV	150GeV	200GeV	350GeV	500GeV	750GeV	1000GeV
Soft	1.2	1.3	1.3	0.9	0.9	0.9	0.7	0.7	0.7
$\text{Hard}_{\tau^+\tau^-}$	0.9	0.9	0.9	0.9	0.7	0.7	0.7	0.5	0.6
$\text{Hard}_{W^+W^-}$	-	0.9	0.9	0.7	0.7	0.7	0.5	0.5	0.5

Table 2: Optimized Half-Cone angle opening value by MDP at 3σ. for each Dark Matter M_χ.

Half-Cone Cut°	50GeV	80.3GeV	100GeV	150GeV	200GeV	350GeV	500GeV	750GeV	1000GeV
Soft	1.2	1.2	1.2	1.2	0.9	0.9	0.9	0.7	0.7
$\text{Hard}_{\tau^+\tau^-}$	1.3	1.2	1.2	0.9	0.7	0.7	0.7	0.5	0.5
$\text{Hard}_{W^+W^-}$	-	0.9	1.2	0.9	0.7	0.7	0.5	0.5	0.5

Table 3: Optimized Half-Cone angle opening value by MDP at 5σ. for each Dark Matter M_χ.

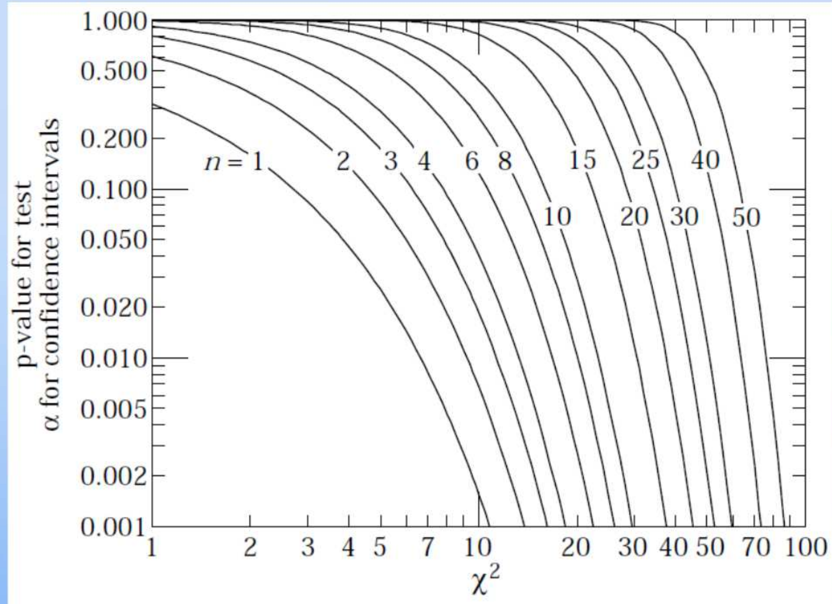
Estimate systematics

If the PSF is approximated by a 2-dim Gaussian, then a median resolution of, say, 3°, means that 3° corresponds to a value of $\chi^2 = \alpha^2/\sigma^2$ with a 50% cumulative probability

The value of χ^2 for the median is 1.38 for 2 degrees of freedom (ROOT function TMath::Prob) so:

$\alpha/\sigma = \sqrt{\chi^2} = 1.17$. Varying by ± 50% this value of $\sqrt{\chi^2}$ the cumulative probability varies by +34%, - 28%.

=> if the cut is on the median, the effect of a 50% total uncertainty on resolution and pointing (small effect) produces a 30% additional uncertainty on efficiency (effective area). The exact values of the cone cuts w.r.t. the median resolutions have to be considered in detail.



One minus the χ^2 cumulative distribution, $1-F(\chi^2; n)$, for n degrees of freedom

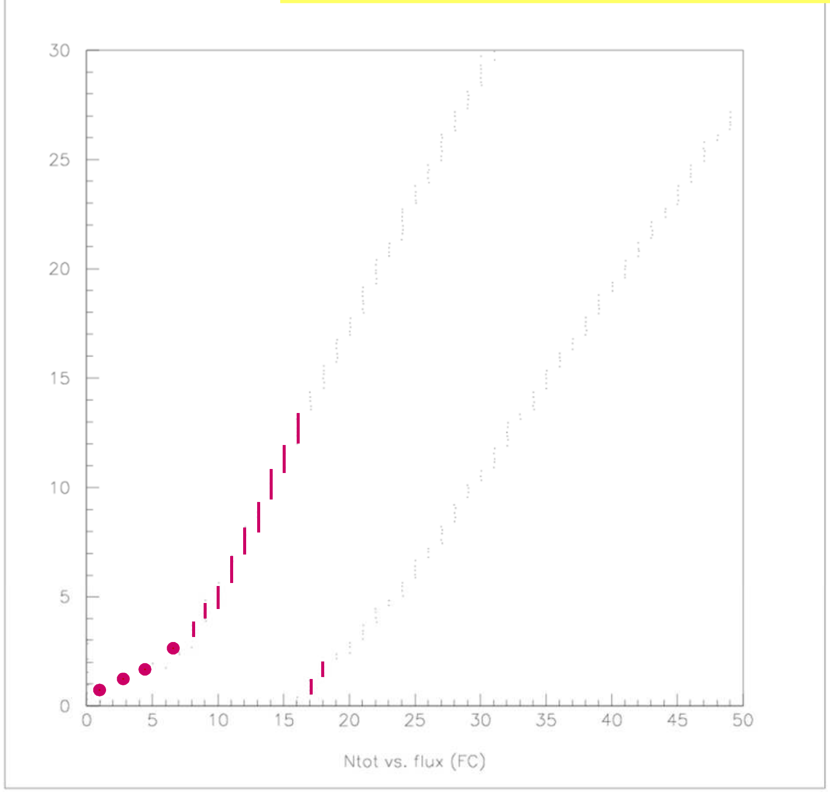
Energy dependence

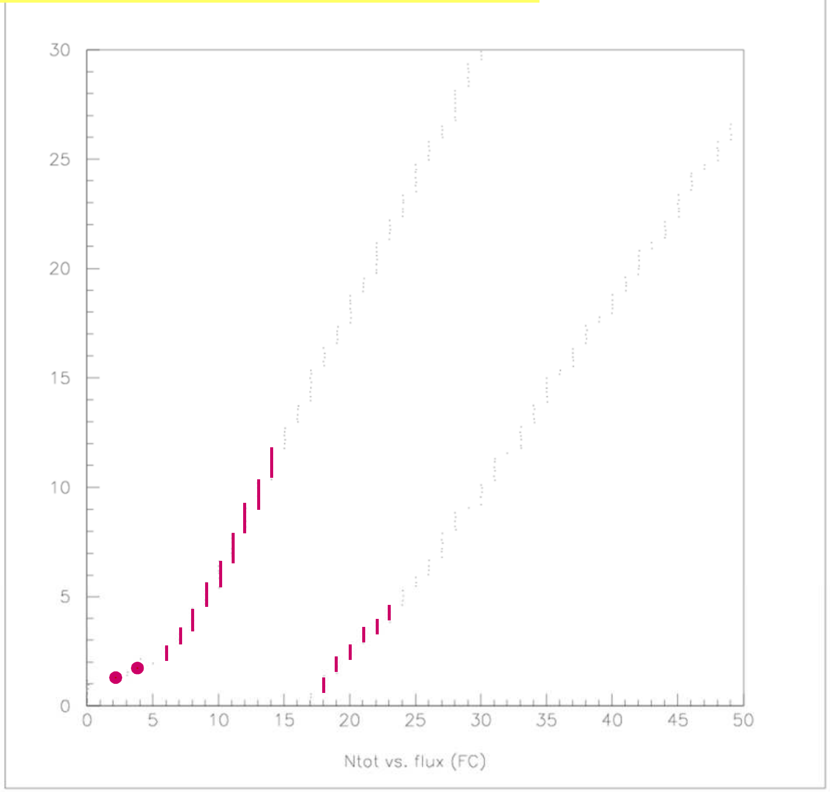
The PS averaged 20% uncertainty which has been obtained mainly for high energy neutrinos is also applicable to low energy ones, or an energy dependence has to be taken into account?
Is there a study as a function of the neutrino energy, especially for E_ν of a few 10s to few 100s of GeV ?

J. Brunner, CERN meeting in February: comparison of two MC samples (provided by Annarita) with modifed absorption length and angular acceptance of PMTs. For a neutrino selection which had been adjusted to look for oscillations:
a 20% effect for multi-line events and a 14% effect for single line events (single line events can be considered as a very low energy test sample).

So: no obvious difference at the lowest energies.

Diffuse flux belts: POLE outputs





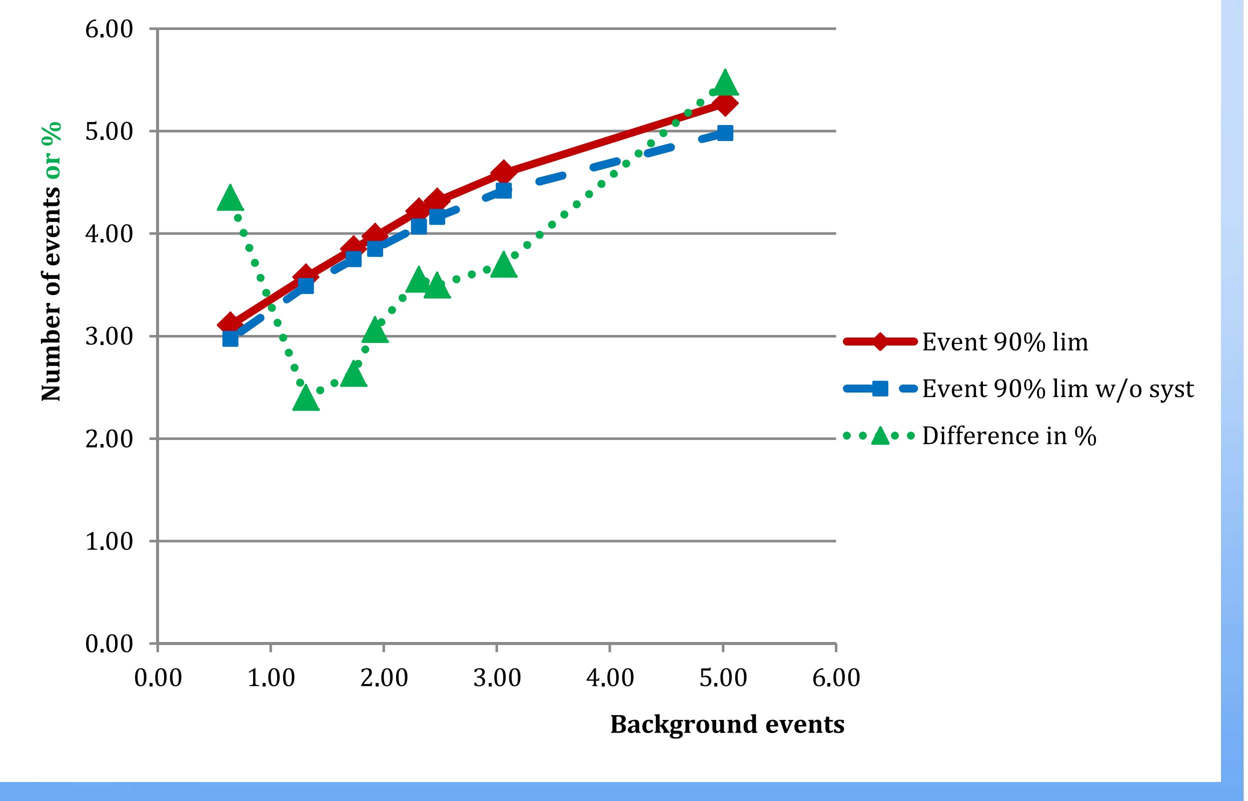
Sensitivity

- The average upper limit (sensitivity) at 90% CL is then computed weighting each limit with the Poissonian probability Poisson ($\mu=n_b$, n_{obs})* .
- There are 8 n_b values corresponding to 8 half-cone angles. An increase of the upper limit between 3% and 6% is found for the 20% efficiency uncertainty.

- *In fact the median should be used instead.

Upper limits on event numbers

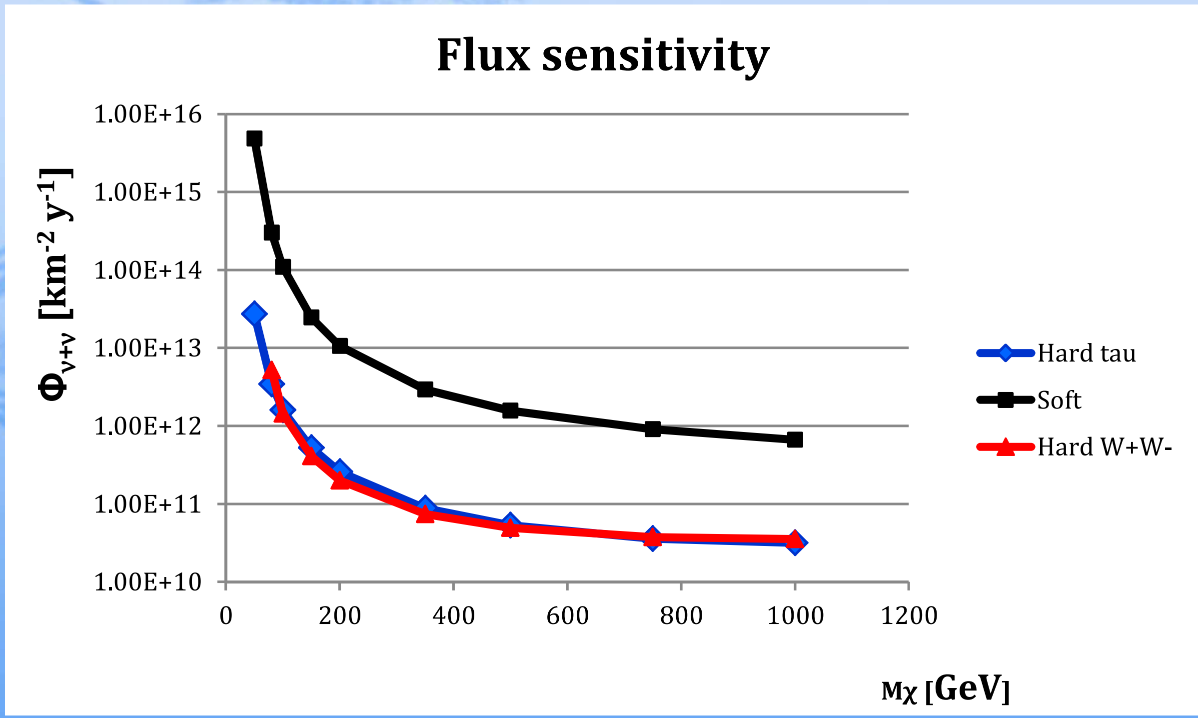
nb	90% UL with syst	90% UL w/o syst
5.02	5.27	4.98
3.06	4.59	4.42
2.47	4.31	4.16
2.31	4.22	4.07
1.92	3.97	3.85
1.73	3.85	3.75
1.31	3.57	3.49
0.64	3.11	2.97



Flux sensitivity

$$\frac{\text{average event number upper limit}}{\text{effective area integrated above 10 GeV * exposure time}}$$

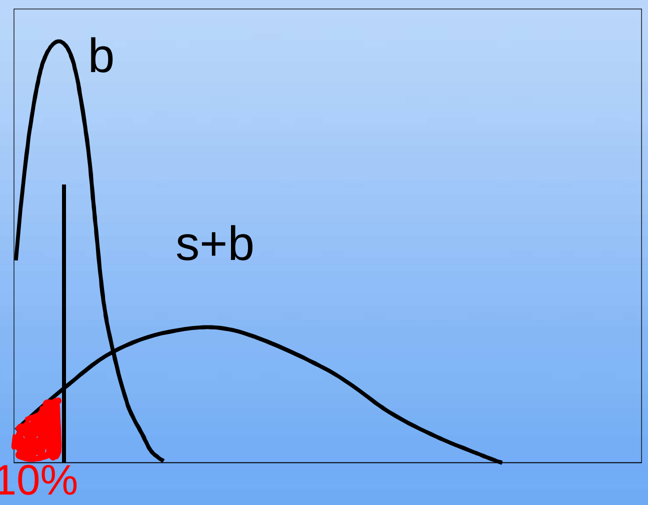
= flux sensitivity as a function of Mχ



The tools

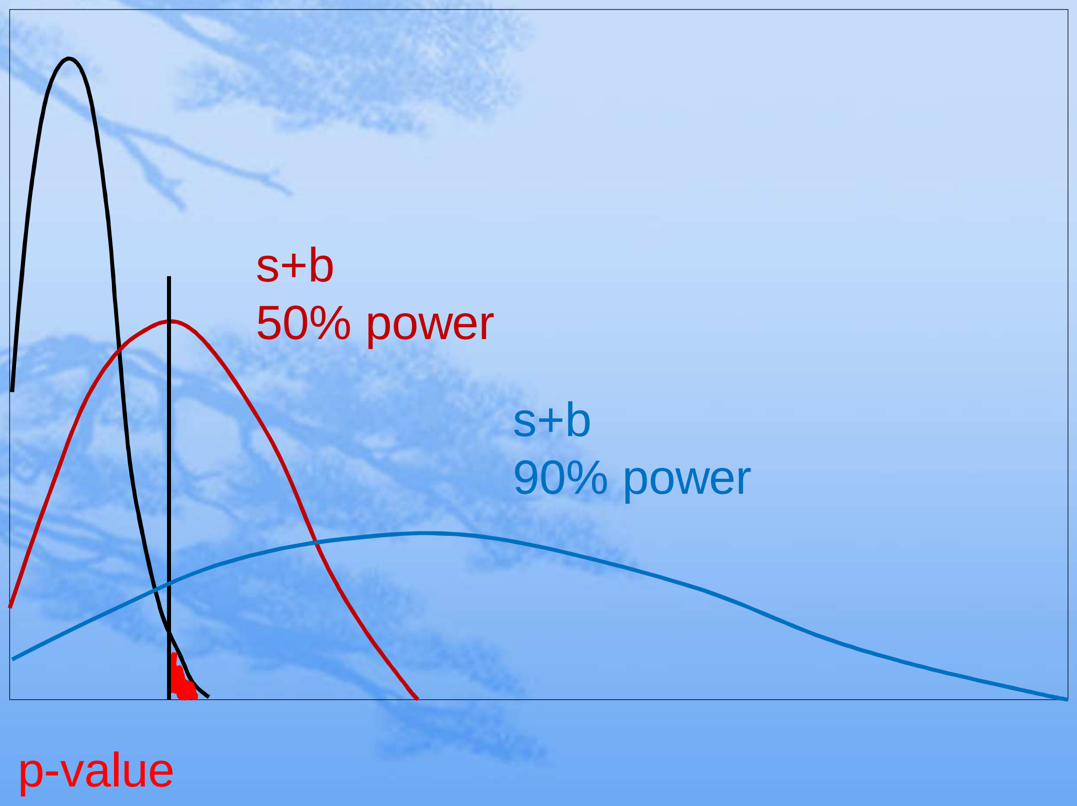
The results shown here are obtained with Pole ++ (2006)* . It integrates formula (1). The program had been first run in its Fortran version. But testing it with systematics=0 it was found that it did not give the Feldman-Cousins result. The Fortran version provides a slight underestimation of upper limits. The C++ version gives somewhat higher upper limits (only a 1% effect added to systematics of the order of 5%) and, when systematic uncertainties of background and of signal efficiency are put to 0, the FC limit is found.

The inclusion of systematics in the upper limit was also computed, for a check against Pole, using Root. In this case a naïve upper limit was computed in Root, corresponding to the figure:
Same shift found with the two methods.



*Installed with the help of F. Tegenfeldt and F. Schussler.

Least Detectable Signal definition



Model Discovery Potential (MDP)

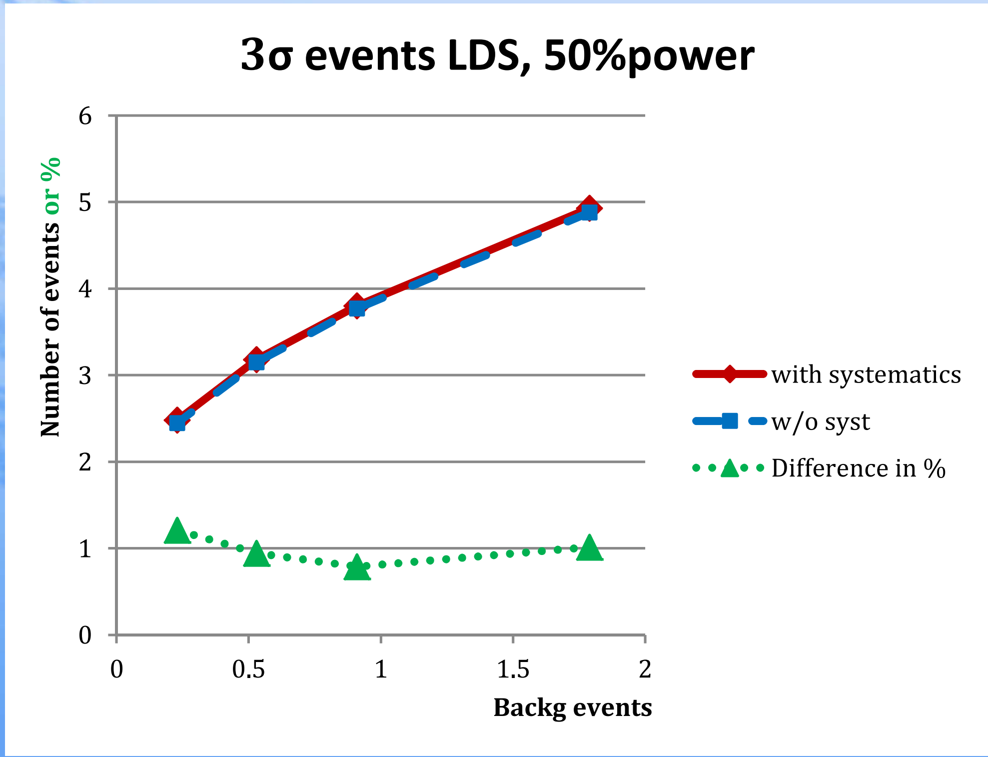
Least detectable signal (LDS) number of events

The minimum probability (p-value) to claim for a discovery is defined and then the best cone search size is optimized in order to minimize the required signal to have such a discovery. The FC construction is not used. A minimum number of events n_{crit} is defined such that the p-value of detecting n_{crit} or more events for an average background n_b should be lower than the chosen one.
Expressed in terms of standard deviations (two-sided, although the problem is one-sided). 3σ, $5\sigma \rightarrow 2.7\times 10^{-3}$, 5.73×10^{-7}

Assuming that the number of observed events nobs is composed by both background (n_b) and signal (n_s), the strength of the observed signal should be large enough so that $n_b + n_s$ produces the desirable n_{crit} with a certain probability (statistical power - SP). Typically 50% or 90% SP's are required. The minimal signal needed to reach the required SP is the least detectable signal (n_{lds}).

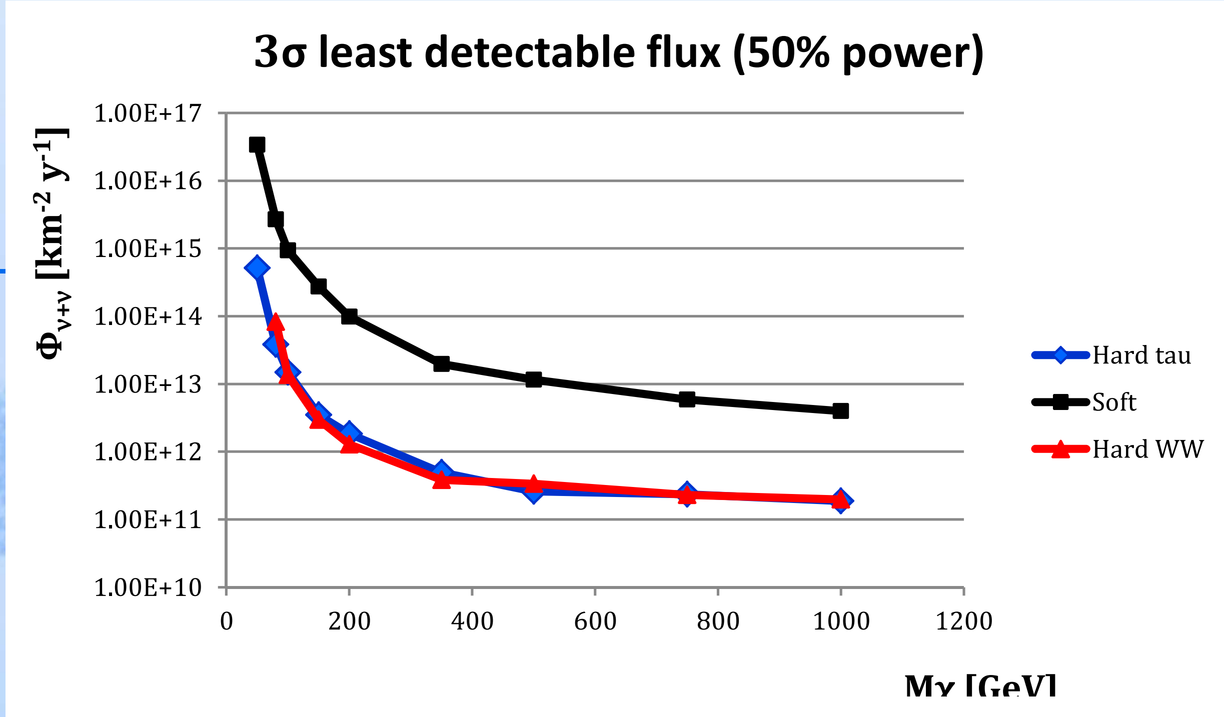
Event number LDS

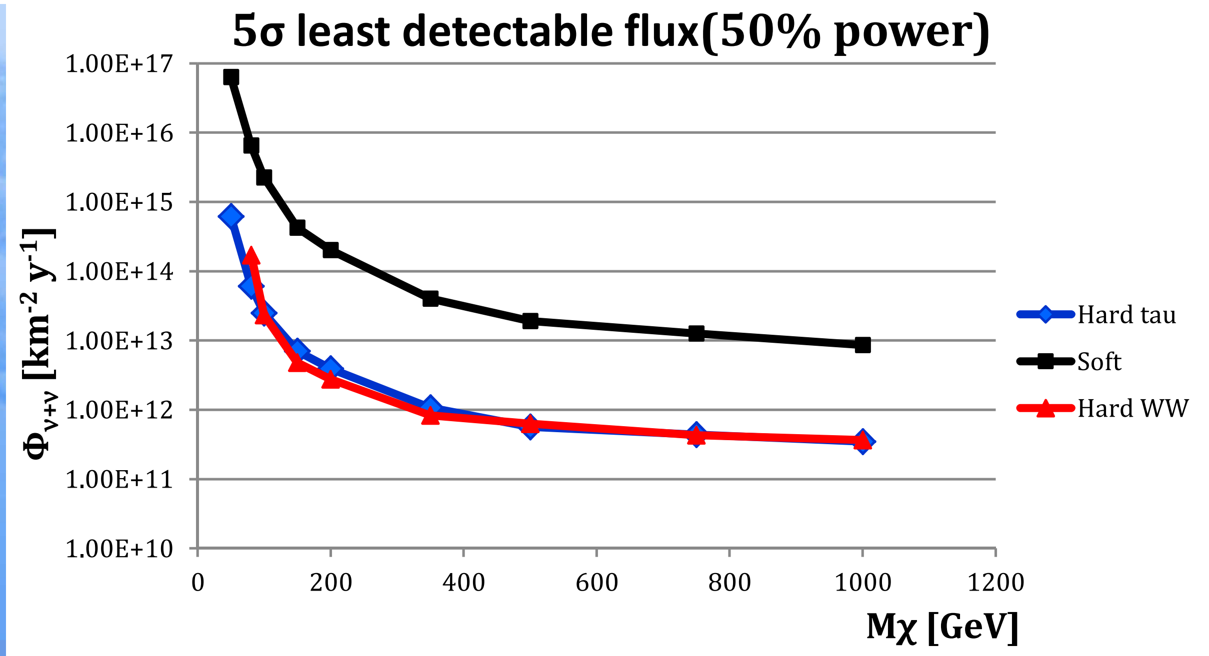
For the inclusion of systematics formula (1) is used again in ROOT, not POLE. There are 4 nb values corresponding to 4 half-cone angles optimized for 3σ discovery. The inclusion of 20% uncertainty in the signal efficiency brings only a negligible effect of ~1%.



Least detectable fluxes

The least detectable flux as a function of Mχ: computed from the event number LDS and the effective area integrated above 10 GeV and over the exposure time.





Summary

- An increase of the upper limit between 3% and 6% is found for a 20% efficiency uncertainty. Smaller effect on discovery potential.
- Inclusion of an additional hypothetical strong uncertainty of 50% on the angular resolution translates to ~ 30% signal efficiency uncertainty → ≤ 10% additional increase of the upper limit. Exact values need:
- - estimate of the angular resolution and its uncertainty
- - final angular cuts

Systematics in interval estimation

- F. Tegenfeldt, J. Conrad A NIM A 539 (2005) 407–413 (Pole C++)

  J. Conrad, et al., Phys. Rev. D 67 (2003) 012002 (older, Fortran)

- Profile likelihood Trolke Root class:
  - Rolke et al,NIM A 551 (2005) 493
  - J. Lundberg et al. Comp. Phys. Comm. 181, (2010) 683.